

MEI

**Innovators in
Mathematics
Education**

**Mathematics in
Education and
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Over 50 years
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of Mathematics
Education



Using
technology to
investigate
differential
equations

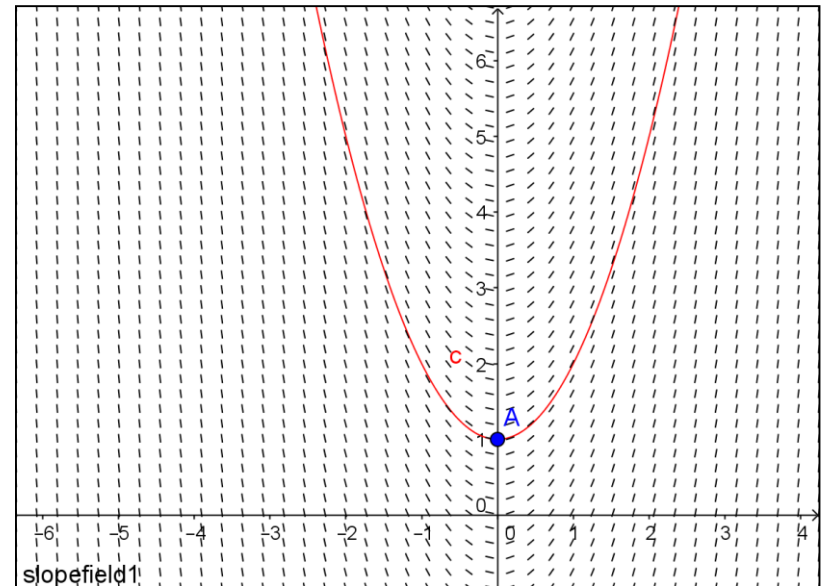
Differential equations on GeoGebra

- **SlopeField[f]**
- **SolveODE[f,A]**
- **CAS: SolveODE[equation]**
NB use y'

Try these for:

$$\frac{dy}{dx} = 2x$$

$$\frac{dy}{dx} = 3y$$



Solving Differential Equations

- 10 (i) Obtain the solution to the differential equation

$$x \frac{dy}{dx} + 3y = \frac{1}{x}, \text{ where } x > 0,$$

given that $y = 1$ when $x = 1$.

[7]

- (ii) Deduce that y decreases as x increases.

[2]

Find a solution passing through some points other than $(1, 1)$ such that y does not always decrease as x increases.

Spreadsheet for the Euler method

For the differential equation $\frac{dy}{dx} = \sqrt{xy}$ with initial conditions $y = 2$ when $x = 2$.

Find an estimate for $y(3)$ when using a step size of $h = 0.1$.

Euler method:

$$y_{n+1} \approx y_n + hf(x_n, y_n)$$

Differential Equations in FPT

Further Pure with Technology is an A level Further Maths Option with three topics:

- Investigation of curves (graphing/CAS)
- Number Theory (programming language)
- Differential Equations (graphing/CAS/spreadsheet)
 - Using slope fields
 - Numerical solutions to DEs (Euler/Runge Kutta)
 - Using CAS to solve DEs

FPT SAM question

- 3 This question explores the family of differential equations $\frac{dy}{dx} = \sqrt{1 + ax + 2y}$ for various values of the parameter a . Fig. 3 shows the tangent field in the case $a = 1$.

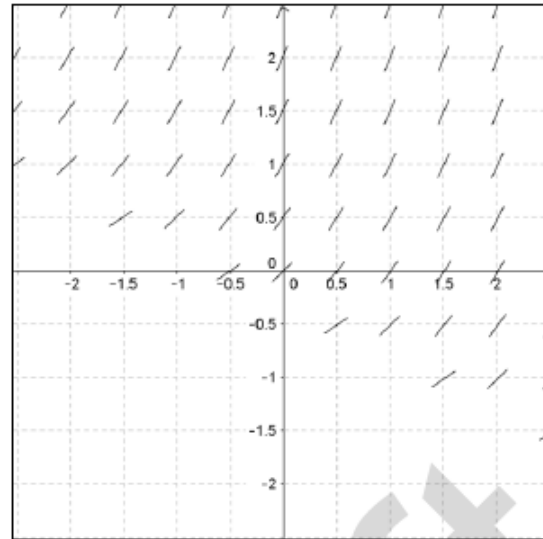


Fig. 3

- (i) (A) Sketch the tangent field in the case $a = -2$.
- (B) Explain why the tangent field is not defined for the whole coordinate plane.
Give an inequality which describes the region in which it is defined.
- (C) Find a value of a such that the region for which the tangent field is defined includes the entire x -axis.

Second Order Differential Equations

In the CAS view use:

SolveODE[<Equation>, <Point(s) on f>, <Point(s) on f'>]

e.g. **SolveODE[$y'' + 2y' + y = \sin(x)$, (0, 1), (0, 2)]**

There are also a lot of good examples online, e.g.

- geogebra.org/m/zGah4NrW
- geogebra.org/m/y4Ercw2b

Or search for “Second Order ODE” at: geogebra.org/materials/