MEI

Innovators in Mathematics Education

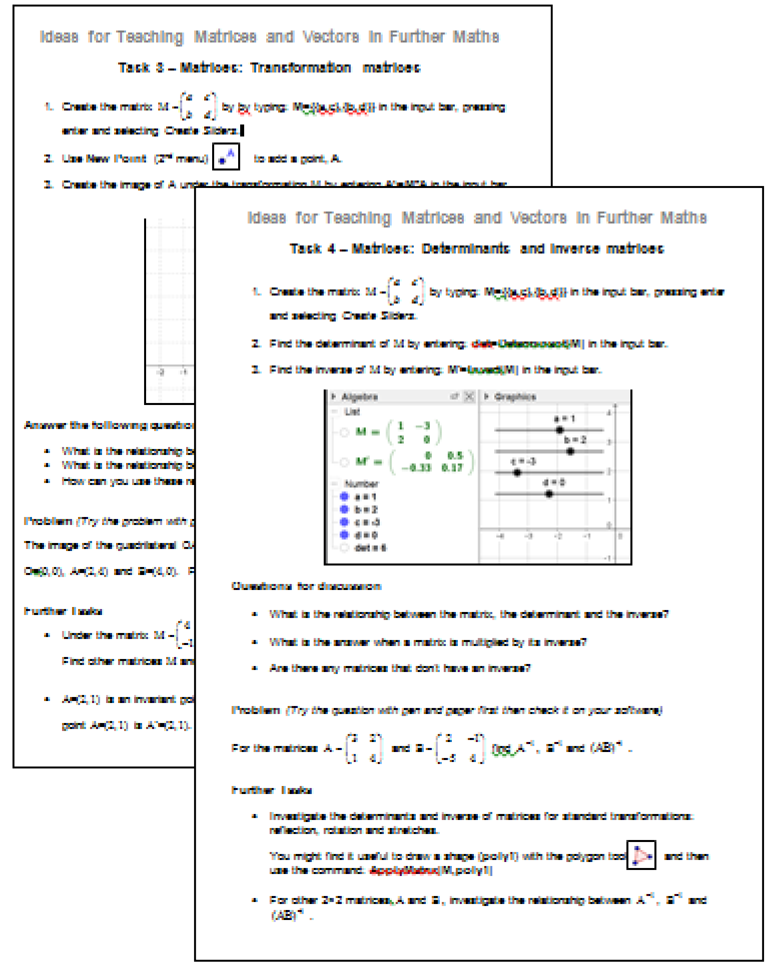
Mathematics in Education and Industry

Over 50 years at the forefront of Mathematics Education

Ideas for teaching matrices, transformations and vectors in Further Maths

Entering Matrices in GeoGebra

- Enter **M={{1,2},{3,4}}**
- GeoGebra Further Pure tasks 3 & 4
- More tasks are available from mei.org.uk/geogebra



Ideas for Teaching Matrices and Vectors in Further Maths

Task 3 – Matrices: Transformation matrices

1. Create the matrix: $M=\begin{pmatrix} a & c \\ b & d \end{pmatrix}$ by by typing: M={{a,c},{b,d}} in the input bar, pressing enter and selecting Create Sliders.
2. Use New Point (2nd menu) to add a point, A.
3. Create the image of A under the transformation M by entering A'=M*A in the input bar.

Ideas for Teaching Matrices and Vectors in Further Maths

Task 4 – Matrices: Determinants and inverse matrices

1. Create the matrix: $M=\begin{pmatrix} a & c \\ b & d \end{pmatrix}$ by typing: M={{a,c},{b,d}} in the input bar, pressing enter and selecting Create Sliders.
2. Find the determinant of M by entering: det=Determinant[M] in the input bar.
3. Find the inverse of M by entering: M'=Invert[M] in the input bar.

Questions for discussion

- What is the relationship between the matrix, the determinant and the inverse?
- What is the answer when a matrix is multiplied by its inverse?
- Are there any matrices that don't have an inverse?

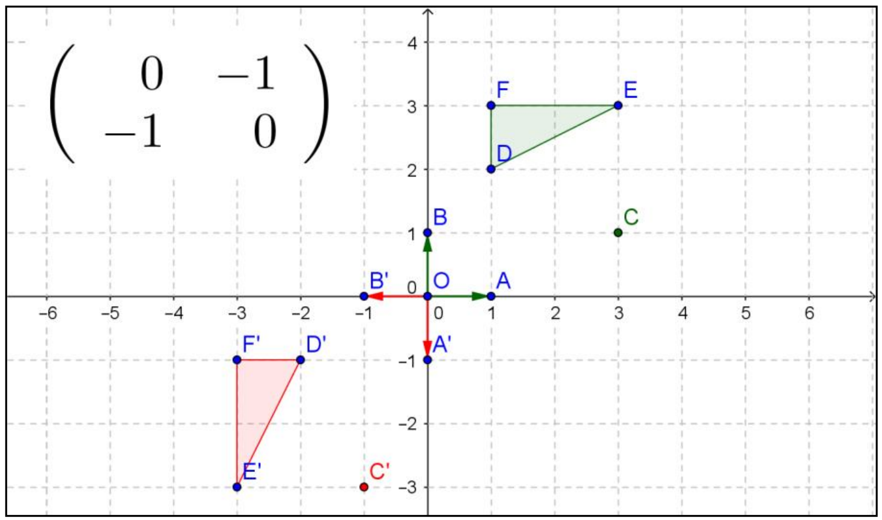
Problem (*Try the question with pen and paper first then check it on your software*)

For the matrices $A=\begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix}$ and $B=\begin{pmatrix} 2 & -1 \\ -5 & 4 \end{pmatrix}$ find A^{-1}, B^{-1} and $(AB)^{-1}$.

Further Tasks

- Investigate the determinants and inverse of matrices for standard transformations: reflection, rotation and stretches.
  You might find it useful to draw a shape (poly1) with the polygon tool and then use the command: ApplyMatrix[M,poly1]
- For other 2×2 matrices, A and B, investigate the relationship between A^{-1}, B^{-1} and $(AB)^{-1}$.

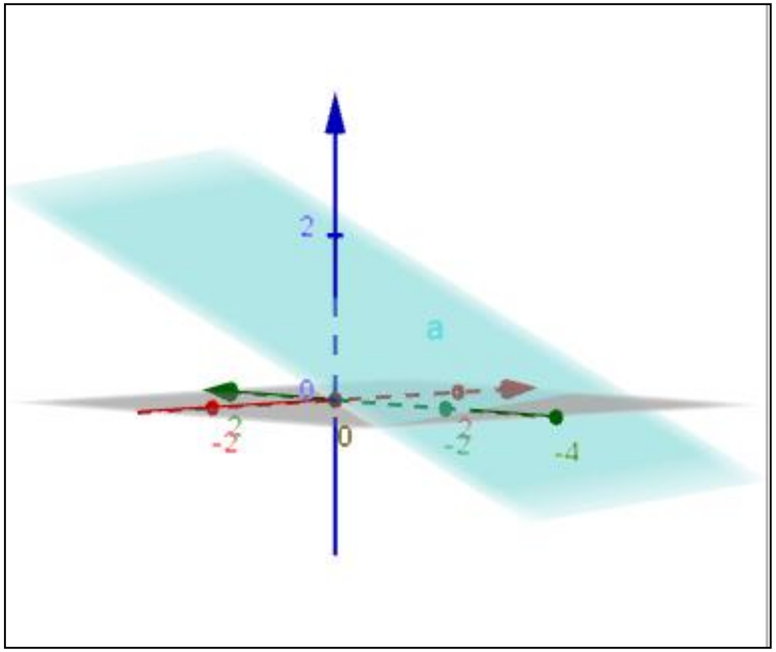
The matrix of transformation



Further ideas in the *Using GeoGebra for A level Further Mathematics* workbook

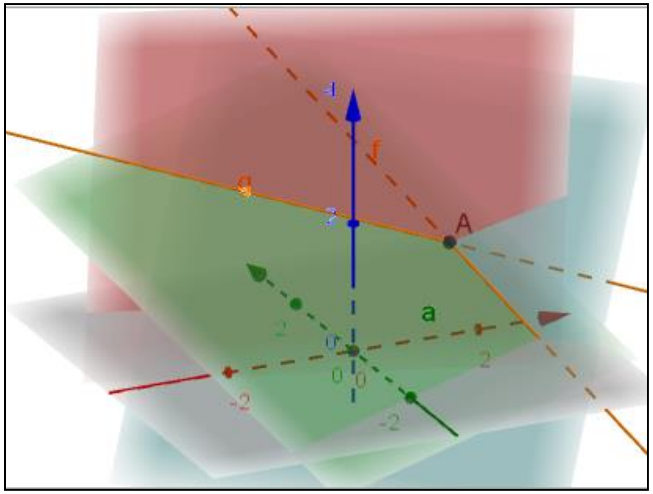
Equation of a plane

- In 3D GeoGebra enter:
  x – y + 4 z = 4
  – Why is this a plane and not a line or a curved surface?
- Change 4 to **k**
  – How does varying **k** vary the plane?
- What information is required to define a plane?



Finding the intersection of 3 planes

- Enter the following planes:
  x - 2y + 4z = 4
  x + y - z = 2
  x + 3y + z = 6
- Find the lines of intersection of two pairs of planes and then the intersection of the lines
- Show that the product of the inverse 3×3 by the RHS is the point of intersection



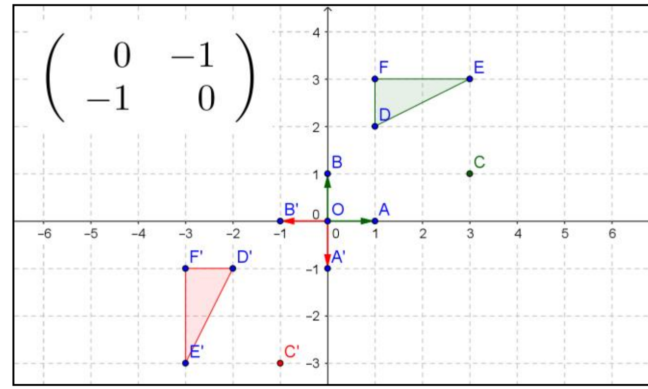
Finding the non-intersection of 3 planes?

- How many distinct cases are there of orienting 3 planes so that don't have a unique point of intersection?
  - For each case give an example and confirm that there is no unique point based on the matrix method for solving

Further Activities

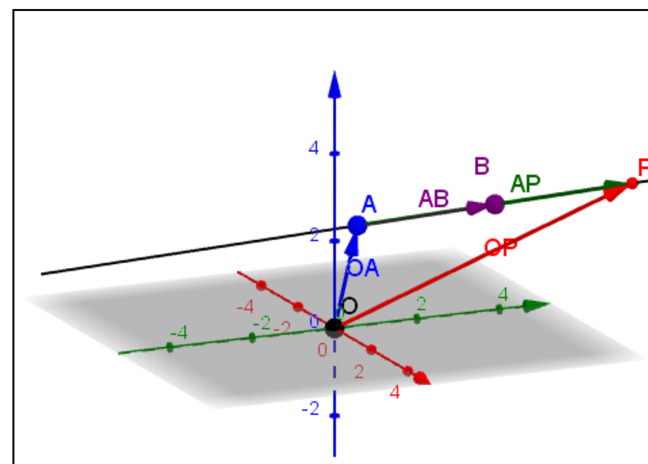
Work through the constructions for:

- Matrix transformations in 2D
- Matrix transformations in 3D
- Vector equation of a line in 3D



Or generate your own files for:

- Angle between two lines
- Intersection/skew lines in 3D
- Distance between two lines in 3D
- Distance from a point to a line



Other information

- Using Autograph for vectors in 2D & 3D
  - http://mei.org.uk/files/pdf/C4_Vectors_Autograph.pdf
  - http://mei.org.uk/files/pdf/FMVectors_Autograph.pdf