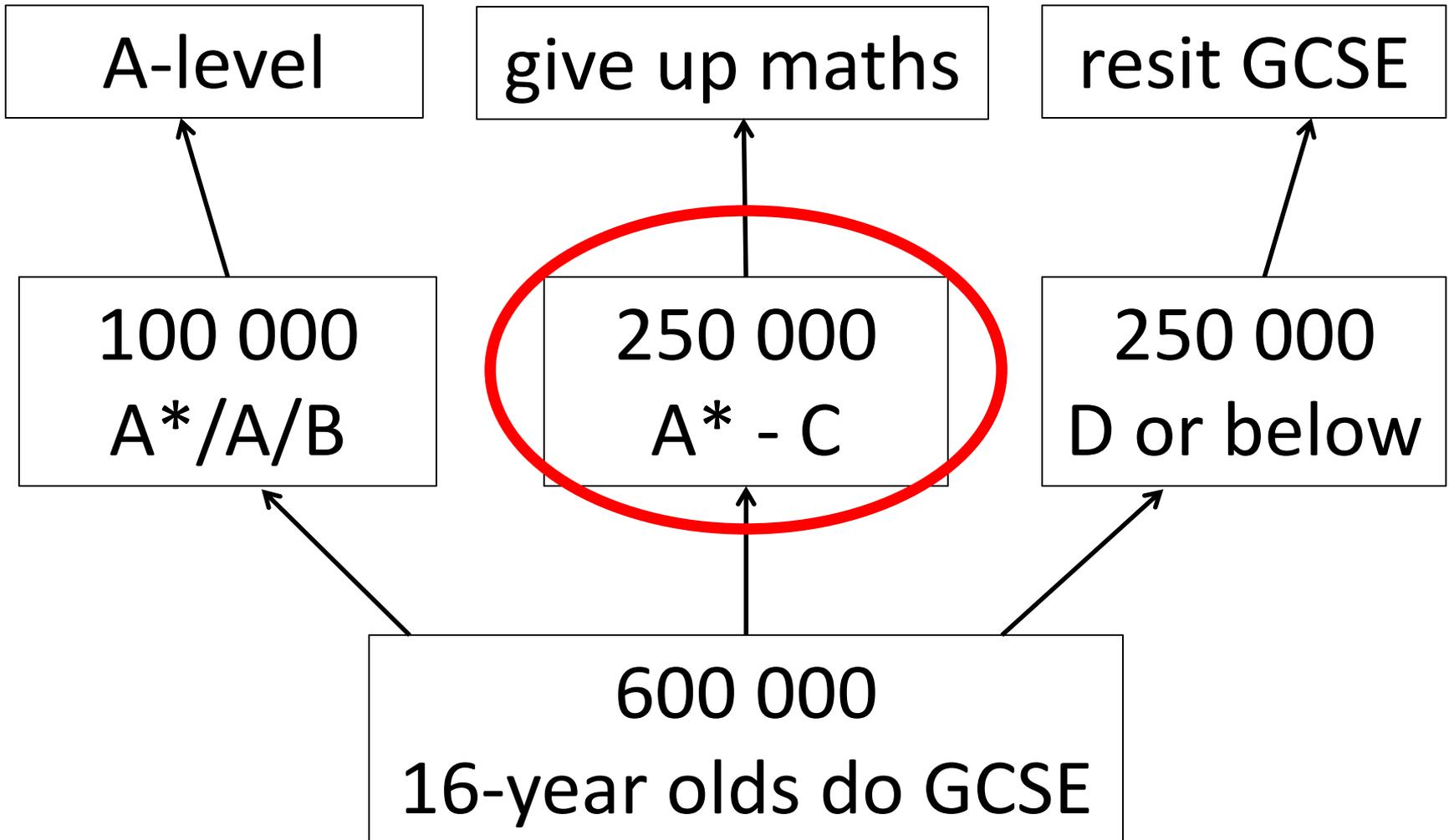


[core:maths]

- Who is it for?
- What's the content?
- Why should you care?
- What's it like to teach?

Who is it for?



What's the content?

Up to 20 UCAS points

Level
3

A/S Maths &
A2 Maths

Complex maths in
straightforward settings

Core Maths

Straightforward maths
in complex settings

Level
2

GCSE

Straightforward maths in
straightforward settings
(but those settings are getting
harder!)

What's the content?

2x 180 hours

180 hours in 2 yrs

**Level
3**

A/S Maths &
A2 Maths
Complex maths in
straightforward settings

20%

Core Maths
Straightforward maths
in complex settings

**Level
2½?**

**Level
2**

GCSE
Straightforward maths in
straightforward settings
(but those settings are getting
harder!)

80%

Straightforward maths in complex settings?

An A-level question

Use the substitution $x = 2\sqrt{2} \sin \theta$ to prove that

$$\int_2^{\sqrt{6}} \sqrt{(8-x^2)} \, dx = \frac{1}{3} (\pi + 3\sqrt{3} - 6).$$

[7 marks]

(Edexcel A2 Mathematics)

A Core Maths question

Estimate the total number of school pupils in the UK. State all your assumptions.

[5 marks]

(OCR Quantitative Reasoning)

Why should you care?

A study into the feasibility of compulsory maths study for all pupils to 18 will be undertaken by statistician and former civil servant Professor **Sir Adrian Smith**. Announced in the Budget, **Sir Adrian** will **review** maths teaching for 16 to 18-year-olds, with a focus on skilling a future teaching workforce. 17 Mar 2016



RECOMMENDATIONS:

- Uptake (of post-16 maths) should be near universal within 10 years
- All schools should be offering Core Maths within x^* years
- There should be no funding disincentives and there should be funding incentives to continue with Core Maths

* x would appear to be a number close to 5

So what's it like to teach?

Depends which '*it*' you mean

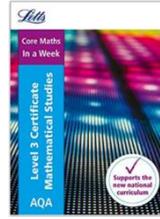
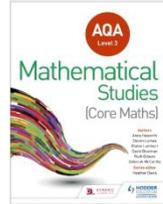
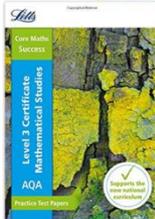
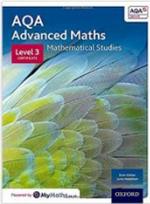
6 Different Qualifications

	AQA	Mathematical Studies
	City & Guilds	Using and Applying Mathematics
	Edexcel	Mathematics in Context
	Eduqas/ WJEC	Mathematics for Work and Life
	OCR	Quantitative Reasoning (MEI) (H866) Quantitative Problem Solving (MEI) (H867)

2016: 2931 entries

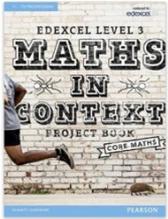
	AQA	Mathematical Studies
---	-----	----------------------

73%



	Edexcel	Mathematics in Context
---	---------	------------------------

6%



 Oxford Cambridge and RSA	OCR	Quantitative Reasoning (MEI) (H866)
		Quantitative Problem Solving (MEI) (H867)

14%

6%

What's in these courses?

Critical Analysis

Do the figures support...?

Use the data to defend...

Why is the tax calculation wrong?

Modelling

(spreadsheets)

'PROBLEM SOLVING'

Financial Maths:

Real rates from real banks

Exchange rates (**real** ones)
commission and buy/sell rates)

Taxation (not Edexcel)

Statistics (Probability):

Stress interpretation (box-plots)

Concerned with the idea of 'risk'

Estimation:

practical approximation (inc bounds)

Fermi estimation (not Edexcel)

Money: a good place to start

Currency Exchange:

Mr McIvor wants to take 500 euros on holiday. He has £420 and is being offered an exchange rate of 1.13 to the £. Does he have enough?



CURRENCY	RATE FROM £1000		TRAVEL MONEYCARD	
	WE SELL	WE SELL	WE SELL	WE BUY
EURO	1.1252	1.1308	1.1303	1.3261
EURO	1.1208	1.1269	1.1297	1.3261
USA	1.2897	1.2167	1.2215	1.4152
USA	1.2059	1.2125	1.2155	1.4152
AUSTRALIA	1.5982	1.6069	1.6109	1.8760
CANADA	1.6150	1.6238	1.6279	1.9824
UAE	4.3234	4.3401	4.3500	5.1990
SWITZERLAND	1.1911	1.1976	1.1986	1.4283
TURKEY	4.2791	4.3826	4.3133	5.1660
CROATIA	8.1343	8.1790		9.8820
BULGARIA	2.1385	2.1422		2.5934
CZECH	29.4018	29.5635		35.6731
HUNGARY	335.4763	337.3294		411.9822

Sainsbury's Rates

	Sell	Buy
Euro	1.1252	1.3261

Mr McIvor plans to change his currency at Sainsbury's. Estimate the commission rate.

Money: a good place to start

CURRENCY	RATE FROM £1000		TRAVEL MONEYCARD	
	CURRENCY WE SELL	WE SELL	WE SELL	CURRENCY WE BUY
EURO	1.1252	1.1308	1.1303	1.3261
EURO	1.1208	1.1269	1.1297	1.3261
USA	1.2897	1.2967	1.2215	1.4852
USA	1.2059	1.2125	1.2155	1.4852
AUSTRALIA	1.5982	1.6069	1.6109	1.8760
CANADA	1.6150	1.6238	1.6279	1.9826
UAE	4.3234	4.3491	4.3599	5.3885
SWITZERLAND	1.1911	1.1976	1.1988	1.4203
TURKEY	4.2791	4.3826	4.3133	5.1640
CROATIA	8.1343	8.1790		9.8820
BULGARIA	2.1385	2.1422		2.5934
CZECH	29.4688	29.5635		35.8731
HUNGARY	335.4763	337.3214		411.9822

Sainsbury's Rates

	Sell	Buy
Euro	1.1252	1.3261

Mr McIvor plans to change his currency at Sainsbury's. Estimate the commission rate.

MODELLING

A SIMPLE APPROACH:

£ to €

€ to £

Pick a sum of money (e.g £100)
convert to euros and back again

$$£100 \times 1.1252 = €112.52$$

$$€112.52 \div 1.3261 = £84.85$$

Over 15% charged across the 2 transactions so about 7.5% each way

Check with multipliers: $100 \times 0.925^2 = £85.56$

Money: a good place to start

but students need to be good with

MULTIPLIERS

ANY METHOD YOU LIKE

USING MULTIPLIERS

1. Calculate 15% of £25
2. Jack sees a book with an original price of £12 but marked 20% off. How much will Jack save?
3. Jane is looking through the Argos catalogue. She sees a pair of earrings originally priced at £87.99 but marked 25% off. How much will she pay for the earrings?
4. Max buys a new car for £12000. Given that cars lose 15% of their value every year, how much will the car be worth after 3 years?
5. In a sale all prices are reduced by 30%. The sale price of a jacket is £70, what was the original price?
6. Olivia puts £1500 in savings account which pays 3% interest per year. How much will she have after 5 years?
7. A phone was reduced in price from £160 to £140.80. What is the percentage discount.
8. A diamond ring goes up in value from £4500 to £5940. What was the percentage increase
9. William got $\frac{32}{70}$ on a test. What was his percentage?
10. 'All prices include VAT at 20%'. If a watch is priced at £29.99, what was the price *before* VAT was added?

Money: a good place to start

but students need to be good with

MULTIPLIERS

ANY METHOD YOU LIKE

USING MULTIPLIERS

1. Calculate 15% of £25
2. Jack sees a book with an original price of £12 but marked 20% off. How much will Jack save?
3. Jane is looking through the Argos catalogue. She sees a pair of earrings originally priced at £87.99 but marked 25% off. How much will she pay for the earrings?
4. Max buys a new car for £12000. Given that cars lose 15% of their value every year, how much will the car be worth after 3 years?
5. In a sale all prices are reduced by 30%. The sale price of a jacket is £70, what was the original price?
6. Olivia puts £1500 in savings account which pays 3% interest per year. How much will she have after 5 years?
7. A phone was reduced in price from £160 to £140.80. What is the percentage discount.
8. A diamond ring goes up in value from £4500 to £5940. What was the percentage increase
9. William got $\frac{32}{70}$ on a test. What was his percentage?
10. 'All prices include VAT at 20%'. If a watch is priced at £29.99, what was the price *before* VAT was added?

Percentages and Multipliers

Find a basic introduction for students here:

https://youtu.be/UqVWmNc_n9A

Income Tax rates and bands

The table shows the tax rates you pay in each band if you have a standard Personal Allowance of £11,000.

Band	Taxable income	Tax rate
Personal Allowance	Up to £11,000	0%
Basic rate	£11,001 to £43,000	20%
Higher rate	£43,001 to £150,000	40%
Additional rate	over £150,000	45%



this is the amount you earn in ONE YEAR
usual abbreviation p.a. (per annum)

Band	Taxable income	Tax rate
Personal Allowance	Up to £11,000	0%
Basic rate	£11,001 to £43,000	20%

£20 000 pa



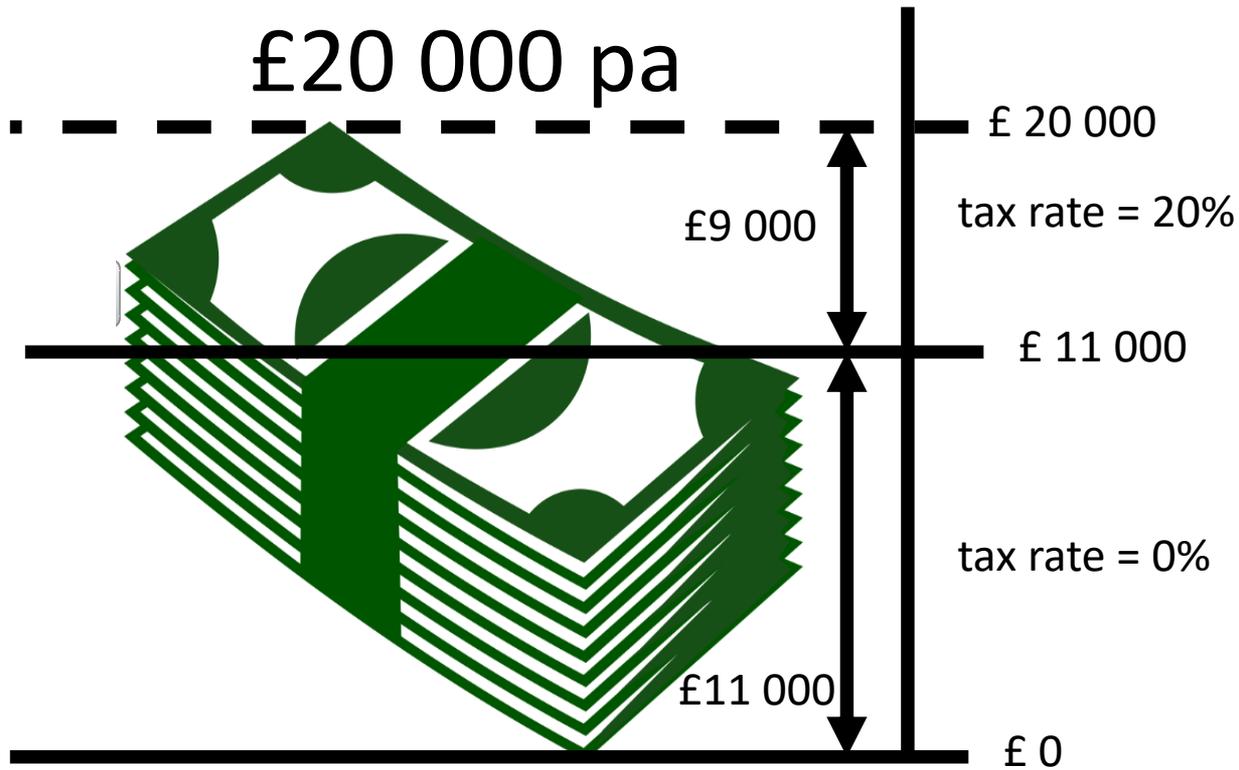
£11 001 - £43 000

Tax rate = 20%

20% of £20 000

= £4 000 **X**

Band	Taxable income	Tax rate
Personal Allowance	Up to £11,000	0%
Basic rate	£11,001 to £43,000	20%



CALCULATION:

£20 000 - £11 000
 = £9000 taxable income

0% of £11 000 = £0

20% of £9 000 = £ 1800

Income tax payable
 = £ 1800



Introductory videos

Find the Income Tax lesson online here: <https://youtu.be/jpgPsNVI2fA>

Find the follow-up National Insurance lesson here: https://youtu.be/dCWDqzOB_28

Money: what next?

RPI/CPI and INFLATION are a new application of compound interest

SPREADSHEET MODELLING of savings plans with regular payments is a decent activity

INCOME TAX and NI are often popular

DON'T DO AER/APR TOO SOON

Fermi Estimation: it's new!

BIG IDEA:

Getting rough answers for hard-to-calculate problems. Often work with orders of magnitude

How many pupils are there in the UK school system?

OCR Specimen Materials

How many 5-18 year olds are there in the UK?

Roughly how many people live in the UK?

A 100 000

B 1 000 000

C 10 000 000

D 100 000 000

Roughly how many people live in the UK?

A 100 000

B 1 000 000

C 10 000 000

D 100 000 000



Population of the UK roughly 100 000 000

What is the approximate lifespan in years of the average person in the UK?

A 1

B 10

C 100

D 1000

Average lifespan of people in the UK roughly 100



Population of the UK roughly 100 000 000

What is the approximate lifespan in years of the average person in the UK?

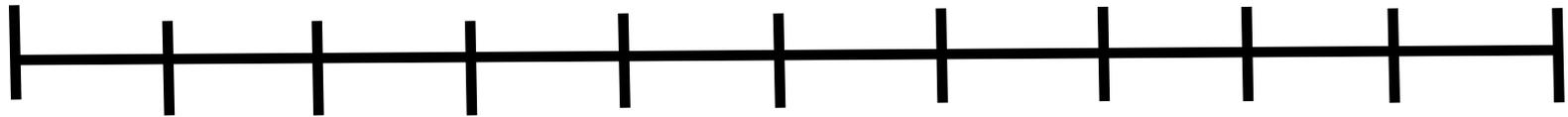
A 1

B 10

C 100

D 1000

Average lifespan of people in the UK roughly 100



Population of the UK roughly 100 000 000

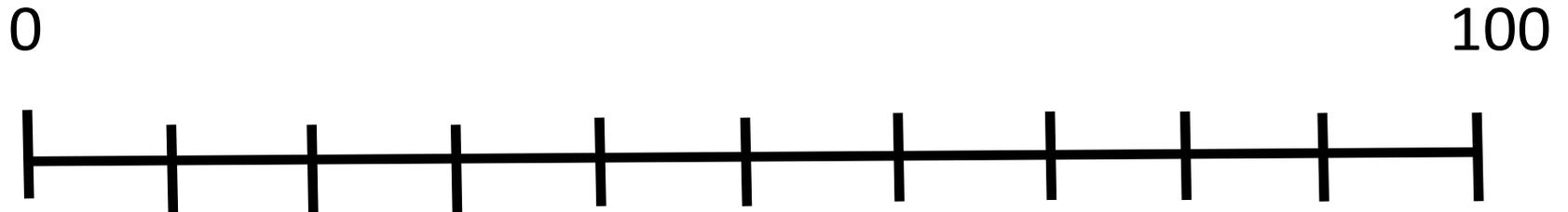
What is the approximate lifespan in years of the average person in the UK?

A 1

B 10

C 100

D 1000



Population of the UK roughly 100 000 000

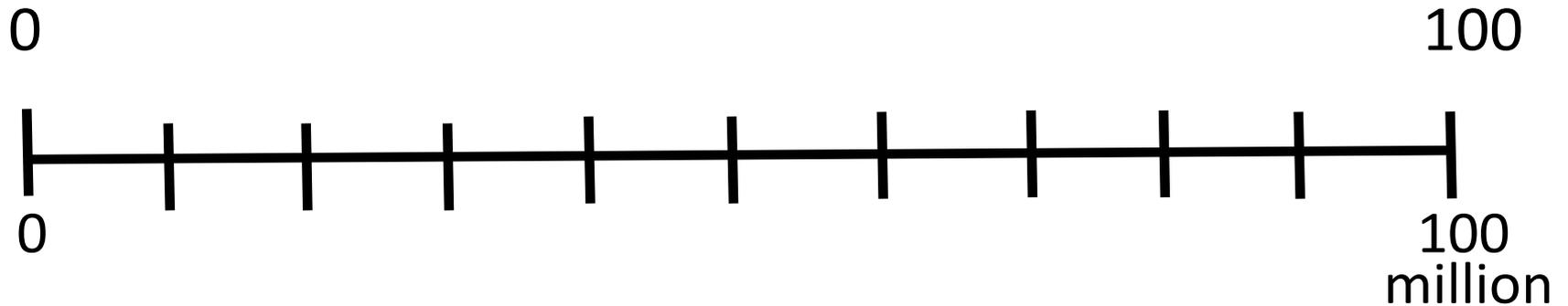
What is the approximate lifespan in years of the average person in the UK?

A 1

B 10

C 100

D 1000



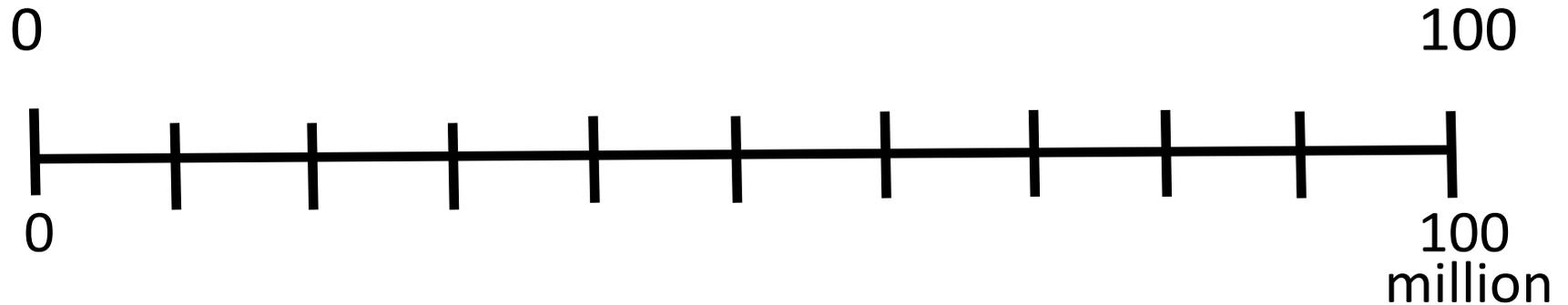
Roughly how many 5 to 18 year olds are there in the UK?

A 10 000

B 100 000

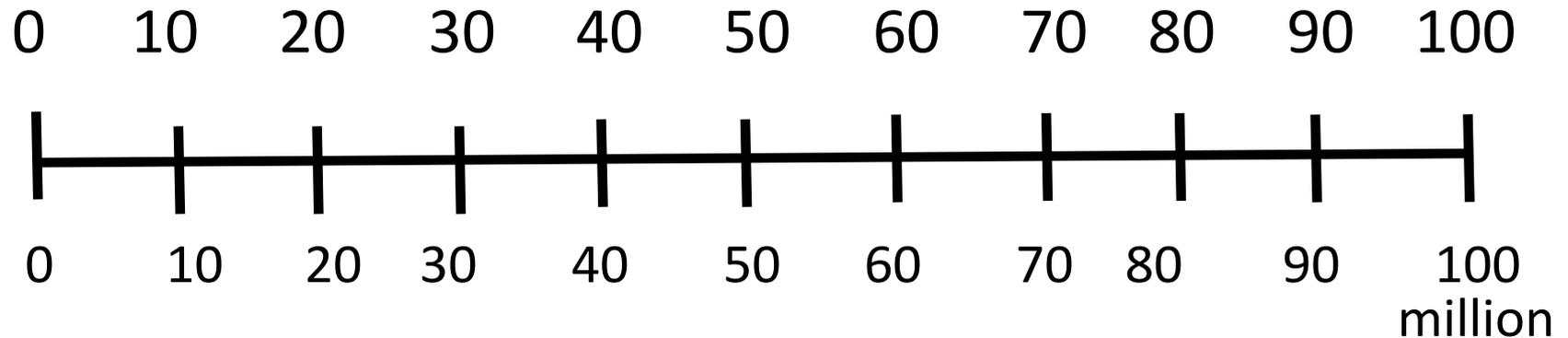
C 1 000 000

D 10 000 000



Roughly how many 5 to 18 year olds are there in the UK?

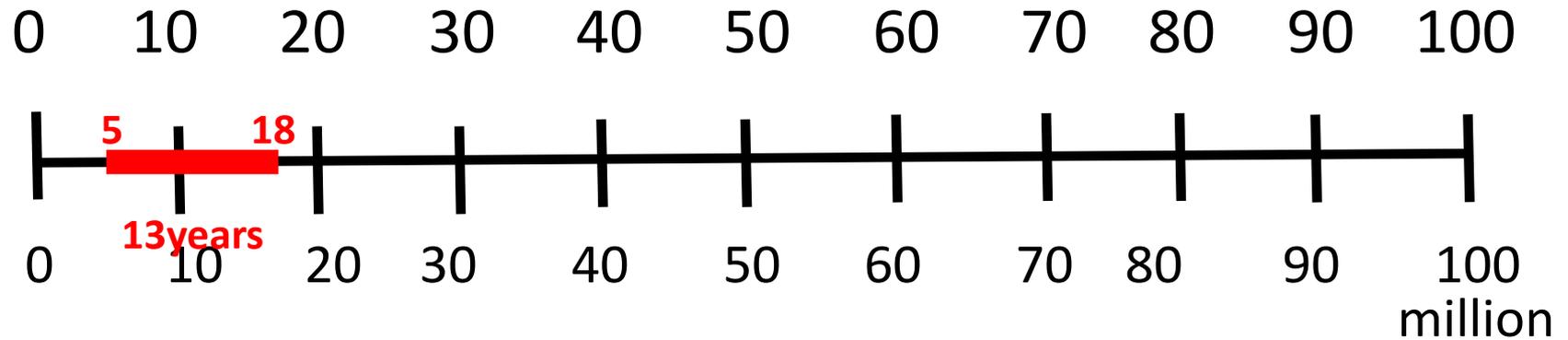
A 10 000 B 100 000 C 1 000 000 D 10 000 000



Roughly 1 million people in every 1 year interval

Roughly how many 5 to 18 year olds are there in the UK?

A 10 000 B 100 000 C 1 000 000 D 10 000 000

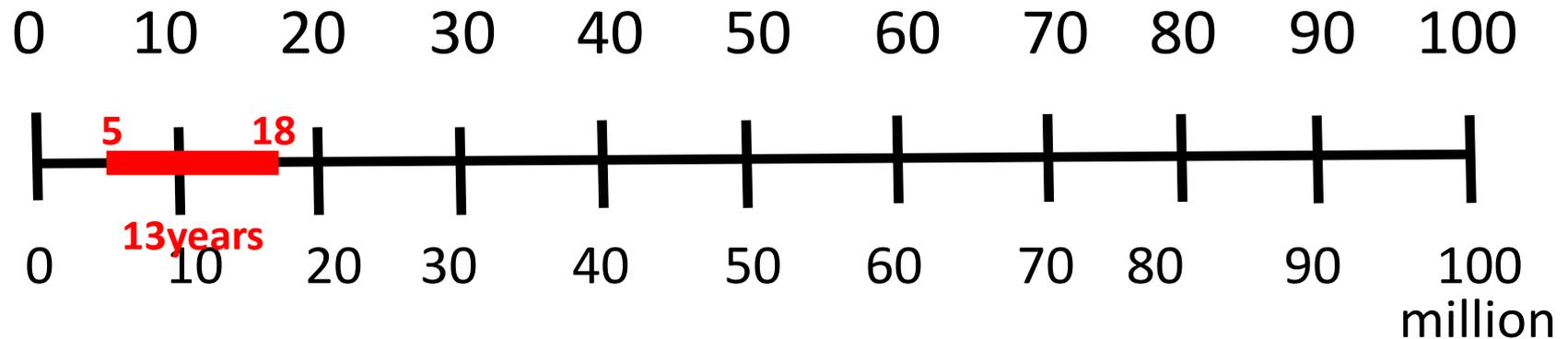


Roughly 1 million people in every 1 year interval

MODELLING AGAIN

Roughly how many 5 to 18 year olds are there in the UK?

A 10 000 B 100 000 C 1 000 000 D 10 000 000



Roughly 1 million people in every 1 year interval

13 year interval corresponds to **13 000 000 people \approx 10 000 000**

A	B	C	D	F	G	I	K	L	N	O	Q	S	T	V	W	Y	AA	AB	AD	AE	AG
Age as at 31 August 2015	State-funded primary schools (1)(2)					State-funded secondary schools (1)(3)(5)					State-funded special schools (4)				Non-maintained special schools						
	Full-time		Part-time		Total	Full-time		Part-time		Total	Full-time		Part-time		Total	Full-time		Part-time		Total	
	Boys	Girls	Boys	Girls		Boys	Girls	Boys	Girls		Boys	Girls	Boys	Girls		Boys	Girls	Boys	Girls		
Pupils aged:																					
2 and under	2,268	2,362	24,041	23,790	52,461	98	73	270	314	755	97	55	190	107	449	3	1	3	2	9	
3 (born April to August)	6,984	6,905	46,249	44,912	105,050	117	101	500	477	1,195	111	75	264	128	578	1	1	5	2	9	
3 (born January to March)	4,696	4,621	27,366	26,796	63,479	67	76	321	308	772	84	51	151	91	377	1	1	4	3	9	
3 (born September to December)	6,923	6,905	37,934	37,102	88,864	104	109	416	399	1,028	155	82	286	146	669	2	1	12	0	15	
4 (born April to August)	137,011	131,356	958	715	270,040	1,699	1,550	21	18	3,288	1,005	408	41	35	1,489	8	4	0	0	12	
4 (born January to March)	79,608	75,846	426	377	156,257	972	907	13	18	1,910	664	235	30	13	942	2	8	2	1	13	
4 (born September to December)	113,605	108,300	543	490	222,938	1,430	1,335	19	14	2,798	1,023	378	32	15	1,448	10	6	1	0	17	
Total under 5	351,095	336,295	137,517	134,182	959,089	4,487	4,151	1,560	1,548	11,746	3,139	1,284	994	535	5,952	27	22	27	8	84	
5	328,425	313,890	0	0	642,315	3,539	3,531	0	0	7,070	3,445	1,317	0	0	4,762	34	20	0	0	54	
6	322,355	309,547	0	0	631,902	2,973	3,026	0	0	5,999	3,860	1,416	0	0	5,276	74	19	0	0	93	
7	322,854	310,925	0	0	633,779	2,912	2,934	0	0	5,846	4,379	1,562	0	0	5,941	76	28	0	0	104	
8	310,798	298,794	0	0	609,592	2,582	2,535	0	0	5,117	4,522	1,606	0	0	6,128	91	38	0	0	129	
9	293,213	282,815	0	0	576,028	8,758	8,657	0	0	17,415	4,792	1,664	0	0	6,456	122	50	0	0	172	
10	285,052	275,969	0	0	561,021	10,009	9,509	0	0	19,518	5,153	1,841	0	0	6,994	155	46	0	0	201	
Total 5 to 10	1,862,697	1,791,940	0	0	3,654,637	30,773	30,192	0	0	60,965	26,151	9,406	0	0	35,557	552	201	0	0	753	
11	670	527	0	0	1,197	284,489	275,653	0	0	560,142	6,840	2,707	0	0	9,547	220	67	0	0	287	
12	119	129	0	0	248	276,462	268,038	0	0	544,500	7,074	2,663	0	0	9,737	256	85	0	0	341	
13	0	1	0	0	1	266,168	258,461	0	0	524,629	7,190	2,663	0	0	9,853	279	94	0	0	373	
14	0	0	0	0	0	266,657	260,378	0	0	527,035	7,376	2,866	0	0	10,242	283	105	0	0	388	
15	0	0	0	0	0	268,104	262,344	0	0	530,448	7,472	3,027	0	0	10,499	311	121	0	0	432	
Total 11 to 15	789	657	0	0	1,446	1,361,880	1,324,874	0	0	2,686,754	35,952	13,926	0	0	49,878	1,349	472	0	0	1,821	
16	0	0	0	0	0	106,228	116,029	32	11	222,300	3,634	1,927	0	1	5,562	304	118	0	0	422	
17	0	0	0	0	0	88,246	101,200	50	23	189,519	3,110	1,651	1	0	4,762	262	114	0	1	377	
18	0	0	0	0	0	11,149	10,314	31	17	21,511	2,344	1,247	0	0	3,591	227	90	0	1	318	
19+	0	0	0	0	0	314	308	1	0	623	32	29	0	0	61	25	14	0	0	39	
Total 16 to 19+	0	0	0	0	0	205,937	227,851	114	51	433,953	9,120	4,854	1	1	13,976	818	336	0	2	1,156	
CSA and above (8)	1,863,486	1,792,597	0	0	3,656,083	1,598,590	1,582,917	114	51	3,181,672	71,223	28,186	1	1	99,411	2,719	1,009	0	2	3,730	
All ages	2,214,581	2,128,892	137,517	134,182	4,615,172	1,603,077	1,587,068	1,674	1,599	3,193,418	71,362	29,470	995	576	105,363	2,746	1,031	27	0	3,814	

UK Government

figure:

7 917 767

**UK Government
figure:**

7 917 767

Fermi estimate:

10 000 000

Building Estimation skills

SINGLE STEP PROBLEMS

in a recent announcement the UK government said it will spend £5.2bn on £5.2 billion seems to be a huge amount of money; how much treatment will this buy you?

The New York Times reported that 30 billion tons of food is imported to the USA annually. Does this figure seem realistic?

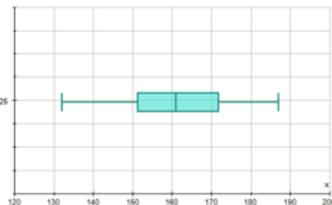
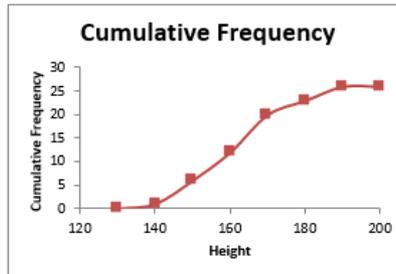
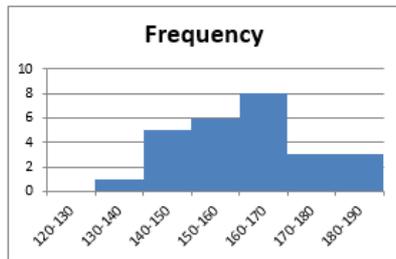
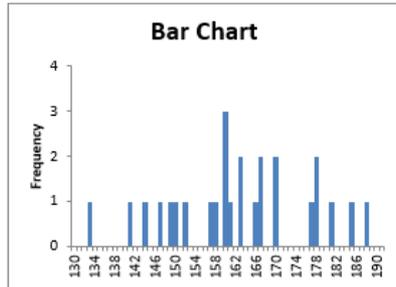
MULTI-STEP PROBLEMS

A multi-billionaire offers to give you £5 billion but only if you count it out in £1 coins and arrange for them to be transported and stored at your home. Could you meet these conditions?

MORE INFO REQUIRED PROBLEMS

A newspaper suggests that 5% of the UK has a car parked on it. Is this figure reasonable?

What about data?



A

B

C

D

	Question	Answer	Which graphs show this	Which graph(s) show this most clearly?
1)	How many people have been measured [Sample size]			
2)	How many measurements have been taken on each person [Number of variables]			
3)	What is the most common height?			
4)	What is the maximum height?			
5)	What is the minimum height?			
6)	What is the median height?			
7)	What is the height below which 25% of people are lower? [Lower Quartile]			
8)	What is the height below which 75% of people are lower? [Upper Quartile]			
9)	What is the height range from where 25% of people are below the median to 25% above the median [Interquartile range]			
10)	What is the mean?			

Then what?

HOSPITAL DATA TASK (from OUP textbook)

For non-emergency treatment, the waiting time to see a consultant should be no more than 18 weeks from referral. The table gives the waiting time results for one hospital.

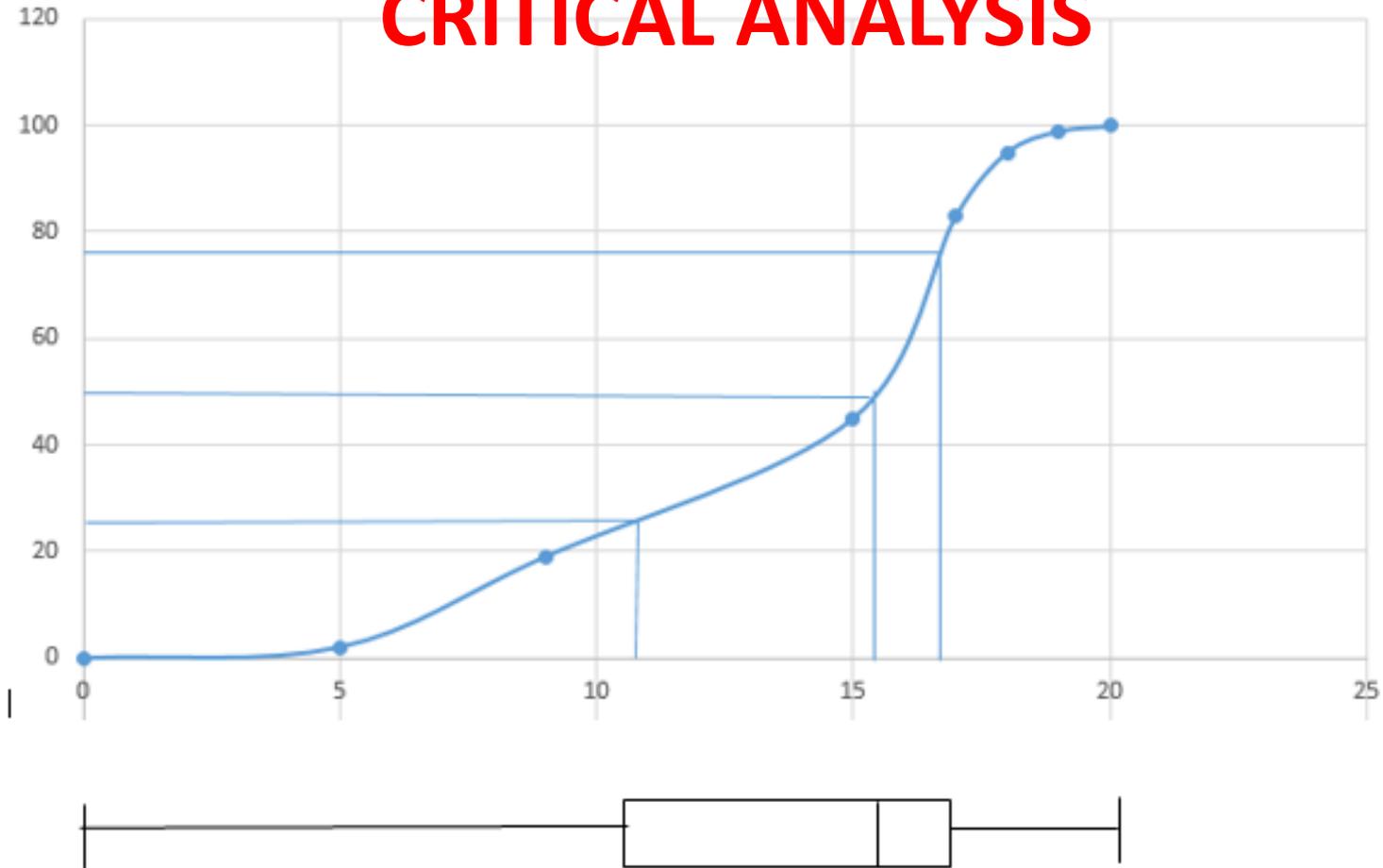
Comment on the hospital's performance.

Use statistical measures and/or measures to support your comments (5)

Waiting time	% of patients
Less than 5 weeks	2
5-9 weeks	17
10-15 weeks	26
16-17 weeks	38
18 weeks	12
19 weeks	4
20 weeks	1
More than 20 weeks	0

95% of appointments were within 18 weeks so the hospital is doing well

CRITICAL ANALYSIS



With a median wait time of around 16 weeks, the hospital appears to be struggling to meet its 18 week target. While it is encouraging to note that only 5% of the wait times are above the target, the half of patients have to wait 12-17 weeks, with only 25% experiencing a wait of under 11 weeks

And data beyond GCSE?

STANDARD DEVIATION is a good 'higher level' place to start (some good approaches for this)

CORRELATION looking formally at regression lines through a mean point and correlation coefficients is often quite successful (lots of opportunities for spreadsheets here)

THE NORMAL DISTRIBUTION IS HARD
AVOID CONFIDENCE INTERVALS UNTIL Y13

Topics to watch out for:

AER/APR

Annualisation causes enormous confusion – don't tackle it until the students are really good with multipliers.

$$r = \left(1 + \frac{i}{n}\right)^n - 1$$

The compound interest formula is probably the most bewildering one they will see

$$C = \sum_{k=1}^m \left(\frac{A_k}{(1+i)^{t_k}} \right)$$

THE NORMAL DISTRIBUTION

They need to be really fluent with this, don't rush to z-numbers.

Start with some real data, put it in a histogram and work out the proportions of a population between particular values. Use the Normal Distribution as a practical way of modelling reality.

Things I wish I'd known before we started to teach Core Maths:

- There's plenty of time to get through the content so don't rush, take the time to develop those higher level thinking skills
- The students need to feel they're learning something new so give them a quick win early on (Financial Maths is good)
- When you assess them, build up to those hard, '10 mark questions' slowly, use carefully constructed multiple choice and short questions at the start
- Take the time to find at least one really good, interesting problem to work on each week – there's a lot more out there now

A Problem Solving Approach

Teaching FOR problem solving

probably has to come before

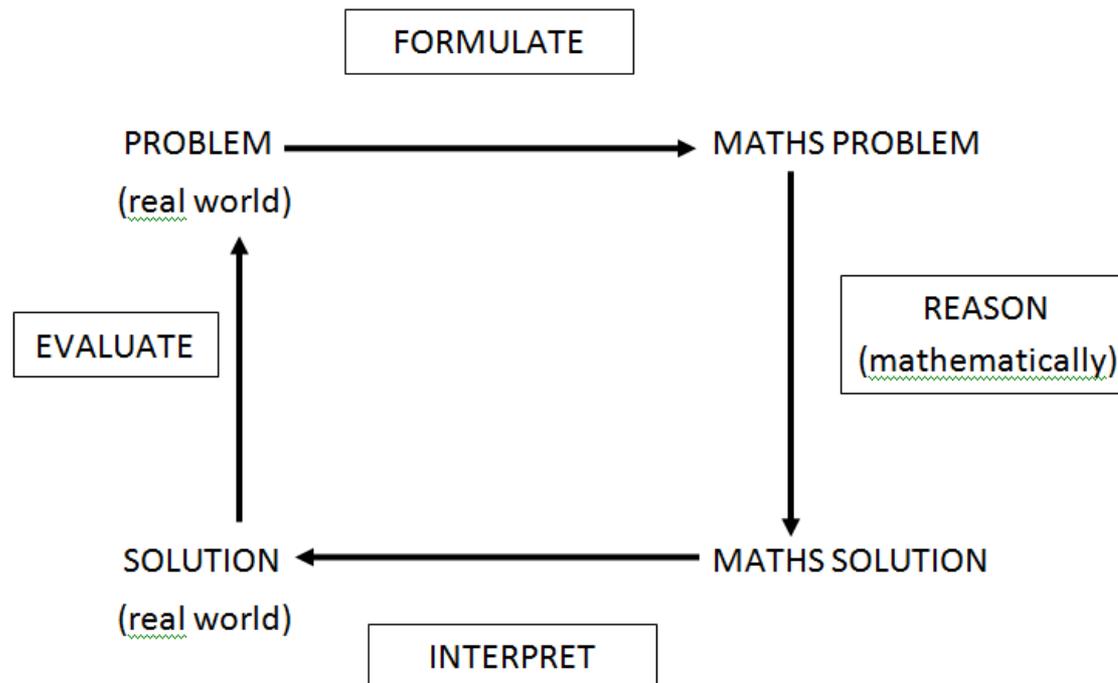
Teaching THROUGH problem
solving

New questions from old

In a box of pens, there are

three times as many red pens as green pens
and two times as many green pens as blue pens.

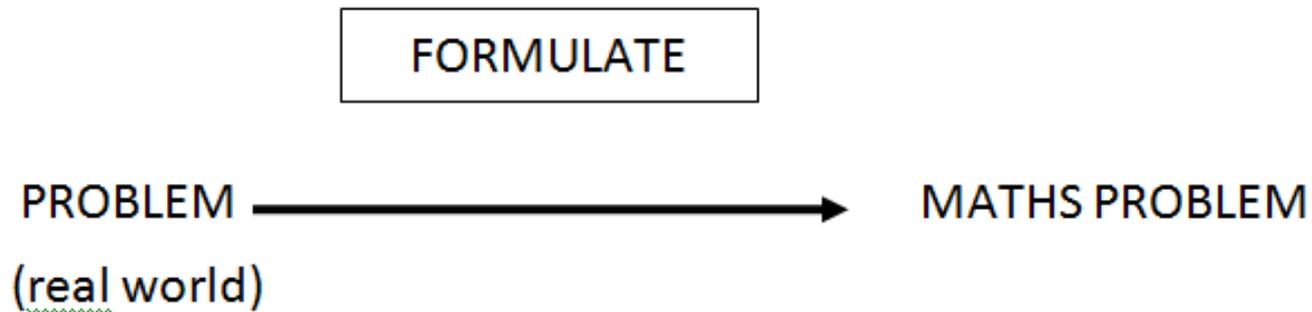
For the pens in the box, write down
the ratio of the number of red pens to the number of green pens to the number of blue pens.



In a box of pens, there are

three times as many red pens as green pens
and two times as many green pens as blue pens.

Draw a picture of what the box might contain



Stuff that has worked

TEACHERS

Have
benefitted
from
experiencing a
lesson then re-
planning it
straight
afterwards

Teaching FOR problem solving
before teaching THROUGH it

Showing teachers how to
scaffold through questioning

Showing teachers how to
create new questions from old

STUDENTS

Develop
their critical
thinking
skills slowly