Making Sense of Statistics

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Making sense of Statistical concepts

- Mean and median
- Standard deviation
- Data analysis
- The value of charts
- Tricky tables
- Probability
- Expected values
- Sampling
- Hypothesis Testing

How would you describe this collection of stones?

Measures – averages

- You have just looked at some of these.
- What about the mode?
- Need it if you want to describe colour.
- All are summary statistics.
- What don’t they tell you about the stones?

Measures – standard deviation

- The challenge
- To calculate the standard deviation of 51 53 51 53 in under 3 minutes
- Knowing nothing except
- Standard deviation measures spread.

Measures – standard deviation

- Measures spread
- 0 2 0 2
- Deviation from what?
- The mean which is 1
- What is the deviation of each of those numbers from the mean?

Measures – standard deviation

- Now look at 1 3 1 3
- Standard deviation measures spread.
- Are the numbers more or less spread out, or have the same spread?
- What's the standard deviation?

1 3 1 3
has a standard deviation of 1

- Add 50 to give
- 51 53 51 53
- what’s the standard deviation?

Measures – standard deviation

- Double the numbers: 2 6 2 6
- Standard deviation measures spread.
- Are the numbers more or less spread out, or have the same spread?
- What's the standard deviation at a guess?

Measures – standard deviation

- What about 6 6 6 6 ?
- Standard deviation measures spread.
- What's the standard deviation?

Measures – standard deviation

- What next ?
- Start with easy numbers: 2, 3, 7
- Attach a meaning and units
- Time spent brushing one’s teeth
- You can think of a better example.

Measures – standard deviation

- Measures spread
- Deviation from what?
- The mean, which is 4 minutes.
- What is the deviation of each of those numbers from the mean?

Measures – standard deviation

- Deviations: 2, 3, 7 each minus 4 gives
- -2, -1, 3
- **Standard** deviation implies an average
- There's a problem!
- The average deviation is 0.
- Coincidence? Good teaching point.

Measures – standard deviation

- To get round this we square the deviations which gives 4, 1, 9
- The sum is 14 and the mean 4·3333
- Hold it – the units are square minutes.
- How does this measure a spread?
- Take the square root

Measures – standard deviation

- $\sqrt{14}$ is 3.74
- Standard deviation is 3.74 minutes.
- Nearly every observation will be within 3 standard deviations of the mean.
- Teeth cleaning most likely takes between 15 seconds and 8 minutes.

Measures – standard deviation

- Use a spreadsheet to see what happens if you change a number.
- 2, 3, 10 minutes for instance gives a standard deviation of 6.16 minutes.
- One value can have a large effect.

Data analysis is about letting the data tells its story

- Use real data that students can identify with eg CensusAtSchool. http://www.censusatschool.org.uk/
- Is closed for time being but resources still there. See examples in my resources.

Data Stories

http://askten.co.uk/newsletter

- A weekly bulletin of 10 interesting facts.
- **Mark Carney to examine soaring credit card debt.** Credit card debt is rising at its fastest rate in more than a decade, says the Bank of England, increasing by 9.3% in the year to February. Households slapped £600m on their cards last month and now owe £67.3bn.
- **How do they know?**

Data Stories

http://askten.co.uk/newsletter

- **Bankers 'to blame for weak UK productivity'.** The Office for National Statistics says that just 5 sectors are responsible for two-thirds of the decline in productivity growth: bankers, telecoms companies, energy producers and management consultants and legal and accounting services. The Independent
How do they know?

More real data

- **Full fact** https://fullfact.org/ explores the facts behind news stories.
- **Significance magazine:** http://www.statslife.org.uk/significance
- **Mori Polls**: www.ipsos-mori.com/
- Neighbourhood Statistics (brill!)

Data should prompt questions

- What would be interesting to know?
- Off syllabus answers should be permissible.
- First of all LOOK at it !

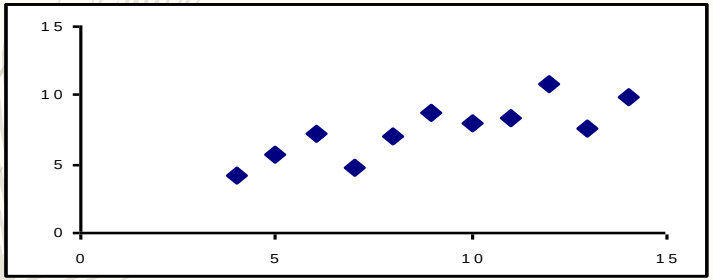
Anscombe’s data sets

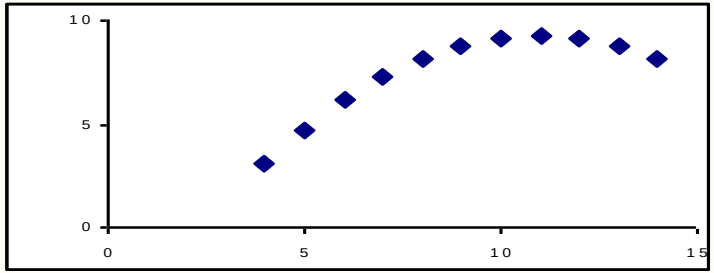
	x1	y1	x2	y2	x3	y3	x4	y4
	10	8.04	10	9.14	10	7.46	8	6.58
	8	6.95	8	8.14	8	6.77	8	5.76
	13	7.58	13	8.74	13	12.74	8	7.71
	9	8.81	9	8.77	9	7.11	8	8.84
	11	8.33	11	9.26	11	7.81	8	8.47
	14	9.96	14	8.1	14	8.84	8	7.04
	6	7.24	6	6.13	6	6.08	8	5.25
	4	4.26	4	3.1	4	5.39	19	12.5
	12	10.84	12	9.13	12	8.15	8	5.56
	7	4.82	7	7.26	7	6.42	8	7.91
	5	5.68	5	4.74	5	5.73	8	6.89
mean	9	7.5	9	7.5	9	7.5	9	7.5
correlation coefficient		0.82		0.82		0.82		0.82

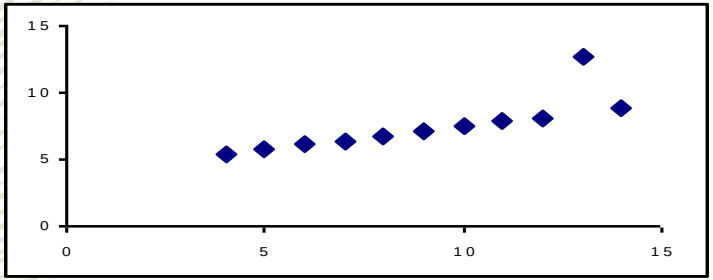
Looked? Now chart.

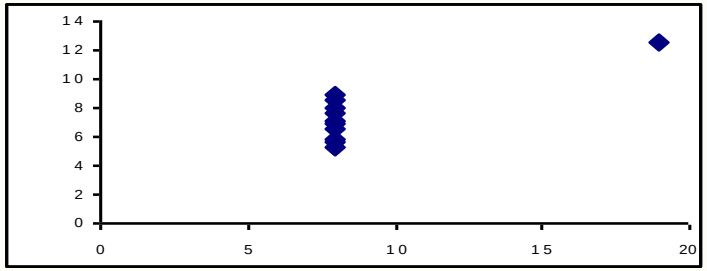
- Why chart?
- To help us understand the data.
- Sometimes a chart tells us far more than just computations.
- Frank Anscombe produced 4 data sets;
- data sets that were described by the **same linear model**

Was this what you expected?









Anscombe had a moral

- Whatever the figures say
- The model may not be appropriate.
- Here is it only appropriate in the first case.
- That’s why we check for a pattern in the residuals.

Regression- an aside

- Do they understand the equation of a straight line?
- Even if they did have an A* at GCSE.

The value of charts.
Graphical excellence consists of

- Complex ideas communicated with
- clarity, precision, and efficiency.

- Gives the viewer the greatest number of ideas in the **shortest** time with the **least ink** in the **smallest space**.

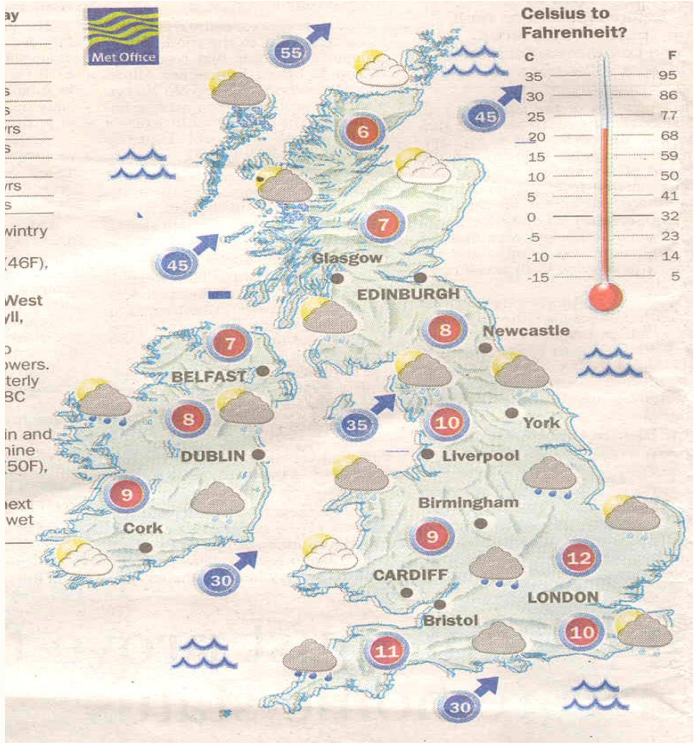
- It tells the **truth**. Edward Tufte

What chart illustrates all this?

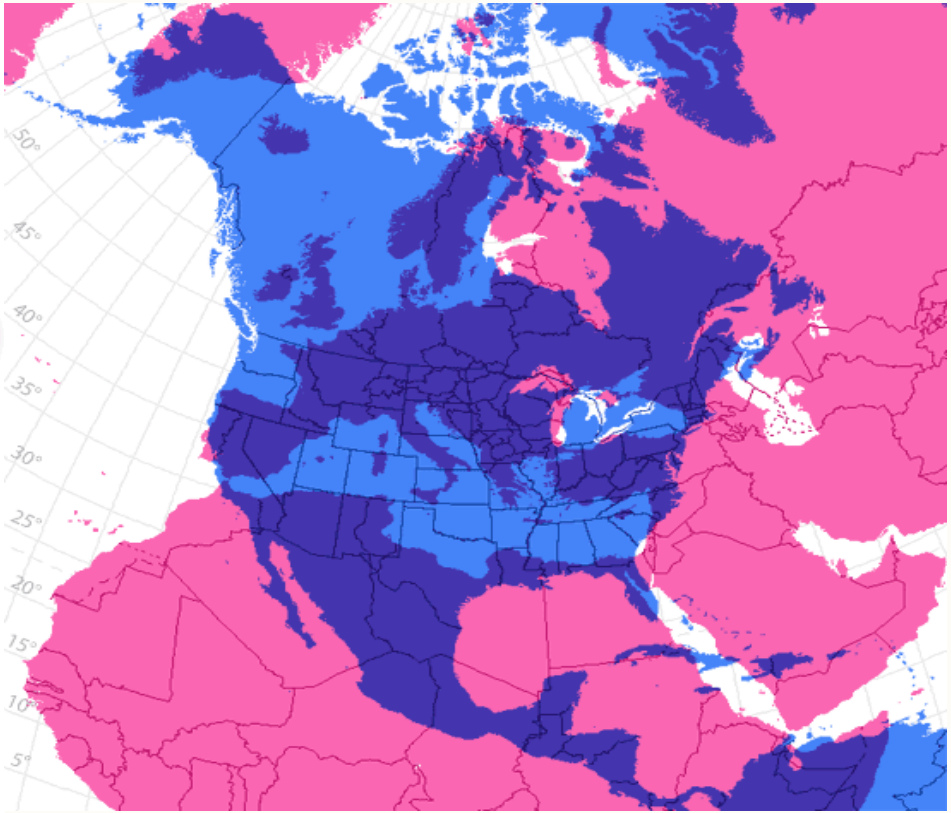
- Found in the newspaper.
- What is it?

Graphical excellence consists of complex ideas communicated with clarity, precision and efficiency.

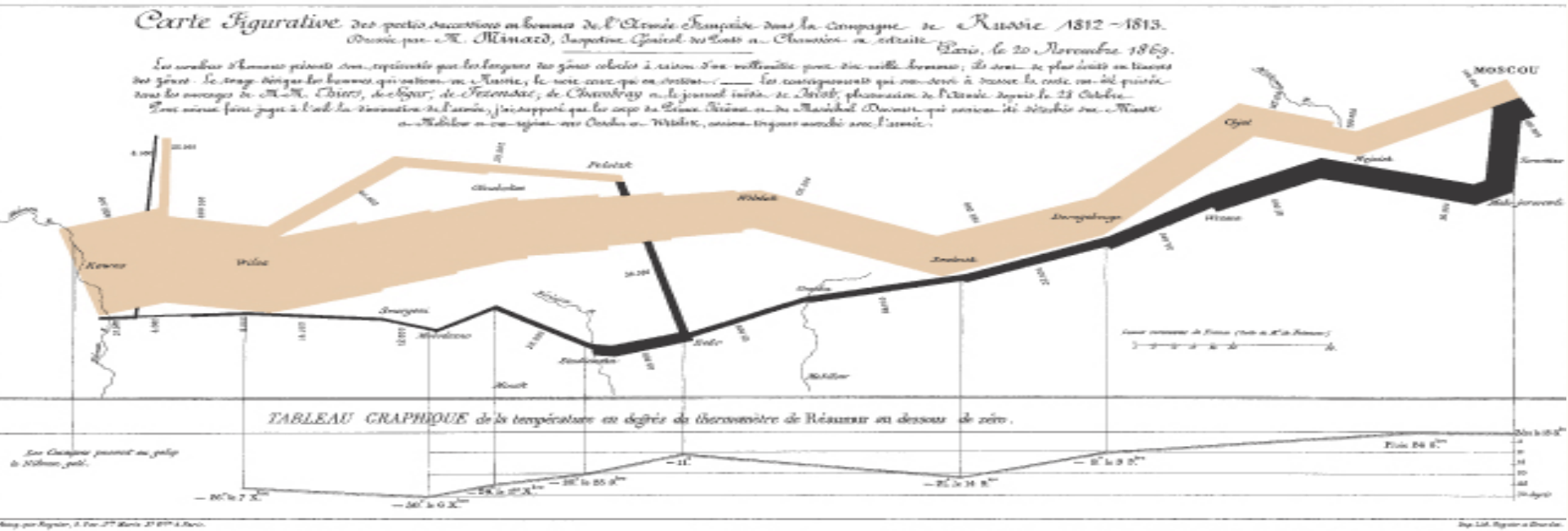
Consider for a moment just how much information is here.



And a few more



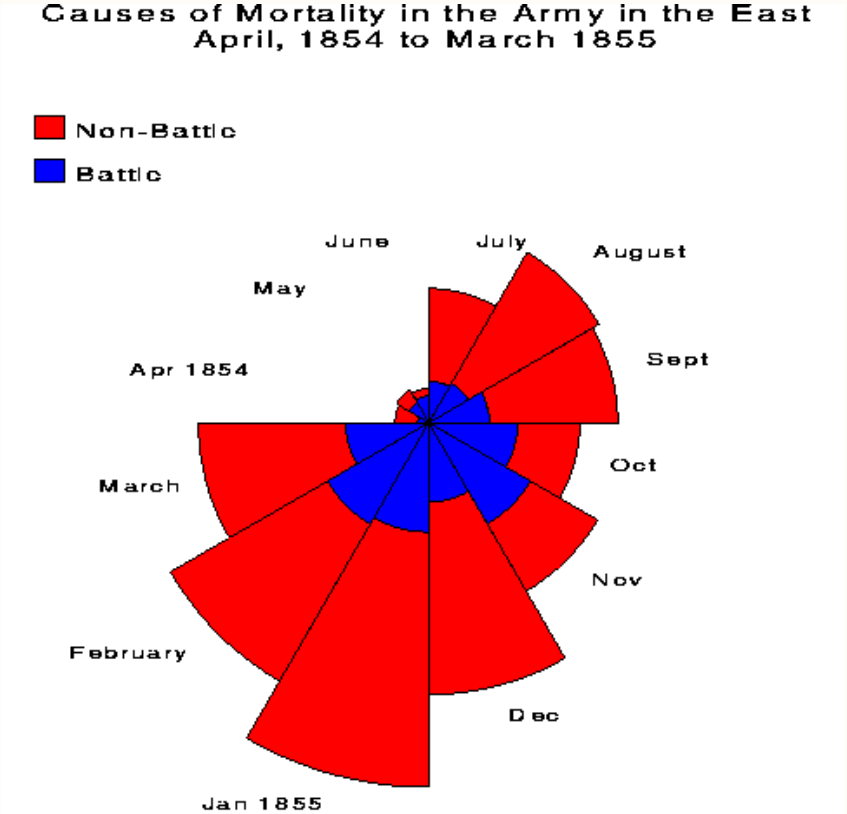
Napoleon’s retreat from Moscow
Minard’s chart



Dr. John Snow's map of cholera deaths 1854.

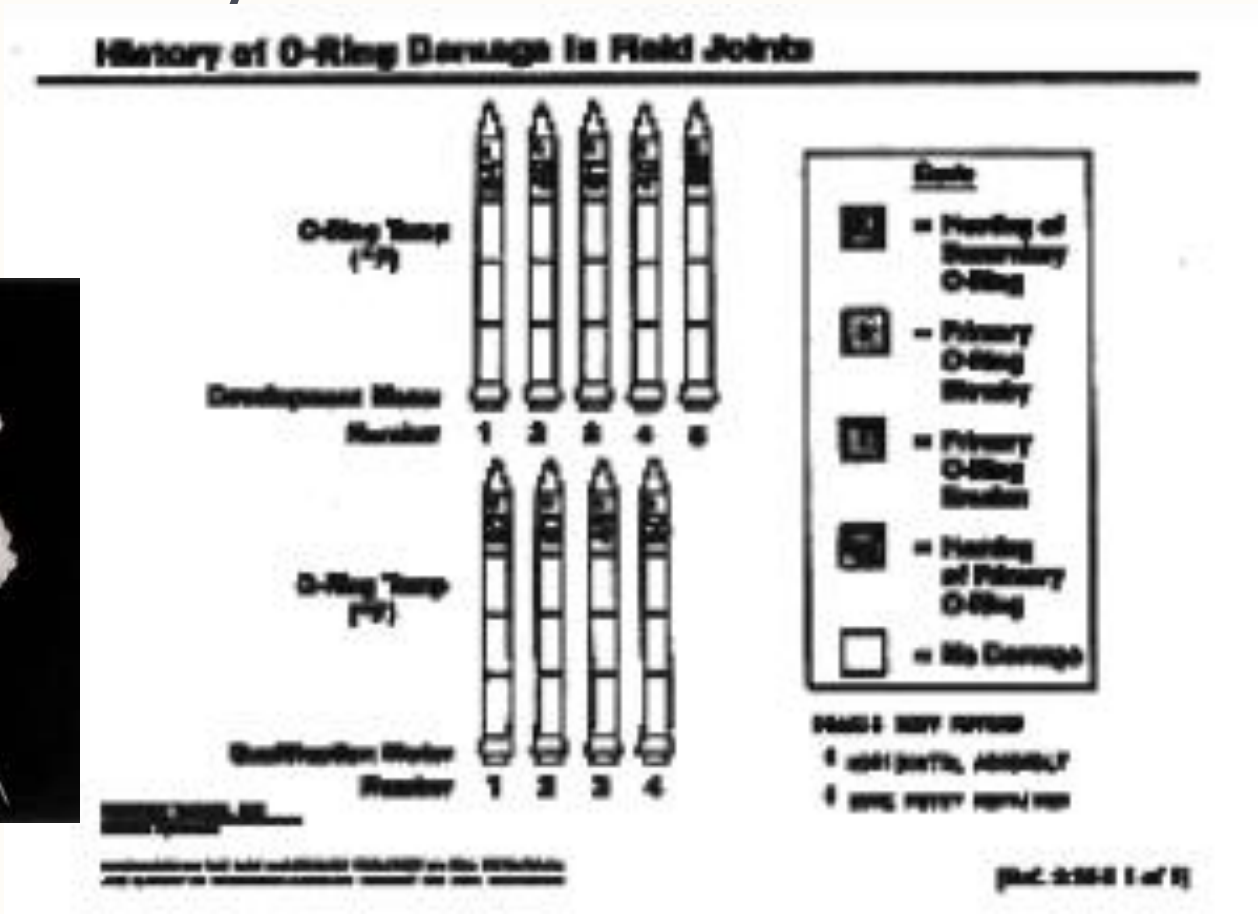
The small lines represent deaths and were centred round the Broad Street pump.

Florence Nightingale Cockscomb chart



From: F. Nightingale, "Notes on Matters Affecting the Health, Efficiency and Hospital Administration of the British Army", 1858

The Challenger Disaster
28 January 1986



Time for a break

- Look at your bottle of water.
- Find the **e** symbol.
- What does this mean?

The E (e) mark

- Weights and Measures (Packaged Goods) Regulations 2006:

- These Directives set out three rules with which packers must comply:

First rule

- the actual contents of the packages should not be less, **on average**, than the nominal quantity;

Rules 2 and 3

- the proportion of packages which are short of the stated quantity by a defined amount (the “tolerable negative error” or TNE) should be less than a specified level; and
- no package should be short by more than twice the TNE.

The Tolerable Negative Error

Nominal quantity in grams and millilitres	Tolerable negative error	
	As a %age of nominal quantity	g or ml
5 to 50	9	-
from 50 to 100	-	4.5
from 100 to 200	4.5	-
from 200 to 300	-	9
from 300 to 500	3	-
from 500 to 1,000	-	15
from 1,000 to 10,000	1.5	-
from 10,000 to 15,000	-	150
above 15,000	1	-

How does the production manager cope?

- Using sampling.
- What’s the general idea?
- How much of the picture are you getting?
- Look at the jigsaw pieces.

The jigsaw

How much did you get?

Compare the jigsaws

- What are the problems with sampling?
- Important for the pollsters!

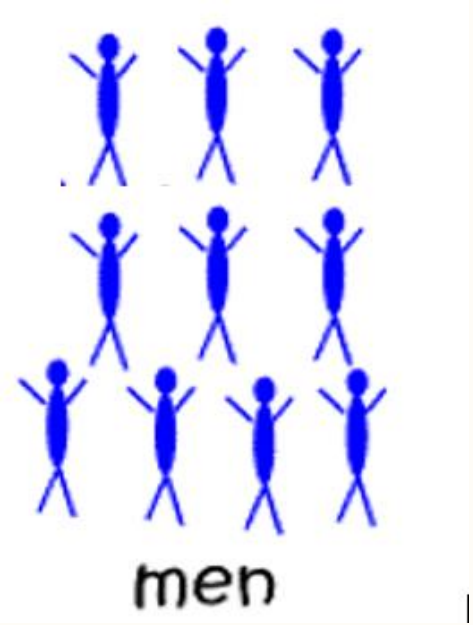
The best sample is a simple random sample

- which means that **every** item has an **equal** chance of being chosen and so does **every subset** of your items.

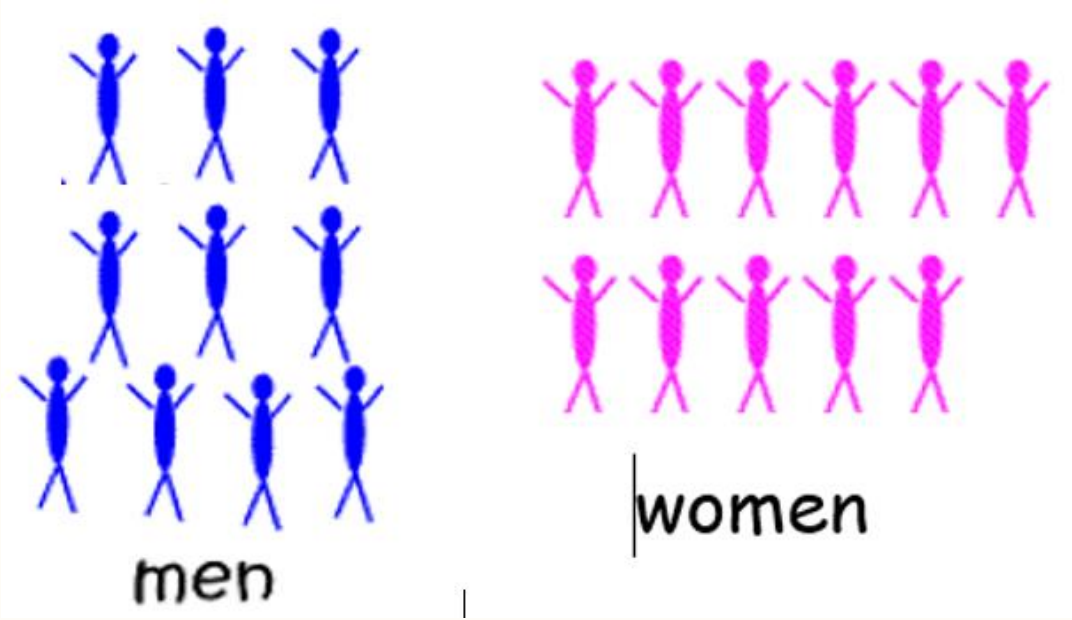
A good example

20 people in a room

We want a random sample of 2 people



A random sample of 2 people would include the possibility of 2 men.



If we randomly select 1 man from the 10 men and 1 woman from the 10 women

- Each member of the population would have an equal chance of being chosen.
- But we would never get a sample of 2 men.

Random numbers

13962	70992	65172	28053	02190	83634
66012	70305	66761	88344	43905	46941
72300	11641	43548	30455	07686	31840
03261	89139	00504	48658	38051	59408

Can also use for **simulation**.

Different types of samples:
Simple, Systematic Simple, Stratified, Cluster,

- Will you get the same answer each time?
- Even if you use the same technique but start in a different place?
- **Unitown exercise.**

Unitown

- Spreadsheet of responses for each address.
- Enabling you to carry out a ‘survey’ of the town using different methods.

I have a hypothesis

- I need to collect some data
- And then evaluate the evidence
- Here goes:

Choose and write down one of the following numbers:

1 2 3 4

Collect the data

The hypothesis was:

- More people choose
- 3
- than any other number.

What do you think?

- What does the evidence suggest?
- How do you make up your mind?

Here’s one collected earlier.

	1	2	3	4
number	16	23	34	12

- What does the evidence suggest?
- How do you make up your mind?

What about this one?

	1	2	3	4
number	16	18	20	18

- What does the evidence suggest?
- Can you make up your mind?

This illustrates the basic steps

- State the hypothesis
- Collect the data
- Evaluate the evidence

The hypothesis test

- Uses the evidence to decide
- whether the null hypothesis
- can be accepted
- or not.
- A good analogy is ….

An English court of law:

The defendant is assumed **not guilty** until there is sufficient evidence to find otherwise.

And there can be

- Mistakes!
- Which is why statisticians are picky
- **Do not accept** rather than reject.
- **Do not reject** rather than accept.

Different tests

- A good example for stats tests is clinical trials data for a weight reduction drug.
- See Excel 2013 **stats tests.xls** for the data and step by step instructions for the tests.

Significance level

- A practical example:
- ESP extra sensory perception using playing cards.

95% Significance level

- Look at the squares.
- Any idea how many?
- 126,000
- The yellow bit of paper is 5%

The Salk Vaccine Trial

- A problem to be solved – a challenge.
- An experiment to test the efficacy of a new polio vaccine in the U.S.A.
- Biggest public health experiment ever: the 1954 field trial of the Salk Poliomyelitis Vaccine.

The Salk Vaccine Trial: test the efficacy of a new polio vaccine

- What would you do?
- Resources almost boundless.
- Difficult to predict the next outbreak, where and when.

The Salk Vaccine Trial: test the efficacy of a new polio vaccine

- How will you set up a control group?
- How large will your sample be?
- Are there any ethical issues?

What happened

Summary of study cases by diagnostic class and vaccination status (rates per 100,000)

		Poliomyelitis Cases											
		All Reported Cases		*Total*		*Paralytic*		*Non-paralytic*		*Fatal polio*		*Not Polio*	
Study Group	*Study Population*	*No.*	*Rate*	*No.*	*Rate*	*No.*	*Rat*	*No.*	*Rate*	*No.*	*Rate*	*No.*	*Rat*
All areas: Total	1,829,916	1,013	55	863	47	685	37	178	10	15	1	150	8
Placebo control areas: Total	749,236	428	57	358	48		36	88	12	4	I	70	9
Vaccinated	200,745	82	41	57	28	33	16	24	12	—		25	12
Placebo	201,229	162	81	142	71	115	57	27	13	4	2	20	10
Not inoculated*	338,778	182	54	157	46	121	36	36	11	-	-	25	7
Incomplete vaccinations	8,484	2	24	2	24	1	12	1	12				
Observed control areas: Total	1,080,680	585	54	505	47	415	38	90	8	11	1	80	7
Vaccinated	221,998	76	34	56	25	38	17	18	8		-	20	9
Controls**	725,173	439	61	391	54	330	46	61	8	11	2	48	6
Grade 2 not inoculated	123,605	66	53	54	44	43	35	11	9	—		12	10
Incomplete vaccinations	9,904	4	40	4	40	4	40	—	—	—	-	—	—

***Includes 8,577 children who received one or two Infections of placebo,**
****First and third-grade total population**

***Source*: Adapted from T. Francis. Jr. (1955), Tables 2 and 3.**

What happened

- Interpreting the results.
- Using tables.
- But tables can be tricky.
- 2 stories might help
- The first: the Titanic survivors

Tricky Tables – Titanic data
what % of survivors were 1st class?

		Class		
Survived	1st	2nd	3rd	Total
No	**15%**	**19%**	**66%**	100%
Yes	**43%**	**26%**	**31%**	100%
Total	**25%**	**21%**	**44%**	100%

Tricky Tables – Titanic data
what % of 1st class were survivors?

		Class		
Survived	1st	2nd	3rd	Total
No	**40%**	**58%**	**81%**	66%
Yes	**60%**	**42%**	**19%**	34%
Total	**100%**	**100%**	**100%**	100%

Not the same questions

- Be clear in your own mind what you want to compare.
- Characteristics of passenger classes?
- Characteristics of the survivor groups?

Do men exercise more than women?

- Put each group in a separate ‘room’ in your mind
- You want %’s to add to 100% for that ‘room’.

Tables are tricky

what % of men are very active?

	men	women
very active	75%	50%
not very active	25%	50%
total	100%	100%

	men	women	total
Very active	86%	14%	100%
Not very active	68%	32%	100%

What % of the very active are men?

	men	women
very active	75%	50%
not very active	25%	50%
total	100%	100%

	men	women	total
Very active	86%	14%	100%
Not very active	68%	32%	100%

This is not the same question

- Be clear in your own mind what you want to compare.
- Characteristics of men v women?
- Characteristics of the activity level groupings?

Look at the tables again

	men	women
very active	75%	50%
not very active	25%	50%
total	100%	100%

	men	women	total
Very active	86%	14%	100%
Not very active	68%	32%	100%

Do men exercise more than women?

- This brings us to probability
- hypothesis testing,
- sampling,
- and the second (simple) story.

Probability

- Corks and long term relative frequency
- Expected values
- Coincidence?

Expected values

- Choose 1 of 3 boxes
- £1 10p 1p (total 111p)
- Expected value is 37p
- What would **you** expect to get?

Coincidence?

a card trick see the end of the slide show

Now look at the beans

- There are 5 bags of a sample of 30 beans.
- In statistics this is considered a

large sample.

Suppose your hypothesis was that in the population of beans 30% were white

- Which sample or samples would support that hypothesis?
- The 10%, 20%, 30%, 40% or 50%?

Would those samples

- also support a different hypothesis?
- This is a problem!
- How can we improve our confidence?
- The answer is to increase the sample size.

Now for the normal distribution

- And
- The Central Limit Theorem
- The knock out of statistics

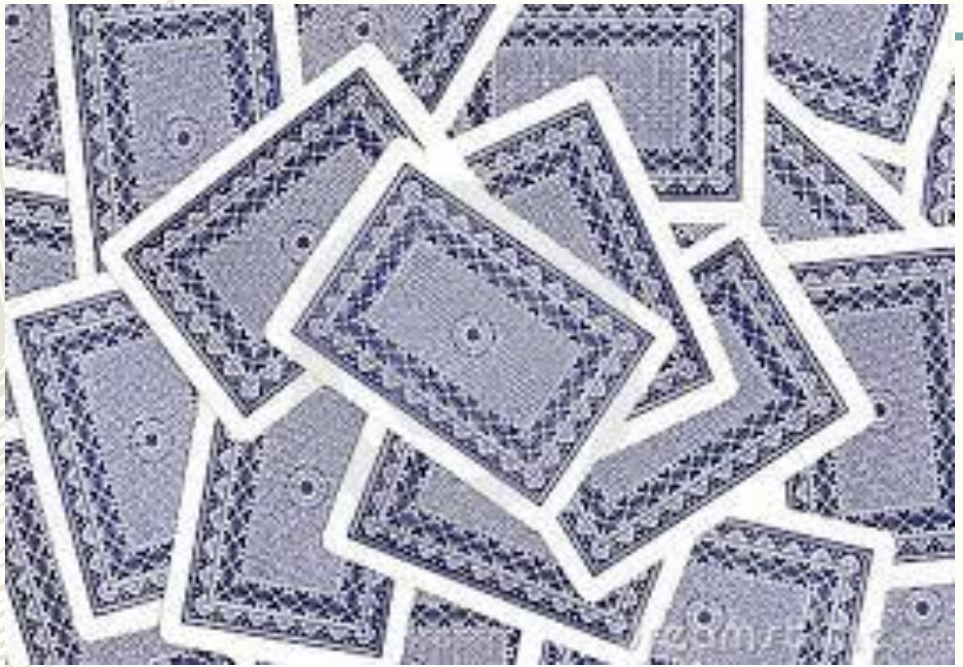
And here

- We used DISCUS sampling which is in the resources.
- The End (but the card trick follows)
- Thank you.

Coincidence?

The card trick follows.
Is it a coincidence?
Algebra shows not!

Spread the 52 cards out face down



Now to use your ESP

- Person 1 picks up 8 cards which they think are black DO NOT LOOK AT THEM
- Person 2 picks up 7 cards which they think are red
- Person 3 picks up 6 cards which they think are black
- Person 4 picks up 5 cards which they think are red

One person

- Takes those cards
- Shuffles them and holds on to them.

- Leave the rest face down on the table.

The person with the shuffled cards

- Turns one over and place face
- If it’s black
  someone else chooses what they think is a black card and places it next to it face down
- If it’s red choose a red card.

Continue turning over the shuffled cards

- Until you have 4 piles

- Red turned over + what you think are red face down
- Black turned over + what you think are black face down

How many ?

- How many **black** cards do you think there are in the face down pile next to the **black** cards?
- How many **red** cards do you think there are in the face down pile next to the **red** cards?

- Count them!
- Coincidence? A good exercise in algebra.