'What is mastery, why does it matter, and what are teachers doing where it's working well?

Debbie Morgan
Director for Primary Mathematics

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What does it mean to master something?

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- I know how to do it
- It becomes automatic and I don't need to think about it- for example driving a car
- I'm really good at doing it – painting a room, or a picture
- I can show someone else how to do it.

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Mastery of Mathematics is more…..

- Achievable for all
- **Deep** and sustainable learning
- The ability to build on something that has already been sufficiently mastered
- The ability to reason about a concept and make connections
- Conceptual and procedural fluency

Why does it matter?

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The Achievement of All

As children progress through primary, the attainment gap widens rather narrows

This is not the case in other countries

Why does the attainment gap widen?

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- Pupil beliefs
- Teacher beliefs
- Differentiation - slow some down
- Teaching/classroom organisation

How does Mastery Close the Gap?

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- The expectation that all will achieve
- Class working together at broadly the same pace
- A small detailed step approach to teaching

What are teachers doing that is working well?

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- Whole class teaching
- Teaching with Variation
- Seeking to develop depth in learning through attention to mathematical structure and relationships
- Developing automaticity with number facts
- Use of language structures

Whole Class Teaching

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Whole Class teaching

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Provides a clear and coherent journey through the mathematics

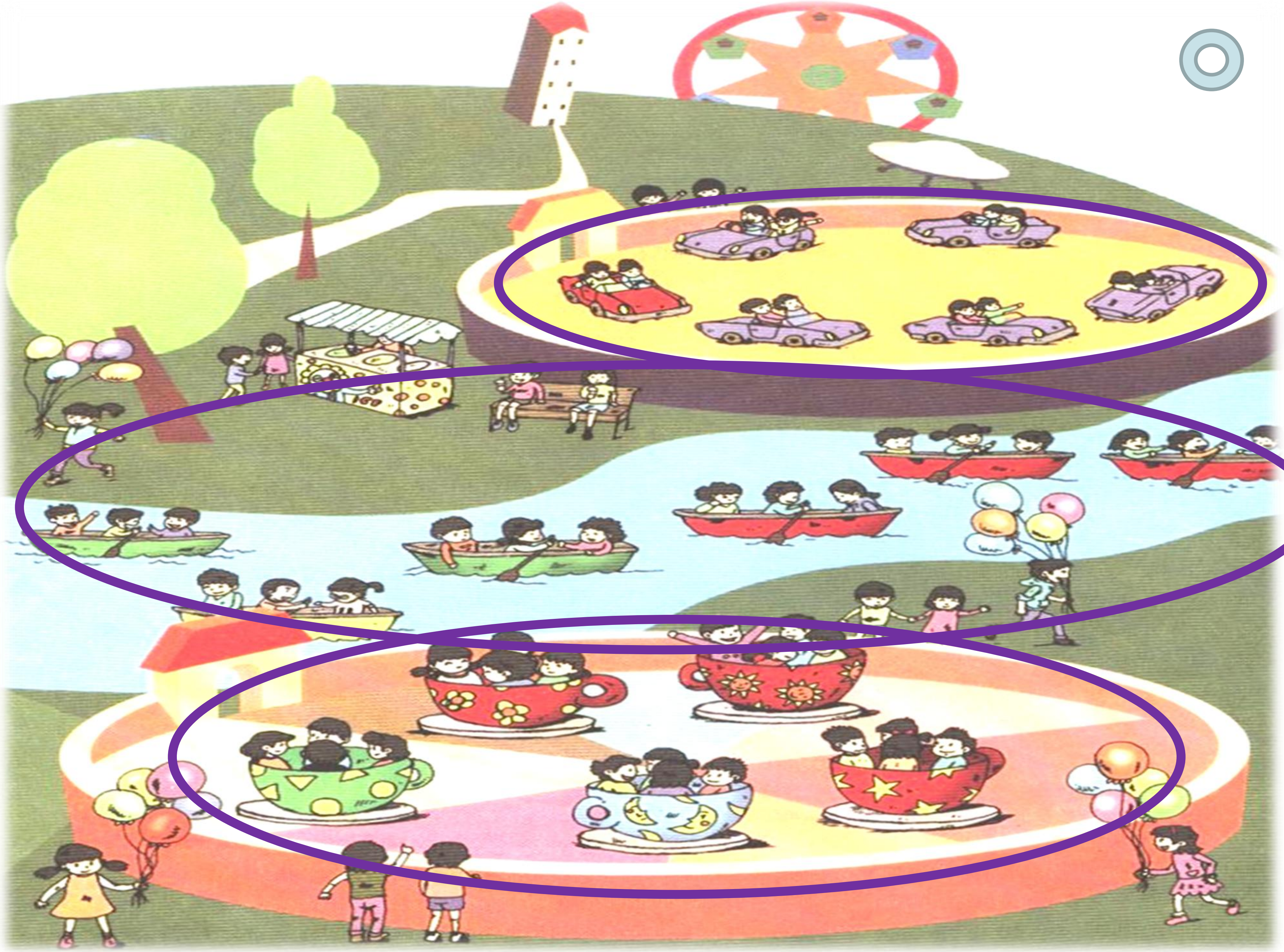
Provides access

Provides detail

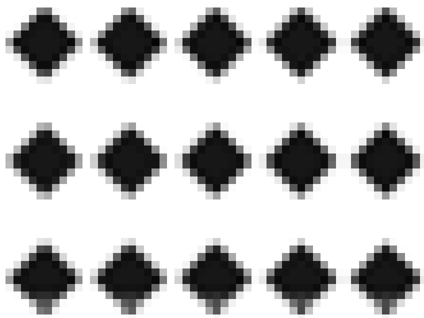
Provides scaffolding for all to achieve

Provides the small steps

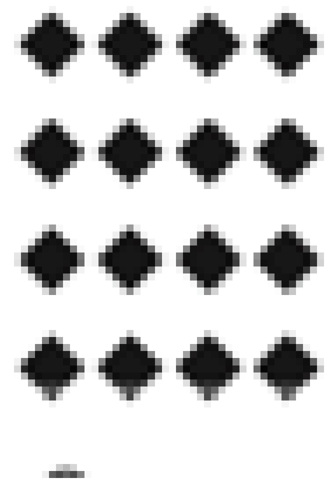
Sharpens the focus



Equal Groups



Groups of 5



Groups of 4

Groups of 6

Groups of 2

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Variation

Non Conceptual Variation

Who made equal groups?

Methods, Answers, Conclusion

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You have just showed me your answer

But I want you to show me your method

Pupil Support

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One of the most important tasks of the teacher is to help his students…

If he is left alone with his problem without any help or insufficient help, he may make no progress at all…

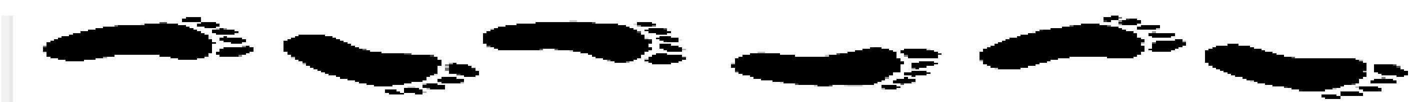
If the teacher helps too much, nothing is left to the student

(Polya 1957)

Providing a Pudian

By putting blocks or stones together as a Pudian, a person can pick fruit from a tree which cannot be reached without the Pudian (Gu 2014 p. 340).

The teacher provides the steps but the child takes and connects the steps, reasoning along the way



Forcing Awareness

"Forcing awareness", according to Gattegno,
has two meanings: "One is concerned with What we do to ourselves, and the other with what can be done to us so that we become aware of what has escaped us, or might escape us" (1987. p. 210).

What are teachers doing that is working well?

Teaching with Variation

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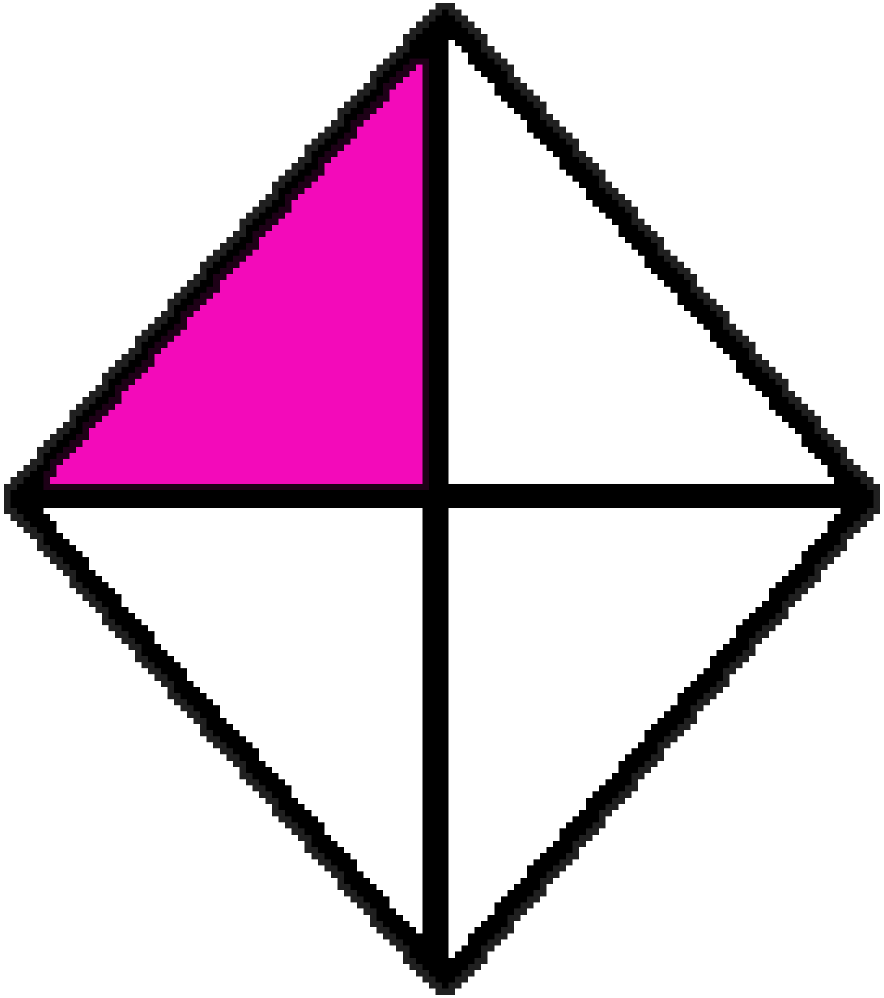
Conceptual Variation

An important teaching method through which students can **definitely master concepts**. It intends to illustrate the essential features by demonstrating different forms of visual materials and instances or highlight the essence of a concept by varying the non essential features. (Gu 1999)



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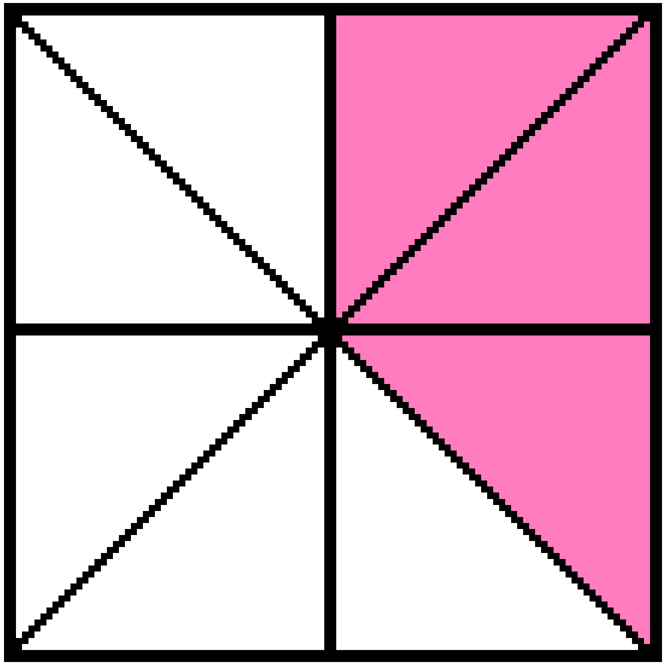

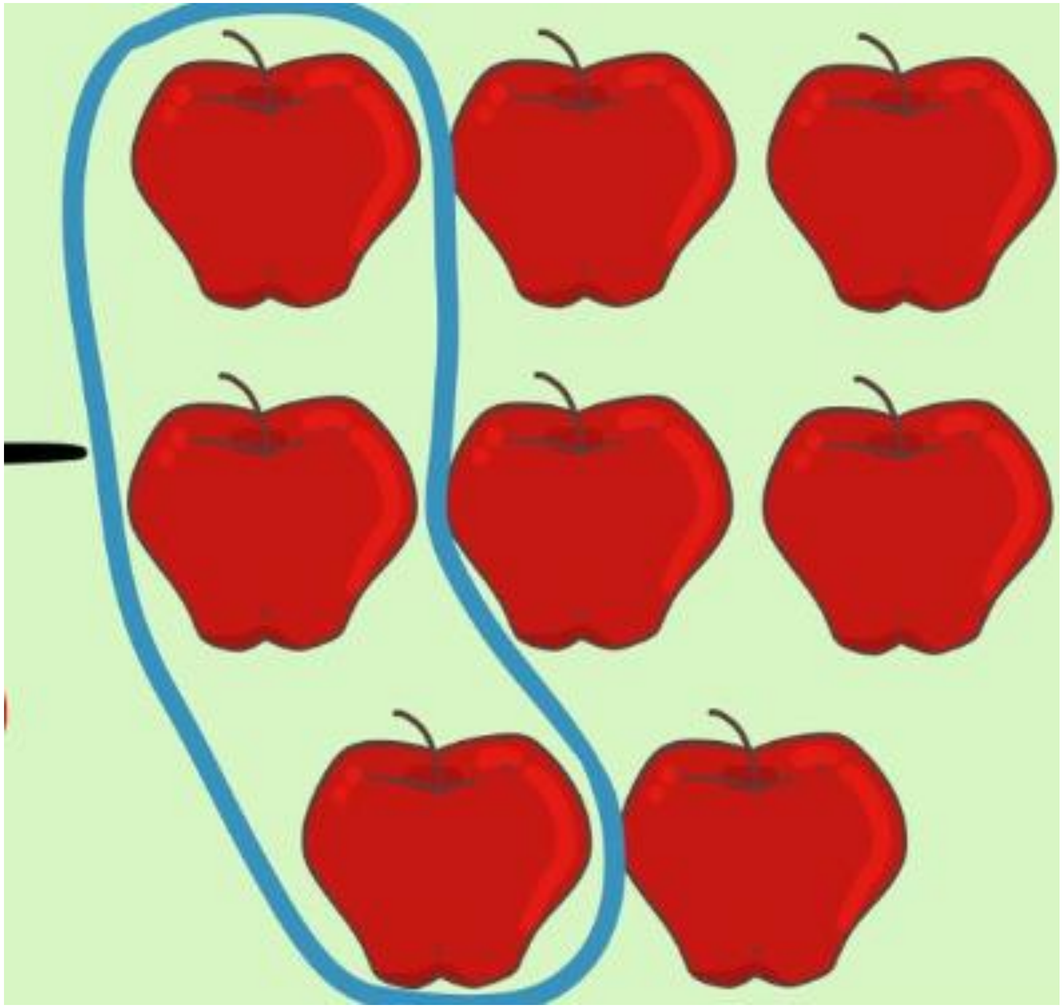

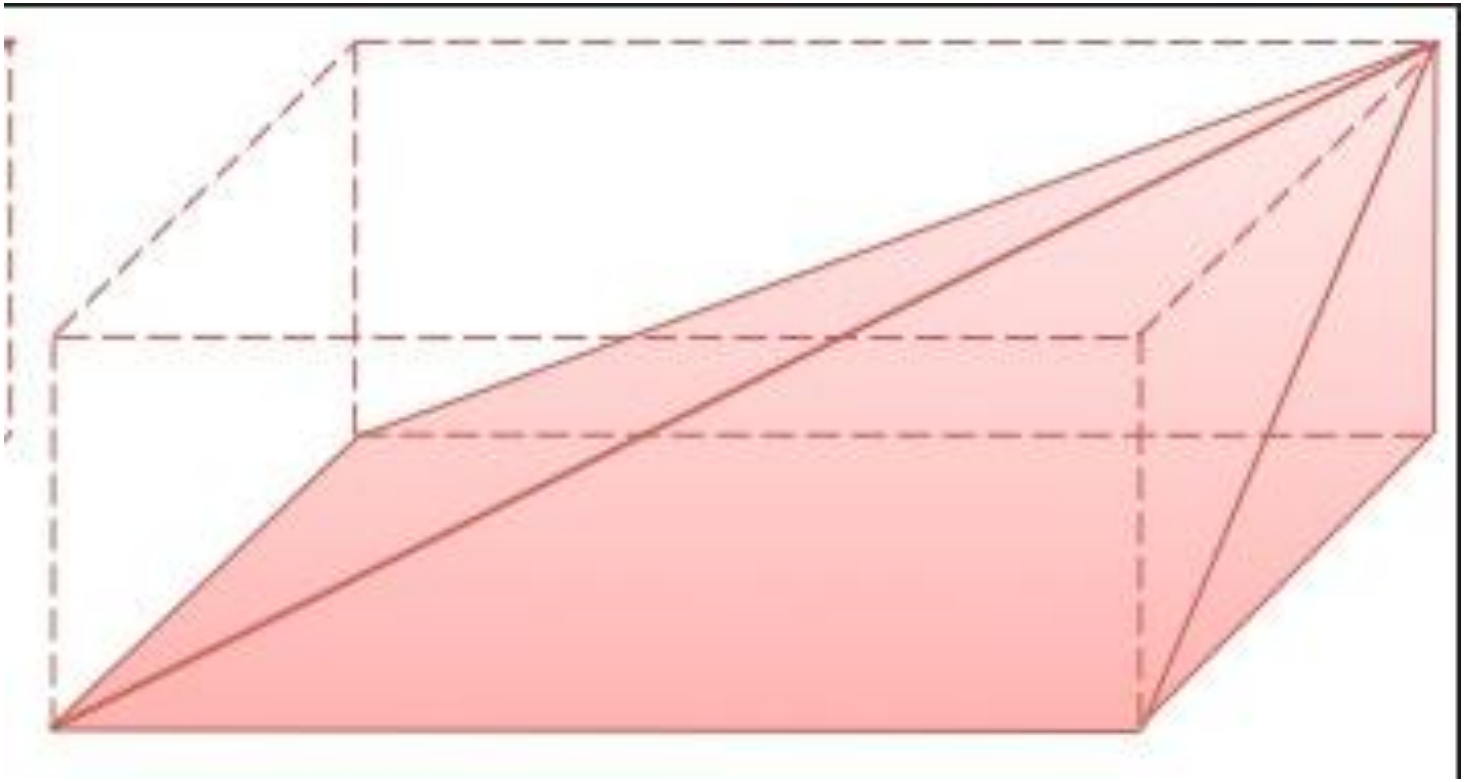

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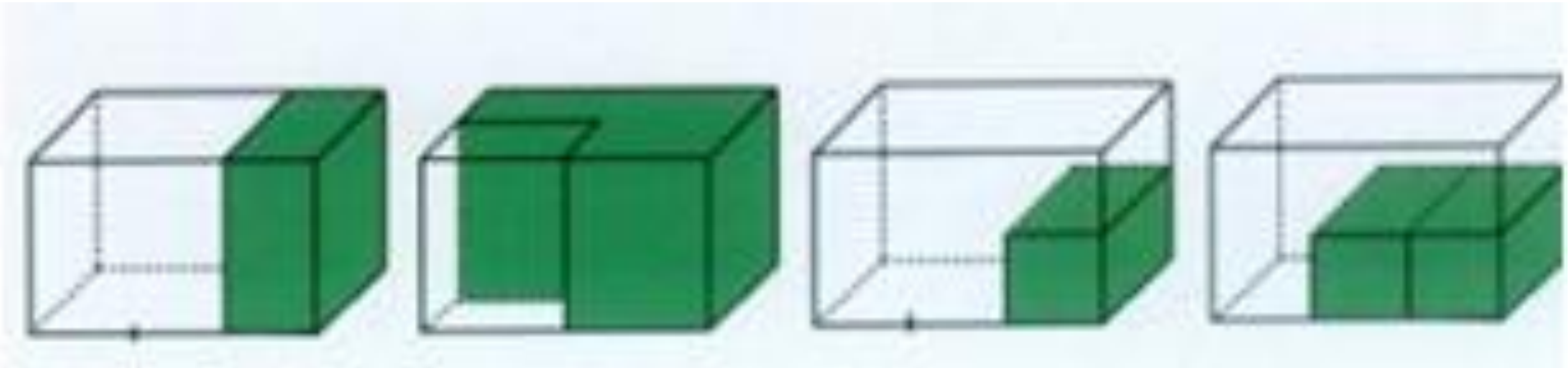
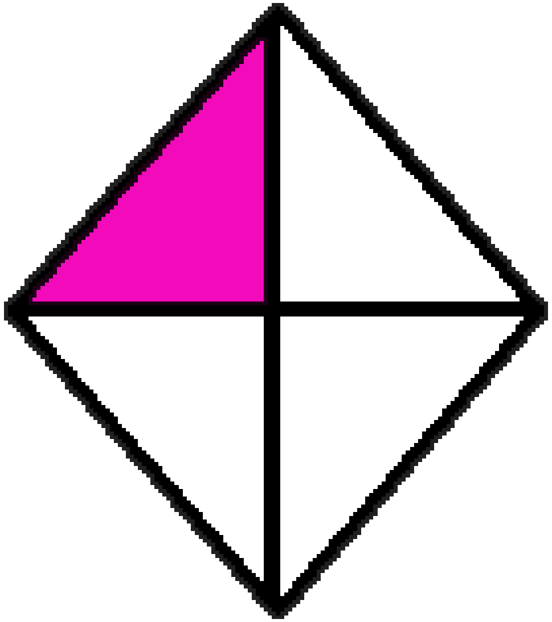


Stem Sentence

The whole is divided into ________ equal parts.

Each part is $\frac{1}{\square}$ of the whole

$\frac{\square}{\square}$ are shaded/identified.



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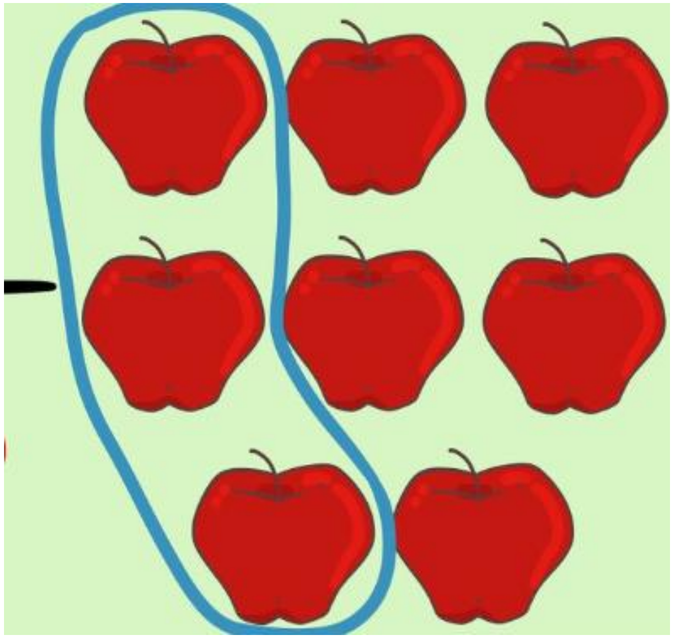
The whole is divided into ____ equal parts.

Each part is $\frac{1}{\square}$ of the whole

$\frac{1}{\square}$ is shaded



The whole is divided into _____ equal parts.

Each part is $\frac{1}{\square}$ of the whole

$\frac{3}{\square}$ are circled.

Repetition and Memorisation

Researchers report that Chinese learners recognise the mechanism of repetition as an important part of the process of memorization and that understanding can be developed through memorisation. Likewise, Hess and Azume (1991) refer to repetition as a route to understanding. Dahlin and Watkins (2000) assert that the traditional Asian practice of repetition can create a deep impression on the mind and enhance memorisation, but they also argue that repetition can be used to deepen and develop understanding.

Cited by Lai and Murray (2012)

Stem Sentences

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1.4

I say 1.4 but I read it as one and four tenths

Stem Sentences

2.6

I say _______ but I read it as ________and ________tenths

Stem Sentences

$$\boxed{1.8}$$

I say _______ but I read it as ________and ________tenths

What are teachers doing that is working well?

Teaching with Variation

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Procedural Variation

the importance of “core connections” in designing procedural variation.

In designing [these] exercises, the teacher is advised to avoid mechanical repetition and to create an appropriate path for practising the thinking process with increasing creativity.

Gu, 1991

Procedural Variation

Provides the opportunity

- To focus on relationships, in conjunction with the procedure
- To make connections between problems
- Using one problem to work out the next

Procedural Variation
Making Connections

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7 + 2 =	9 + 6 =
17 + 2 =	10 + 6 =
7 + 12 =	11 + 6 =
17 + 12 =	13 + 6 =
9 – 5 =	9 – 7 =
8 – 5 =	11 – 7 =
7 – 5 =	13 – 7 =
6 – 5 =	15 – 7 =

6 + 2
8 + 3
9 + 4
5 + 6

5 – 4
7 – 3
8 – 5
7 - 6

12 + 3
11 + 5
15 + 4
13 + 6

15 – 2
17 – 3
16 – 5
18 - 6

What are teachers doing that is working well?

Attention to mathematical structure and relationships

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Part Part Whole Structure

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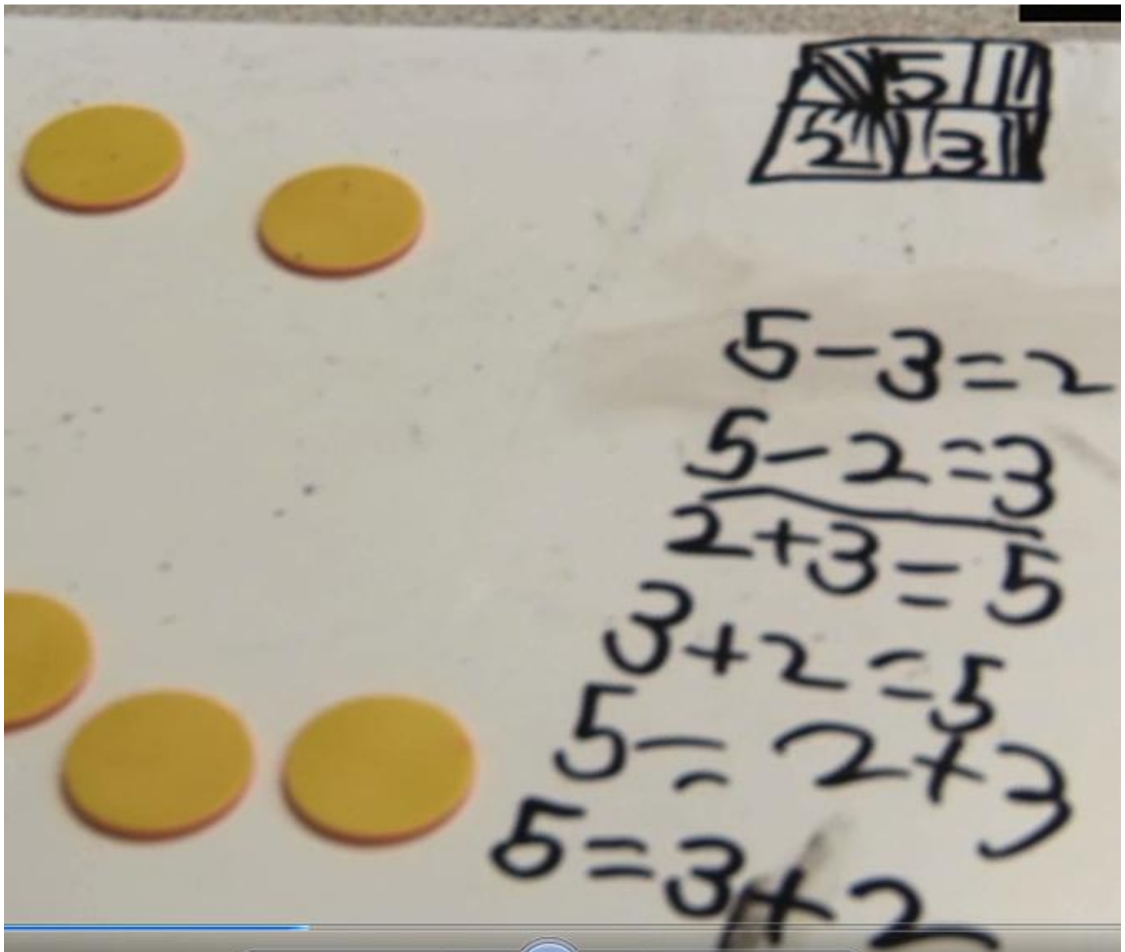

5	
3	2

Numberblocks

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Developing Depth/Simplicity/Clarity

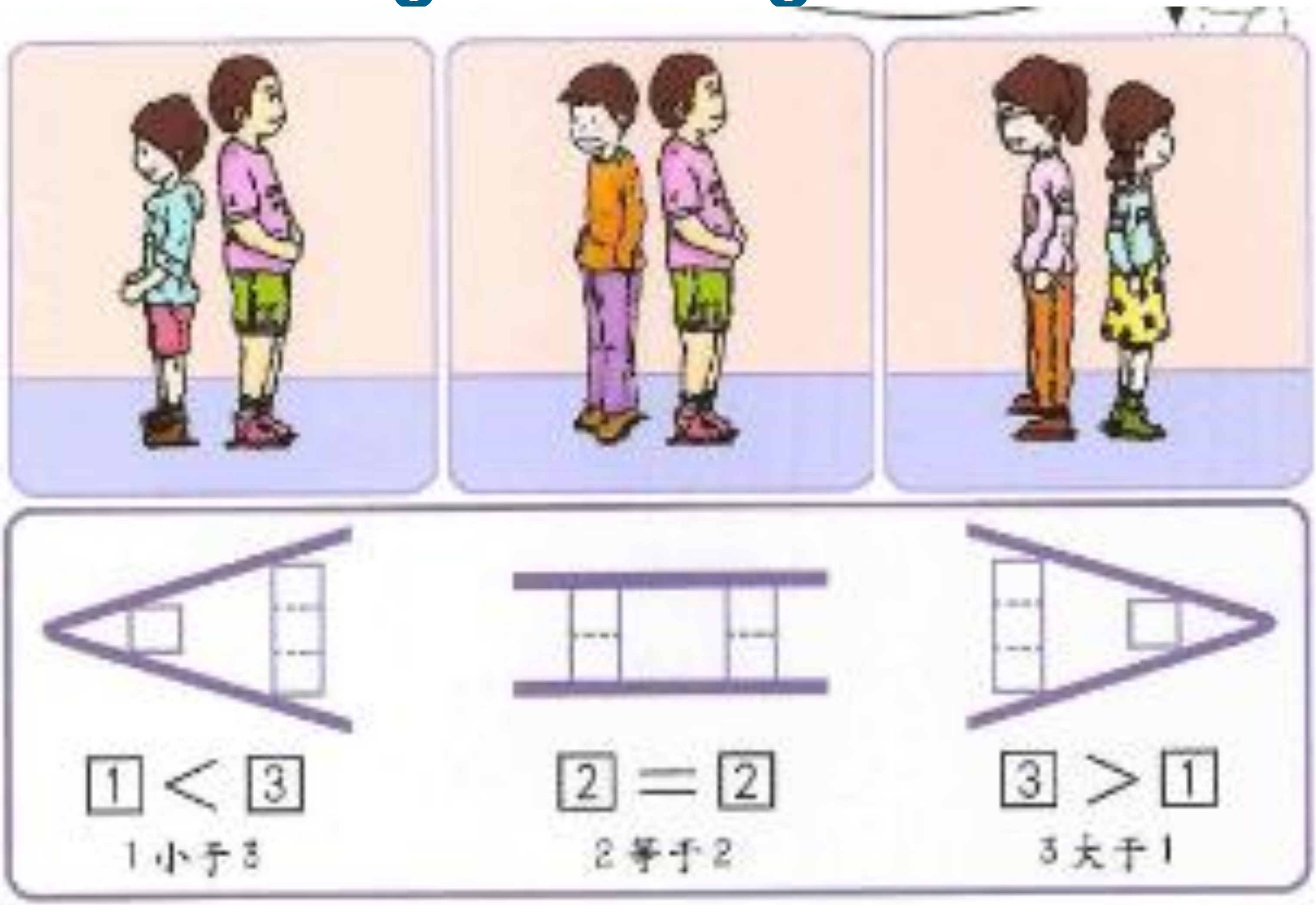
7	
2	5

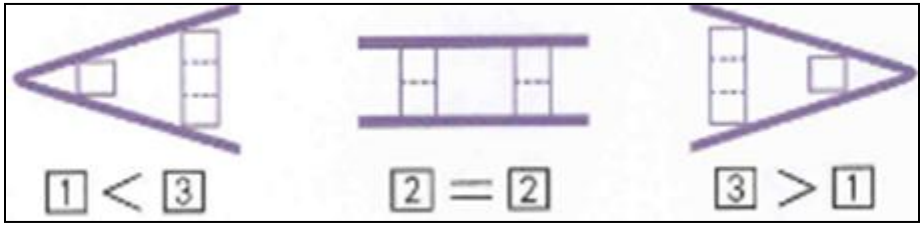
7	
1.9	5.1

7.4	
1.7	5.7

c	
a	b

Connecting Learning





Intelligent Practice

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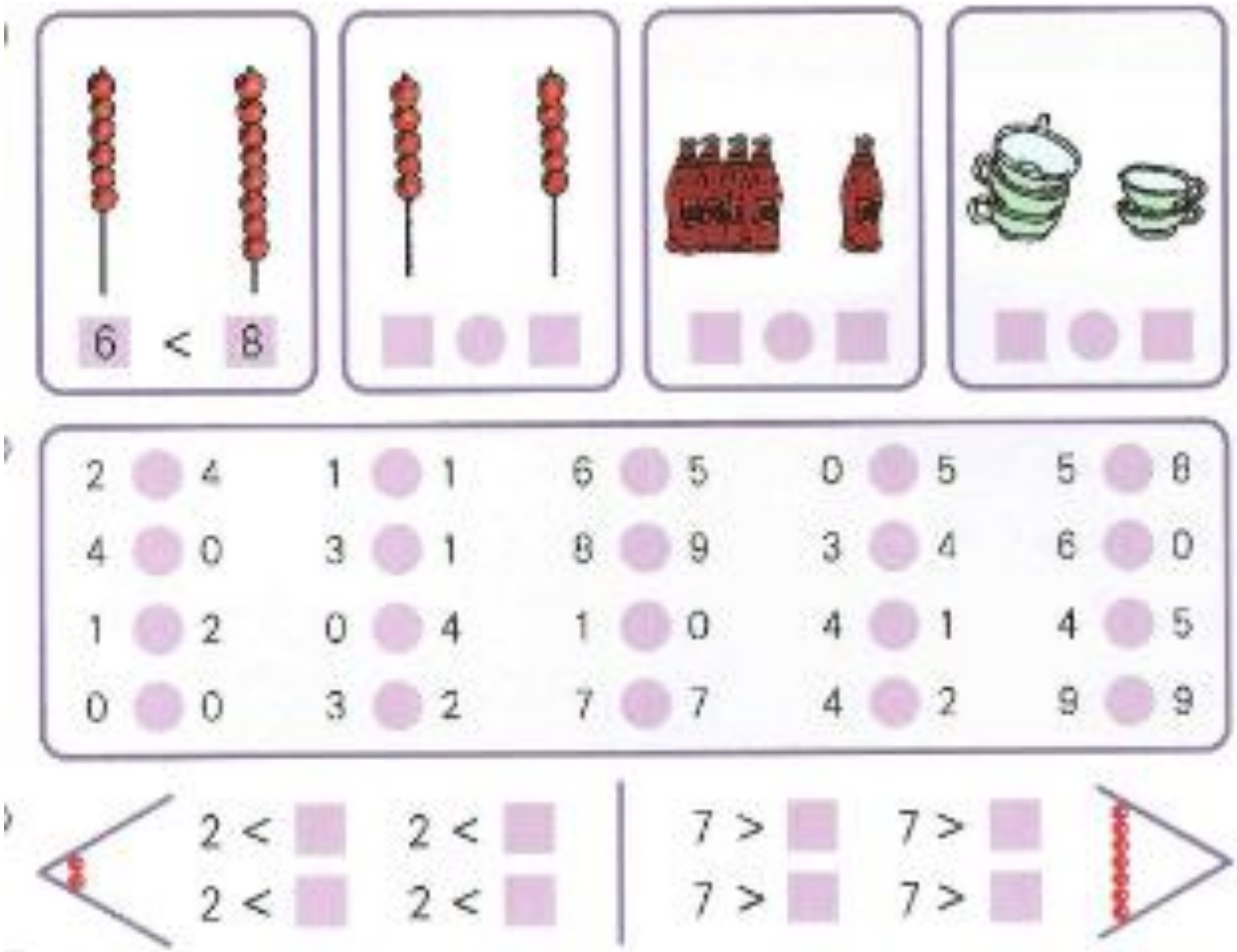
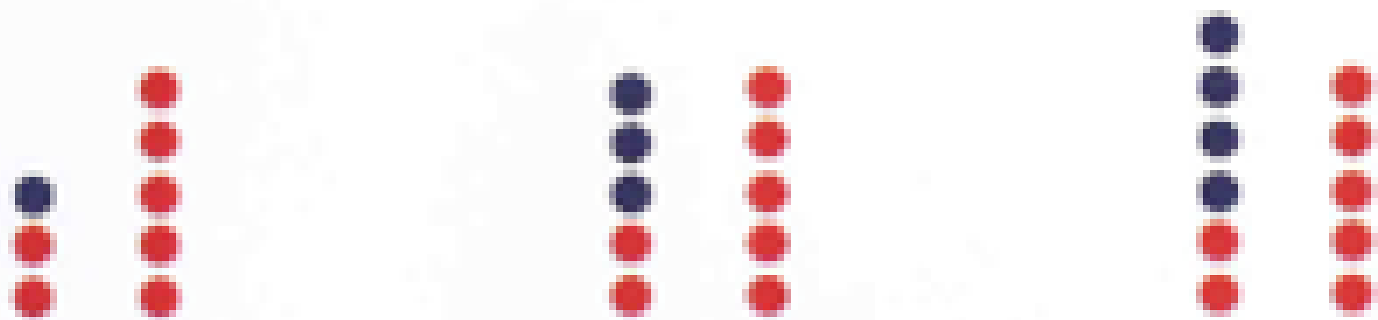


< > or =

10 ○ 7	15 ○ 10	13 ○ 15	2 ○ 20
10 ○ 10	15 ○ 15	5 ○ 15	12 ○ 20
10 ○ 13	15 ○ 30	1 ○ 15	20 ○ 20

3+ 3 ○ 5	3 + 2 ○ 5
1 + 4 ○ 5	4 + 1 ○ 5
0 – 5 ○ 5	6 + 0 ○ 5



$2 + 1 < 5$ $\qquad$ $2 + 3 = 5$ $\qquad$ $2 + 4 > 5$

$4 + 6 \bigcirc 6 + 3$

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<, > or = ?

$8 \times 6 - 5 \bigcirc 7 \times 6 + 4$

$\downarrow$

$7 \times 6 + 1 \bigcirc 7 \times 6 + 4$

Mastery is the fluency to transform equations and see the relationship between them.

$< \quad > \quad =$

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$3.08 \div 0.5 \bigcirc 3.08$

$0.83 \div 1 \bigcirc 0.83$

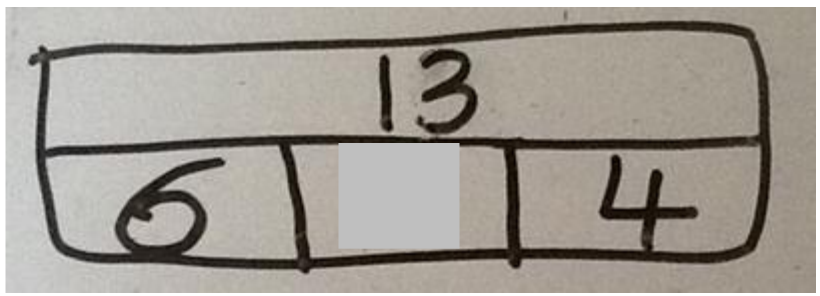
$4.12 \div 1.2 \bigcirc 4.12$

$0.46 \div 0.21 \bigcirc 0.46$

What do I want pupils to attend to?

Amy

$$6 + \square + 4 = 13$$



$$6 + \boxed{3} + 4 = 13$$

$100 + 100 = 200$

$100 + 100 + 100 = 300$

Calculate

(4/5 + 1/6) + (5/6 + 1/7) + (6/7 + 1/8) +
(7/8 + 1/9) + (8/9 +2/10) =

4/5 + **1/6 + 5/6** + **1/7 + 6/7** + **1/8 + 7/8** +
1/9 + 8/9 + **2/10** = 5

Thinking about relationships

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21 5,542 ÷ 17 = 326

Explain how you can use this fact to find the answer to **18 × 326**

17 x 326 = 5,542
18 x 326 = 5,542 + 326

How might children respond to this question?
What is the best response?

What are teachers doing that is working well?

Attention to mathematical structure and through images

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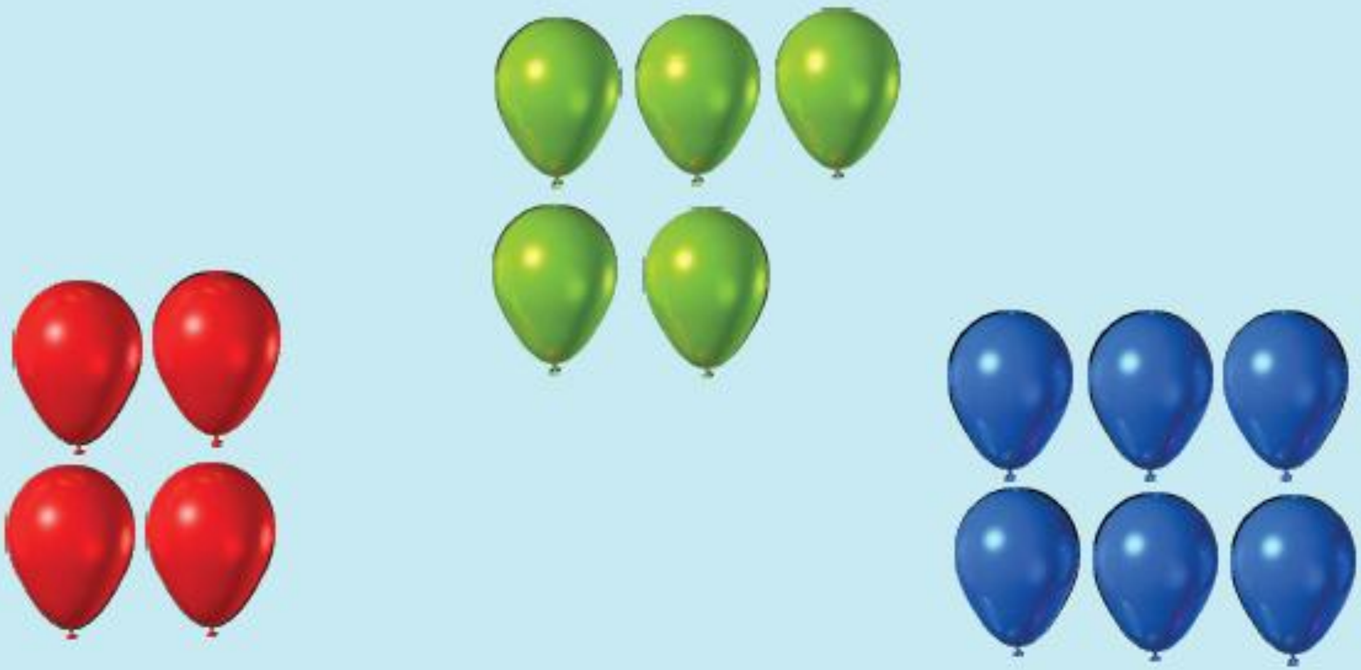
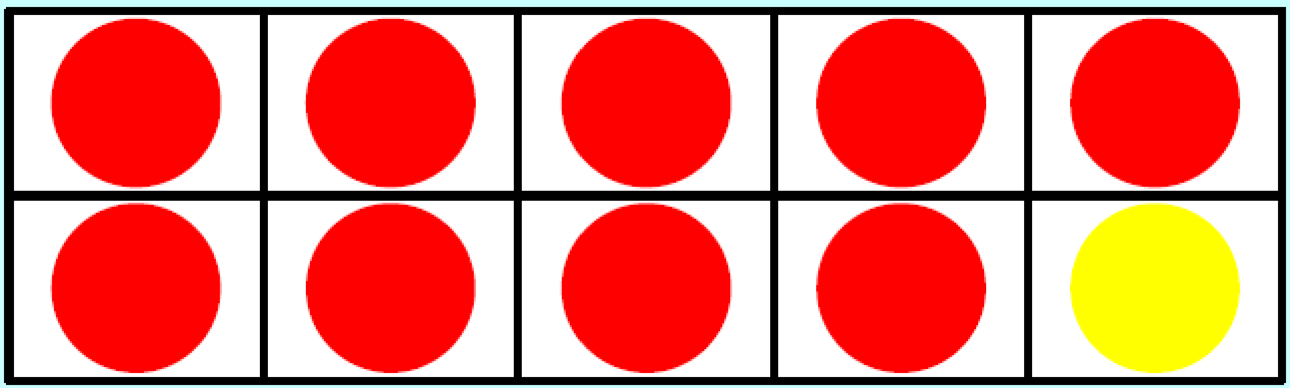
Using Structure to draw attention to

There are _____ and ______ and _____.



Using structure to draw attention to

There are _____ and ______ and _____.



Say: ______ plus ______ equals ten

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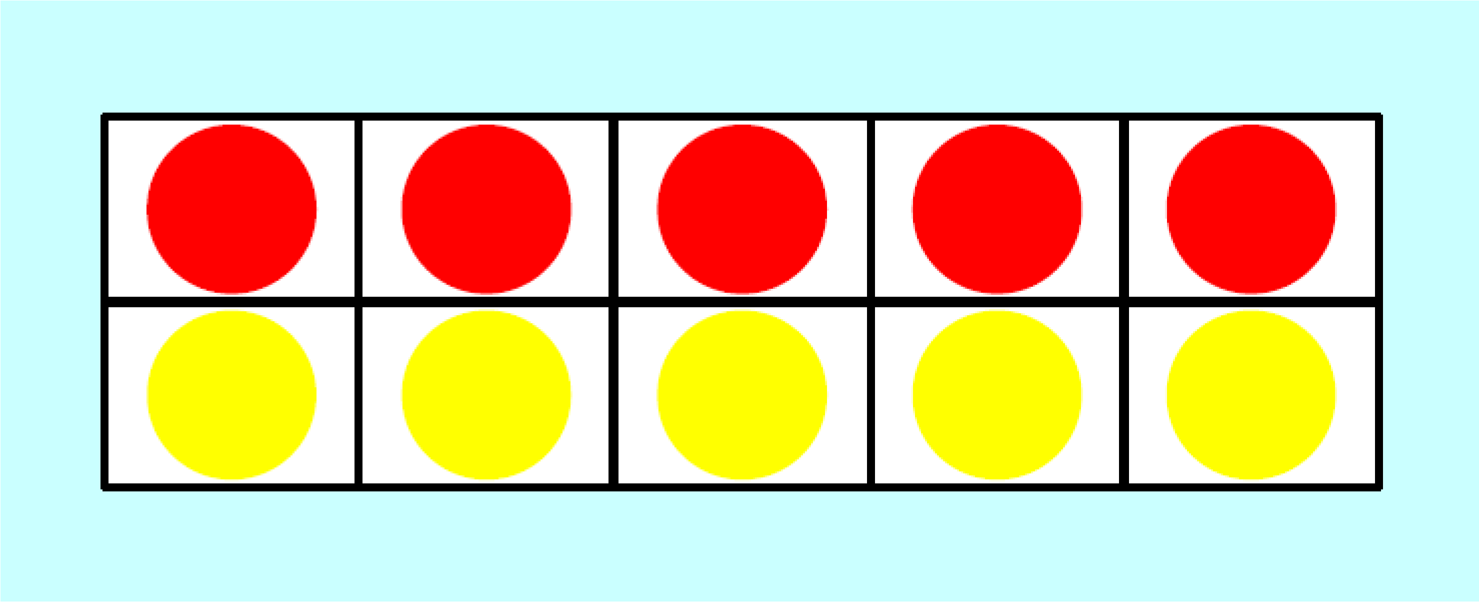

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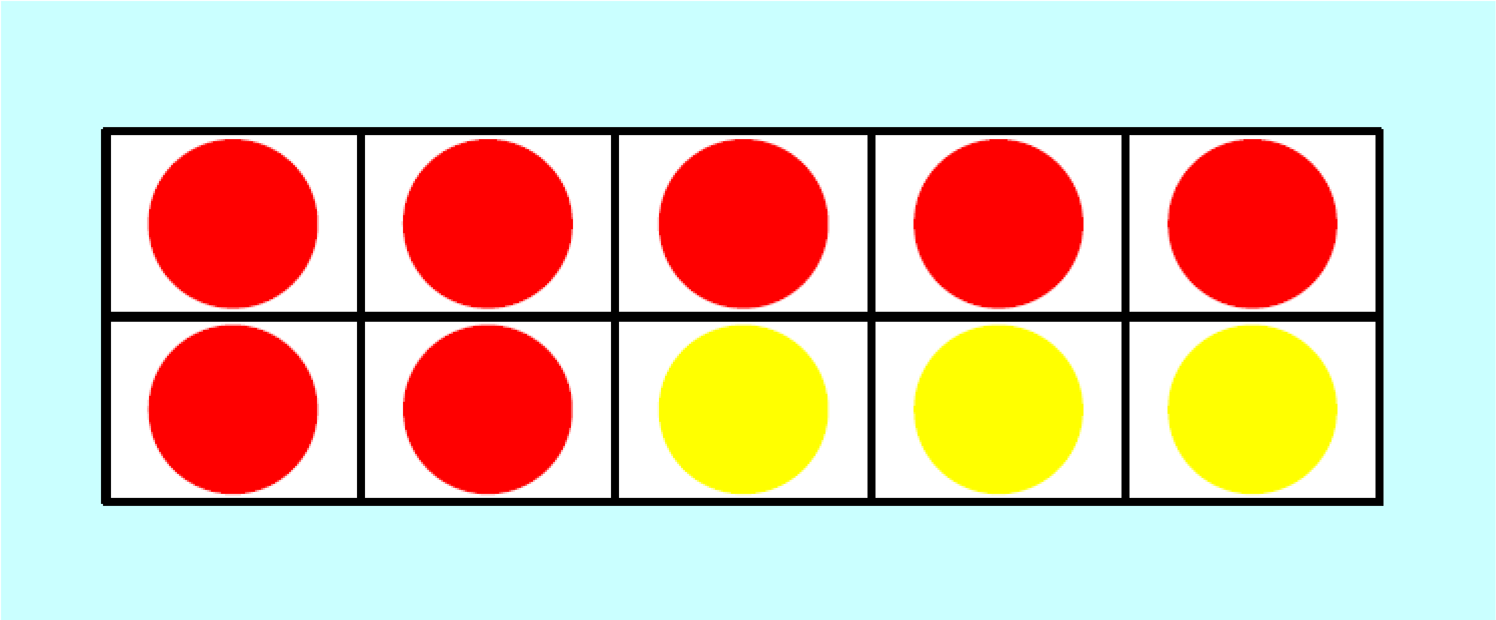

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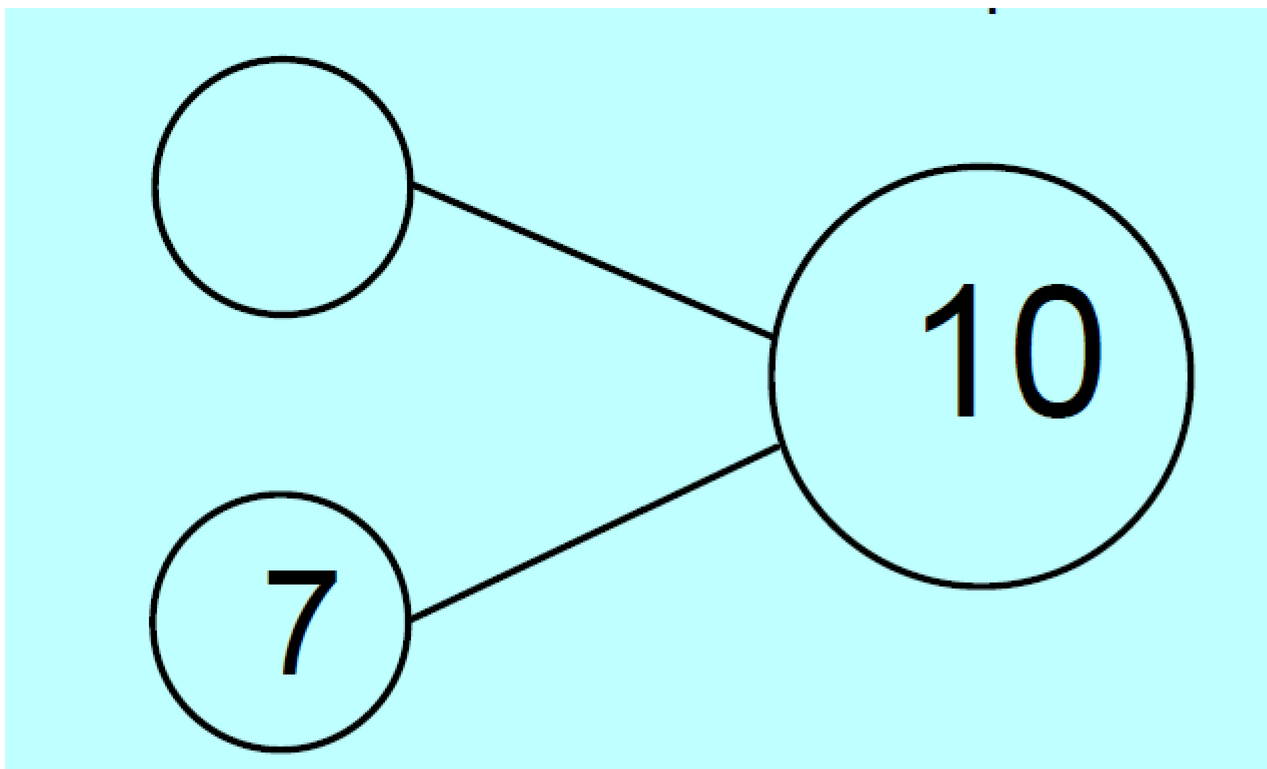

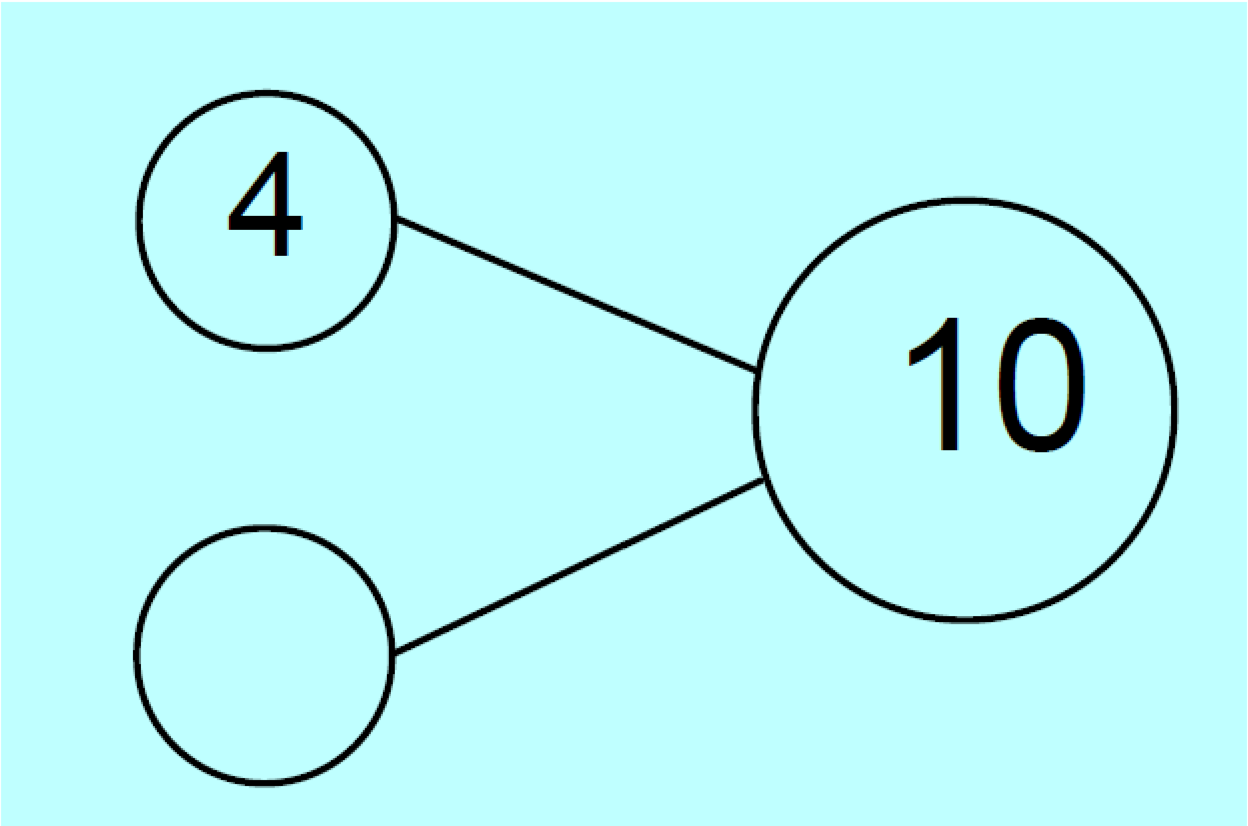
4
10

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$8 + 2$

Memorisation of Facts

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- Reduces cognitive overload
- Supports flexibility with number – using what I know – to work out what I don't know (making connections)
- Develops confidence

We are seeing evidence of this taking place

What are teachers doing that is working well?

Dong Nao Jin
Think more deeply

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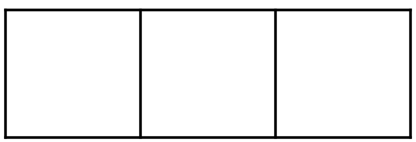
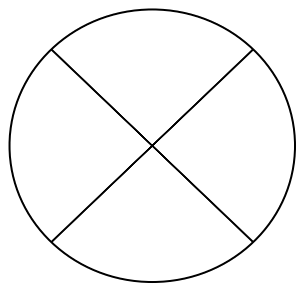
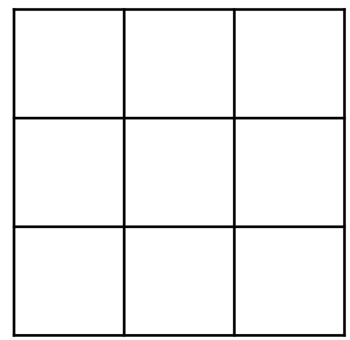
Illustrate these fractions on the diagram.



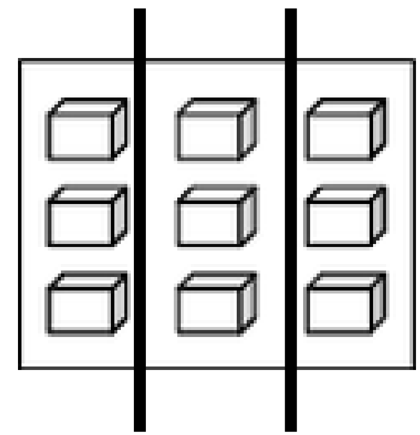
$$\frac{2}{3}$$



$$\frac{1}{4}$$



$$\frac{4}{9}$$

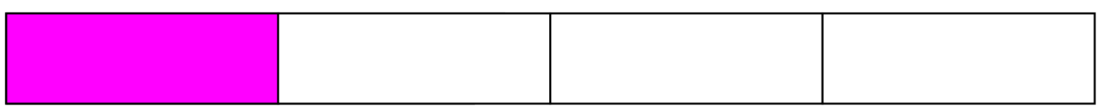


$$\frac{2}{3}$$

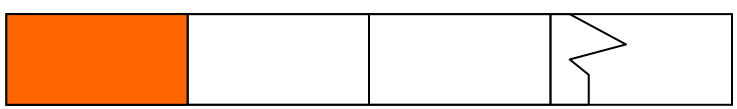
2 paper tapes were broken, can you guess which original paper tape is longer?

Why? How do you get your answer?

$\frac{1}{5}$



$\frac{1}{5}$



Thinking masterfully

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$$0.62 \times 37.5 + 3.75 \times 3.8$$

$\cancel{0.6}$

$= 0.62 \times 37.5 + 37.5 \times 0.38$

$= (0.62 + 0.38) \times 37.5$

$= 1 \times 37.5$

$= 37.5$

Personal Reflection

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- The approach is very structured and ordered
- Teacher led – but that does not result in passive learning
- The result is deep understanding with a propensity to look for relationships
- Supports fluency and flexibility – leading to creativity
- Supports the ability to solve problems

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Thankyou for Listening