December 2022

YBMA News

Vol. 32 No. 1

The Newsletter of the Yorkshire Branch of the Mathematical Association

Wednesday, December 7th

at 7.30pm

by Zoom

Our famous Christmas Quiz and Buffet

led by Tim Devereux

and Alan Slomson

As we are again online his year, you will need to provide your own seasonable food and drink, but you can enjoy it in good company! Persuade your colleagues and friends to come and enjoy themselves too.

To receive the Zoom link for this event, please email

a.slomson@leeds.ac.uk

A date for your diary

Key Stage 5 Maths Day
Wednesday, 29th March 2023

including the

W P Milne Lecture for Sixth Formers

Further details will be made available nearer to the date of this event.

YBMA Officers 2022-23
President: Bill Bardelang (rgb43@gmx.com)
Secretary: Alan Slomson (a.slomson@leeds.ac.uk)
Treasurer: Jane Turnbull (da.turnbull@ntlworld.com)

Obituary

Peter Johnson

We are very sorry to report the death in September of Peter Johnson.

Peter was a stalwart member of our Branch, serving as our Secretary from 1992 to 1998 and as our President from 2002 to 2004.

Peter served as Head of the Mathematics Department at Cockburn High School. One of our members recalls observing Peter several times at work in the classroom while doing her PGCE and was a bit in awe of how he managed to get a bunch of unruly 16 year olds to actually enjoy doing some maths on a Friday afternoon.

After retirement from Cockburn, Peter also worked at Leeds University as a part-time Maths PGCE tutor. He did a great job with many students helping them to become inspiring teachers of mathematics.

It was always a pleasure to see Peter at our meetings, and he will be sadly missed. We send our condolences to the members of his family

See the next page for *Mathematics in the Classroom*

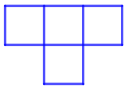
Our web page is

https://www.m-a.org.uk/branches/yorkshire

where you will find earlier issues of our Newsletter.

Mathematics in the Classroom

T pieces



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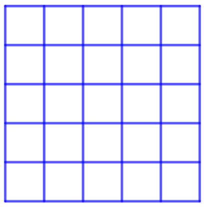
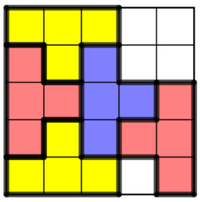


In the previous News letter we asked “What is the maximum number of T-pieces which can be placed i the 5 × 5 grid shown, without overlapping, and with their edges along the lines of the grid?”

We also suggested investigating the corresponding problem for square grids of other sizes.

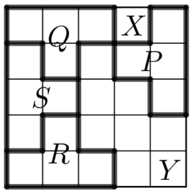
An Answer



It is possible to place five T-pieces in a 5 × 5 grid but not six.

The diagram on the right shows one way to fit five T-pieces. There are lots of others.

We now show that although a 5 × 5 grid has 25 cells, and 25 ÷ 4 is greater than 6, it is not possible to fit 6 T-pieces in a 5 × 5 grid.

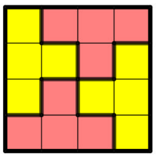


To fit 6 T-pieces we would need to cover all but one of the cells. Therefore at least three of the corner cells must be covered.

If two corners are cover by T-pieces whose rows of three squares are perpendicular, as shown by the T-pieces P and Q in the diagram, they leave a cell, here the cell marked X, which cannot be covered.

The only way two corner cells and all the cells between them can be covered is with T-pieces arranged like Q, R and S in the diagram. However with these T-pieces arranged as shown, a third corner can only be covered by a T-piece such as P which leaves a cell, X, that cannot be covered and a fourth corner cell, Y, which cannot be covered.

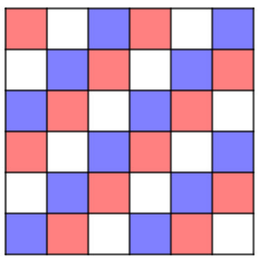
Therefore it is not possible to cover 24 cells of the grid by non-overlapping T-pieces. Hence the maximum number of T-pieces that can be fitted in the grid is five.



It is easy to see that four non-overlapping T-pieces may be placed in a 4 × 4 grid, and in essentially one way, as shown on the right. It follows that for each positive integer k, a $4k \times 4k$ grid may be completely covered by $4k^2$ non-overlapping T-pieces.

Most of the other grids are not so easy to deal with. Investigating these makes a good classroom exercise. It is easy to do experiments to try to see what is possible. You are then soon confronted with the question: how many unsuccessful attempts do you need to make to fit a given number of T-pieces in a grid, in order to *prove* that it is not possible to do this? Pupils can learn about the need for mathematical proof by thinking about this question.

To prove that n is the maximum number of T-pieces that can be fitted in a grid, you first need to show that there is one way to do this. Since each grid is finite, the number of ways to fit a given number of T-pieces in a grid is finite. So, to complete the proof, all you now need to do is to check *all* the ways of fitting n T-pieces in the grid, showing that in each of these cases it is not possible to fit in one more T-piece.



The snag is that there will be a large number of ways to fit n T-pieces in the grid and unless you can find a short argument to deal with all the cases, the problem becomes unmanageable.

Sometimes a colouring argument will work.

Can you use the colouring of the cells shown on the right to prove that a 6 × 6 grid cannot be completely covered by nine T-pieces?