## A return to in person meetings!

We are pleased to say that we are now able to return to meeting in person. We therefore hope to see you at our meeting in May. The meeting is open to non-members, so please encourage colleagues and friends to come.

Saturday, May 20th 2023 at the School of Mathematics, University of Leeds, at 3pm

The Presidential address:

> Fermat, Euler, Pollard and a $\$ 100$ prize

Bill Bardelang
YBMA President

"Factorising numbers is easy in principle but not so in practice. We look at some of the methods devised over the centuries and why this is a subject that matters to all of us."

This talk will be preceded by a short AGM and refreshments.

The agenda and other papers for the AGM will be circulated nearer to the date of the meeting.

## A date for your diary

## Our Christmas Quiz will take place in

 person on Wednesday, December 6th 2023.Would you like to provide a round of questions for this quiz?

Please let us know if you would like to contribute in this way.

## Vacancy for the position of Secretary

"I became Secretary of the YBMA in 1968. I hasten to add that I haven't been the Secretary for the whole of the past fifty-five years! My first period of office lasted just three years. I resumed this role in 2016. I now think that it is time to hand on the position to a younger person.
"If you are interested, please feel to contact me to find out what is involved."

Alan Slomson

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https://www.m-a.org.uk/branches/yorkshire where you will find previous
Newsletters.
Please see the next page for Mathematics in the Classroom

## Mathematics in the Classroom

## Equiangular and Equilateral Pentagons

The regular polygons are both equiangular (all their internal angles are equal), and equilateral (all their sides have the same length).
As everyone knows, when it comes to triangles these two properties are equivalent. An equilateral triangle is equiangular and an equiangular triangle is equilateral. All equilateral, and hence equiangular, triangles are similar.
For quadrilaterals things are a little more interesting.
An equiangular quadrilateral is a rectangle. Two rectangles are similar if the ratios, $w: h$, of the length of their shorter sides to that of the longer sides are equal. Hence there is a one-parameter family of similar rectangles.


An equilateral quadrilateral is a rhombus. A rhombus has two pairs of equal opposite angles with sum $180^{\circ}$. Two rhombuses are similar if their smaller angles are equal. Therefore there is one-parameter family of similar rhombuses given by real numbers $\alpha$ in the range $0<\alpha \leq 90$.

## Equiangular Pentagons

If we begin with a regular pentagon and make a cut parallel to one of the edges we obtain a new pentagon that is still equiangular, but no longer equilateral.
As this process can be repeated, at first sight we have complete freedom
 when it comes to constructing equiangular pentagons.

However, if we consider the equiangular pentagon $P Q R S T$, we see that once we have fixed the positions of the vertices $P, Q, R$ and $S$, there is no choice about the fifth vertex $T$. It must be the point where the line through $P$ making an angle $108^{\circ}$ with $Q P$ meets the line through $S$ making an angle $108^{\circ}$ with the line $R S$.


Therefore there is a two-parameter family of equiangular pentagons. The similarity class of an equiangular pentagon is determined by the values of the ratios $P Q: Q R$ and $R S: Q R$. If we fix the length of $Q R$ to be 1 , these parameters are the length $a$ of $P Q$ and the length $b$ of $R S$. The lengths of $P T$ and $S T$ are determined by the values of $a$ and $b$.

In an equiangular pentagon that is also equilateral all the ratios of the side lengths are equal to 1 . What can be said about these ratios in general? This leads us to the following question:

Does there exist an equiangular pentagon whose sides are all of different lengths, but such that all the ratios of the side lengths are rational numbers?

## Equilateral pentagons

An equilateral pentagon is determined by the values of two adjacent internal angles. The values of the angles $\alpha$ and $\beta$, as in the diagram, fix the positions of the vertices $P$ and $S$. The vertex $T$ must be such that $T P=P Q=Q R=R S=S T$. Provided that $\alpha$ and $\beta$ are chosen so that the line segments $Q P$ and $R S$ do
 not meet and the length of $P S$ is less than twice the length of $Q R$, there will be two possible positions for $T$, only one of which gives rise to a convex equilateral pentagon. This leads us to ask:

Does there exist an equilateral pentagon whose internal angles are all different but with each of their sizes being a whole number of degrees?

This note was inspired by Question 15 of the UKMT Intermediate Mathematical Challenge 2023.

