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YBMA News

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The Newsletter of the Yorkshire Branch of the Mathematical Association

Our Christmas Quiz

We very much regret that in the current circumstances we are unable to hold our traditional Christmas Quiz, accompanied by mulled wine, mince pies and cheese.

As a slight compensation we offer in this newsletter some mathematical puzzles that we hope will help to entertain you.

So please open your favourite bottle and settle down to enjoy them.

We wish you

A very merry Christmas and a happy and healthy New Year

Obituary

We regret to have to report the death Joyce Johnson, a long standing and stalwart member of the YBMA. We will all miss her enthusiasm for mathematics which she showed at our meetings.

Our condolences go to her husband Peter and the other members her family.

Officers of the Yorkshire Branch of the Mathematical Association 2020-21

President: Bill Bardelang (rgb43@gmx.com)

Secretary: Alan Slomson (a.slomson@leeds.ac.uk)

Treasurer: Jane Turnbull (da.turnbull@ntlworld.com)

Make numbers out of 2021

Using all the digits 2, 0, 2, 1 *in this order*, and mathematical symbols such as $+$, $-$, $\times$, $\div$, $\sqrt{\ }$, ! , . , (,) , but not letters or other numerals, make the integers from 1 to 20.

As a starter

$$1 = 2 + 0 - 2 + 1,$$
$$2 = 2 + 0 \times 2 \times 1.$$

When you reach 20, pour yourself another glass as a reward, and keep going until the bottle is empty!

Sum and Product

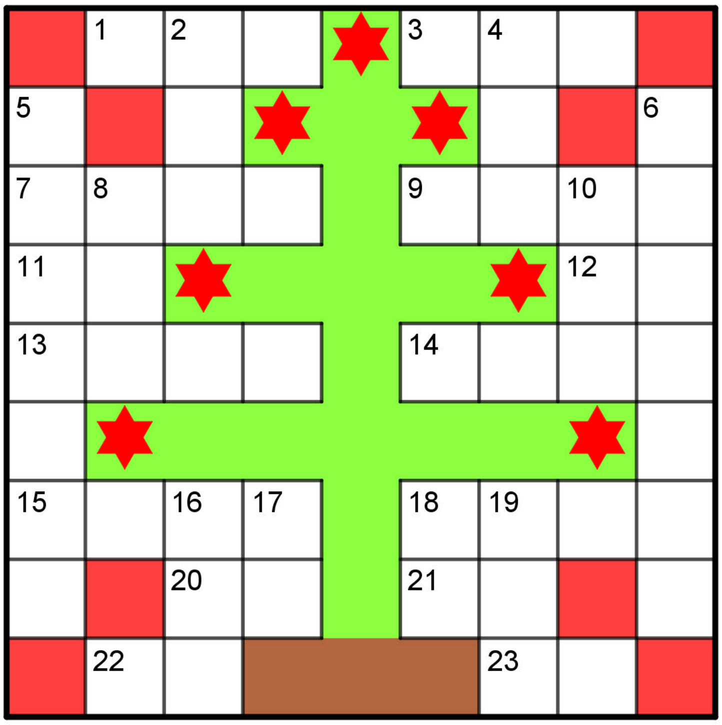
In some cases the sum of two positive integers is a factor of their product. For example $4 + 12 = 16$ is a factor of $4 \times 12 = 48$.

Can you find more examples? Are there infinitely many examples?

Based on an article *Sum and Product* by B.S.Beevers and T.F.Devereux, Maths in Schools, volume 23, March 1994, pages 17-19.

See below for our Christmas Crossnumber and Mathematics in the Classroom

A Christmas Crossnumber



Solve the clues and fit the answers in the grid

Across

a) The number of pounds in an English ton (4)
b) $7^3 + 8^3 + 9^3 + 10^3$ (4)
c) An integer whose digits are all the same (4)
d) A power of 2 (4)
e) The number of square yards in an acre (4)
f) The fourth power of a prime (4)
g) The smallest three-digit Fibonacci number (3)
h) The sum of the cubes of two different primes (3)
i) A prime (2)
j) The product of three different primes (2)
k) A fifth power (2)
l) The square of a prime (2)
m) Twice a prime (2)
n) Twice a perfect number (2)

Down

o) The eighth power of a prime (7)
p) 1111^2 (7)
q) $7^0 + 7^1 + 7^2 + 7^3$ (3)
r) A non-prime Fibonacci number with two digits that are the same (3)
s) The sum of four consecutive primes (3)
t) The product of two consecutive primes (3)
u) $2! + 3! + 4! + 6!$ (3)
v) A fifth power (3)
w) Written in base 3 this number is 1202 (2)
x) Both a triangular number and a Fibonacci number (2)

Hint: The answers to these clues are numbers in standard form, so they cannot begin with the digit 0.

Christmas is the time for old chestnuts:

1. Which is the longest word in the English language?
2. Naughty child "Why are you going to tan me?" What was the reply?

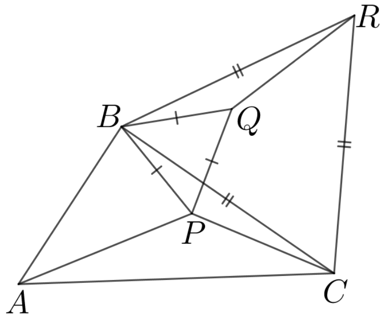
(Answers to these chestnuts are at the bottom of the next page.)

Mathematics in the Classroom

In the last issue we asked you to construct a point P in a triangle ABC that minimizes the total length $PA + PB + PC$.

Solution

Let P be a point in the triangle. Let Q and R be the points such that BPQ and BCR are equilateral triangles as shown.



In the triangles CBP and RBQ we have

$CB = RB$ and $PB = QB$.

We also have

$$\begin{aligned} \angle CBP &= \angle QBP - \angle QBC \\ &= 60^\circ - \angle QBC \\ &= \angle RBC - \angle QBC \\ &= \angle RBQ. \end{aligned}$$

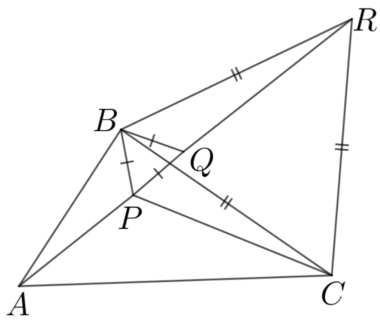
It follows that the triangles CBP and RBQ are congruent. Therefore $PC = QR$.

We also have $PB = PQ$.

Hence $PA + PB + PC = AP + PQ + QR$.

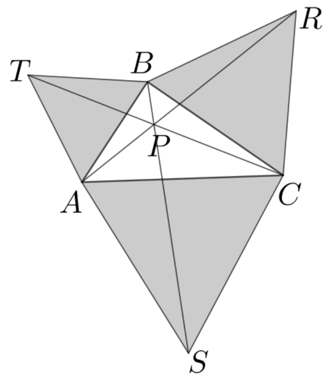
Now the point R is fixed by the triangle ABC. Hence the minimum value of $AP + PQ + QR$, and hence the minimum value of $PA + PB + PC$ is equal to the length of AR and occurs when the position of the point P is such that the points P and R lie on the line AR.

From the diagram on the right we see that P and R lie on the line AR when P is the point inside the triangle that lies on the line AR and is such that $\angle BPA = 120^\circ$.



This proof is taken from the book *The Geometry of the Triangle* by Gerry Leversha, UKMT, 2013, page 145

According to this book, the problem of locating the point P was posed by the French mathematician Pierre de Fermat (1601-1665) and solved by the Italian mathematician Evangelista Torricelli (1608-1647). Although Torricelli solved the problem, the point P is usually referred to as the *Fermat point* of the triangle.



Note that it follows from the above argument that if equilateral triangles CRB, ASC and BTA are constructed on the three sides of the triangle ABC, then the lines AR, BS and CT meet at the point P.

This makes it easy to construct the point P using a straight edge and a pair of compasses.

Note also that all the three angles $\angle APC$, $\angle CPB$ and $\angle BPA$ are 120°, and $AR = BS = CT$.

Answers to Christmas Chestnuts

1. "smiles" because there is a mile between its first and last letters.

2. Parent to naughty child " 'cos of your sin".