Identification of Mathematically Gifted Pupils

by **Anita Straker,** Mathematics Adviser, Wiltshire

This article is one of a series contributed by members of the "Mathematically Gifted Children" Sub-committee of the Teaching Committee of the Mathematical Association.

Nearly all discussion about able children starts by focusing on two issues: who are these children and how are they recognised?

What is a "Gifted" Child?

During the last 10 years the notion of "giftedness" has broadened considerably. In fact, in 1973 a noted educational journal asked the question "Are the gifted coming back?". One difficulty in attempting a definition is that researchers themselves have differing ideas about the subject. Some terms that have been used include:

The top 2% of the ability range.
A general attribute, closely related to exceptional intelligence as measured on a reliable test such as the full range WISC.
Outstanding talent in music, dance, drama, art, gymnastics, etc.
Possession of certain behavioural characteristics, such as considerable initiative, persistence, curiosity, and so on.
Particular aptitude in certain fields, e.g. creativity, social leadership, mechanical ingenuity.

In the way that the word "giftedness" is used more generally nowadays it is not intended to indicate a potential Newton or Galileo. On the other hand, neither is it meant to indicate all the children in the A stream. HMI in their survey (1977) of *Gifted Children in Middle and Comprehensive Schools*[1] used a working definition covering those children

who are generally recognised by their schools as being of superior all round intellectual ability, confirmed where possible by a reliable individual intelligence test giving an IQ of 130 or more,
or who exhibit a markedly superior development of performance and achievement which has been reasonably consistent from earlier years,
or where confident predictions are being made as to the child's continual rapid progress towards outstanding achievements, either in traditional academic areas or in others less easily measured such as sport, dance and art,
and where the child's abilities are not primarily attributable to purely physical development *or excessive environmental influences* (my italics).

What is Mathematical Ability?

Mathematical ability, like musical ability, often appears early and tends to persist. There is the well known story of the young Gauss, whose primary school master had asked all his class to add up the numbers from 1-99 in order to keep them busy for a long time, while he did a job outside. Before he could reach the door, Gauss slammed down his slate showing $50 \times 99 = 4\,950$, saying "There it lies".

Mathematical talent may well be accompanied by ability in other fields (Leibnitz, Euler and Hamilton were equally good at other things) or may be the single outstanding ability. Certainly most teachers will have met a child who was, in their view, outstanding in mathematics, but average in other fields.

However, it is easy to mistake mathematical ability for the ability to compute, particularly in young children. The ability to compute is obviously a talent which must not be ignored, but on the other hand many successful mathematicians will unhesitatingly say that they are "No good at arithmetic". (Perhaps there is some food for thought here for those schools where a child's performance in an arithmetic test is the sole criterion used for placing that pupil in a particular mathematics set or group).

Mathematical ability may also reveal itself in two quite different personalities. Perhaps this is because mathematics itself has two facets which can attract two different kinds of thinking. Firstly, there is mathematics as presented in the Euclidean way, which appears as a systematic deductive science attracting the convergent thinker, and possibly detected more easily within the traditional approach to the subject. Secondly, there is mathematics in the making, i.e. in the process of being invented, which appears as an experimental, inductive science attracting the divergent thinker and which may be far more difficult to detect.

Krutetskii in his book *The Psychology of Mathematical Abilities in School Children*[2] contends that there is such a thing as a "mathematical caste of mind" — a tendency to interpret the world mathematically — which can be seen clearly in pupils who are especially gifted in mathematics. He suggests that inclinations of this sort may be present at birth, but can be developed by appropriate environment and applied resource. He identifies three basic types of mathematical mind: the analytic (who tend to think in verbal, logical terms), the geometric (who tends to think in visual, pictorial terms), and the harmonic (who combine characteristics of the other two).

Who Recognises the Gifted Child?

The first and the most obvious factor here is that any ability will not be recognised unless it is being sought. Ernest Bevin was said to be "the one bright diamond on a heap of coal". If we are not looking for diamonds on a heap of coal then it is hardly surprising if the diamonds are missed!

It is not always the teacher in school who is the first to recognise exceptional ability.

Parents sometimes are the first to point out that their child may have mathematical talent. This may be because of characteristics noted by the parents, or the interests and hobbies the child pursues at home. On other occasions, screening procedures initiated by a school, or by the LEA, detect a mathematically able pupil. Just occasionally, it is the child's contact with a mathematics specialist from outside the school that provides the first clue. (After all, it took Hardy to recognise Ramanujan.) From time to time exceptional ability is detected when a child has been referred to the School Psychological Service for a different reason such as disruptive behaviour in the classroom.

What Will Help the Recognition of Mathematical Ability in School?

The Development of Attitudes

When the staff of a school as a whole are aware of and broadly sympathetic to the problem, then the identification of gifted or talented pupils and the problem of providing for them becomes less difficult. Nevertheless, educationists, schools and individual teachers can vary tremendously in their awareness and acceptance of giftedness. Some may suppress the topic for fear of having to cope with pupils who may be much more intelligent or capable than themselves. Others might consider it to be, for example, an elitist activity, or insignificant in relation to all the other problems in school (particularly the problem of the less able pupils). Still others may be thinking of the genius, not likely to occur in the ordinary school, while in other schools there is a complacent attitude, "We deal with the full ability range here" or "We are accustomed to this sort of pupil".

Encouraging positive attitudes can be difficult but talking about the problem and bringing any prejudices on the part of teachers into the open can help. If there is general acceptance that gifted pupils must exist and their needs ought to be identified then there may be more determination to spot such children. General willingness to accept non-conformity, divergent thinking, possibly unconventional behaviour, questions and responses on the part of children would be a great step forward. (The child in a secondary school second year class who asked a teacher following a demonstration of the construction of the perpendicular bisector of a line "Why is that, sir? Why does it work?", and received the reply "Because it's very scientific" could hardly have been encouraged.)

Examination of Organisation and Teaching Methods

It could be argued that certain situations or forms of organisation in school make the identification of the most able pupils more difficult. Examination of how and why these situations might lead to the gifted pupil being missed, or being able to conceal deliberately his or her power, can help to heighten teachers' awareness. Examples worth considering could include the start of formal schooling three times a year, large classes, mixed ability groups, rigid streaming/banding, etc.

In much the same way, it is worthwhile examining the methods we use to teach children, and the demand we make on pupils. If there is insufficient pace and challenge in the teaching material, the able young mathematician could well become bored. It is obviously important for us to set suitably high targets and expectations for all pupils at every stage. The school that sets out to aim for "O" level with 40% of its all-ability intake will almost certainly do better than a similar school that says "Our usual pass-rate is about 25%".

Within the classroom, other points need consideration. For example, is there a good balance between large group or class work, and individual assignments, so that the individual pupil has the opportunity to excel? Do we expect too much to be written down on paper, particularly with primary and lower secondary pupils? Do we over-direct children's activities in mathematics, and insist on prescribed methods? (After all, who wants to sit through a long laboured explanation of the long division algorithm or how to solve simultaneous equations, when one knew about them all the time.) Do we provide sufficient opportunity and encouragement for children to use intuitive guess-work, to "have a go", or do we dampen incentive by saying too often "No, that's not right" and by putting too many crosses in books? Do we give pupils, including those in primary schools, the opportunity if they wish to research, make "discoveries", and tackle a sustained piece of work in depth?

The Development of Good Classroom Practices

The *National Primary Survey*[3] emphasised the general need for development of good classroom practice to aid the high-lighting of aptitude and potential of all pupils. It was suggested that we should observe systematically for each child his way of learning, his knowledge and skills, and his ability to apply and use these.

We need to make full use of records from previous teachers. We need to maintain these records keeping details of children's progress and noting the effects of the provision we are making. When we are observing children, we also need to consider whether possibly frustrated talent could be the cause of any disruptive behaviour, boredom, or rejection by the peer group. At the same time, allowances need to be made for such things as age in relation to the rest of the group, insecurity, adverse home circumstances, reaction to a new environment on changing school, and so on.

If we could develop within schools consistently good classroom practices for all children, perhaps we would find the recognition of able young pupils less difficult.

Using Checklists

When attempting to identify gifted children it is sometimes helpful to consider the characteristics that they might have. However, it is perhaps worth mentioning that although there may not be very many "gifted" children, their characteristics will vary far more widely than those of "normal" pupils. For example, if IQ scores are examined then most pupils will fall roughly within the fairly narrow range 85-115, whereas the range of giftedness, like that of educational subnormality, is considerably greater. It is therefore to be expected that the characteristics of very able pupils are not easily generalised.

Some studies have, nevertheless, concentrated on seeking any similarities in the characteristics of the exceptionally able. There are, for example, longitudinal studies which identify a group of pupils and monitor their progress from that time. In her study (1973) *In Search of Promise*[4] Hitchfield described the characteristics of 138 children aged 11 years, who had been born during a particular week in 1958, and who had been identified as being gifted. The favourite subjects of the most intelligent boys were mathematics, art and science, whereas the girls liked art, English and mathematics. Although mathematics was enjoyed by the children, this was the only subject in the curriculum which presented difficulty to some of them. It did so to 21% of the girls and 11% of the boys. (More research is apparently needed into why, even by the age of 11 years, more boys than girls are seemingly good at the subject.)

Other research has helped the development of a number of checklists, attempting to illustrate behavioural characteristics and designed to aid teachers and others in identification procedures. Such lists, whether they are of a general or a specific nature, can be helpful in promoting discussion about identification, but they do need to be used with discretion. There can be danger in type-casting very able children.

The Schools Council project on *Gifted Children in Primary Schools*[5] drew up a list of 38 behavioural criteria. These describe gifted children as being:

Intensely curious, sometimes in one special field
Superior in quantity and quality of vocabulary, but not always keen to write
Compulsively perfectionist about their own achievements and impatient with second best
Able to listen to only part of the teacher's explanation, and then withdrawing into private thought, or thinking on ahead
Possibly arrogant (but at the other extreme, sometimes shy, reserved and unassertive)
Able to work for long periods with persistence and total absorbtion
In need of less sleep than the average

and so on.

Sometimes lists are given relating to a particular area of the curriculum. For example, high ability in mathematics often manifests itself in the youngest children in:

A liking for numbers, including use of them in stories and rhymes.
An ability to argue, question and reason using logical connectives: if then, so, because, either, or . . .
Pleasure in jigsaws and other constructional toys
Pattern making revealing balance or symmetry
Precision in positioning toys, e.g. cars set out in ordered rows, dolls arranged in order of size
Use of sophisticated criteria for sorting and classification.

Krutetskii[2] contends that there are three basic stages of mental activity in solving a mathematical problem, i.e. gathering the information needed to solve the problem, processing the information to obtain a solution and retaining information about the solution. To each of these stages, Krutetskii suggests there corresponds one or more abilities, and pupils in school who are especially capable in mathematics:

Are better able to grasp the essence of a problem at once, or relate one problem to another quite different one
Can generalise mathematical material rapidly and easily
Tend to skip over intermediate steps in a logical argument
Switch easily to another solution method
Strive for an elegant solution where possible
Reverse their train of thought if necessary
Tend to remember the relationships in a problem and the principles of a solution, whereas less capable pupils tend to remember only specific details, if anything, about a problem.

Some of the behavioural characteristics which mathematically able pupils may display can include:

Persistence in a search for the best and simplest solution to a problem. Children who have a gift for mathematics do not easily tire when they are occupied with it. Eleven or 12-year-old pupils can work continuously for up to three hours before showing any deterioration in their performance
Self confidence in a new mathematical situation, where they are prepared to use initiative in tackling something different
Open mindedness in that mathematically capable children will weigh evidence and be prepared to change a point of view according to this evidence
Laconicism, i.e. a drive towards economy of thought and the compression of arguments, although the same pupils may talk avidly with others about mathematics. This may show itself particularly in written work in English, but also in mathematics where there may be a dislike in the early stages of writing out solutions to problems which can be solved mentally.

(This last characteristic shows itself in a piece of writing from a 10-year-old boy following an activities day arranged at a University for able young mathematicians and scientists.

"I arrived with my mum and gran at 9.00 a.m. on Saturday and while there was no-one else there we looked over the premises which we were in, which was the teaching block at the University. Eventually the other parents and children arrived, and as soon as everyone was there the parents had to be split up from the children. I was left to my destiny in a room with 26 other children. The first thing we did was miming, in which we all pretended to be eating a cake, or opening a box, or swimming in some water.
The next thing I chose to do was astro-navigation which I half liked because we went outside to make readings on the sun, and half disliked because it started to rain just when we started.
After this, there was a half-hour lunch interval, during which I had lunch.
When lunch was over I had a go at hexagonal chess, which is ordinary chess, except that it is played in different moves, and on a hexagonal shaped board. I didn't like it because I had very little time to understand it, and so I couldn't play it.
The last thing I did that afternoon was a general knowledge quiz, which I enjoyed because it was the nearest thing to what I am used to doing.
While I did this my mum and grandma felt at home by listening to a talk on African Violets.
Then at 4.00 p.m. we went home".)

Using Group Tests

Much of what has been said already is concerned with suggesting ways in which schools can determine, by a subjective process, which of their pupils are mathematically gifted. In this, there will be much valid argument and debate between those teachers who are making assessments.

It is also possible to use a standardised objective test as a group screening procedure in preliminary identification programmes. This may be something which is initiated by the LEA, but might also be part of a school's own systematic procedure.

Caution is needed here in the choice of the test, in its administration, and in the interpretation of scores. The LEA Psychological Service will be able to offer advice to schools who may wish to use this form of testing as an additional aid to the identification of their most able pupils. One of the reasons why scores obtained by pupils in standardised tests need to be treated with some reservation is that the tests themselves contain "answer only" items, which militate against detection of the process of solution and depth of thought used by the pupil. Also, some standardised tests contain multi-choice questions, and the divergent mathematical thinker may well select the "wrong" answer. Where a test is used it is important to select one which tests mathematical attainment rather than arithmetical competence, and relational rather than instrumental understanding. It is also important to remember that an IQ score taken in isolation is not necessarily indicative of mathematical ability. (Of the 54 mathematically able sixth-formers from maintained schools in one LEA who attended a conference arranged for them at a university between 1974 and 1977, 39 had been recorded as taking the LEA's 11+ examination six years previously. Eight of these 39 students had VRQ scores of below 115 at the age of 11 years, but all were extremely successful in mathematics both at school and later.)

Probably the most valuable use of a group test is in helping to detect the able child who is under-achieving. Although many of the test items are related to the mathematics that has been taught, nevertheless children with high mathematical ability will reason intuitively to obtain their answer. Certainly if a pupil produces a high score on a mathematics test, particularly one which is designed for a slightly older age group, this usually indicates potential ability in the subject. The AH series of tests designed by Alice Heim, and available from the NFER, contain sections which could be useful here.

General Investigation

It may be appropriate, if a pupil is thought to be very able, to confirm first impressions by investigating further.

People involved in this secondary assessment might include not only the pupils' present teachers but also LEA advisory staff and other specialists (e.g. teachers or lecturers from other educational establishments). Any "outsiders" would probably continue with similar techniques of identification that the school has been using previously, i.e. discussion with the child's teachers, examination of present school work, and of any school records concerning the pupil. Observation of the child working with a teacher whom he or she knows well can also be helpful if it is possible to arrange this without inhibiting either the teacher or the pupil. Sometimes it may be sensible (taking into account the reservations already expressed) to extend any previous objective testing to administering a reliable individualised test. Discussion with the parents may throw some light on a child's out-of-school activities and interests.

Discussion with the individual child also needs to take place, and might centre around his or her general interests, hobbies, hopes or expectations. The child may have made some personal mathematical "discoveries" and be prepared to talk about these. Other pupils may be able to talk about and give reasons for their preferences for branches of mathematics, and the satisfaction gained from the subject. Older pupils may be able to judge what motivated them towards and what has been achieved in a particular solution.

Probably the best way of all to detect the degree of a pupil's ability in mathematics is to present him or her informally with a problem and ask for "thinking out loud" as he or she is solving it. Some young children find this hard to do, and will first give their solution, but afterwards they may be prepared, given sensitive questioning, to discuss their methods of strategy and thinking. This can then be extended by asking questions like:

What would happen if . . . ? (we changed numbers for letters, used a parallelogram instead of a square, looked at it under a microscope, changed the rules. . . .)
How else could we tackle this?
How many more ways are possible of . . . ?
Is it possible to . . . ?
Which is the most likely way of . . . ?
Can we use this to solve . . . ?
Will it work?
Can we draw a general conclusion from this?
How do you know that? What evidence is there?

and so on.

Teachers who are looking for suitable problems and puzzles to discuss with their pupils in this way will find a wealth of both in two totally contrasting books: Krutetskii[2] and H. E. Dudeney's *Amusements in Mathematics*[6]. The first contains a whole chapter (approximately 80 pages) of problems classified in 26 ways, e.g. problems with an unstated question; direct and reverse problems; mathematical sophisms, etc. The second contains entirely problems (and some solutions) this time classified according to their content, e.g. points and lines problems; moving counter problems; measuring, weighing and packing puzzles.

Older children in the upper secondary school could be asked to work on pencil and paper solutions to problems first, and then to discuss these afterwards. The problem solving examination is, after all, the method of identifying gifted students to take part in Mathematical Olympiads. It is also the preliminary method used in Russia and elsewhere to select students for the Mathematical Schools.

It is within such discussion of problems and their solution that really exceptional ability may reveal itself, perhaps in the unusual remark, in the flash of insight, the speed of thought, the sophistication of technique, the ability to see beyond the immediately obvious, and in the maturity of judgement.

Practical Steps for the Secondary School

If a mathematics department in a comprehensive school wanted to set about identifying their most mathematically able children in their intake year, what action could be taken which would help in achieving this? Practical steps perhaps could include some of the following:

- Provide resources (materials, books, a maths club, special assignments, films, school competitions, producing a mathematics magazine, mounting special displays, special talks, etc.) to cater for such pupils. If the challenge and the stimulation are right then the most able children should begin to excel and reveal themselves.
- Designate, if possible, a member of the department to take overall responsibility for co-ordinating work with the most able lower school pupils. Every member of the department will necessarily be involved, but having a co-ordinator helps.
- Hold a departmental meeting and discuss:

(a) personal attitudes of the staff about providing for the very able;

(b) how many, statistically, of such pupils are there likely to be in the intake year;

(c) whether school and class organisation, and the general teaching methods in the department, could militate in any way against identification (so that they can be taken into account).

- Check the department's systems of observing children, recording their progress, effort, apparent interest, and how information from others is incorporated. Design a profile card on which to record particular information about the intake children for six months or so.
- Ask each contributory Primary School to nominate lists of their top 5% of mathematically able children who will transfer to the school. Try, if possible, to discuss these children with their Primary teachers, and to look at their current exercise books before they arrive. (Guard against being over-influenced by neatness, and straightforward computation, resulting in a preponderance of ticks!)
- If the school administers a reliable IQ test, e.g. AH2/3 include also on the lists any children with high numerical/perceptual scores.
- Develop a checklist, within the department, of characteristics of children in the 10-12-year-old age group which might indicate exceptional ability in mathematics and which can be looked out for as clues.
- Teach the children for a while, following the general first year curriculum, observing pupils particularly carefully in oral/discussion situations from the point of view of the checklist. Include also, for the children on the list, a selection of problems from Krutetskii[2] or Dudeney[6] or elsewhere, which lend themselves to informal discussion and/or informal pencil and paper working.
- Show all the children the mathematics section of the school library and ask for an (oral) review of something they have discovered that interested them.
- Discuss the children with their parents at the first possible open evening, to discover whether the pupils on the list have any mathematically related outside interests or hobbies.
- Try some short written questions with no time limit. Give, say, three or four problems which have alternative solutions. Accept correct solutions with no working equally with those in which working is shown. Read any such questions to the pupils before they start.

Using all the entries on the profile cards, it should now be possible to decide which children, at this stage, warrant particular provision involving extra pace, challenge, different homework and coverage of topics outside the "O"-level syllabus.

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6. Dudeney, H. E. (1970) *Amusements in Mathematics.* Dover.

Training and retraining to teach

Readers will be well aware of the shortage of mathematics teachers but not be clear either as to the extent of that shortage or about efforts being made to deal with it.

In January 1979 LEAs reported vacancies for 463 mathematics teachers, 431 physical science teachers and 294 craft, design and technology teachers. It was estimated that to replace those teachers who are teaching these subject areas without a qualification 4 300 mathematics teachers, nearly 2 000 physical science teachers and 2 200 craft, design and technology teachers will be needed.

The arrangements made by the Government for training and retraining teachers for shortage subjects include two elements:

(a) Retraining courses at a number of selected teacher training institutions to retrain qualified teachers as secondary school teachers of mathematics, the physical sciences, craft, design and technology and business studies.

(b) Awards and grants from the Manpower Services Commission, both for these retraining courses and for one year teacher training courses for those aged 28 and over who are not already teachers.

Further details may be obtained from the Information Division, DES, Elizabeth House, York Road, London SE1 7PH.