A Turkish interlacing pattern and the golden ratio

Whirling dervishes and a geometry lecture in Konya

by John Rigby

The main topic of this article is an elaborate interlacing pattern on a wall in the Karatay Medrese in Konya, Turkey (Fig. 1): how and why do the elements of the pattern fit together, what are the precise measurements of the tiles that make up the pattern, and how might a 13th century Seljuk craftsman have adapted the pattern to produce something less complicated. This main topic is interlaced with accounts of mathematical events in my life during my stay in Turkey more than thirty years ago.

Events in Turkey, Mainly Mathematical

I spent the academic year 1972–73 at Hacettepe University in Ankara, Turkey, as a 'foreign expert'. The name, pronounced *Hadjet tépé*, means *Necessity Hill*, which sounds like a topographical difficulty encountered in *The Pilgrim's Progress*. Whilst I was at Hacettepe I gave lecture courses in English on group theory and projective geometry.

I wrote letters to my parents from Ankara at least once a week. My mother kept them all, so I have interspersed this article with a few relevant quotations concerning mathematics or numbers. Even at the British Embassy Church I found mathematical connections.

27 September 1972. The chaplain [Stephen Skemp] is Archdeacon of the Aegean in the Diocese of Gibraltar, and his brother has written a book on The Psychology of Learning Mathematics (Skemp, 1971), which I have partly read.

A friend and colleague at Hacettepe University, Seyfettin Aydın, lent me a fascinating book, *Exercices de Géométrie* by Frère Gabriel-Marie, from which I learnt about Kiepert's hyperbola and other matters; I wrote about this in the *Mathematical Gazette* thirty years ago (Rigby, 1973). He also planned a visit to the ancient city of Konya in December 1972, so that we could see the whirling dervishes, and he arranged for me to give a lecture there in April 1973, at a conference on geometry at the Teacher Training College where a friend of his was Principal.

The dervishes of Konya, with their long white full-skirted robes and tall white almost-cylindrical hats, carry out their whirling ceremonies not in the old mosque but in a large sports hall with the inevitable basketball markings on the floor, in order to accommodate the large number of spectators and tourists. The performers belong to the Religious Order of Mevlevi. 'The conical hats represent their tombstones, their coats their tombs, and their skirts their shrouds.'

On my second visit to Konya, the geometry 'konferans' turned out to be just me giving a lecture. I had planned a technical talk for a few mathematics students, on axioms for Euclidean and hyperbolic geometry, with my friend Seyfettin Bey giving a simultaneous translation. But we were shown into the main hall of the college, and my memory tells me that we were confronted with the Principal and all the academic staff in the front row, and the entire student body – it looked like several hundred at first sight – stretching into the distance as far as the eye could see! Somehow I survived the next hour by reshaping the talk as I went along, to suit the unexpected audience; my letter home shows that I must have pitched it right for at least some of them.

6 April 1973. There were about 200 students, all training to be maths or science teachers, and the audience clapped when I wrote some Turkish words on the pocket-handkerchief-sized blackboard. There were a number of questions at the end, which shows that they must have grasped something of what I was saying. ... I think I shall be both singing in & conducting a quartet on Easter morning! So many of the singers will be away on holiday.

An Interlacing Pattern in Konya

After the lecture I remained in Konya for two more days to study some of its famous architecture, especially the tiling patterns on the walls of mosques and other buildings. I was particularly intrigued by an interlacing pattern on the walls of a hall in the Karatay Medrese, a former Islamic theological college built in 1251, now a museum of tiles; the pattern is one of the most complicated that I have come across either in reality or in a photograph or drawing.

Islamic interlacing patterns consist of interlacing braids or ribbons weaving alternately under and over each other; they occur throughout the Muslim world from India to Morocco



Fig. 1 The Karatay interlacing pattern, with a change of colouring [©]John Rigby 2004

and Southern Spain, in woodwork or stonework, and in books, but frequently on tiled walls. I am interested in Islamic patterns from a geometrical and artistic viewpoint; for an account of their spiritual significance see, for instance, Critchlow (1976). In the Karatay pattern the braids are black or very dark blue, against a turquoise blue background, with a thin line of white plaster between the black and turquoise tiles; but colour preferences vary between countries. In Spain, for instance, it seems much more common to have white braiding against a multicoloured background. Escher copied some Spanish patterns (Schattschneider, 1990), and they can also be found in guidebooks to Granada and Seville. Other examples of the same type of colouring occur on the front covers of Critchlow (1976) and Bourgoin (1971). I recently produced the white-and-multicoloured version of the Karatay pattern shown in Figure 1 in order to show that there is some kind of order and logic in the intricate crisscrossing of the braids: at first sight the original black-andturquoise pattern looks alarmingly chaotic. I shall refer to Figure 1 as 'the pattern', even though the complete pattern in the Karatay Medrese consists of Figure 1 repeated several times horizontally and vertically.

Whilst I was in Konya I made a freehand sketch of this pattern: a skeletal version in which the braid is reduced to a single line (Fig. 2). Back in Ankara I did lots of calculations – I had no pocket calculator in those days – and produced some accurate drawings; Figure 3 is a skeletal version of one quarter of the pattern. Note that there is a centre of 2-fold rotational symmetry at the centre of Figure 3. Since regular pentagons and ten-pointed stars occur in the pattern, it is not surprising that the golden ratio crops up everywhere; we shall discuss this in more detail later.



Fig. 2 My original sketch



Fig. 3 A skeletal version of the top right quarter of the pattern

Easter Day, 22 April 1973. I have had a lovely birthday today. ... After breakfast I went to church; ... we sang an anthem and two descants for hymns, one of which I had composed for the occasion.

My birthday has three times occurred on Easter Day, and twice I was in exotic locations: once in Ankara, and once in a National Park in the Malaysian Jungle. The old *Book of Common Prayer* of the Church of England contains instructions for finding the date of Easter, which provided me as a choirboy with opportunities for mental arithmetic during dull sermons; they begin: 'To find the Golden Number, or Prime, add one to the Year of our Lord, and then divide by 19', but there is no connection here with the golden ratio or prime numbers! I recently calculated that the next occasion on which I have an Easter birthday will



Fig. 4 Rosettes from Afghanistan and Spain

be in 2057; I expect then to be in an even more exotic location.

I had to analyse the Karatay pattern in order to produce the accurate drawings, but it is only in the last year or so that I have taken the analysis one stage further and asked how the pattern might have been built up from small motifs. The most obvious motif is the rosette at the centre of Figure 1. Rosettes form prominent parts of many interlacing patterns, with six, eight, ten, twelve, sixteen or twenty petals, or occasionally nine or fourteen; examples from Afghanistan and Spain are shown in Figure 4. A closer examination of Figure 3 reveals other motifs, in particular the 'fivediamond' motif in Figure 6a and the 'large pentagon' motif in Figure 6c; the reason for the circles and circular arcs in Figure 6 will soon become clear.

11 June 1973. I have been commissioned to do two more [flowerpaintings] to sell at the church fete. I have drawn an orchid, but the puce colour is impossible to produce from my paint-box, so I must see what the art-shops of Ankara can provide.

The tube of violet paint which one of the art shops of Ankara provided was used thirty years later in my recent colouring of Figure 1.

Building up the Pattern

Here is my suggestion as to how the pattern can be built up from the small motifs, and why the motifs fit together. Based on the figure of a regular decagon surrounded by ten regular pentagons, Figure 5 shows part of a tiling of the plane by regular decagons and pentagons and non-regular hexagons. All the edges in Figure 5 have the same length, 2r say. Draw circles with radius r, centred at each vertex in Figure 5. Only some of these circles are shown in the figure. Inside each circle in Figure 5, draw a copy of Figure 6a, or Figure 6b if the circle occurs on a vertical line of symmetry, taking care



Fig. 5 The underlying grid

that three of the vertices of the regular ten-pointed star (in 6a or 6b) are situated at points where the circle touches other circles. In each curvilinear pentagon draw Figure 6c. In each curvilinear decagon draw Figure 6d. In each curvilinear hexagon draw Figure 6e. This process will produce the skeletal version of the Karatay pattern.

Figure 6e contains some narrow shapes that do not match any other part of the pattern and seem to have crept in because nothing else would quite fit, so as an alternative – what I might have done if I had been designing the pattern – I devised Figure 6f without the central diamond. Some time later I discovered that the central part of Figure 6f does in fact occur in 17th century window-shutter designs in Istanbul, as described in the final section of this article.

The Width of the Braids

Next, how wide should the braids be? There is a certain amount of flexibility in the size of the diamonds and of the pentagon in Figure 6a, as shown in Figure 7a, b, c. Let us decide to make the braids as wide as possible (with the skeletal single line always running down the centre of the braid), and compare what happens in 7a, 7b and 7c. If the diamonds are too big or too small, some very awkward background shapes occur; but if the size of the diamonds is just right, three acute angles in the braid all come together at a single point, producing three triangular regions in the background rather than the awkward non-convex regions. By a happy coincidence, this size of diamond produces a central pentagon between the five diamonds that is the same size as the other pentagons in the pattern. My original rough sketches do not indicate whether this choice of the widest possible braid was actually used in the Karatay pattern, but it would certainly make the cutting of the turquoise





Fig. 6 Various motifs in the pattern



Fig. 7 The maximum braid-width for different sizes of diamond

background tiles much easier during the process described below.

Detailed Measurements

In order to make an accurate drawing of this pattern, or to cut tiles as part of the physical process of constructing the pattern, we need a large number of detailed calculations to find the measurements of the various background shapes. This is where the golden ratio comes in, and this section is more technical than the rest of the article. The golden ratio occurs in various branches of mathematics and science, and is used in architecture and aesthetics. Its basic definition is very simple: the *golden ratio* is the ratio of the lengths of a diagonal and a side of a regular pentagon. There are two notations for the golden ratio; I shall follow Coxeter and use τ , but other writers prefer ϕ .



Fig. 8 A regular pentagon

In Figure 8, suppose the regular pentagon ABCDE has sides of length 1; then the diagonal AC has length τ . All the angles marked with a dot are angles of 36°, so the triangles ACD and DFC are similar isosceles triangles with angles of 36°, 72°, 72°. But AF = 1 because AFDE is a rhombus, so $FC = \tau - 1$. Hence, from the similar triangles, $\tau/1 = 1/(\tau - 1)$, so $\tau^2 = \tau + 1$, or $\tau^2 - \tau - 1 = 0$, whence we deduce that $\tau = (1 + \sqrt{5})/2$.

Another useful fact is $\tau^{-1} = \tau - 1 = (-1 + \sqrt{5})/2$. We shall refer to triangles with angles of 36°, 72°, 72°, and edges in the ratio $\tau : 1$, as golden triangles.

All the angles in the pattern are angles of 36° , 72° , 108° or 144° ; during the calculations we shall have occasion to bisect

angles, so angles of 54° will occur also. We need a suitable unit of measurement, so let us suppose that the 'slant width' of the braids, measured at an angle of 72° to the edge (as shown for instance by the dotted line at Z in Figure 9), is one unit. Because the diamonds in Figure 9 are all congruent and have four equal sides, the shaded golden triangles are all congruent, with sides of length π and x say. The regular ten-pointed star, some of whose edges are indicated by a thick line, has all its edges equal, so $x + 1 = \pi x$, from which we deduce that $x = \tau$; so the shaded golden triangles have sides of lengths τ^2 and τ .

That part of the pattern made up of the pentagons **P** and **Q**, the diamond in between them, and the golden triangles surrounding the diamond, has two perpendicular lines of symmetry; hence **P** and **Q** are congruent, so the pentagon in the centre of the five-diamond motif has the same size as the other pentagons. The larger golden triangle XYZ has edges of length $\tau^2 + 1$ and y say; hence $\tau^2 + 1 = \tau y$, so $y = \tau + \tau^{-1} = 2\tau - 1 = \sqrt{5}$. This is the edge-length of the pentagons.

Here is just one further example: finding the length HD in Figure 9 – a more awkward calculation this time. We have a choice as to the exact size of the pentagon **R** at the centre of the large pentagon motif. We shall assume it has the same size as **P** and **Q**; then the broken line AB is a line of symmetry of that part of the pattern lying between **Q** and **R**, so it bisects the angle of 144° at H. Also CD is a line of symmetry of the large pentagonal motif, so it bisects the angles of 108° at G and D. Hence, by reflection in AB, EF bisects the angle of 108° at E. It is now easy to see that the triangles HFD₃ HEF and AFG are congruent isosceles triangles, so HD = HF = HE = AF = AG = z say. The golden triangle KAH has side-lengths $\tau^2 + 1 + z$ and 2z. Hence $\tau^2 + 1 + z = 2z\tau$, so

$$\tau^2 + 1 = (2\tau - 1)z = (\tau + \tau^{-1})z = (\tau^2 + 1)\tau^{-1}z.$$

Hence $z = \tau$.

Readers may enjoy the challenge of finding ingenious ways of calculating all the lengths in the pattern. It is interesting that all of them are of the form $\alpha \tau + \beta$ where α and β are integers. The use of trigonometry is not recommended: for example, I calculated one of the lengths *d* trigonometrically and obtained the formula

$$2d \cos 18^\circ = (2\tau - 1) \tan 54^\circ$$
,

where $\tan 54^\circ = \tau /\sqrt{3} - \tau$ and $\cos 18^\circ = \sqrt{4\tau} + 3/2\tau$. It turns out that $d = \tau$, but this is not immediately obvious!

Note that we also have a choice as to the exact width of the flower petals in the central rosette motif. Readers may like to



Fig. 9 Calculating various lengths

decide how this width was determined, by using a straight edge to investigate what happens in Figure 3 when the edges of the petals (in two corners of the figure) are extended into the pattern.

The Physical Construction of a Tile Pattern

What about the physical process of constructing such a pattern with tiles on a wall? I quote from an English summary in a Turkish magazine (Kerametli, 1973, p. 46). "The technique most commonly employed in Anatolia during the Seljuk period was that of tile mosaic. Here large monochrome tiles were fired separately and then coloured and glazed. Pieces were then cut from these panels in accordance with the design to be produced and fitted together on a coloured pattern laid out on the floor. The sides of each piece of mosaic were cut at an angle towards the back so that they fitted exactly in front. White plaster was then poured into the spaces at the back of the tiles and the whole panel fitted onto the wall." This technique from the 11th, 12th and 13th centuries is still in use today: here is part of a letter I received from Michael Fox after I had mentioned the technique at the annual conference of the Mathematical Association in 2003. "Earlier this year I was in Marrakesh. ... We did learn how the patterns are made, and it confirms what you put in the hand-out: the individual pieces are cut or sawn from uniformly coloured square tiles; and are then assembled face down on a flat surface (presumably some kind of template). Plaster, or mortar, is applied to the top, i.e. the back of the pattern, and when it has set, the whole unit is lifted and stuck to the wall."

Did the craftsmen use patterns or templates? Were they passed around from one to another to be reused or adapted? All I can find is a quotation (El-Said and Parman, 1976, p.7) that suggests the answer is not known: "To our knowledge, no record has survived to instruct us in the theory of designing Islamic geometric patterns. In this chapter, we will attempt to illustrate how the craftsmen at different times and places in the Muslim world proceeded to apply the geometric principles to the practical problems of making geometric patterns." A motif similar to our motif 6a, but with only four of the diamonds (Bourgoin, 1971, p.182), occurs in a pattern that is described by Critchlow (1976, p.96) as "a much revered pattern ... [that] can be found from Turkey to India and from India via Saudi Arabia to Egypt, Morocco and Spain, as well as in the mosques of Persia." A pattern much more closely resembling the Karatay pattern can be found in Bourgoin (1971, p.190), but unfortunately this Dover reprint does not include the text of the original French edition, "which consisted of an art-historical statement of no particular value", so there is no indication of where the pattern comes from.

Creating a Simpler Pattern

At this point in my investigations I decided to cast myself in the role of a 13th century designer craftsman of the Seljuk period and see how the process of adaptation of an existing design might have been carried out. I constructed a pattern that is not so complicated as the Karatay pattern but still retains its intricacy and most of its basic motifs. Instead of starting from the tiling of decagons, pentagons and irregular hexagons in Figure 5, I started from Figure 10. My unfavourite motif 6e crept in again. Then I discovered a simpler method of obtaining my new pattern. Figure 11 (which is the same as Figure 3: the top-right quarter of the skeletal version of the original pattern) contains two



Fig. 10 The underlying grid for the new pattern



Fig. 11 'Cutting and joining' the Karatay pattern

identically-shaped zig-zag lines running across the pattern. Cut out the intervening part of the pattern, and join the jagged edges together. If we then rotate the remaining pattern through 90° , and introduce the braiding, we end up with Figure 12a.

This new quarter-pattern contains two horizontal braids that appear to run right across the pattern. This is an illusion, but it detracts from the overall effect by chopping up the pattern into strips. So let us replace motif 6a (inside the circles in Figure 12a) by motif 6b, to produce Figure 12b. We have now lost the attractive five-diamond motif 6a, which is a pity; but new decagonal stars have appeared (almost), into which we can put back 6a, and this has the extra effect of getting rid of that narrow shape in motif 6e. One quarter of the final new pattern is shown in Figure 12c. I hope readers will agree that the complete new pattern in Figure 13, produced as a Christmas card in 2003, is worthy of the hard work that went into it.

A Stained-glass Window of Golden Triangles

Figure 13 was intended to be the conclusion – mission accomplished – but here is a postscript, or perhaps a cadenza and coda.

A golden triangle with angles of 36° , 72° , 72° will now be called an *acute golden triangle*, and a triangle with angles of 108° , 36° , 36° is an *obtuse golden triangle*; both types of triangle occur in the regular pentagon of Figure 8. The acute and obtuse golden triangles with side-lengths 1 and τ^{-1} will be denoted by \mathbf{A}_0 and \mathbf{B}_0 ; those with side-lengths τ and 1 by \mathbf{A}_1 and \mathbf{B}_1 , and those with side-lengths τ^2 and τ by \mathbf{A}_2 and \mathbf{B}_2 . These are the *six basic golden triangles*. Figure 14 shows that \mathbf{A}_2 can be dissected into one copy of \mathbf{B}_1 and two copies of \mathbf{A}_1 , which may be expressed as $\mathbf{A}_2 = 2\mathbf{A}_1 + \mathbf{B}_1$; also $\mathbf{B}_2 = \mathbf{A}_1 + \mathbf{B}_1$. Similarly $\mathbf{A}_1 = 2\mathbf{A}_0 + \mathbf{B}_0$ and $\mathbf{B}_1 = \mathbf{A}_0 + \mathbf{B}_0$. Hence $\mathbf{A}_2 = 5\mathbf{A}_0 + 3\mathbf{B}_0$ and $\mathbf{B}_2 = 3\mathbf{A}_0 + 2\mathbf{B}_0$.

We mentioned earlier that all lengths in the pattern are of the form $\alpha \tau + \beta$, where α and β are integers; but we can be more precise and say that all lengths are of the form $\lambda \tau^{-1} + \mu$, where λ and μ are non-negative integers. There is a corresponding result about the background shapes in the pattern: all the background shapes in the pattern are either basic golden triangles themselves, or can be made up from (or dissected into) copies of two or more of the basic golden triangles. Since A_1 , B_1, A_2, B_2 can be made up from copies of A_0 and B_0 , it follows that all the background shapes can be made up from copies of A_0 and B_0 .

This idea can be pursued in various different directions.

(i) Dissection puzzles. As an example, Figure 15 shows a



Fig. 12 Stages in the construction of the new pattern



Fig. 13 A new interlacing pattern adapted from the Karatay pattern [©]John Rigby 2004



Fig. 14 Golden triangles dissected into smaller golden triangles

regular pentagon with side-length $\tau + \tau^{-1}$ dissected into 20, 15 and 11 copies of the six basic golden triangles; find a dissection into only 10 such triangles. How many different dissections are there of this pentagon into copies of \mathbf{A}_0 and \mathbf{B}_0 only, having rotational symmetry?

(ii) Alternative geometrical proofs. Earlier we investigated the irregular hexagon DHEGLM in Figure 16, with angles of 108° at D, E, G, L, and angles of 144° at H and M, where DH = HE = LM = MD; see also Figure 9. Given that GE = $\tau + \tau^{-1}$, find the length DH. Once we have conjectured, or shown by another method, that DH = τ , there is a very simple dissection method of confirming the result: it suffices to prove the converse, and it is easily seen that if DH = τ then the hexagon can be dissected into six copies of **B**₂, together with an **A**₁ and a **B**₀, so GE = $\tau + \tau^{-1}$.

(iii) A different style of mosaic tiling. Instead of cutting each background tile individually, we can start with a large number of precut \mathbf{A}_0 and \mathbf{B}_0 tiles (or perhaps \mathbf{A}_0 , \mathbf{B}_0 , \mathbf{A}_1 and \mathbf{B}_1 , or even all six basic golden triangles) from which each section of the background is built up as a mosaic. This

produces a stained-glass-window effect. Instead of carrying out this idea in Figure 1 or Figure 13, I have created a new pattern based on the five-diamond motif, shown in Figure 17.

From that flight of fancy we return finally to Turkey, to a



Fig. 15 Dissecting a regular pentagon



Fig. 16 Dissecting a background shape



Fig. 17 A pattern with a background of multicoloured golden triangles [©]John Rigby 2004



Fig. 18 The skeletal pattern from a door in the New Mosque

pavilion at the New Mosque in Istanbul. A pair of doors from this pavilion is illustrated on the front cover of another issue of the magazine Türkiyemiz already mentioned (No. 6, 1972). They date from the 17th century, four hundred years after the tiled Karatay pattern. The braiding is of wood, and the background shapes are cut from mother-of-pearl and tortoiseshell alternately. This time the braiding is not 'as wide as possible', so the tortoiseshell has to be cut into intricate non-convex shapes. The basic rectangle of the Karatay pattern (Fig. 1), and of my adapted pattern (Fig. 13), has sides in the ratio tan 54° : 1; each of the door panels consists of a single basic rectangle with sides in the ratio tan 72° : 1. Yet the door panels are so similar to the Karatay pattern, built up from five-diamond and large pentagon motifs (Figs 6a and 6c) and ten-petalled rosettes, that I was able to make a drawing of one of the skeletal patterns by tracing, cutting and pasting, and otherwise adapting Figure 3. The ability to build up background shapes from small golden rectangles is also a great help in such adaptations.

This skeletal pattern, turned on its side, is shown in Figure 18; readers may like to analyse it for themselves, and to create an underlying grid for it. The other door panel differs in detail from Figure 18, and both panels are asymmetrical. It has been suggested to me that slight asymmetries of this type are introduced intentionally, because only Allah is perfect; but I have also recently found illustrations of other similar examples of doors and windowshutters, from the Topkapı Palace, Istanbul, in which pairs of panels exactly match each other and each is symmetrical. In one of these examples the centre part of Figure 6f occurs.

This article may be coming to a close, but the subject is just beginning to open up: there is much more to be investigated.

Notes

This article is an expanded version of a talk given at the Mathematical Association Conference in York, April 2004. Apart from Figure 17, which shows my first attempt at using the software '*Paintshop Pro*', all the original drawing and painting was done by hand. I am grateful to Michael Kennard, who scanned and edited the colour figures; and my special thanks go to Bill Richardson, who spent much

time enhancing and redrawing the scanned versions of my line drawings in order to show them to their best advantage when printed.

Figure 4(a) shows a photograph by Antony Hutt, reproduced from El-Said and Parman (1976), of a pattern from Qal'ah-I-Bist, Afghanistan. We have been unable to find an address in order to ask permission to use this copyright material.

Readers who are interested in repeating patterns in general may like to consult Brian Wichmann's CD, with accompanying booklet (Wichmann, 2001), which contains more than 4500 patterns, about 300 of them Islamic.

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