



in the learning of
MATHEMATICS


 games to stimulate
 mathematical discussion

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Many mathematical games are valuable for stimulating and encouraging mathematical discussion. Collaborative games like the Cube Game¹, and Puzzle-type games like Blind Alley¹ provide examples, as does the following game.

3-DICE GAME

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30

This is a game for two players. Each player has a number grid with numbers 1-30 on it. Player A throws three dice. The numbers thrown may be combined in any way using the four rules, to get a number to cross off on her grid. For example $(3 \times 5) - 4 = 11$, so 11 could be crossed off with a throw of a 3, 4, and 5. Player A may cross off up to 5 different numbers from her one throw, but for each must explain to player B how the result is obtained, if player B challenges her. Player B then has a turn. The aim is to be the first person to cross off all of their numbers.

Using this game I have often found a lot of discussion evoked regarding what is or is not possible, with the combination of dice thrown. For example “You can’t get 21 (with those 3 numbers) but you can get 26 by ...”, and *occasionally* gems like “you can’t get in the 20’s with those three numbers because the biggest number possible is by multiplying them, and $2 \times 3 \times 3 = 18$ ”.

However, since mathematicians are frequently concerned with definitions, perhaps we should clarify the following:

What Do We Mean by Discussion in Mathematics?

Sometimes the term has been equated with the sort of question and answer commonly deployed between teacher and pupil(s) but I believe that that severely limits and undervalues the term. Rather, it will often be between pupils, and will invariably be in the more widely accepted sense of the word, involving reason, challenge, counter-example, and consideration of different suggestions and ideas. Thus in the game described above, pupils will sometimes need to explain how they reached an answer for the partner to check and perhaps challenge. In the Cube game pupils will be involved in discussing whether or not a shape is different from those that have gone before.

It is worth noting that such work is likely to help girls to feel more comfortable and confident in their mathematics², which supports the argument for including this use

of games in the mathematics curriculum. There are several other interrelated reasons why mathematical discussion is important in the learning of mathematics, though.

Reasons Why Mathematical Discussion is Important

HMI describe the ability to reason as a fundamental one which pupils need at all stages in mathematics.³ Concluding an examination of the relationship between language and thought, Dickson *et al.* wrote:

“... it is clear that a central aim of mathematical education must be to enable children to express their mathematical ideas verbally. This includes the ability to listen and to talk about mathematics, as well as to read and to write about it.” (Dickson *et al.*, 1984, p. 333)

I suggest that there are at least four reasons why mathematical discussion is important. These pertain to communication, mathematical thinking, the relationship between discussion and understanding, and the practical value for the teacher. They are outlined briefly below.

1. Mathematics as Communication

The National Curriculum Council has highlighted the importance of pupils being enabled to communicate their mathematics. Whilst the communicative ability of pupils was not directly assessed in the first APU secondary survey, the testers found many cases where pupils were struggling to express what they were doing or how they had arrived at a result. This underlines the need to provide more opportunities for developing pupils' ability to communicate in mathematics.

2. Mathematical Thinking

The National Curriculum proposals for mathematics included the following:

“... for all pupils, efforts are needed to help them to develop or make fuller use of their language as a means of mathematical thinking. This needs to be done through the discussion of mathematical ideas and problems, both with other pupils and with the teacher ...” (DES 1988 para. 8.26 — my underlining)

Such a use of language is also referred to in the Cockcroft Foundation list of mathematical topics for secondary pupils.

3. The Relationship Between Discussion and Understanding

To articulate one's understanding helps to clarify one's understanding — or to point to one's lack of it! HMI put it this way:

“... The quality of pupils' mathematical thinking as well as their ability to express themselves are considerably enhanced by discussion.” (DES 1985 para. 4.8)

Theory and research support an intimate link between thought and language and suggest that concept formation and language development go hand in hand.⁶ Thus when pupils are stretching the limits of their ability to discuss mathematically (e.g. IF ... THEN ... BECAUSE ...) they are also deepening their conceptual understanding. The role of language in the learning of mathematics has come to be seen as one of vital importance.

4. Practical Value for the Teacher

Discussion can be of immense value to the teacher in **illuminating the learner's mathematical understanding and thinking**. This is often kept hidden from the teacher unless the teacher makes a specific attempt to uncover it. Teachers need to know what pupils find difficult, and why they find it difficult, in order to decide on the appropriate next stage for those pupils.

An additional value to the teacher is in terms of **classroom management**. There is much evidence to indicate that pupils rarely work collaboratively — particularly in mathematics. Individualised schemes are common, and these make considerable organisational and management demands on the teacher. No wonder that a concern currently expressed by many teachers in relation to the National Curriculum is how they will find time to observe, listen to, and thereby assess children! The careful use of good mathematical games is one strategy that commends itself to addressing this concern. Indeed a move towards more group work can actually alleviate some of the pressure on the teacher.

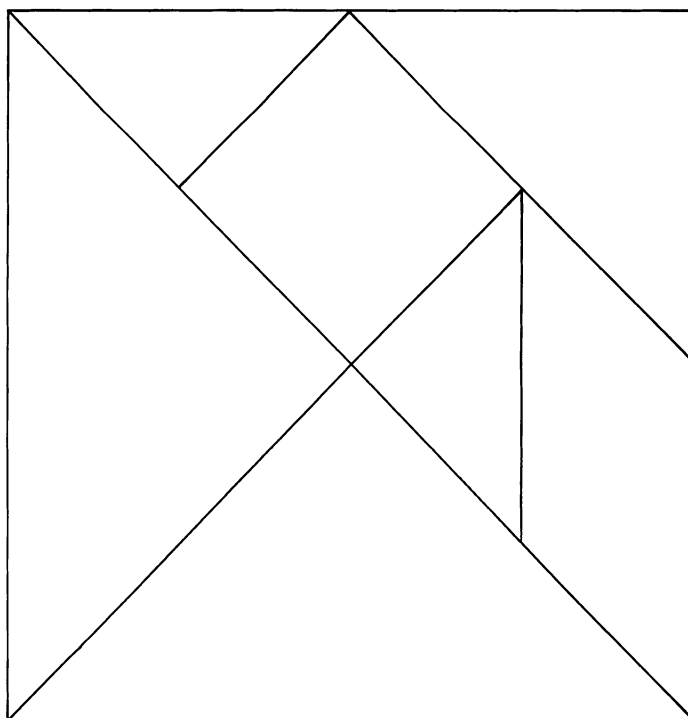
It is worth remembering that the ability to work cooperatively is recognised as an aim of mathematics teaching.^{3,5} Ernest has already pointed out that in the process of encouraging pupils to work cooperatively, using mathematical games, we also encourage discussion.⁷

A further encouraging factor for the teacher is that the use of mathematical games which stimulate discussion does not just help pupils' mathematical development. Discussion in the terms described above also addresses the requirements of the English National Curriculum programmes of study for Speaking and Listening for all key stages.

More Games Which Can Stimulate Mathematical Discussion

TANGRAM GAMES

The traditional TANGRAMS can be used as a puzzle-type game which lends itself to discussion.



A task would be set, like “make a different candle shape from everyone else in the group”, or “make as many different mathematical shapes as you can, recording results”.

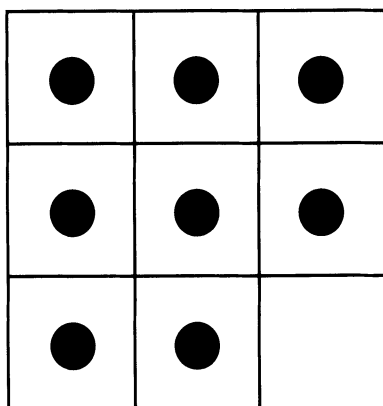
Discussion is likely to arise regarding what actually counts as different — for example, is a mirror reflection “different”? Do all pieces have to be used? What if one piece — like the candle-stem — is just rotated through 180 degrees?

An alternative type of game involves working in pairs. All players have a set of tangram shapes. Player A makes a mathematical shape (e.g. a triangle, or a parallelogram) and has to explain to their partner how to make it, without their partner being able to see the original! If two or three pairs are involved, it could be a race to see which pair gets the copy done first! The teacher would probably want to give each pair the same shape to communicate to partners, in this case.

People Games

In these, a group of people have to work out a solution to a problem. Such games invariably stimulate much discussion!

CORNER-TO-CORNER



Nine chairs are set out in a square with 3 rows of 3. Eight people sit down on separate chairs, leaving one corner chair empty. The task is to arrange for the chair in the opposite corner to become free. Only one person may move at a time, and they can only move forward or back, or right or left, one space, to an empty chair — viz. no diagonal moves are allowed.

Various questions then arise — how many moves did it take? Could you repeat it? Is that the least number of moves possible? Are there other ways which have the same number of moves? What about 16 chairs? 25 chairs? Is there a pattern? How could you record the moves? Is there a better way of tackling the problem than getting people to move around?

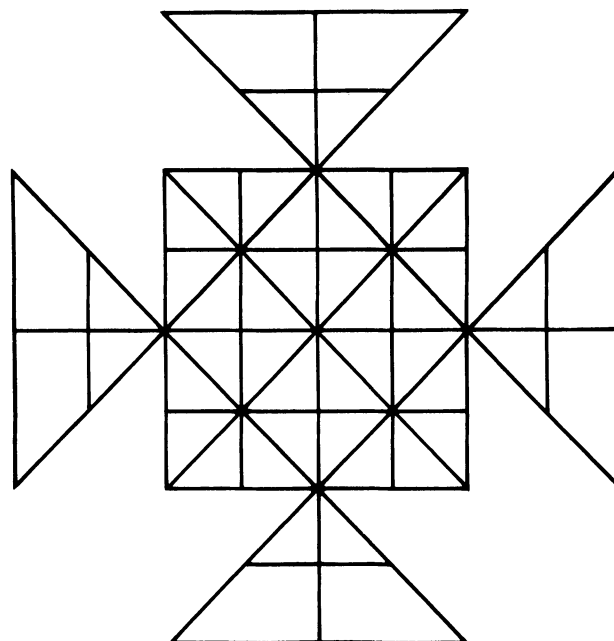
The game could become competitive in that different groups of eight could work on the same problem — which group will find the “best” solution?

LEOPARDS and COWS

Games which involve strategies are particularly valuable for stimulating mathematical discussion. NIM¹ is a good example. Another is Leopards and Cows:

The game comes from Sri Lanka and is played on a board as shown below:

Two black counters (leopards) and 24 white counters (cows) are required. One player has the leopards and the other the cows.



The aim for the leopards is to kill all the cows; the aim for the cows is to trap the leopards (cows cannot kill leopards!).

The board starts empty, and a leopard is placed on any junction (where lines meet or cross), followed by a cow and then the second leopard. From then onwards, a cow is placed on the board after each move by a leopard. Cows are not allowed to move until all 24 have been placed on the board (it is not unusual for some cows to have been killed before all 24 have been placed).

Cows and leopards always move along a line from one junction to the next one, which needs to be empty. A leopard kills a cow by jumping over it to the next junction; that cow is then removed from the board.

The value of many mathematical games can be considerably enhanced by suitable questioning. In the above game, for example, one might ask where the best place for the leopard to start is, or the safest/most dangerous positions for the cows. Answers would be expected to be backed up with reasoning!

One of the great benefits of large outdoor chess sets and other suitable mathematical games in the playground is that they are wonderful stimulators of discussion over what players should and should not do, and why! A mathematical game is an embodiment of a mathematical structure. It provides a concrete representation which can be manipulated to support and demonstrate the mathematics behind it. It thereby lends itself to supporting mathematical discussion. If the child's language or logic falter s/he can immediately resort to the concrete to check and support their thinking.

References

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3. DES (1985) *Maths. from 5-16-Curriculum Matters 3* (HMI), London: HMSO.
4. Dickson, L., Brown, M. and Gibson, O. (1984) *Children Learning Mathematics*, London: Cassell for the Schools Council.
5. DES (1988) *Mathematics for ages 5-16: Proposals of the Secretary of State for Education and Science and the Secretary of State for Wales*, London: HMSO.
6. Brissenden, T. (1988) *Talking about Mathematics — Mathematical discussions in Primary Classrooms*, Oxford: Blackwell Ltd., p. 8.
7. Ernest, P. (1986) Games — A Rationale for their Use in the Teaching of Mathematics in School, *Mathematics in School*, 15 (1), p. 3.