An Everyday Example

A couple in a carpet store are discussing whether they could afford a particular make of stair carpet costing £7.75 a metre. They know they need a length of 22 metres. Rounding the figures to 8 x 20 they reckon this would set them back something like £160. The store is offering interest free credit with 12 monthly repayments. They quickly estimate that they would need to find something between £10 and £15 a month and decide to go ahead. The salesman then works out the exact price on a calculator and looks up the monthly repayments on a chart. The customers were happy that the final figure of £14.21 per month is about what they had expected.

This is typical of how calculations are actually carried out in the real world, yet bears very little resemblance to the typical approach to teaching arithmetic that we encounter in many of our schools.

The Importance of Estimation

The important part that estimation in particular plays in the everyday calculations of a numerate person, as illustrated in the above anecdote, is now being recognized in influential statements about the school mathematics curriculum. The Cockcroft Report (paragraphs 257-262) stresses the importance of estimation skills both in employment and in everyday life, and contrasts this with the fact that very little attention is actually given by mathematics teachers to the development of these skills in their pupils. In the USA reports by the National Institute of Education and the National Council of Supervisors of Mathematics both include computational estimation and the ability to judge the reasonableness of results as basic goals for the teaching of mathematics. In the past we have perhaps assumed that pupils will just pick up such skills, but it is now surely time that the stated importance of estimation as part of numeracy is matched by the specific inclusion of this topic in our mathematics syllabuses, and with a concomitant willingness by mathematics teachers to undertake the necessary conceptual and pedagogical analysis.

Clearly, the availability of calculators should lead mathematics teachers to put more emphasis on such goals, as correspondingly less emphasis is put on algorithmic processes in arithmetic.

A Starting Point

The first thing we need to develop in our pupils is a sense of which situations require an exact answer and for which ones an approximate answer will suffice. If an exact answer is required then it can be obtained by one of three approaches: (a) using a standard paper and pencil algorithm; (b) using an informal method appropriate to the particular question — what we have termed an “adhocorithm” — this might be mental, written or a combination of the two; (c) using a calculator. In the past our school mathematics schemes have emphasised the first of these, and for a large number of pupils this has led to confusion and failure in mathematics. Plunkett even goes as far as questioning whether we need teach algorithms at all any longer. Certainly it is the case that the ready availability of calculators and the recognition of the ways in which people in real life actually do the calculations that are required of them have led in the 1980’s to an increasing emphasis on teaching the use of ad-hocorithms and calculators. These two factors should also lead us to give due recognition to the need to teach estimation skills — partly because of their importance in the real world and their relationship to informal methods for precise calculations, and partly because of their importance when using a calculator as a checking procedure.

What is required now is that mathematics teachers become as consciously aware of the techniques of estimating
and ways of teaching them as they are for the methods of precise calculations. Textbooks, curriculum materials and syllabuses have really overlooked this aspect of numeracy.

Time given to teaching estimation will pay considerable dividends. Not only do pupils acquire genuinely useful skills — particularly if estimation is taught in applied contexts — but also in our experience they become more adept at reasoning with numbers, more flexible in their thinking, more aware of the relationships between different operations and generally develop a greater feel for number.

As we shift our emphasis in teaching arithmetic in the late 1980’s away from algorithms and more towards the understanding of the underlying structure of the operations and how and when they should be applied, then the teaching of estimation skills becomes even more important.

Strategies Used by Good Estimators

The National Assessment of Education Progress, found that in the USA estimation skills of students were very weak. Few research studies had focused directly on computational estimation. Some of the problems in researching this area would clearly be that the processes involved are difficult to identify, that the criteria for good estimates are ambiguous and that performance in estimating is invariably confounded with students' computational ability.

However Reys’ has undertaken a significant study in the USA which showed that estimation skills are indeed surprisingly poorly developed and that they do not automatically develop with maturation nor from the study of more mathematics. His study also highlighted a greater proficiency amongst students in estimating successfully in an applied context than in straight computational items. Having tested students in order to identify good estimators he conducted a series of interviews to determine the strategies and processes used by these good estimators.

As would be expected with informal methods of computation a variety of different and often idiosyncratic strategies were observed. Nevertheless some general strategies and processes used frequently and regularly by good estimators did emerge.

Three key processes can be identified, although it is stressed that these are not mutually exclusive and would often be used in combination.

Translation. This is the process whereby a student might change the mathematical structure of the problem to a more manageable form. For example, (347 x 6)/43 might be changed mentally to 347/(43/6), i.e. roughly 350/7, giving an estimated answer of 50. In conjunction with rounding, of course, translation might involve the use of the associative and commutative properties, or the relationships between various operations such as addition and multiplication or multiplication and division.

Reformulation. This is the process whereby a student may change a computation to a more manageable form for mental computation, but leaving the structure of the problem basically unchanged. Two particular techniques employed here are (a) the use of leading digit (front end) arithmetic — for example, 32 x 4.12 is reformulated as approximately 30 x 4, and (b) the use of compatible numbers — for example, the 43 in (347 x 6)/43 is spotted as being compatible with the 6 if it is replaced by 42, and then, after cancelling, the 347 is replaced by 350 to make it compatible with the 7. Clearly such mental manipulations as this require considerable familiarity with basic number relationships.

Compensation. This is where the student makes adjustments to an estimated answer to compensate for errors introduced by the approximations used. For example, having estimated 32 x 4.12 to be about 120 by front end arithmetic the good estimator would be inclined to add a bit on to compensate for having reduced both of the original numbers. Such adjustments often reflect an intuitive feeling for number and are often characterised by an inability to verbalise a specific rationale for the adjustment (why do we feel that adding 10 would be about right in this example, but 50 would be far too much?)

There are several strategies which good estimators will use within each of these processes, — such as averaging, grouping manageable numbers, compensation of the second place value, use of the associative and distributive laws, rounding one multiple, interchanging fractions, decimals and percentages — but undoubtedly the most common is the use of front-end arithmetic, although this is very rarely taught overtly in schools as a genuine technique. Probably the second most important strategy would be rounding, which, of course, is a technique conventionally taught in schools.

Prerequisite Mathematical Skills

Clearly to be a good estimator the student will need to have developed confidence and flexibility in handling numbers and number relationships. In particular, there are certain specific skills which might be required. These include:

(a) the technique of rounding numbers.
(b) a thorough grasp of place value, including decimal places.
(c) instant knowledge and recognition of basic number bonds for addition, subtraction, multiplication and division, and corresponding results for multiples of 10, 100 etc.
(d) familiarity with the relationship between addition and subtraction, and between multiplication and division.
(e) knowledge of where the distributive, associative and commutative properties apply and where they do not.
(f) the ability to recognize equivalent reformulations of a calculation.
(g) knowledge of simple number patterns in, for example, sets of multiples.
(h) knowledge of common equivalences between decimals, fractions and percentages.

Teaching Computational Estimation

The processes, strategies and skills outlined above will form the basis for any scheme for teaching computational estimation. But there is a lot to do here — and we may need to be selective in tackling particularly such key strategies as front-end arithmetic and rounding and compensation.

Students can also be led to articulate explicitly their thoughts about the range within which the answers to calculations might lie. For example, why must 724 x 5.8 lie
between 700 × 5 and 800 × 6, whereas 724/5.8 must lie between 700/6 and 800/5.

In one series of nine one hour lessons with a class of 12-13-year-olds, for example, the teacher concentrated on the following, using both "abstract" calculations and calculations drawn from real-life situations (Poulter*).

- When do we need exact answers and when will estimates be sufficient — looking at examples from everyday life.
- The technique of rounding.
- Using rounding to make estimates and to check answers obtained on a calculator.
- Front end arithmetic — the importance of the leading digit. Discussion of how this approach differs from conventional arithmetic processes.
- Taking the second digit into account — combining front-end arithmetic and rounding.
- Consideration of the range within which the answer to a calculation might lie, for each of addition, subtraction, multiplication and division.
- Compensation — including both final and intermediate compensation.
- Spotting compatible numbers in order to simplify calculations.
- Review of place value ideas, particularly for decimals. Application of earlier techniques to more examples using decimals.

The above teaching programme was evaluated by means of a pre-test and post-test, with the pupils’ performances compared to those in a parallel class not taking the estimation course. The results indicated that estimation is teachable! (No difference in mean scores on the pre-test, but a highly significant difference on the post-test, with p < 0.001). By explicit attention to some of the techniques and strategies mentioned above it was found that the pupils definitely improved in their ability to make good estimates in the real situations which we face in everyday life.

Conclusions

Clearly if we are to take seriously the teaching of computational estimation then time must be found for it in our mathematics syllabuses. The techniques must be introduced gradually and systematically over a period of time, but also they should be given validity by explicit reference whenever we come across calculations in mathematics or other lessons. All the research evidence (such as Edwards4; O’Daffer1; Trafton12; Atweh1; Reys8) suggests that the first hurdle will be developing proper attitudes to estimation. But also there is sufficient evidence that this is something which can be taught, and need not be left for pupils to pick up in their own time and in their own way.

The place of computational skills in our teaching of mathematics must now be seen in the light of the contribution which they make to one’s ability to use mathematics in the real situations which we face in everyday life. The real world in which a calculator costs no more than a gallon of petrol, and in which machines generate most of the exact answers that are needed, demands that the numerate person become less dependent on paper and pencil algorithms. The really important skills are surely the ones which we have outlined in this article. Our argument that it is time that teachers got down to teaching these is no more than a recognition of the pressing need to prepare our pupils for the genuine and realistic demands of the world in which they live.

References