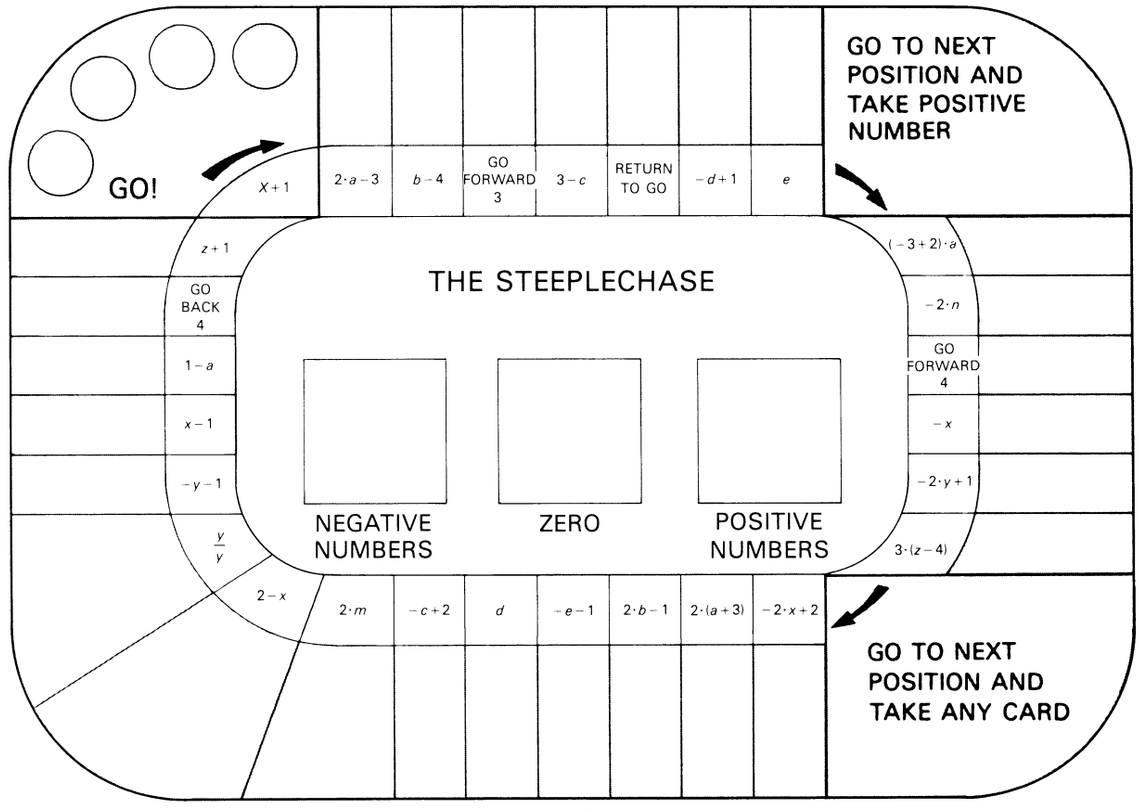

Fig. 1
The Game Board for "The Steeplechase"

GAMES

A Rationale for their Use in the Teaching of Mathematics in School

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Why use games in the mathematics classroom? Can games help children to learn mathematics, or are they just a form of entertainment or "time-filler"?

Many teachers bring out games, puzzles and recreations around Christmas time, or at the end of the Summer term. These activities help to create a light hearted and "fun" atmosphere in the classroom which pupils and teachers enjoy. But pupils will often say "that was fun and I enjoyed it, but it wasn't real mathematics". Are they right? Are games just an enjoyable interlude or can games be used to actually teach mathematics?

The aim of this article is to show that games are a valuable addition to our repertoire of methods for teaching mathematics; that there is much to be gained from including games in our mathematics teaching.

Games have many Advantages

We have all experienced the enthusiasm generated by games such as Chess, Cards, Darts, Monopoly, Scrabble and so on, by playing ourselves or by watching others play. Games generate enthusiasm, excitement, total involvement and enjoyment.

This is the first and most striking advantage of introducing games into the mathematics classroom. Pupils become strongly motivated, they immerse themselves in the activity, and over a period of time should enhance their attitude towards the subject. In addition to being motivating in themselves, games also add variety to the overall mathematics curriculum, by bringing another varied approach into the teaching of the subject.

The celebrated paragraph 243 of the Cockcroft Report[1] stressed that children need to discuss mathematics, as well as learning it in other ways. When groups of children play mathematical games they need to talk over moves and discuss the correctness of answers and different strategies. Thus mathematical games encourage discussion; between groups of children and also between pupil and teacher.

In recognition of the importance of discussion to the learning of mathematics, the new GCSE mathematics

examinations[2] will include oral assessment. Thus it is vitally important that any means of encouraging this neglected area of mathematics learning, such as games, is used to help prepare our students.

The recent publication *Mathematics From 5 to 16*[3] stresses co-operative working as one of the aims of teaching mathematics. Children playing a mathematical game as a team quickly learn that they need to co-operate to play effectively. Even children competing against each other are co-operating in playing the game. Thus introducing games into the mathematics classroom can be a way of encouraging co-operation. Through attaining this aim we also encourage discussion.

The success of all mathematics teaching depends to a large extent on the active involvement of the learner. Children learn mathematics by doing and by making the concepts and skills of mathematics their own. Playing games demands involvement. Children cannot play games passively, they must be actively involved, the more so if they want to win. Thus games encourage the active involvement of children, making them more receptive to learning, and of course increasing their motivation.

Active involvement not only enhances learning, but according to some psychologists is essential for learning to take place at all. For this reason psychologists including Piaget, Bruner and Dienes suggest that games have a very important part to play in learning, particularly in the learning of mathematics.

Of these three, Zoltan P. Dienes goes furthest by suggesting that all mathematics teaching should begin with games. Although Dienes may be overstating his case, he is a man well worth listening to. Dienes has not only carried out an extensive programme of classroom research, he has also developed some of the best apparatus available for teaching mathematics, including the multi-base arithmetic blocks, the algebraical experience materials, logic blocks and the number balance. Thus teaching mathematics through games is not only psychologically sound, it is also psychologically desirable.

This section suggests that there can be many advantages to teaching mathematics through games. However all that it has shown is that *if* we can teach mathematics through games, then there are many desirable by-products including pupil motivation, active involvement, co-operative working and discussion. The outstanding question remains; can mathematics be taught effectively by using games?

Games Teach Mathematics Effectively

We teach mathematics so that our pupils will attain mathematical objectives. Of course we also have more general aims such as giving enjoyment, encouraging co-operation, discussion and so on. Leaving these aside, the major purpose of teaching mathematics is the attainment of objectives. This section focuses on three types of objective, and on the use of games in attaining them. These objectives concern helping children to;

- *Gain new concepts and develop them,*
- *Practice and reinforce skills, and*
- *Develop problem solving strategies.*

Each of these objectives is considered in turn, but in an order which reflects the frequency with which games are used to attain them.

1. The Reinforcement and Practice of Skills

Much of mathematics teaching revolves around giving children practice in newly acquired skills, or in reinforcing and further developing skills. Games provide a way of taking the drudgery out of the practice of skills, and indeed of making the practice more effective.

Steeplechase[4] is a game specially developed to give pupils practice at algebraic substitution. The **Steeplechase** board is shown in Figure 1. The game is played by two to four players who put their counters on the "Go" position to start. On the board there are three piles of cards shuffled and placed face down. Negative number cards have the values -1 to -6, and the positive number cards have the values 1 to 6.

Players take turns choosing a pile of cards. They then substitute the number on the top card of the pile into the algebraic expression written on the board square where their counter is positioned. The resulting value is the size of their move. The first player twice around the track wins, although variations include finishing exactly on "Go".

The game was used with first, second and third year secondary pupils in Israel.[5,6] It was found that more able pupils achieved mastery at algebraic substitution whether they played the game or not. Among the less able pupils it was found that those who played the game scored considerably better on skills than those that did not, (as well, in fact, as the more able pupils).

Overall it can be said that playing this game allowed pupils to practice substitution until they mastered it, and that the learning, particularly of the less able pupils, was enhanced by playing the game. There were other benefits as well, which will be discussed later.

A team of American researchers Bright, Harvey and Wheeler have carried out over a dozen studies of the use of games to teach mathematics.[7] Two of the studies[8] involved the use of games to reinforce basic multiplication and division facts (with single digit factors) among 14 classes of 9, 10 and 11 year olds in 1976 and among 10 classes of 10 and 11 year olds in 1977.

The classes played the games for 15 minutes daily for a total of 7 days. Gains in test performances showed that the games treatment was an effective way to retrain and reinforce children's skills with basic number facts. A further study[9] by the same team concerned the use of games to develop further the skill of ordering common fractions. The study showed that the games treatment was effective.

These four studies show that appropriate mathematical games incorporated into the teaching programme are an effective way of practising and reinforcing skills.

2. The Acquisition and Development of Concepts

One of the main objectives of mathematics teaching is to aid children in acquiring new concepts. A further objective is to assist children in developing and extending their concepts. Mathematical games can be an effective way of attaining both of these objectives. The following studies illustrate this.

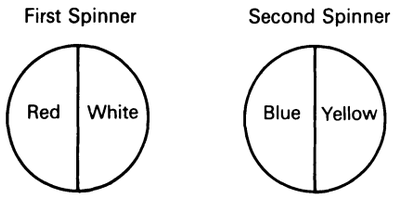
The American team of Bright, Harvey and Wheeler Carried out two studies,[10,11] with 11 year old and 13 year old pupils. The studies concerned the concepts of fairness and unfairness with regards to probability. The pupils played 8 pairs of games one of which is fair and the other unfair, in that one of two players is favoured.

The pupils were tested with 14 items like those shown in Figure 2 both before and after the game playing. Some pupils were also given the results of a computer simulation of playing the test games for 50 trials. These pupils showed appreciable gains in their test scores, which the others did not, in both studies. The games helped the pupils acquire the probabilistic concepts of fairness and unfairness, which they could successfully apply in a situation if sufficient information was given.

Edith Biggs carried out a research project[12] teaching both slow learning and able pupils from 7 to 13 years old in twelve London Schools (six first schools, six middle

The following set of problems ask you to choose the game which gives you the best chance of winning. If the chances are the same you are to identify that it doesn't make any difference.

Which game gives you the best chance of winning or doesn't it make any difference?



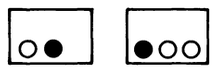
A. Game 1 You spin the first spinner once and you win if you get red.

B. Game 2 You spin both spinners once and you win if you get white and yellow, or white and blue.

C. It doesn't make any difference.

The following boxes contain black and white marbles. To play this game you pick a marble from one of the two boxes. *You win if you choose a black marble.*

If you can play this game only once, do you have a better chance of winning if you pick from Box A or Box B, or doesn't it make any difference?



A. Box A

B. Box B

C. It doesn't make any difference.

Fig. 2 Sample Test Items for Fair/Unfair Games

schools) over a period of two terms. Games, especially dice games, made up a large part of the teaching programme for all the pupils, except for the older more able pupils. Although games were included, the teaching programme for these pupils concentrated more on problem solving and investigatory work.

With the slow learners Dr Biggs used games first as a diagnostic tool and then for teaching purposes. She reports that most of these children acquired the concepts of place value and an understanding of the four number operations. The children began by showing a prejudice against games, saying that they were not "proper maths". During the experiment the children slowly changed their attitudes and began to enjoy the work. As reported there were substantial gains in the conceptual understanding of number.

Dr Biggs' study does not have the same careful experimental design of some of those reported above. Thus the real conceptual gains of the children cannot be attributed solely to the inclusion of games in the programme. It is rather that games played a central part in an effective teaching programme. The results of Dr Biggs' work with more able children will be reported in the next section.

It has been suggested that the game **Steeplechase** contributes to the reinforcement of algebraic skills. Furthermore playing the game can be said to have deepened pupils' understanding of the concepts involved.

"1. *On the whole, the pupils who played the game did not regard the expression $-x$ as giving negative outcomes only, whereas non-playing pupils did.*

2. *When asked to choose a number to substitute into a given expression, the pupils who played chose positive and negative numbers equally, whereas pupils who did not play chose positive numbers only.*

3. *Given the number resulting from a (unknown) substitution, playing pupils succeeded to a far greater extent than non-playing pupils in finding the number to be substituted to obtain the given result (that is, in performing the reverse process before they had learned to solve equations).*"

(Ref. 6 page 224-5)

These results indicate that playing the game **Steeplechase** not only deepened players understanding of certain algebraic concepts but also contributed to the development of higher level problem solving skills in algebra, providing evidence for the next section.

In an experimental study[13] Dienes successfully used games as a teaching vehicle for conceptual acquisition and development. Dienes believes that "*the rules of games represent realistic restrictions on possible mathematical operations and thus help to guide and shape children's mathematical understanding.*"[14]

The sample studies discussed above show how games can play a vital part in aiding children to first acquire and then to further develop mathematical concepts.

3. The Development of Problem Solving Strategies

In addition to gaining and improving skills and understanding, an important objective of mathematics teaching concerns the acquisition of problem solving strategies and skills. HM Inspectorate[3] have gone so far as to specify the following problem solving strategies as distinct objectives of mathematics teaching.

- **Trial and error methods**
- **Simplifying difficult tasks**
- **Looking for pattern**
- **Making and testing hypotheses**
- **Reasoning**
- **Proving and disproving**

Mathematical games can foster the development of most, if not all, of these strategies and higher-level skills, as the following studies illustrate.

Edith Biggs used games in a teaching experiment with more able children aged 7 to 13 years. Dr Biggs observed not only that these children's conceptual understanding deepened, but that their problem solving abilities grew. Thus this study suggests that the inclusion of games in a teaching programme can aid the development of problem solving strategies, certainly among more able pupils.

Two American studies[15,16] found that a game involving mathematics can be used to elicit problem solving behaviour from both inexperienced and experienced problem solvers. One of these studies[16] found that it was possible to use the responses of pupils to games to classify them into groups according to the problem solving strategies they used. Thus these two studies suggest that games can be used both to diagnose the problem solving strategies employed by students and to encourage the acquisition and development of problem solving strategies among students.

The Shell Centre has produced a multi-media package[17] which shows (on video) children and teachers learning problem solving strategies through playing "*Pirates*" and other games. Although this is not a research study, it is an illuminating and accessible example of the use of games in developing problem solving strategies. In conjunction with the studies reported above it serves to illustrate the vital role that games have to play in the fostering of problem solving.

4. The Motivational Effect of Games

The evidence reviewed above indicates that including mathematical games in mathematics teaching aids the achievement of a full range of mathematical objectives. It is quite likely that this success is related to the positive effect of games on motivation and attitudes reported in some of

the studies[6,12]. This hypothesis is confirmed by the result of a study[18] involving nine classes of first and second year secondary pupils taught by two teachers in an inner-city school in Detroit.

The experimental programme lasted for two terms. The classes had games included in their mathematics curriculum either both terms, or the first or second term only or in neither term. In summary, the results of the study were as follows. The absentee rate in the non-games classes were at least twice as high as in the games classes, irrespective of which teacher taught the classes or whether the pupils had played mathematical games the previous term. In the games classes the absentee rates for pupils never reached 20%. In the non-games classes the absentee rates exceeded 20% before half term, and approached 35% to 40% by the end of term.

The study was designed to minimise the effect of factors other than the influence of the games. The findings confirm, therefore, that games have a strong motivational effect, and this is particularly striking in an inner city, and presumably demotivated, context.

Games should be included in the Mathematics Curriculum

The evidence reviewed above, which is only a sample of the evidence available, suggests that games are an effective way of teaching mathematics, both in terms of attaining mathematical objectives and in terms of motivating students to learn.

The evidence strongly suggests that if games are to contribute to the effective teaching of mathematics they must be fully incorporated into the mathematics curriculum. During the teaching of a specific topic, or directed at a particular objective, games should be

1. selected on the basis of the desired objectives,
2. incorporated into the teaching programme.

Used in this way mathematical games have a vital part to play in aiding pupils' achievement and success in mathematics.

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Cutting the Flapjack

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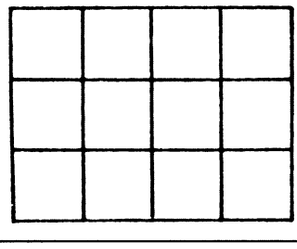
The following simple game for two players can lead to an interesting mathematical investigation for the whole class.

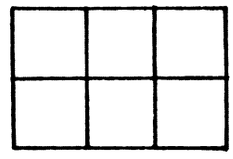
Game Description

A freshly baked tray of flapjack, represented by the rectangular grid of squares in Figure 1, is to be cut along the grid lines by the two players. Player A can cut any rectangle into two smaller ones along grid lines running North–South whilst player B can cut any rectangle along a line from East to West. When one player is unable to make a move, that player is the loser.



Illustration

Suppose the flapjack had been in the form of a 2×3 rectangle as in Figure 2.



Initially it may be thought that because there are more North–South lines than East–West lines then player A should win. However after his first cut he must leave a 2×1 rectangle and a 2×2 square. If player B cuts the 2×1 rectangle then player A is faced with the 2×2 square which is clearly a losing position.

Had player B made the first cut in the 2×3 rectangle then this would place player A in a winning position.

Extension

After playing the game with some different rectangles the children might be encouraged to examine the set of results for the whole class. The discussion could be developed into a consideration of the winning strategies from given starting shapes. What sizes of flapjacks are the most interesting for the game situation?