## SPC March 2019

## Student Problems

Students up to the age of 19 are invited to send solutions to either or both of the following problems to Stan Dolan, 126A Harpenden Road, St Albans, Herts., AL3 6BZ.

Two prizes will be awarded - a first prize of $£ 25$, and a second prize of $£ 20$ - to the senders of the most impressive solutions for either problem. It is not necessary to submit solutions to both. Solutions should arrive by May 20th 2019. Please give your School year, the name and address of your School or College, and the name of a teacher through whom the award will be made. Please print your own name clearly! The names of all successful solvers will be published in the July 2019 edition.

The MA and the Gazette comply fully with the provisions of the 2018 GDPR legislation. Submissions must be accompanied by the SPC permission form which is available on the Mathematical Association website https://www.m-a.org.uk/the-mathematical-gazette
Note that if permission is not given, a pupil may still participate and will be eligible for a prize in the same way as others.

## Problem 2019.1 (Paul Stephenson)

A unit circle has centre at the origin $O$. Two perpendicular chords of the circle intersect at the point $P(x, y)$, where $x$ and $y$ are both positive.

One of the four regions into which the chords divide the circle has area $\frac{\pi}{2}$.

Find the area of the diagonally opposite region in terms of $x$ and $y$.


## Problem 2019.2 (Chris Starr)

For which integer values of $n$ is

$$
\left(2 \cos \frac{\pi}{7}\right)^{n}+\left(2 \cos \frac{3 \pi}{7}\right)^{n}+\left(2 \cos \frac{5 \pi}{7}\right)^{n}
$$

an integer?

## Solutions to 2018.5 and 2018.6

Both problems were solved by Andrew Ng (Tonbridge School). Problem 2018.6 was solved by Ngo Hang Chan (Charterhouse), Sae Koyama (King's College London Mathematics School) and a student for whom no GDPR form was submitted.

