A Patchwork of Squares

The aim is to fill a rectangle with squares, no two the same size.

The dimensions of the rectangle and the sizes of the squares to fill it depend on how we arrange the squares. This will emerge at the end.

Our arrangement of squares leads to the smaller of Zbigniew Moroń's two examples of a "squared rectangle" first given in his 1925 paper.

A simple method, usually successful.

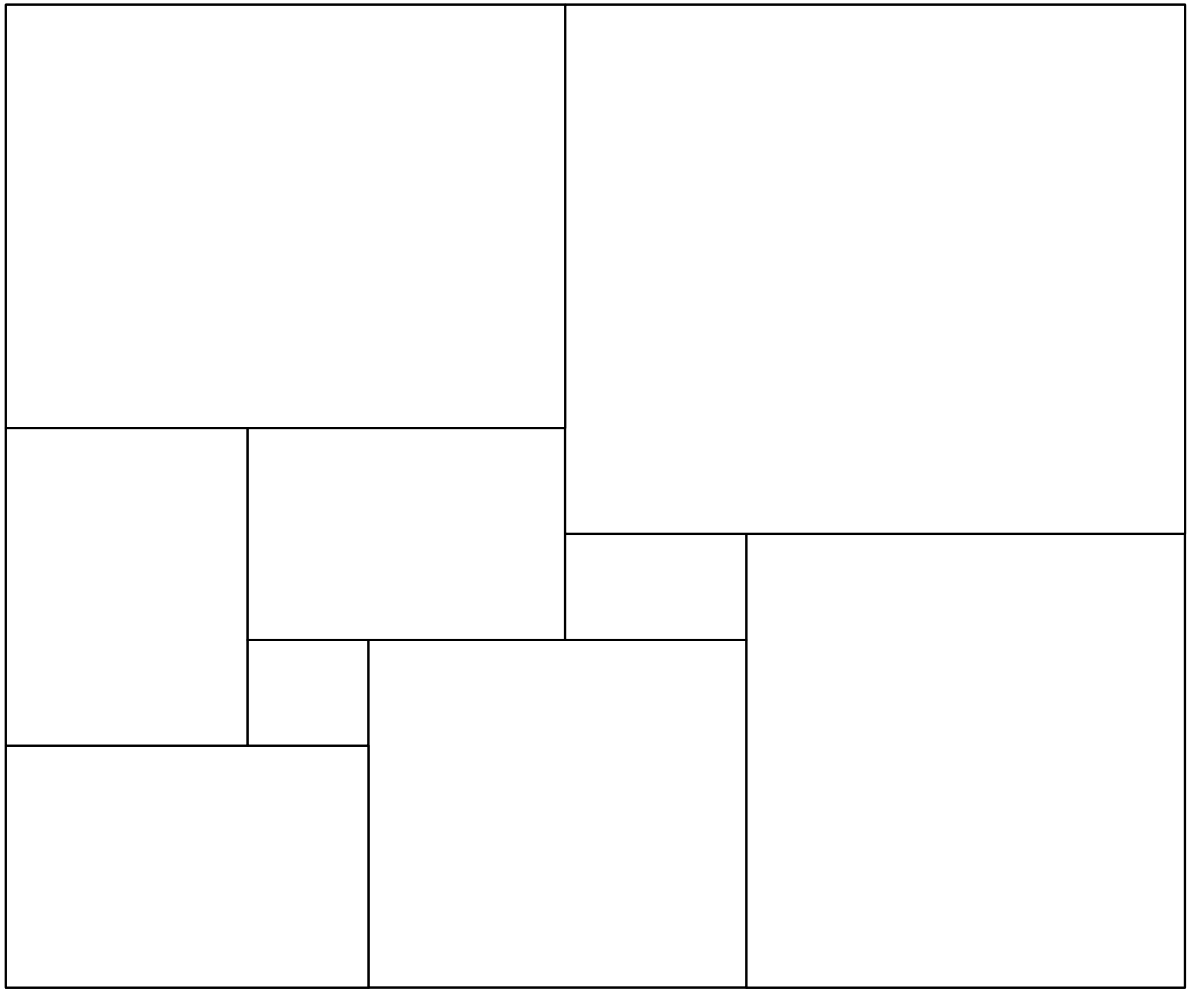
Draw a rectangle.

Starting at one corner, add squares one by one.

Adjacent squares must never have the whole of one side in common.

Avoid introducing symmetry.

If some of the squares don’t look like squares, don’t worry, it’s only a sketch!



All
done!

Labelling the squares

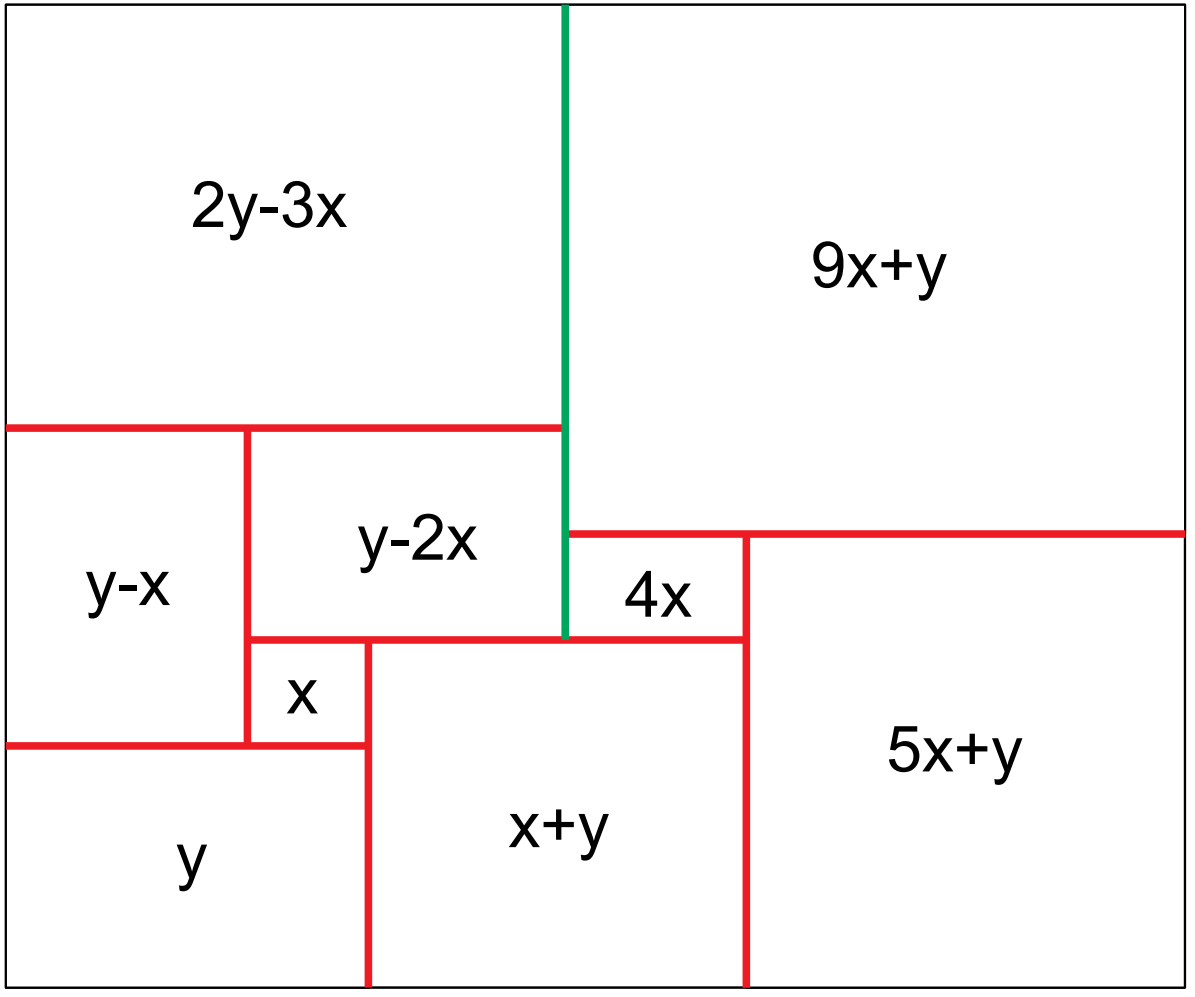
Enter x into one of the smaller squares. This means each side is x units long.

Enter y into an adjacent square.

If the sizes of nearby squares can now be found in terms of x and y, label them with their sizes.

With more complex arrangements of squares it may be necessary to introduce further unknowns: z, w, …

Continue in the same manner until the sizes of all squares have been found in terms of the unknowns.

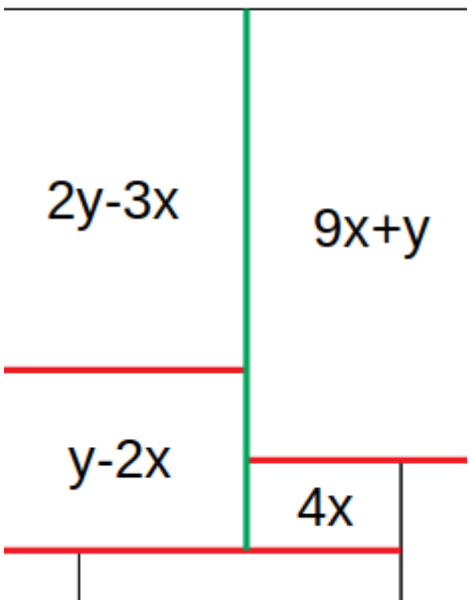


All squares have been labelled.

One inside edge has not been used.

The length of this edge can be found in two ways, viz. using the squares to the left and to the right.

This gives us an equation relating x and y.



$$(2y-3x) + (y-2x) = (9x+y) + 4x$$

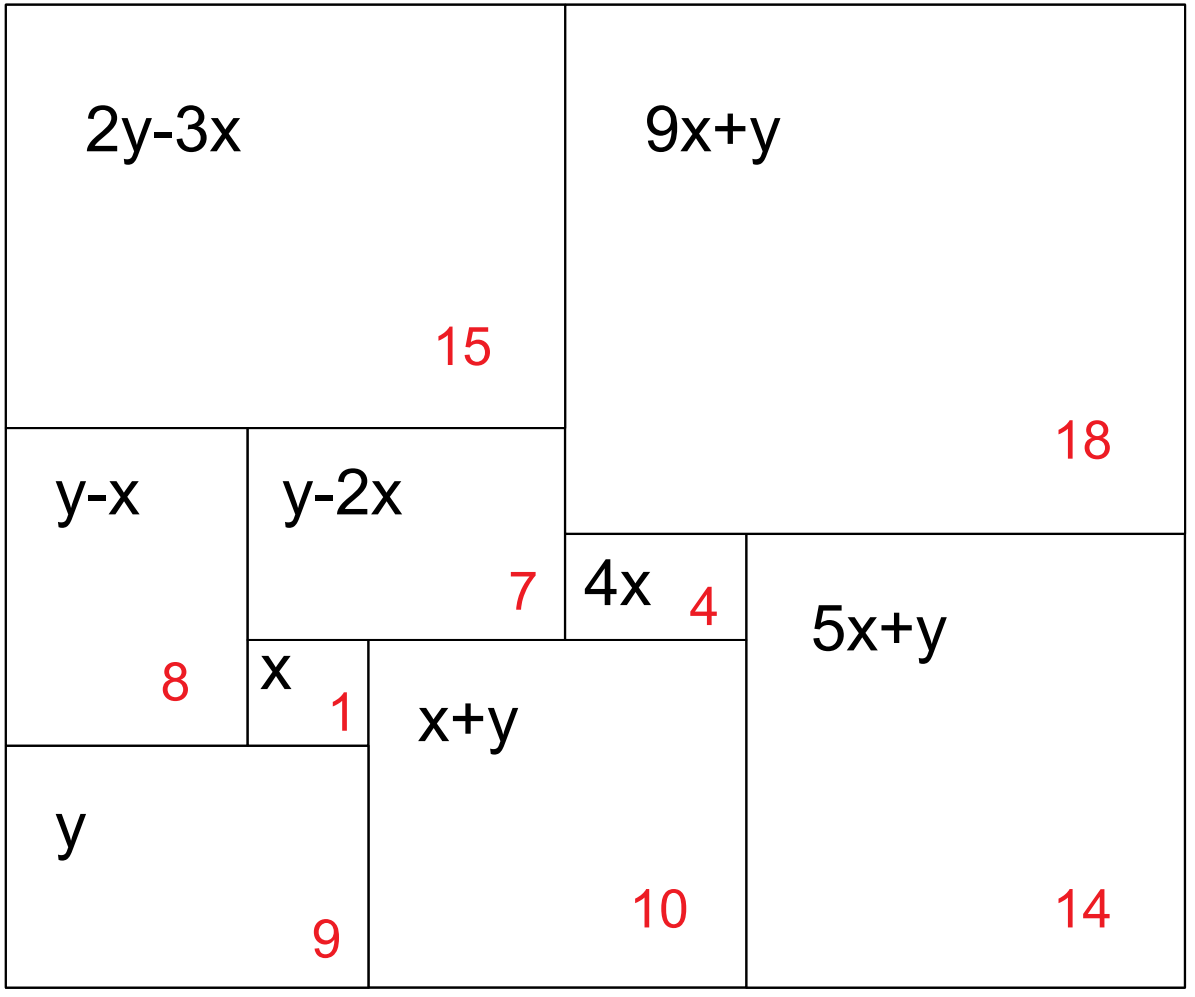
$$3y - 5x = 13x + y$$

$$2y = 18x$$

$$y = 9x$$

Least positive integer solution:

$$x = 1,\ y = 9$$



Substituting

x=1

y=9

Rectangle:

33 × 32

Mission accomplished!