# INDEX TO ANSWERS

# ANSWERS FROM ISSUE 60

## SOME TRIANGLE NUMBERS – 2

Many thanks to Andrew Palfreyman who found ***five***, not four solutions!

| | | |
|---|---|---|
| | 7 | |
| 1 | 0 | 5 |
| | 3 | |

Grid A

| | | |
|---|---|---|
| | 7 | |
| 3 | 0 | 0 |
| | 3 | |

| | | |
|---|---|---|
| | 7 | |
| 4 | 0 | 6 |
| | 3 | |

| | | |
|---|---|---|
| | 7 | |
| 9 | 0 | 3 |
| | 3 | |

| | | |
|---|---|---|
| | 7 | |
| 9 | 4 | 6 |
| | 1 | |

| | | | |
|---|---|---|---|
| | 6 | | 6 |
| 1 | 3 | 2 | 6 |
| 2 | 0 | 1 | 6 |
| 0 | | 0 | |

Grid B

## CROSSNUMBER

Many thanks again to Andrew Palfreyman who pointed out that 1 Down and 8 Across do not give unique answers so there are four possible solutions.

| | | | | | |
|---|---|---|---|---|---|
| [1] 1 | 4 | 4 | | [2] 8 | |
| 3 | | | [3] 9 | 1 | 9 |
| [4] 2 | 3 | [5] 1 | 0 | | |
| | | [6] 1 | 0 | 9 | [7] 8 |
| [8] 3 | [9] 6 | 1 | | | 0 |
| | 5 | | [10] 3 | 4 | 3 |

| | | | | | |
|---|---|---|---|---|---|
| [1] 1 | 4 | 4 | | [2] 8 | |
| 8 | | | [3] 9 | 1 | 9 |
| [4] 2 | 3 | [5] 1 | 0 | | |
| | | [6] 1 | 0 | 9 | [7] 8 |
| [8] 3 | [9] 6 | 1 | | | 0 |
| | 5 | | [10] 3 | 4 | 3 |

| | | | | | |
|---|---|---|---|---|---|
| [1] 1 | 4 | 4 | | [2] 8 | |
| 3 | | | [3] 9 | 1 | 9 |
| [4] 2 | 3 | [5] 1 | 0 | | |
| | | [6] 1 | 0 | 9 | [7] 8 |
| [8] 9 | [9] 6 | 1 | | | 0 |
| | 5 | | [10] 3 | 4 | 3 |

| | | | | | |
|---|---|---|---|---|---|
| [1] 1 | 4 | 4 | | [2] 8 | |
| 8 | | | [3] 9 | 1 | 9 |
| [4] 2 | 3 | [5] 1 | 0 | | |
| | | [6] 1 | 0 | 9 | [7] 8 |
| [8] 9 | [9] 6 | 1 | | | 0 |
| | 5 | | [10] 3 | 4 | 3 |

## TREASURE HUNTS 12, 13

12
This is a rostral column in St Petersburg, Russia. Built in 1811 it follows the example of the Romans who used the prows (rostrum) of conquered ships to decorate their monuments.

13
The school with the plaque shown is Trinity-House School in Newcastle. It came into being in 1505 and teachers included the renowned mathematician Robert "Beau" Harrison in 1756 and John Fryer, the land surveyor famed for his maps of both Newcastle and Northumberland, in 1771. Edward Riddle was Schoolmaster between 1814 and 1821, when he was appointed Master of the Royal Naval College, Greenwich. There is more about other mathematics teachers at [http://www.british-history.ac.uk/no-series/newcastle-historical-account/pp443-445](http://www.british-history.ac.uk/no-series/newcastle-historical-account/pp443-445)

**SYMKEN - 5**

Easy

| | | | | |
|---|---|---|---|---|
| 2x 1 | 5 5 | 24x 3 | 2 | 4 |
| 2 | 1 | 160x 4 | 15x 3 | 5 |
| 15x 3 | 2 | 5 | 4 | 1 |
| 5 | 48x 4 | 2x 2 | 1 | 6x 3 |
| 4 | 3 | 6+ 1 | 5 | 2 |

Moderate

| | | | | |
|---|---|---|---|---|
| 6+ 5 | 7+ 3 | 9+ 4 | 3+ 2 | 1 |
| 1 | 4 | 5 | 5+ 3 | 2 |
| 4+ 3 | 1 | 7+ 2 | 5 | 12+ 4 |
| 11+ 2 | 5 | 9+ 1 | 4 | 3 |
| 4 | 2 2 | 3 | 1 | 5 |

**NOORTS AND CROXXES - 2**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| o | o | x | o | x | o | x | x | o | x |
| x | o | x | o | o | x | o | x | x | o |
| o | x | o | x | x | o | x | o | o | x |
| x | o | x | x | o o | o | x | o | x | o |
| x | x | o | o | x | x | o | x | o | o |
| o | o | x | x | o | x | o | o | x | x |
| x | x | o | x | o | o | x | x | o | o |
| o | o | x | o | x | x | o | x | x | o |
| o | x | o | o | x | x | o | o | x | x |
| x | x | o | x | o | o | x | o | o | x |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| o | x | x | o | x | o | o | x | x | o |
| o | x | x | o | o | x | o | o | x | x |
| x | o | o | x | o | x | x | o | o | x |
| x | x | o | x | x | o | o | x | o | o |
| o | o | x | o | x | o | x | o | x | x |
| x | o | o | x | o | x | x | o | x | o |
| x | x | o | o | x | x | o | x | o | o |
| o | o | x | x | o | o | x | x | o | x |
| x | x | o | x | o | o | x | o | x | o |
| o | o | x | o | x | x | o | x | o | x |

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| o | o | x | x | o | o | x | x | o | o | x | x | o | x |
| x | o | x | o | o | x | x | o | x | x | o | o | x | o |
| o | x | o | x | x | o | o | x | x | o | o | x | o | x |
| o | o | x | x | o | x | x | o | o | x | x | o | o | x |
| x | x | o | o | x | x | o | o | x | x | o | o | x | o |
| x | x | o | o | x | o | x | x | o | o | x | x | o | o |
| o | o | x | x | o | o | x | x | o | o | x | o | x | x |
| x | x | o | o | x | x | o | o | x | x | o | x | o | o |
| x | x | o | o | x | o | o | x | o | x | o | x | o | x |
| o | o | x | x | o | x | x | o | x | o | x | o | x | o |
| o | x | x | o | o | x | o | x | x | o | o | x | x | o |
| x | o | o | x | x | o | o | x | o | x | x | o | o | x |
| o | o | x | x | o | o | x | o | o | x | x | o | x | x |
| x | x | o | o | x | x | o | o | x | o | o | x | x | o |

# ANSWERS FROM ISSUE 61

**NEW KID ON THE BLOCK**

One solution is {1, 2, 3, 5, 6, 7, 8, 11, 12, 14, 15, 20, 21, 24, 28, 30, 34, 37, 38, 40} and

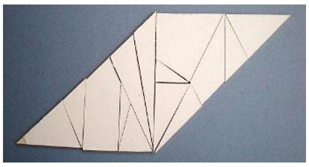
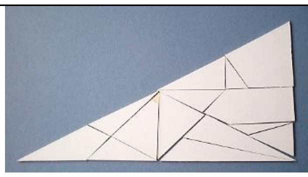
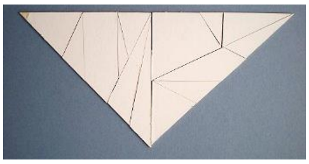
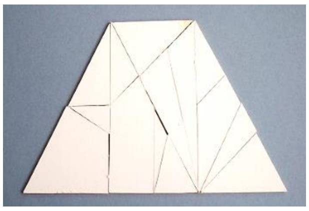
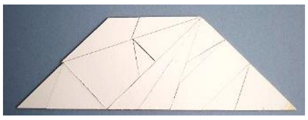
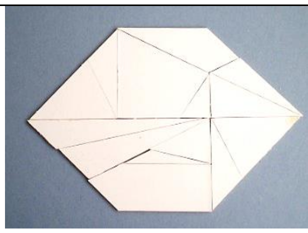
{4, 9, 10, 13, 16, 17, 18, 19, 22, 23, 25, 26, 27, 29, 31, 32, 33, 35, 36, 39}

**A 2-DIMENSIONAL PROBLEM**

Area of $EFGH = 90\text{cm}^2$.

If $P$ divides $EH$ in the ratio $1{:}x$ then the area of the large square is $\dfrac{10(2x-1)^2}{(x-1)^2}$.

**STOMACHION**

| parallelogram | rectangle | right-angled triangle | right-angled isosceles triangle |
|---|---|---|---|
|  |  |  |  |
| trapezium | different trapezium | hexagon | |
|  |  |  | |

**PLACING NUMBERS**

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 9 | 4 | 1 | 5 | 2 | 6 | | 3 | | 5 | |
| 0 | | 3 | | 5 | | 4 | 7 | 1 | 8 | 0 |
| 8 | 2 | 7 | 9 | 4 | 1 | 9 | 2 | | 2 | |
| 8 | 1 | 3 | 4 | 6 | | 6 | 9 | 6 | 4 | 7 |
| 7 | 2 | 8 | 7 | | 3 | 8 | 0 | 1 | 5 | |
| | 9 | | 8 | 7 | 1 | 4 | | 9 | | 8 |
| 6 | 1 | 5 | 0 | | 4 | | 7 | 0 | 2 | 4 |
| 7 | | 3 | | 3 | 2 | 5 | 1 | 3 | | 3 |
| 5 | 0 | 4 | 5 | 9 | 8 | | 9 | 0 | 3 | 6 |
| 6 | | 9 | | 6 | 5 | 7 | 7 | 8 | 5 | 0 |
| 4 | 8 | 8 | 4 | 3 | 7 | | 8 | | 7 | |
| 9 | | 3 | | 6 | | 2 | 5 | 1 | 6 | 0 |
| 4 | 6 | 2 | 7 | 1 | 8 | 9 | 0 | | 6 | |
| | 7 | 5 | 6 | 2 | 5 | 0 | | 9 | | 1 |
| 5 | 8 | 2 | 1 | | 5 | 6 | 6 | 2 | 3 | 4 |
| | 2 | | 3 | 4 | 4 | 3 | | 2 | | 1 |
| 3 | 0 | 1 | 0 | | 2 | 9 | 8 | 1 | 7 | 0 |

**QUICKIE 30**

If the sides of the rectangle are $x$ and $y$ with $x < y$, then $4y + 5x = 264$ and $3x = 2y$.
This gives $x = 24$ and $y = 36$, so the perimeter is 120cm and the area is 864cm$^2$.

**QUICKIE 31**

$$1014492753623188405797 \times 7 = 7101449275362318840579$$

**SYMKEN - 6**

Easy

| | | | | | |
|---|---|---|---|---|---|
| 5x<br>5 | 12x<br>2 | 3 | 6÷<br>1 | 6<br>6 | 12x<br>4 |
| 1 | 72x<br>4 | 2 | 6 | 11+<br>5 | 3 |
| 3 | 6 | 12x<br>4 | 2<br>2 | 1 | 5 |
| 48x<br>4 | 3 | 1 | 5<br>5 | 12x<br>2 | 6 |
| 6 | 5x<br>1 | 30x<br>5 | 12x<br>3 | 8x<br>4 | 2 |
| 2 | 5 | 6 | 4 | 4+<br>3 | 1 |

Moderate

| | | | | | |
|---|---|---|---|---|---|
| 9+<br>3 | 4-<br>2 | 6 | 5+<br>4 | 5÷<br>5 | 1 |
| 6 | 12+<br>3 | 5 | 1 | 2÷<br>2 | 4 |
| 6÷<br>1 | 6 | 4 | 10+<br>5 | 3 | 2 |
| 3+<br>2 | 1 | 4+<br>3 | 10+<br>6 | 4 | 8+<br>5 |
| 4<br>4 | 5<br>5 | 1 | 3÷<br>2 | 6 | 3 |
| 9+<br>5 | 4 | 1-<br>2 | 3 | 5-<br>1 | 6 |

**MALCOLM AND HIS FLIGHTS**

The three consecutive odd numbers are 53, 55 and 57. The total is 165 minutes.

**PI PIX**

We have $\pi r^2 = 2r + \pi r$ leading to $r = \dfrac{2+\pi}{\pi}$.

**TREASURE HUNTS 14, 15**

TH 14 was York railway station, designed by Thomas Prosser and William Peachey. It was opened in 1877. The girder indicated by a yellow line appears to be part of an ellipse.
TH 15 is the grave of Florence Nightingale in the churchyard of St Margaret of Antioch, Wellow, Hampshire
Adam Mallis was the first name drawn out of the hat in each case and so won the prizes.

# ANSWERS FROM ISSUE 62

**DSL HZMT GSRH HLMT? (WHO SANG THIS SONG?)**

Queen sang it: it is the opening words from Bohemian Rhapsody.

Is this the real life?
Is this just fantasy?
Caught in a landslide
No escape from reality
Open your eyes
Look up to the skies and see
I'm just a poor boy, I need no sympathy
Because I'm easy come, easy go

The letter that appears in position $n$ in the alphabet has been replaced by the letter at position $27 - n$ i.e. the alphabet has been reversed.

**CROSSWORD**

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 D | E | 2 C | I | 3 M | A | 4 L | | 5 T | O | W | E | 6 R |
| A | | W | | E | | A | | H | | | | H |
| 7 I | N | T | E | G | E | R | | 8 R | A | 9 T | I | O |
| L | | | | A | | G | | E | | O | | M |
| 10 Y | A | R | 11 D | | | 12 E | 13 V | E | N | T | | B |
| | | | I | | | | A | | | A | | U |
| 14 S | I | 15 D | E | | 16 N | E | T | | 17 P | L | U | S |
| P | | A | | | O | | | | E | | | |
| H | | 18 T | E | 19 N | T | 20 H | | | 21 G | I | L | 22 L |
| E | | U | | I | | O | | 23 R | | | | I |
| 24 R | O | M | A | N | | 25 U | N | I | F | 26 O | R | M |
| E | | | | T | | R | | N | | N | | I |
| 27 S | O | U | T | H | | 28 S | E | G | M | E | N | T |

**SYMKEN – 7**

Easy

| | | | | |
|---|---|---|---|---|
| 24x 2 | 4 | 15x 3 | 1 | 5 5 |
| 3 | 6x 1 | 5 | 2 2 | 8+ 4 |
| 9+ 5 | 2 | 4 4 | 3 | 1 |
| 4 | 3 | 10x 1 | 5 | 1- 2 |
| 4- 1 | 5 | 2 | 4 4 | 3 |

Moderate

| | | | | |
|---|---|---|---|---|
| 36x 3 | 3+ 1 | 32x 4 | 2 | 15x 5 |
| 1 | 2 | 50x 5 | 4 | 3 |
| 4 | 3 | 2 | 5 | 1 |
| 7+ 2 | 15x 5 | 3 | 10+ 1 | 4 |
| 5 | 4 4 | 1 1 | 3 | 2 |

**SCORES IN THREE SUBJECTS**

By thinking systematically, and creating tables like those below, where each row represents one or more students with identical sets of scores across the subjects, you can be sure that you have accounted for all possible scenarios.

With at least 4 students, we have 2 possibilities that are consistent with the first two given graphs. These correspond to the first two of the six graphs shown above, and are numbered accordingly.

**1**

| Art | Geography | Chemistry |
|---|---|---|
| 1 | 3 | 1 |
| 2 | 1 | 1 |
| 2 | 3 | 2 |
| 3 | 2 | 3 |

**2**

| Art | Geography | Chemistry |
|---|---|---|
| 1 | 3 | 2 |
| 2 | 1 | 1 |
| 2 | 3 | 1 |
| 3 | 2 | 3 |

With at least 5 students, we have 4 more possibilities that are consistent with the first two given graphs. These correspond to the last four of the six graphs shown above, and are numbered accordingly.

**3**

| Art | Geography | Chemistry |
|---|---|---|
| 1 | 3 | 1 |
| 1 | 3 | 2 |
| 2 | 1 | 1 |
| 2 | 3 | 1 |
| 3 | 2 | 3 |

**4**

| Art | Geography | Chemistry |
|---|---|---|
| 1 | 3 | 1 |
| 1 | 3 | 2 |
| 2 | 1 | 1 |
| 2 | 3 | 2 |
| 3 | 2 | 3 |

**5**

| Art | Geography | Chemistry |
|---|---|---|
| 1 | 3 | 1 |
| 2 | 1 | 1 |
| 2 | 3 | 1 |
| 2 | 3 | 2 |
| 3 | 2 | 3 |

**6**

| Art | Geography | Chemistry |
|---|---|---|
| 1 | 3 | 2 |
| 2 | 1 | 1 |
| 2 | 3 | 1 |
| 2 | 3 | 2 |
| 3 | 2 | 3 |

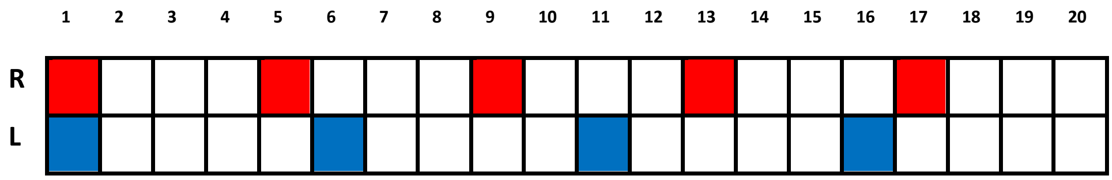
So overall there are 6 possibilities, leading to the 6 graphs shown.

**RHYTHM AND POLYRHYTHM**

**1** Phase length is 20

**2** 20 is the least common multiple of 5 and 4

**3**




**SYMdoku 2016-2017**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **2** | **0** | **1** | **6** | 4 | 3 | 7 | 5 |
| 7 | 4 | 3 | 5 | 0 | 1 | 6 | 2 |
| 0 | 1 | 2 | 7 | 5 | 4 | 3 | 6 |
| 3 | 6 | 5 | 4 | 1 | 7 | 2 | 0 |
| 6 | 3 | 0 | 2 | 7 | 5 | 4 | 1 |
| 5 | 7 | 4 | 1 | 6 | 2 | 0 | 3 |
| 1 | 2 | 7 | 0 | 3 | 6 | 5 | 4 |
| 4 | 5 | 6 | 3 | **2** | **0** | **1** | **7** |

**QUICKIE 32**

$$105263157894736842 \times 2 = 210526315789473684$$

**TREASURE HUNTS 16, 17**

TH16 This is the Garabit Viaduct, a railway arch bridge spanning the River Truyère near Ruynes-en-Margeride, Cantal, France. The bridge was constructed between 1882 and 1884 by Gustave Eiffel.

TH17 18 x 3 x 8 x 8 = 3456 plants in the red trapezium.
No entries were received for any of these treasure hunts.

# ANSWERS FROM ISSUE 63

**SYMKEN – 8**

Very easy

| | | | |
|---|---|---|---|
| 6+ 4 | 5+ 3 | 2 | 2- 1 |
| 2 | 4÷ 1 | 4 | 3 |
| 1 1 | 1- 4 | 3 | 2- 2 |
| 6x 3 | 2 | 1 1 | 4 |

Easy

| | | | |
|---|---|---|---|
| 6+ 4 | 3- 1 | 1- 2 | 3 |
| 2 | 4 | 36x 3 | 8X 1 |
| 3÷ 1 | 3 | 4 | 2 |
| 3 | 1- 2 | 1 | 4 |

Quite easy

| | | | |
|---|---|---|---|
| 3+ 2 | 2- 3 | 1 | 7+ 4 |
| 1 | 8+ 4 | 16x 2 | 3 |
| 3 | 1 | 4 | 2 |
| 2- 4 | 2 | 2- 3 | 1 |

Moderate

| | | | |
|---|---|---|---|
| 2- 1 | 3 | 2÷ 4 | 2 |
| 24x 4 | 1 1 | 5+ 2 | 3 |
| 3 | 2 | 1 1 | 3- 4 |
| 2 2 | 7+ 4 | 3 | 1 |

**TWO SPECIAL FRAMES**

Q1 The area of a trapezium is the sum of the parallel edge lengths, divided by 2, multiplied by the width.
Join corresponding points on each shape to get a set of trapezia.

To sum their areas, you need not do a separate calculation for each, you can just add all the edge lengths, divide by 2 and multiply by the width.

But you know from what you are given that the edge length total is the same for $A$ and $B$.
And you are also given that the blue area is the same for $A$ and $B$.

Therefore their widths must be the same.

Q2 Equating areas: $(k^2-1)s^2 = \frac{3\times 4}{2}(l^2-1).$ (**I**)

Equating perimeters: $4(k+1)s = (3+4+5)(l+1).$ (**II**)

Using the width information: $\frac{(k-1)s}{2} = 2.$ (**III**)

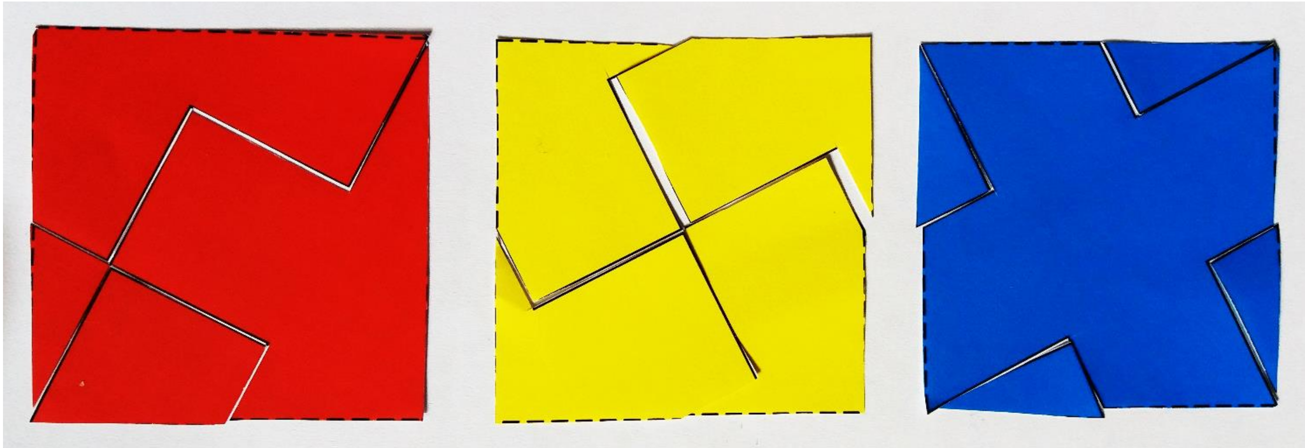
Dividing (**I**) by (**II**) and substituting for $s$ from (**III**), you find $l = 3$. This gives you the required dimensions.

If you do not believe the answer to Q1, you can check by substituting back in (**I**) to find the area, and equating to $B$'s area written as the sum of its trapezia, i.e. the perimeter total $\times\frac{w}{2}$.

**AREA AND PERIMETER PROBLEM**

Small triangle: area $\frac{1}{12}$ perimeter $\frac{5+\sqrt{13}}{6}$. Large triangle: area $\frac{1}{3}$ perimeter $\frac{5+\sqrt{13}}{3}$.

**SQUARES FROM CROSSES**



Now you can make a rectangle by putting the three squares together.

Use Pythagoras' Theorem to work out the perimeter of each piece by concentrating on the dotted lines.

**QUICKIE 33**

$105264 \times 4 = 410526$

## ANSWERS FROM ISSUE 64

**SYMKEN – 9**

Easy

| | | | |
|---|---|---|---|
| 2- 4 | 1 1 | 1- 2 | 3 |
| 2 | 1- 3 | 5+ 1 | 4 4 |
| 2- 3 | 2 | 4 | 2x 1 |
| 1 | 1- 4 | 3 | 2 |

Quite easy

| | | | |
|---|---|---|---|
| 1- 1 | 2- 3 | 10+ 4 | 2 |
| 2 | 1 | 1- 3 | 4 |
| 24x 3 | 4 | 2 | 1 1 |
| 4 4 | 2 | 2- 1 | 3 |

Moderate

| | | | |
|---|---|---|---|
| 3- 4 | 1 | 1- 2 | 2- 3 |
| 2- 2 | 4 | 3 | 1 |
| 7+ 1 | 3 | 2- 4 | 2 |
| 3 | 8x 2 | 1 | 4 |

Moderate

| | | | |
|---|---|---|---|
| 5+ 3 | 1 | 8+ 2 | 4 |
| 1 | 1- 3 | 4 | 2 |
| 32x 4 | 1- 2 | 3 | 3x 1 |
| 2 | 4 | 1 | 3 |

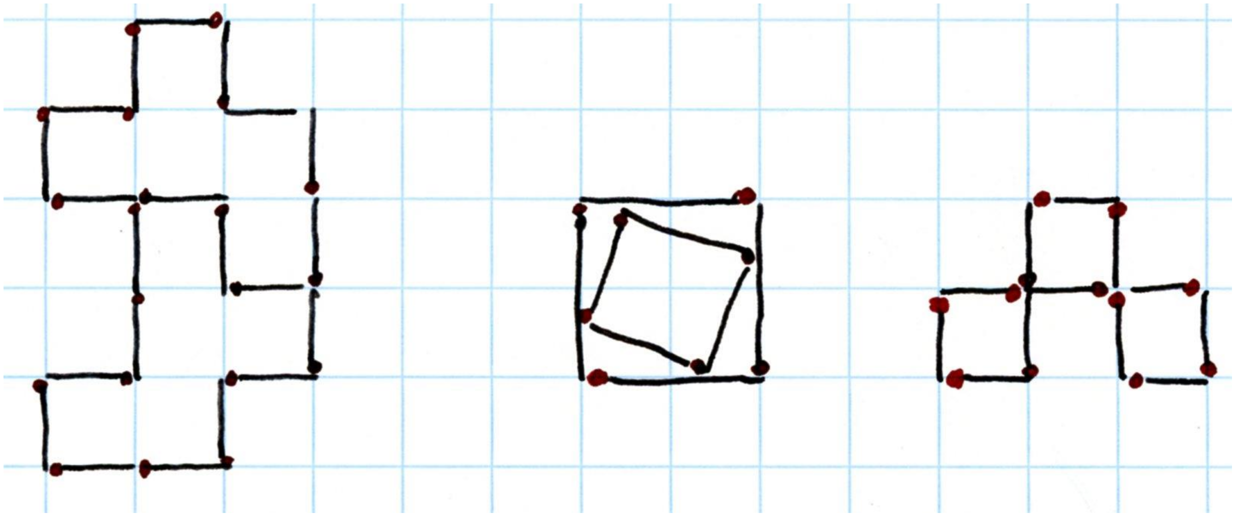
**MORE MATCHSTICK PUZZLES**

There's always one that slips through the net! Unfortunately Puzzles 2 and 3 were not meant to be the same and I missed noticing that until I came to do the answers.

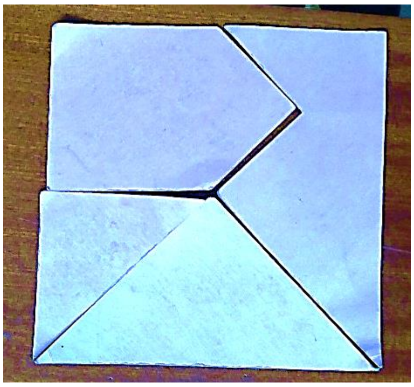
**Puzzle 1**

**Puzzles 2 and 3**

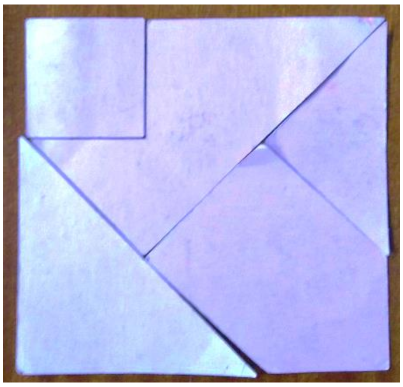
**Puzzle 4**



**A DOUBLE DISSECTION**



Area 16 squares



Area 18 squares

**MUYB R'A HRNJP-FCKG** **WHEN I'M SIXTY-FOUR**

When I get older losing my hair
Many years from now
Will you still be sending me a valentine
Birthday greetings, bottle of wine?
If I'd been out till quarter to three
Would you lock the door?
Will you still need me, will you still feed me
When I'm sixty-four?

The coding used was as follows:

| | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S | T | U | V | W | X | Y | Z |
| S | I | X | T | Y | F | O | U | R | V | W | Z | A | B | C | D | E | G | H | J | K | L | M | N | P | Q |

The alphabet is written out and then SIXTYFOUR, the codeword, is written below the first letters of the alphabet, followed by the missing letters in alphabetical order from the last letter of SIXTYFOUR.

The message is then coded, so the first letter of the message, W, becomes M etc.

**TEST YOUR DIVISION!**

$498 \div \boxed{3} = 166$ $682 \div \boxed{22} = 31$ $952 \div \boxed{7} = 136$ $2555 \div \boxed{5} = 511$ $6669 \div \boxed{19} = 351$ $56144 \div \boxed{8} = 7018$

**QUICKIE 34**

$1016949152542372881355932203389830508474576271186440677966 \times 6 =$
$6101694915254237288135593220338983050847457627118644067796$

Of course, $1016949152542372881355932203389830508474576271186440677966 \times (10^{58} + 1)$ multiplied by 6 also works. In fact, there are an infinite amount of numbers found by placing $1016949152542372881355932203389830508474576271186440677966$ in front of each of each solution you find.

# ANSWERS FROM ISSUE 65

**TWIN PRIMES**

1997 and 1999 are the 61st prime pair

**TREASURE HUNT 18**

The speed limit represents 14 kmh$^{-1}$. Its location is at Tortworth Court, Wotton Under Edge, Gloucester, GL12 8HH.

**CROSSNUMBER**

| | | | | | |
|---|---|---|---|---|---|
| 9 | 6 | 1 | | 2 | |
| 9 | | | 3 | 7 | 7 |
| 1 | 0 | 2 | 4 | | |
| | | 6 | 5 | 6 | 1 |
| 1 | 4 | 3 | | | 3 |
| | 9 | | 4 | 2 | 9 |

**TRIANGLE SEARCH**

**1** 12 triangles
**2** 15 triangles
**3** $3n + 3$ triangles
**4** 33 triangles
**5** 24 triangles
**6** 30 triangles
**7** $6n + 6$ triangles
**8** 66 triangles
**9** 40 triangles
**10** 50 triangles
**11** $10n + 10$ triangles
**12** 110 triangles
**13** 75 triangles
**14** Your own explanation
**15** $\frac{(n+1)(n+2)(m+1)}{2}$

| | | Number of internal lines from top vertex | | | | |
|---|---|---|---|---|---|---|
| Number of internal parallel lines | | 0 | 1 | 2 | 3 | 4 |
| | 0 | 1 | 3 | 6 | 10 | 15 |
| | 1 | 2 | 6 | 12 | 20 | 30 |
| | 2 | 3 | 9 | 18 | 30 | 45 |
| | 3 | 4 | 12 | 24 | 40 | 60 |
| | 4 | 5 | 15 | 30 | 50 | 75 |
| | $n$ | $n + 1$ | $3n + 3$ | $6n + 6$ | $10n + 10$ | $15n + 15$ |

**PUZZLE PAGE**

Easy

Medium

**A GOOD AGE**

Aneeta was born in 2008.

| | | | |
|---|---|---|---|
| 2 | 1 | 2 | 4 |
| 3 | 4 | 3 | 1 |
| 5 | 1 | 5 | 2 |
| 2 | 4 | 3 | 1 |
| 3 | 1 | 5 | 2 |

| | | | |
|---|---|---|---|
| 1 | 2 | 3 | 4 |
| 3 | 4 | 1 | 2 |
| 1 | 2 | 5 | 4 |
| 5 | 4 | 3 | 2 |
| 3 | 1 | 5 | 1 |

**TEST YOUR MULTIPLICATION**

There are many solutions. Here are three: $4093\times 7=28651$, $5694\times 3=17082$, $7039\times 4=28156$.

Harder

**NOTHING 2 IT** 520 zeroes if you include 2018

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 1 | 3 | 1 | 3 | 5 | 1 | 2 |
| 5 | 4 | 5 | 4 | 5 | 2 | 4 | 3 | 4 |
| 1 | 3 | 2 | 3 | 1 | 3 | 1 | 2 | 1 |
| 5 | 4 | 1 | 5 | 2 | 4 | 5 | 4 | 3 |
| 2 | 3 | 2 | 4 | 3 | 1 | 2 | 1 | 2 |

Very hard

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 4 | 2 | 4 | 1 | 4 | 1 | 4 |
| 4 | 5 | 1 | 5 | 3 | 5 | 3 | 5 | 3 |
| 1 | 2 | 3 | 2 | 1 | 2 | 1 | 2 | 1 |
| 3 | 5 | 1 | 4 | 3 | 4 | 3 | 5 | 4 |
| 1 | 2 | 3 | 2 | 5 | 1 | 2 | 1 | 2 |

**CLUES FROM VARIOUS CROSSWORDS**

**1** Rule of three **2** Quadrilateral **3** Thousand **4** Euler **5** Algebra

**6** Acute-angle **7** Turin **8** Its Greek to me **9** Archimedean **10** Threescore

**11** Tetrahedra **12** Eleven **13** Mathematician

**DAVOOD'S DICE**

4

**PYRAMID NETS**

None of them are nets of a pyramid. Net A is totally flat, some of the sides of net D that meet are of different lengths, and although you can pair up equal length sides in nets B and C, you cannot get them all to meet at a vertex.

**QUICKIE 36 – Pecking chickens**

36 – Note that for a chicken not to be pecked, the chicken on the right must peck to the right and the chicken on the left must peck to the left, so the probability a chicken is not pecked is $\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}$. So, the expected number of unpecked chickens is $\frac{1}{4}\times 144=36$. If the chickens are in a straight line, then the chicken at each end will be unpecked with probability $\frac{1}{2}$. So the expected number of unpecked chickens is $2\times\frac{1}{2}+142\times\frac{1}{4}=36\frac{1}{2}$.

Eagle-eyed readers might have noticed that this should have been Quickie 35 – oops!

**ARRAS GIANTS**

Taking the height of the human as 180 cm and mass as 70 kg, then L'ami Bidasse is about 5 metres high and would have a mass of about 1.5 tonnes. Dédé would be about 4 metres tall and have a mass of about 850 kg.

# ANSWERS FROM ISSUE 66

**MAGIC SQUARES WITH A DIFFERENCE**

| | | |
|---|---|---|
| 24 | 3 | 18 |
| 9 | 15 | 21 |
| 12 | 27 | 6 |

| | | |
|---|---|---|
| 2 | -3 | 4 |
| 3 | 1 | -1 |
| -2 | 5 | 0 |

| | | |
|---|---|---|
| 2 | 17 | 11 |
| 19 | 10 | 1 |
| 9 | 3 | 18 |

| | | |
|---|---|---|
| 7 | 2 | 3 |
| 0 | 4 | 8 |
| 5 | 6 | 1 |

**RED TRIANGLE**

The amazing result is that the area is always $\frac{1}{2}x^2$, no matter what the size of the squares are.

**NUMBER POLITENESS**

Jonathan Tolan let me know on August 2 that the definition of polite numbers should have been defined more carefully! They should be defined as numbers that are the sum of two or more consecutive positive integers. I had let that slip slightly as it had been left as being two or more consecutive whole numbers and whole numbers include 0, which is not a positive number. Some sources include 0 as a natural number, some do not so there is no agreement on the set of natural numbers.

1. 1, 2, 4, 8
2. The sequence continues 16, 32, 64, … The impolite numbers are exactly powers of 2, i.e. $2^n$.
3. 6
4. 10
5. 15
6. 5050, this is the sum of all numbers from 1 to 100.
   The sequence of numbers which are sums of consecutive integers are referred to as triangular.
   The expression for triangular numbers is $n(n + 1)/2$.
7. 18 = 5 + 6 + 7 = 3 + 4 + 5 + 6
8. There was no question here!

9. One way of calculating the politeness of a positive number is to decompose the number into its prime factors, taking the powers of all the prime factors greater than 2, adding 1 to all of the powers, multiplying the numbers thus obtained and subtracting 1.
e.g. $15 = 3^1 \times 5^1$.
(1 + 1) (1 + 1) – 1 = 3
Therefore, the number of ways of expressing 15 as the sum of two or more consecutive natural numbers is 3.
Here they are: 15 = 1 + 2 + 3 + 4 + 5 = 4 + 5 + 6 = 7 + 8.

**10.** 30 has a politeness of 3, 75 a politeness of 5, 70 a politeness of 3, 135 a politeness of 7 and 8192 is impolite as it is a power of 2.

**Polite Friends**

$$42 = 3+4+5+6+7+8+9 = 9+10+11+12$$
$$65 = 2+3+4+5+6+7+8+9+10+11 = 11+12+13+14+15$$
$$70 = 12+13+14+15+16 = 16+17+18+19$$
$$99 = 4+5+6+7+8+9+10+11+12+13+14 = 14+15+16+17+18+19$$

and continues with 105, 117, 133, 135, 154, 175, 180.

**Polite Neighbours**

$$30 = 4+5+6+7+8 = 9+10+11$$
$$42 = 9+10+11+12 = 13+14+15$$
$$75 = 3+4+5+6+7+8+9+10+11+12 = 13+14+15+16+17$$
$$90 = 16+17+18+19+20 = 21+22+23+24$$

and continues with 105, 135, 147, 165, …

Interestingly, 30, 42, 105, 135 are both polite friends and polite neighbours!

**FIVE CARD MAGIC TRICK**

The hidden cards are the Jack of Clubs, the 7 of clubs and the King of Hearts.

**PUZZLES**

1. The largest sum is 981 = 736 + 245. Wil Ransome, the editor of Mathematical Pie, informed me on June 22 that you can also have 981 = 627 + 354. Of course, there are other answers (by switching the digits in the same position (HTU), but not between positions) that give 981, but that is the largest sum. The smallest difference is 124 = 783 – 659.

2. 48 right-angled triangles. From each vertex there are three ways using two edges and three ways using an edge and a face. (Eagle-eyes will have noticed that there are two questions numbered 1 and no question 2.)

3. 48 people

4. Here are examples where $T = 20$ and where $T = 18$ (which is the minimum possible value for $T$.)

<table>
<tr><td>10</td><td>5</td><td>2</td><td>3</td></tr>
<tr><td>6</td><td colspan="2">$T = 20$</td><td>9</td></tr>
<tr><td>4</td><td>7</td><td>1</td><td>8</td></tr>
</table>

<table>
<tr><td>8</td><td>3</td><td>5</td><td>2</td></tr>
<tr><td>9</td><td colspan="2">$T = 18$</td><td>10</td></tr>
<tr><td>1</td><td>4</td><td>7</td><td>6</td></tr>
</table>

5. Andrew Palfryman informed me on July 12 that there was a mistake in the following answer.

There are 22 possibilities. If the stamps are labelled ABC on the top row and DEF on the bottom row, then if 3 stamps remain they can be ABD, ABE, BCE, BCF, DEA, DEB, EFB, EFC. If four stamps remain they can be ABCD, ABCE, ABCF, DEFA, DEFB, DEFC, ABEF, BCDE. If five stamps remain they can be ABCDE, ABCDF, ABCEF, ABDEF, ACDEF, BCDEF. This problem was posed by Brian Stokes in *The New Zealand Mathematics Magazine* of November 2008.

The question mentioned that two or more stamps were torn off and the above solution includes just one stamp torn off (which is the case if five remain). Therefore, the correct answer to the question as set is that there are just 16 possibilities.

6. $123+456+78+9=666=9+87+6+543+21$

## ANSWERS FROM ISSUE 67

**GRAECO-LATIN SQUARES**

Some solutions are given here: they are not unique.

**Part 1**

| | | | |
|---|---|---|---|
| 1 | 2 | 3 | 4 |
| 4 | 3 | 2 | 1 |
| 2 | 1 | 4 | 3 |
| 3 | 4 | 1 | 2 |

| | | | |
|---|---|---|---|
| A | B | C | D |
| C | D | A | B |
| D | C | B | A |
| B | A | D | C |

| | | | |
|---|---|---|---|
| A1 | B2 | C3 | D4 |
| C4 | D3 | A2 | B1 |
| D2 | C1 | B4 | A3 |
| B3 | A4 | D1 | C2 |

**Part 2**

| | | | | |
|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 |
| 2 | 3 | 4 | 5 | 1 |
| 3 | 4 | 5 | 1 | 2 |
| 4 | 5 | 1 | 2 | 3 |
| 5 | 1 | 2 | 3 | 4 |

| | | | | |
|---|---|---|---|---|
| A | D | B | E | C |
| B | E | C | A | D |
| C | A | D | B | E |
| D | B | E | C | A |
| E | C | A | D | B |

| | | | | |
|---|---|---|---|---|
| A1 | B4 | C2 | D5 | E3 |
| B2 | C5 | D3 | E1 | A4 |
| C3 | D1 | E4 | A2 | B5 |
| D4 | E2 | A5 | B3 | C1 |
| E5 | A3 | B1 | C4 | D2 |

**Part 3**

In the 18th century, Leonard Euler proved that it was possible to make Graeco-Latin squares of order *n* for any odd number or any number that was a multiple of 4. However, the other even numbers 2, 6, 10, 14 would not cooperate and the corresponding Graeco-Latin squares remained elusive.

He suggested that squares of the form $4n+2$ could not be done. These numbers are known troublemakers – they are the only ones that are *not* the difference of two squares.

In 1959 some counterexamples to Euler's conjecture were discovered. An example of a square of the order 10 was published in the *New York Times* and caused renewed interest.

In the same year, Bose, Shrikhande and Parker showed that Euler was on this occasion wrong - there are Graeco-Latin squares of any order greater than 2 except for $n = 6$.

**BAB'S BRICKS**

Least volume of a brick is $2.5\times3.5\times4.5=39.375\,\text{cm}^3$, greatest volume is $3.5\times4.5\times5.5=86.625$ 86.625 $\text{cm}^3$.

To find out how many she could definitely store you need to work with the maximum size brick and the minimum internal dimensions, trying to pack in the best possible way. This turns out to be as follows:

$45.5 \div 4.5 = 10.11\ldots$, $44.5 \div 5.5 = 8.09\ldots$, $35.5 \div 3.5 = 10.14\ldots$, so giving $10 \times 8 \times 10 = 800$ bricks.

To find out how many she could possibly store you need to work with the minimum size brick and the maximum internal dimensions, trying to pack in the best possible way. This turns out to be either of the following:

$45.5 \div 2.5 = 18.2\ldots$, $36.5 \div 4.5 = 8.11\ldots$, $46.5 \div 3.5 = 13.2\ldots$, so giving $18 \times 8 \times 13 = 1872$ bricks, or

$45.5 \div 3.5 = 13$, $36.5 \div 4.5 = 8.11\ldots$, $46.5 \div 2.5 = 18.6\ldots$, also giving $13 \times 8 \times 18 = 1872$ bricks.

**STOP THE CLOCK!**

If $a$ diners ate between 6pm and 7 pm, and the increase each hour was $d$ more diners, then the number of diners each hour was:

| *Time* | *Cost (£)* | *Number of diners* |
|---|---|---|
| 6 pm – 7 pm | 6 | $a$ |
| 7 pm – 8 pm | 7 | $a + d$ |
| 8 pm – 9 pm | 8 | $a + 2d$ |
| 9 pm – 10 pm | 9 | $a + 3d$ |
| 10 pm – 11 pm | 10 | $a + 4d$ |
| 11 pm – 12 am | 11 | $a + 5d$ |
| 12 am – 1 am | 12 | $a + 6d$ |

So the mean cost of a meal is $\frac{6a+7(a+d)+8(a+2d)+9(a+3d)+10(a+4d)+11(a+5d)+12(a+6d)}{a+(a+d)+(a+2d)+(a+3d)+(a+4d)+(a+5d)+(a+6d)} = \frac{63a+217d}{7a+21d}$,
and this must be equal to 9.96. A bit of algebra shows that this simplifies to $\frac{d}{a} = \frac{6}{7}$, or $a : d = 7 : 6$,
giving solutions such as $a = 7$, $d = 6$ or $a = 14$, $d = 12$, etc.

Checking, with $a = 7$ and $d = 6$, we find a total spend of £1743 with 175 diners, giving a mean of £9.96.

I received two comments about this issue, concerning the mathematics in it, so have included these here, under the appropriate titles.

**AN INCLINED PLANE IN THE OLD QUAY AT NANTES, FRANCE**

Graham Hoare quite rightly pointed out the statement in the article 'If the angle of the slope is $\alpha$, then the acceleration down the slope is $g\sin\alpha$', is questionable. He is quite correct. He goes on to say 'If the block of mass *m* slides down such a plane then the acceleration is $g(\sin\alpha - \mu\cos\alpha)$, *m* being the coefficient of friction. For a sphere, rolling without sliding, down a similar slope, the acceleration is $\frac{5}{7}g\sin\alpha$ and for a cylinder the corresponding result is $\frac{2}{3}g\sin\alpha$. In the real-life situation there are, as the note indicates, various constraints; the examples mentioned above represent ideal cases. Perhaps it would be more accurate to say that the force due to the weight down the plane is reduced by a factor $\sin\alpha$.'

**Some reflections on Symmetry-Plus No.67 Autumn 2018 – comments from Bob Burn**

**Graeco-Latin Squares**

These are the 25 points of a plane using coordinates modulo 5.

| | | | | |
|---|---|---|---|---|
| (0, 4) | (1, 4) | (2, 4) | (3, 4) | (4, 4) |
| (0, 3) | (1, 3) | (2, 3) | (3, 3) | (4, 3) |
| (0, 2) | (1, 2) | (2, 2) | (3, 2) | (4, 2) |
| (0, 1) | (1, 1) | (2, 1) | (3, 1) | (4, 1) |
| (0, 0) | (1, 0) | (2, 0) | (3, 0) | (4, 0) |

Lines of the form $y = mx + c$ and $y = mx + d$ are parallel, so if we choose a particular gradient $m$ and then enter the value of $c$ for the line $y = mx + c$ through each point, we get a Latin square, for that value of $m$. Each table below gives the "$c$" value for the line through each point, working modulo 5. The different squares below are for different values of $m$: 1,2, 3 and 4. If one square is Latin and another one is Greek, they may be superimposed to give a Graeco-Latin square.

| | | | | |
|---|---|---|---|---|
| 4 | 3 | 2 | 1 | 0 |
| 3 | 2 | 1 | 0 | 4 |
| 2 | 1 | 0 | 4 | 3 |
| 1 | 0 | 4 | 3 | 2 |
| 0 | 4 | 3 | 2 | 1 |

$m = 1$

| | | | | |
|---|---|---|---|---|
| 4 | 2 | 0 | 3 | 1 |
| 3 | 1 | 4 | 2 | 0 |
| 2 | 0 | 3 | 1 | 4 |
| 1 | 4 | 2 | 0 | 3 |
| 0 | 3 | 1 | 4 | 2 |

$m = 2$

| | | | | |
|---|---|---|---|---|
| 4 | 1 | 3 | 0 | 2 |
| 3 | 0 | 2 | 4 | 1 |
| 2 | 4 | 1 | 3 | 0 |
| 1 | 3 | 0 | 2 | 4 |
| 0 | 2 | 4 | 1 | 3 |

$m = 3$

| | | | | |
|---|---|---|---|---|
| 4 | 0 | 1 | 2 | 3 |
| 3 | 4 | 0 | 1 | 2 |
| 2 | 3 | 4 | 0 | 1 |
| 1 | 2 | 3 | 4 | 0 |
| 0 | 1 | 2 | 3 | 4 |

$m = 4$

This idea also works modulo 3 or 7 (and indeed for any prime). The four by four Graeco-Latin squares can be developed this way using the field $\{0, 1, a, a^2\}$ with $a^2 = a + 1$ (modulo 2). And for eight by eight Graeco-Latin squares using the field
$\{0, 1, b, b + 1, b^2, b^2 + 1, b^2 + b, b^2 + b + 1\}$ with $b^3 = b + 1$ (modulo 2).

The non-existence proof for six by six is really hard.

**Take your seat**

Since the seats come in rows of four (two each side) one might expect the difference between similar seats to be a multiple of 4.

**Georg Cantor**

There is mention here of both Dedekind's and Cantor's construction of the real numbers.

Each of these methods faced up to two unproved claims in Cauchy's 1821 exposition of Algebraic analysis. Dedekind worked on the proposition that an increasing sequence of rationals with an upper bound has a limit (separating the reached from the unreached rationals), while Cantor worked on the proposition that a sequence which is eventually as close as you like to its own terms has a limit. Cantor's method was refined in the 20th century using equivalence classes of such sequences to define a real number.

**Cantor's flight of imagination**

At the beginning of Cantor's paper of 1891 in which he gives the diagonal argument for uncountability, he starts with two different things $a$ and $b$. He makes lists, using just these two symbols, and asks whether a full collection of such lists, might be countable, that is whether such a list can be matched with $1, 2, 3, \ldots$ . This approach to diagonalisation is not troubled by suffices. [Rather incidentally, Archimedes did use infinitesimals in "The Method", but in his Introduction he said that these did not provide proofs.]

# ANSWERS FROM ISSUE 68

**CROSSNUMBER**

| | | | | | |
|---|---|---|---|---|---|
| 1 | 4 | 7 | | 9 | 7 |
| 2 | | 2 | 1 | 0 | |
| 4 | | 9 | | 1 | 3 |
| 8 | 1 | | 3 | | 2 |
| | 4 | 8 | 4 | | 3 |
| 2 | 4 | | 5 | 1 | 2 |

**CRACK THE CODE (CLUE: 2)**

I included this since I reckon the song has a link with probability! (and I like ABBA songs)
It uses what is known as a 'rail fence cipher', in this case with two rails.
Each line was written on two rails: e.g.

| | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| E | | C | | L | | N | | W | | S | | R | | T | | E | | O | | T | | O | | A | | L | |
| | A | | H | | I | | E | | A | | W | | I | | T | | N | | N | | W | | R | | I | | S |

Then the letters are written as one line, without spaces: ECLNWSRTEOTOALAHIEAWITNNWRIS
To decipher the message, with the clue there are two rails, split the line into two
ECLNWSRTEOTOAL AHIEAWITNNWRIS
and write as a block with two rows
ECLNWSRTEOTOAL
AHIEAWITNNWRIS
and write the letters, alternating from each row.

The decoded article is

<u>Take a chance on me</u>
If you change your mind / I'm the first in line / Honey I'm still free / Take a chance on me / If you need me / Let me know / Gonna be around / If you got no place to go / If you're feeling down / If you're all alone / When the pretty birds are flown / Gonna do my very best and it ain't no lie / If you put me to the test / If you let me try

**CIRCLE PARTS AND REGIONS**

Diameters

| Number of diameters ($d$) | Number of regions ($R$) |
|---|---|
| 1 | 3 |
| 2 | 5 |
| 3 | 7 |
| 4 | 9 |
| 5 | 11 |
| 6 | 13 |

$R = 2d + 1$

Chords

| Number of chords ($c$) | Number of regions ($R$) |
|---|---|
| 1 | 3 |
| 2 | 5 |
| 3 | 8 |
| 4 | 12 |
| 5 | 17 |
| 6 | 23 |

$R = \frac{n(n+1)}{2} + 2$

Tangents

| Number of tangents ($t$) | Number of regions ($R$) |
|---|---|
| 1 | 3 |
| 2 | 6 |
| 3 | 10 |
| 4 | 15 |
| 5 | 21 |
| 6 | 28 |

$R = \frac{(t+1)(t+2)}{2}$

Secants

| Number of secants ($s$) | Number of regions ($R$) |
|---|---|
| 1 | 4 |
| 2 | 8 |
| 3 | 13 |
| 4 | 19 |
| 5 | 26 |
| 6 | 34 |

$R = \frac{(s+2)(s+3)}{2} - 2$

**MISSING PIECE PUZZLES**

Don't assume that the diagrams are all the same side length.

If a small square in the first diagram has length $x$ units, the side of the top square is $2\sqrt{2}x = 2.828 \ldots x$ and that is less than the side of the second diagram which is $3x$.

Similarly, the second diagram of the second pair has a larger side.

**PUZZLE PAGE**

Easy

| | | | |
|---|---|---|---|
| 5 | 4 | 1 | 4 |
| 1 | 2 | 5 | 3 |
| 4 | 3 | 1 | 2 |
| 1 | 5 | 4 | 3 |
| 3 | 2 | 1 | 2 |

Medium

| | | | |
|---|---|---|---|
| 3 | 1 | 2 | 1 |
| 2 | 4 | 3 | 4 |
| 5 | 1 | 2 | 1 |
| 2 | 4 | 5 | 4 |
| 1 | 3 | 2 | 3 |

Harder

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 2 | 4 | 2 | 1 | 2 | 1 | 3 | 1 | 4 |
| 1 | 3 | 5 | 3 | 4 | 5 | 2 | 5 | 2 |
| 5 | 2 | 1 | 2 | 1 | 3 | 4 | 3 | 1 |
| 1 | 4 | 5 | 4 | 5 | 2 | 1 | 5 | 2 |
| 3 | 2 | 3 | 1 | 3 | 4 | 3 | 4 | 1 |

Very hard

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1 | 5 | 2 | 1 | 3 | 5 | 4 | 3 | 4 |
| 2 | 3 | 4 | 5 | 2 | 1 | 2 | 5 | 2 |
| 4 | 1 | 2 | 1 | 4 | 5 | 3 | 1 | 3 |
| 2 | 5 | 4 | 5 | 2 | 1 | 2 | 5 | 4 |
| 3 | 1 | 3 | 1 | 3 | 4 | 3 | 1 | 2 |

**SIMPLY SOLVE**

| | | | |
|---|---|---|---|
| Q1 231 | Q2 31 | Q3 11 | Q4 496, the third perfect number |
| Q5 30 | Q6 28 | Q7 47 | Q8 20 |
| Q9 3135 | Q10 12 | Q11 3 | Q12 2 |
| Q13 48 | Q14 1997 | Q15 £5.25 | Q16 26 birds, 17 beasts |

**STUDENT PROBLEMS**

### *Problem 1992.1*

$x$, $y$ and $z$ are positive real numbers with $xy + yz + zx = 1$. Show that

$$x + y + z \geqslant \sqrt{3}.$$

*Solution*

(a) The following approach was generally taken:

$$\begin{aligned}(x+y+z)^2 - 3 &= (x+y+z)^2 - 3(xy+yz+zx) \\ &\qquad\qquad\qquad\text{(since } xy + yz + zx = 1\text{)} \\ &= x^2 + y^2 + z^2 - (xy + yz + zx) \\ &= \tfrac{1}{2}\left[(x-y)^2 + (y-z)^2 + (z-x)^2\right] \geqslant 0.\end{aligned}$$

The required result now follows easily.

### *Problem 1992.2*

What are the last two (right-hand) digits of the number $19^{92}$ when written in the usual decimal notation?

*Solution*

The response was a little disappointing to problems 1992.2 and 1992.3. Brian Fulthorpe (Prudhoe County High School), Justin Woo (Stonyhurst College), Matthew James (Repton School) and Jan Gitowski (Stonyhurst College) correctly found the final two digits of $19^{92}$ which are 61; Jan by noting the final terms of the expansion of $(20 - 1)^{92}$, while the others worked modulo 100 along the following lines:

$$19^1 \equiv 19,\ 19^2 \equiv 61,\ 19^3 \equiv 59,\ 19^4 \equiv 21 \text{ and } 19^5 \equiv -1 \pmod{100}.$$

Thus $19^{10} = (19^5)^2 \equiv (-1)^2 = 1$ and $19^{92} = (19^{10})^9.19^2 \equiv 1^9.61 = 61$, so that $19^{92}$ ends … 61.

### *Problem 1992.3*

Give, with working, the prime factorisation of the twenty-digit number 12 318 876 808 768 112 319. You may quote in your solution the results of using a non-programmable pocket calculator.

For 1992.3, Brian Fulthorpe appeared to be the only student to have reached the correct answer, although (sadly) it seems only as a result of commendable perseverance on a pocket calculator. It should be expected that all GCSE and A level students can test (with the aid of a pocket calculator) divisibility by primes up to 100, but (realistically) not much further. If the given twenty-digit number is $N$ then, testing for divisibility by such low primes gives $N = 3^4.11.41^2.97.M$, where $M$ is the 11-digit number 84 791 820 637 – still large enough to exceed most calculators' displays. Since further direct attack is unrealistic, something more subtle might be appropriate. The £20 award was delayed in the hope of receiving further efforts for the second problem and the following hint was given:

$N$ = 12 318 876 808 768 112 319. Let $n$ = 12319. Then $N = \ldots$