Student Problems − Corrected Version

Students up to the age of 19 are invited to send solutions to either or both of the following problems to Tuya Sa, SCH.1.17, Schofield Building, Loughborough University, Loughborough, LE11 3TU. Two prizes will be awarded − a first prize of £25, and a second prize of £20 − to the senders of the most elegant solutions for either problem. It is not necessary to submit solutions to both. Solutions should arrive by 13th May 2024 and will be published in the July 2024 edition.

The Mathematical Association and the *Gazette* comply fully with the provisions of the 2018 GDPR legislation. Submissions **must** be accompanied by the SPC permission form which is available on the Mathematical Association website

https:www.m-a.org.uk/the-mathematical-gazette

Note that if permission is not given, a pupil ***may still participate and will be eligible for a prize in the same way as others****.*

Problem 2024.1 **(Nick Lord)**

For positive real numbers α, β, γ, consider the inequality

$$\alpha p^2 + \beta q^2 > \gamma r^2. \qquad (*)$$

(a) Show that, if $\frac{1}{\gamma} \geqslant \frac{1}{\alpha} + \frac{1}{\beta}$, then (*) holds whenever p, q, r are the sides of a non-degenerate triangle.

(b) Is the converse of (a) true?

Problem 2024.2 **(Himadri Lal Das)**

For a positive integer n, $d(n)$ denotes the number of digits in n. Prove that

$$\frac{20}{19} + \frac{30}{29} + \ldots + \frac{100}{99} \leqslant \sum_{n=1}^{\infty} \frac{10^{(d(n))^2 - 4d(n) + 3}}{n^{d(n) - 1}} \leqslant \frac{10}{9} + \frac{20}{19} + \ldots + \frac{90}{89}.$$