The

Mathematical Gazette

A JOURNAL OF THE MATHEMATICAL ASSOCIATION

Vol. 72 October 1988 No. 461

On examinations

The 1988 Presidential address

MARGARET RAYNER

It is easy to imagine that there has never been a time like the present when long term objectives and the immediate aims of education have been under such scrutiny and when the means of achieving these have been so controversial. But it would be extremely difficult to produce evidence to

support such a conjecture. Many of the educational issues of today were hotly debated well over a century ago; many of the solutions now proposed were put forward in the nineteenth century by fervent supporters and were attacked by an equally fervent opposition.

Our own association had its origins in a pressure group of teachers and mathematicians who were dissatisfied with the nature of the geometry taught in secondary schools and were determined to bring about reform. After the association began in 1870, as the *Association for the improvement of geometrical teaching*, the first woman to become a member in 1874 was Dorothea Beale of the Ladies College in Cheltenham. She had a life-long interest in mathematics. Her ability in the subject was noticeable when she was still a young girl; she learned geometry on her own having borrowed a Euclid; without any help she read the first six books. She also tried her hand at algebra and calculated for herself the distance to the moon. She received her first formal training in mathematics as a student at Queen's College in Harley Street and became the first woman mathematics tutor there in 1849, when she was only 18. But it was as headmistress and reformer in education for girls and in teacher training that she achieved greatest fame. One of her enterprises was the establishment in Oxford of a centre for women's higher education and, what would now be called, inservice training for schoolmistresses and it was this institution which developed into St Hilda's College of which I have been a tutor and a fellow for many years. I am very proud of this, maybe tenuous, link with the early days of the Mathematical Association. Miss Beale kept her interest in the teaching of mathematics and wrote two fascinating articles on the teaching of arithmetic and of mathematics in *Work and play in girls' schools* published in 1898. She starts her article on arithmetic by saying that no child properly taught, would be in sympathy with the lines:

> Multiplication is vexation
> Division is as bad,
> The rule of three doth puzzle me,
> And practice drives me mad.

The advice she gives on how to avoid these feelings is not inappropriate 90 years later, with some rather nice asides which indicate she has first-hand experience of teaching arithmetic. For example, in supporting the need to move from the concrete to the abstract, she suggests that pupils use actual things, for example 'stones, coloured beads—anything but marbles (which one of H.M. Inspectors recommends) or things which run about freely.' Her article includes a sketch of a lesson on the history of numbers with delightful touches of humour and clearly shows her very great skills as a teacher. She was very aware of developments in teaching in other parts of the world and took account of them in her own teaching methods. Her lessons were aimed at catching the interest and enthusiasm of her pupils and not feeding them with definitions. The picture which emerges of Dorothea Beale is of a lively, inventive, sympathetic teacher; it is very unlike the unsmiling, formidable

autocrat which has been handed down by tradition. I do not believe she really was 'so different from us' after all.

Given her own interest in mathematics, it is not surprising that she considered it an essential element in the curriculum of her school. When she arrived at Cheltenham in 1858, no science or mathematics was taught there. Despite her inclination to introduce Euclid straightaway, she says 'Had I done so, I might have seen the death of the College, so I had to wait for the tide. I began my innovations with the introduction of scientific teaching, and under the name of physical geography I was able to teach a good deal. This subject was unobjectionable, as few boys learned geography'.

But if Dorothea Beale led a reform in the curriculum in girls' schools, in one aspect of education she was eventually defeated. She was totally opposed to her pupils entering for public competitive examinations. She was not worried about examinations within the school and indeed the school was visited annually by examiners. Miss Beale was not entirely satisfied with some of the examiners—she suspected that one German examiner who taught at a local school was a waiter who came to England in the off-season. Whether it was for that reason or not, the examiners were later sought in the University of Oxford and Lewis Carroll was the first university examiner in arithmetic and mathematics in 1863. This kind of examining was totally acceptable to Miss Beale. She also believed very firmly that the introduction of compulsory examinations for teachers would raise the level of their performance in the classroom. But she disliked the idea of external examinations which, by the 1860s, were made available by the Oxford and Cambridge Locals. She believed that they would determine the curriculum within schools; Miss Beale had too many ideas about that subject to accept dictation from an external board of examiners. We are less likely to be in sympathy with her other reasons for distrusting examinations which are concerned with the difference in the social status between her own pupils and that of pupils in other schools which were, by then, taking the Oxford and Cambridge Local examinations. Eventually, Miss Beale had to succumb to the wishes of others and the pupils of Cheltenham entered for external public examinations. By the 1860s, external examinations were widely respected and sought after. I propose to spend a little time investigating the events which lead up to this stage and look at some of the qualities and skills which it was believed they encouraged or were able to assess. I shall also show the important role which mathematics played in the birth of examinations. (I am deeply indebted to Janet Howarth and Mark Curthoys for their advice and comments on the early history of examinations.)

The model for public examinations in schools was the Mathematical Tripos in the University of Cambridge, an examination which before 1824 was called the Senate House Examination. John Gascoigne in his article: *Mathematics and meritocracy* says that 'Like the British Empire, the Senate House Examination appeared to have been acquired by the university in a fit of absent-mindedness'. But even if the Tripos was initially introduced in a rather

ad hoc fashion, it grew to have such importance in the thinking of the University that the Master of Trinity—again quoting from Gascoigne—said 'Cambridge men have gone into every part of the empire as professors, teachers, and officers of various kinds of educational bodies; and they have carried with them the conviction in favour of paper examinations and their habit of conducting such examinations'. Before the introduction of the Senate House Examination, an undergraduate obtained a degree on the completion of a certain number of 'disputations' or exercises which tested his ability in oral Latin and his mastery of scholastic logic. In the early eighteenth century, these exercises had become extremely routine and undemanding. The syllabus content and the skills required to complete the exercises on longer seemed important. It is suggested that in order to fend off an unwelcome visitation from Parliament, a cause for complaint could be removed by introducing a stiffer method of testing. Within the University was a regulation which permitted any candidate for a bachelor's degree—after completing all disputations—to be questioned in the 'public schools' by any M.A. It had not been of any great significance before but now it emerged as an additional method for assessing students and, probably, for putting them into some order of seniority. The new Senate House Examination, as it was called, was going to be quite a good tool for putting candidates in order of merit. But there were difficulties. The form of this examination was quite strenuous. Candidates, in their last term, sat in the Senate House for three days, from eight in the morning till five at night. The first two days were taken up by questioning in Latin by any M.A. who wished to take part and the third day to examination in groups of six by a Moderator (i.e. an examiner paid by the University) for as long as he wished and in whatever science he chose. By the middle of the 18th century, the formal disputation or exercises had become insignificant but the Senate House Examination—henceforth I shall call it the Tripos—was the really important test. By the late 1740s, the Tripos lists were first published. Competitive examinations had arrived and by 1753, candidates were divided into Wranglers, Senior and Junior Optimes. The competition for a place high among the Wranglers began to increase. Apart from the satisfaction enjoyed by those who did very well, appointments to College Fellowships began to go to the top Wranglers, and Fellowships in Colleges could lead to high positions in the Church as well as to comfortable livings after a few years in Cambridge. It was therefore a matter of some importance to the candidates that they did well and growing pressure showed up flaws in the system. Colleges began to take an interest in the process; Colleges helped their own undergraduates by sending along M.A.'s to ask particularly difficult questions of other Colleges' candidates. By the late eighteenth century, the examination lasted four days in Lent term. Monday and Tuesday were concerned with mathematics and natural philosophy, Wednesday with questions in moral philosophy; on Thursday the examiners put up the list in which those roughly equal were bracketed together in groups of six. Any one who thought he deserved something better could challenge a candidate in a higher bracket to a

mathematical duel. The final order list went up on Friday. Up to 1826, examination questions were not always written down—indeed they were sometimes dictated. More able candidates had been given the chance to answer problems on a printed sheet since the end of the eighteenth century and the practice of giving marks for particular questions on this had probably been a convenient additional tool for putting candidates into an order.

The emphasis on mathematics in the Tripos had grown steadily until the philosophical content became relatively insignificant. By 1772, the examination began with Euclid and basic algebra, then on to mechanics, hydrostatics, astronomy and optics. For the most able student, there were questions on toughter parts of algebra and Newton's Principia. The examination in philosophy was very light and a superficial knowledge was adequate. In fact, the questions on mathematics became tougher as the competitive character of the examination grew and ranking in the final list depended on success in answering them.

By the early 19th century, Cambridge had perfected a system for putting candidates in order. Competition for the highest places was extremely important in the minds of undergraduates. In 1832, Professor Baden-Powell in Oxford, where no competitive examination of the Tripos kind existed, bemoaned the fact that Oxford had no comparable stimulus and he attributed the high standing of Cambridge in the mathematical world to the good effect of the Tripos. (As a comment, G. H. Hardy in Oxford in 1930 said 'In any case, we need not distress ourselves about Cambridge and its Tripos. If we cannot rival those faded glories, so much the better; it is not possible to found a school of learning on an examination'.) Employers of Cambridge graduates were accepting the assessment of young men by the Tripos as a guide to their potential in areas remote from mathematics or philosophy; to be a Wrangler was to be the best—whatever that might mean. But not everyone in Cambridge seemed to find the system so reliable or sensible, as this piece of doggerel shows. (The Vulture is the tutor turned Examiner.)

> The papers they had finished lay
> In piles of blue and white,
> They answered everything they could
> And wrote with all their might,
> But, though they wrote it all by rote,
> They could not write it right.
>
> 'O Undergraduates, come up'
> The vulture did beseech
> 'And let us see if you can learn
> As well as we can teach.'

About sixty years after the introduction of the Senate House Examination in Cambridge, the University of Oxford decided to change the conditions it required to be satisfied for the award of a degree; up to 1800 candidates were

interviewed—rather like candidates for a doctorate today—sometimes by just one examiner. There was no public scrutiny of the proceedings. Hence, these vivas became rather desultory affairs and it is very likely that considerations other than the strictly academic were important. This was thought to be an inadequate procedure for giving degrees to young men—most of whom would become clergymen. So instead, a public examination was instituted; members of the university could go and sit in on the vivas (this is still true for doctoral vivas) but, in fact, undergraduates were obliged to witness a viva for some other undergraduate before their turn came. Without a certificate from the Proctor to show a previous attendance, an undergraduate could not enter for the examination himself. Two reasons for this practice have been suggested: firstly it ensured that the examination was conducted correctly and, secondly, it gave undergraduates a clear idea of what would be expected of them. Oxford decided to put candidates into classes in the published lists; it was believed that this would provide motivation for serious study. It is not clear where the inspiration for this came from; Sir William Hamilton cited a similar arrangement operating at Louvain. In fact Oxford intentionally did not adopt Cambridge's system of drawing up a strict order of merit—on the grounds that this would encourage undesirable pride in its graduates—particularly unfitting perhaps amongst those intending to become clerics. At Oxford, the examinations perpetuated the traditional emphasis upon the classical curriculum and this did not lend itself so readily to a numerical scheme of marking. Numerical marking was all right for mathematics at Cambridge, it was said at Oxford, but this was not well suited to other subjects.

By the middle of the nineteenth century, Oxford and Cambridge had developed a system of public classified examinations which was widely considered to be enormously successful; despite the fact that these examinations were designed on a local scale and conducted in two small communities, the examination system was taken as a suitable model for examinations on a quite different scale. The first examinations to be taken at a number of different places on a general syllabus were the tests used in teacher training colleges established by the Church but increasingly supported by state finance. The results of the tests were used as a basis for the allocation of grants to the colleges' students; it is more than likely that the tests therefore had the effect of encouraging industry amongst the students and that the contents of the general syllabus influenced the programmes of work developed in the colleges. With rather a different objective in mind, the College of Preceptors was formed by a group of schoolmasters to provide a professional standard of qualifications for teachers, to be administered by the teachers themselves. It was expected that these qualifications would raise the reputation of schoolmasters, particularly those in private schools. As a group, these teachers were badly paid and did not enjoy high social esteem. Part of the armoury which the College of Preceptors used from 1850 was an examination; certificates were issued to those men, either already teaching or wishing to teach, who were successful in the examination. A successful

performance in an examination was seen as a means to the recognition of teachers' professional claims. Unfortunately the proposal did not catch on and in the early 1850s, the College was running examinations for school boys instead. Even this remained small scale, probably because the College itself lacked the prestige necessary to make its qualifications sufficiently attractive. On a much larger scale, but for a rather different purpose, in 1853, an entrance examination became the only way into the Indian Civil Service; it was introduced as a means of removing the effects of patronage in the selection of a candidate and in the hope of improving the quality of those in the Service. Although based on the pattern of University examinations at Oxford and Cambridge, it differed in a marked way in the number of subject areas it covered. Oxford and Cambridge concentrated on a very narrow area, studied in depth; the Indian Civil Service examination went for spread. Examinations were also introduced as a preliminary to the purchase of a commission in the army; this again reflected a dissatisfaction with promotion by patronage and a move to find 'the best man'.

The development of external examinations in secondary schools grew out of a deep concern in the early nineteenth century about the standard of education available to children of middle class parents, such as farmers and prosperous tradesmen. The aristocracy, lawyers and many clergy (amongst others) were sending their sons to public schools; these schools provided an education in keeping with the demands of the society they supported and gave ready access to Oxford and Cambridge and thence to the higher ranks of the army, the church and the state. The public schools were growing in size and number in order to meet an ever-increasing demand for places. The fees and the incidental expenses for a boy at a public school were well beyond the means of most families in the middle class. They had traditionally sent their sons to local grammar schools or to private schools; grammar schools had been the earlier route to the Universities but by the end of the eighteenth century, a great many had dwindled in size and in educational standards. Many private schools had inadequately trained staff and not too much interest in education. Education of middle-class boys was largely in the hands of teachers who had no external standards to guide them, no external reward to expect for excellence and were, very often, at the mercy of local opinion. But the need for an improved standard of education for middle-class boys was obvious if they were to be better farmers than their predecessors or if they were to take a route which would lead them to the positions which were opening up as a consequence of the introduction of examinations into selection processes. It was argued by Thomas Arnold (the headmaster of Rugby) that only a national system of education, through government action, could provide the middle class with the opportunities available to the upper, richer, classes through public schools and the universities. But the thought of government interference did not find favour either with the independently-minded middle class or the headmasters of private schools. Moves to establish new and better schools were left to individuals or to groups of individuals, churches etc, the

schools generally depending on the fees of parents. And these fees had to be kept down if the boys were to come to the new schools. For a variety of reasons, of which the financial one was an important factor and the social standing of teachers was another, this movement to change existing schools and to found new ones was not a great success in solving the problem and, by the late 1840s, the idea of public examinations in secondary schools emerged, as a means of improving the standards of education.

At this point, I shall quote from an admirable book by John Roach called *Public examinations in England 1850–1900* on which I have depended very heavily in writing about the early history of examinations. Dr Roach writes of James Booth who in 1847 expressed the opinion that it was 'the duty of the state to promote education if necessary and this should be done by the enactment of the rule that, after a certain date, no-one should be eligible for office under the Crown or for any public appointment who had not either taken a degree or been through one of the military colleges or obtained a certificate from the Government Board of Examiners. Such a certificate should not itself entitle a man to such a post but should place him in the cadre from which all holders of such posts were to be drawn. The country would be divided into educational districts each with its board of examiners holding annual examinations and awarding certificates of three different grades.' The boards of examiners would devise the syllabus and so influence what was taught in the secondary schools; teachers would have guidance and motivation for improving standards. The certificate would create, quickly and inexpensively, a uniform system of education for middle class children and the award of a certificate would encourage parents to let their sons stay longer at school. It was believed that examinations elsewhere had already shown that they encouraged the habit of application and industry in young men. Even the less able boy had something to aim for.

Although a great deal of James Booth's argument was accepted by reformers and others in the educational world, the idea of a government system of examinations was not, and an approach based on examination by voluntary, independent bodies was much more warmly received.

As I have remarked earlier, the College of Preceptors was already in the field of examining and had moved into the area of examining pupils in the early 1850s but like a number of other bodies which set up examinations it did not carry sufficient national prestige to take them beyond a locality or a particular area of education. But one particular venture succeeded where others floundered, and this is where the Universities of Oxford and Cambridge again come into the picture.

As a consequence of discussion between Joseph Lloyd Brereton, who held the living at West Buckland in Devon, and a local landowner, Viscount Ebrington, a scheme was devised to improve the education of farmers' sons in Devon. Both these men were already much concerned with educational and social reform in the country as a whole and Viscount Ebrington in particular was in close contact with other educational reformers of his time. The original

draft of their scheme was much changed in detail by subsequent events but the main proposal survived unchanged. In 1855, Viscount Ebrington announced he would give a prize of £20 for the best performance in an examination by the son or relative of a Devon farmer, aged between eighteen and twenty-three. The examination would be in the subjects of English language, the history and geography of the British Empire, and practical mathematics. The examination was held in Exeter in Easter week in 1856 but very few candidates presented themselves. But in the same year, another Devon landowner, T. D. Acland, with a very great interest in the improvement of educational standards for middle-class children became enthusiastically involved, called together a committee by January 1857 and revised the original scheme so that the examination was now for boys rather than young men and, at the same time, the syllabus was changed.

It was now to be a system of examinations and prizes for boys being educated for employment in agriculture, arts, manufacture and commerce, rather than just for farmers' sons. The age limits were changed and the candidates were put into two divisions depending on whether they were less than fifteen years old or less than eighteen years old. A large range of subjects was on offer: apart from religious knowledge, English, Latin, modern languages, geography, history, mathematics (including arithmetic, algebra, trigonometry and general principles of natural philosophy), there was practical science and art, mechanics, chemistry, physiology, engineering, surveying, book-keeping, architecture, drawing, music. Every candidate had to take the paper in religious knowledge or get a certificate from a minister of religion to put in its place. Prizes were offered for good performance in the examination. All candidates had to sit a preliminary examination in writing and arithmetic and those who failed this, failed the whole examination. Although the candidates were classified in two divisions, they all took the same subject papers which contained questions of varying level of difficulty. Before the examination, schools were consulted about their use of text-books and material and efforts were made to find practical applications appropriate to the needs of business. The examination started on 16 June 1857 in two rooms in the Clarence Hotel in Exeter. There were 160 candidates; they were permitted to bring their parents and other well-wishers to the examination room in order to remove 'all air of mystery and to satisfy public interest'. Of the 160 who entered, only 38 passed, a great many falling at the first hurdle of the preliminary examination. The report on the examination picked out mathematics as the best taught subject but stated that the boys generally had more idea how to work calculations than how to apply their knowledge to solving problems. How little things change!

T. D. Acland perceived that in order to launch the new examination successfully, backing was needed from the Universities of Oxford or Cambridge, or preferably both. Roach suggests that it was quite a good time in 1857 to put such a proposal to these Universities when they were sensitive of their wealth and privileges and the suspicion of some that they did not do

enough to justify them. He also points out that Acland was himself a distinguished Oxford man and that he was in touch with many in Oxford who had instigated a lot of the ideas of educational reform which were now taking shape. Acland moved very quickly after the formation of his committee in January 1857 and wrote to Frederick Temple, one of Her Majesty's Inspectors of Schools, who in his turn enlisted the support of the Master of Balliol. Temple suggested that the University should be invited to appoint a Board of Examiners who would, themselves, examine all the candidates presented to them. The examination would be divided into groups to cover the subjects most needed by boys in middle-class schools and each candidate would be required to pass the preliminary examination and in one group of subjects. The University would give the title of *Alumnus in artibus* (A.A.) or *Scholaris in artibus* (S.A.) to all boys who passed the examination. The University was assured that it would not have to put money into the scheme, the cost of which would be covered by the fees charged to candidates. The examination would be held once a year in Oxford and in the country 'when the local gentry chose to make arrangements for that purpose'. Temple suggested that to put 'S.A. of the University of Oxford' after one's name would be a right eagerly coveted. Here was the prestige which would make the examination a success and so bring about the changes in schools which its initiators wished.

The proposal was made formally to the University early in 1857; although there was some concern about the suggestions concerning the examination of religious knowledge, a real storm broke out about the suggestion that the University should allow successful candidates to use the letters A.A. or S.A. after their names. It was suggested by members of the University that the letters would be read by many to mean a degree of the University. Others thought that A.A. sounded like the first step of a ladder consisting of A.A., B.A., M.A. and that this was totally misleading in a situation when A.A. was give to non-members of the University but B.A. and M.A. was restricted to those who had followed a course at Oxford. While Oxford was getting very excited about the name of the award, schools outside Devonshire and, in particular in cities, were getting very excited about the examination itself and a great deal of support was sent to Oxford by headmasters who wished to enter their pupils. The final decision in Oxford to adopt the examination was taken just two days after the examination itself started in Exeter; the University accepted the scheme of examinations by a large majority and even accepted the title of A.A., but with considerably less enthusiasm. The University laid its responsibility for the new examination upon the Delegacy for Local Examinations, local here meaning local to the candidate and not local to Oxford. At the same time as Oxford was taking the important and far-reaching decision to link itself with the education system in secondary middle-class schools, Cambridge was considering the same invitation to run local examinations. Again, the main issue was the name given to the award. A classicist—who sounds as if he should have been a college bursar—enquired what the great difference was between A.A. and B.A. that parents should pay

an extra £1000 for their sons to gain the latter. Was it not possible that boys would gain an A.A. from Cambridge and go straight into the army (for example) without ever taking a degree? When the University made its decision in February 1858 to establish a syndicate for local examinations for schools, it did not agree that successful candidates would have the right to an A.A. In one other respect, that of examining religious knowledge, the two Universities took different paths and any idea that the Universities might act together on this examination was then out of the question.

I end this section by gain quoting from Roach: 'Indeed the whole development of secondary education in England during the past century has been deeply influenced by the decisions made during these few days in Oxford. The idea of examination was in the air and almost bound to be applied to secondary education. It was the only scheme of improvement which avoided the bogies of state interference, of religious controversy, of local dissensions between parents and teachers, of the crushing expense of new schools. Of all the ideas for reform in secondary education which had been discussed it alone appeared to make the best use of the resources which were available and, according to the philosophy of the time, to provide people with the means to help themselves'.

The subsequent history of public examinations is fascinating: whether it is a success story and the nature of its legacy will hang over many of our discussions in the next few days. Perhaps legacy was the wrong word to use in the context of public examinations for they are very far from dead. They continue to show a remarkable resilience and they still tempt reformers to use them in situations and for purposes for which they were never designed and for which there is very little evidence that they will succeed.

When the older universities accepted the invitation to assume a responsibility in the continuous process of education, they took a most important step on the way to ensure that the education of young people is not cut up into disjoint sections. I cannot pretend that the situation is now perfect and I am very aware of the difficulties which are experienced by young people when they enter higher education after A-levels. It is quite essential that higher education builds on whatever pattern of A-levels emerges from current discussions; to allow a gap to develop between school and higher education or to treat first year studies in higher education as 'remedial' are both totally unacceptable. On the other hand, the needs of young people going into higher education are as legitimate as those of young people going into immediate employment and it is essential that ideas which enter higher education courses from new developments and research should be sensitively and appropriately adopted into A-level courses.

The Mathematical Association has a most important responsibility in the task of maintaining a unified view of mathematics. With a membership which includes teachers of pupils of all ages, from all types of school and institutions, the Association is particularly well placed to develop that unified view of mathematics and to represent it to others. The Association has been greatly

strengthened by the increase in the number of teachers from primary schools in its membership. By strength I mean not only actual membership number and the financial security this brings but also experience and wisdom. The need for a unified view of our subject has been made abundantly clear by the new proposals for a National Curriculum where every stage of education is slotted in with every other stage. It is with the greatest confidence that I shall hand over my office to Professor Howson as the next President, knowing that the reputation of the Association has never been higher and that its advice has never been so carefully listened to.

MARGARET RAYNER

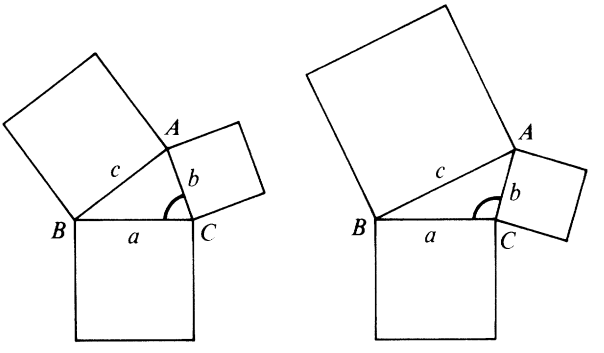
St Hilda's College, Oxford OX4 1DY

Pythagoras extended

A geometric approach to the cosine rule

NEIL BIBBY AND DOUG FRENCH

The motivation for this article was the desire to make the cosine rule in some way geometrically "obvious". What we have ended up with is a mixture of inductive and deductive approaches which we hope goes some way to illuminating the cosine rule, and to enhancing students' relational understanding of it. The germ of these ideas can be sown soon after students have met Pythagoras, with the question "O.K., suppose the triangle ABC is not right-angled: what can we say about a^2, b^2 and c^2 now?" The association of acuteness with the case $c^2 < a^2 + b^2$ and obtuseness with the case $c^2 > a^2 + b^2$ should soon follow:



This conjures up some nice geometric imagery already; the Pythagorean case is seen to be special in that as C increases, it corresponds to a critical