MAthematical PiE

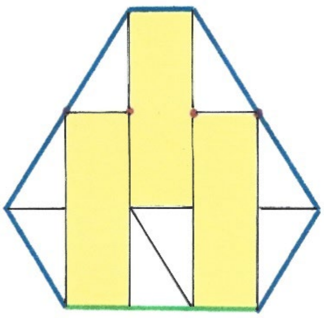
Summer 2022 **Notes for . . . . No 216**

The Smallest Swap Puzzle

Change all the 7s and 8s to zero, as well as the 5 and 6 in the centre column. That's 8 cards in all.

By changing 5 cards to zero it is possible to get a different repetitive total ***xxxx.*** Which five cards, and what is ***x***? Suppose the cards are labelled with 4,5,6,7 instead of 5,6,7,8. Find two ways of getting 1111. Are there any repetitive totals for 6,7,8,9?

On a Higher Level



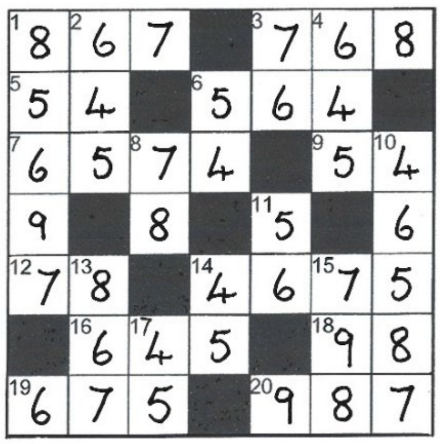
The white area consists of 8 congruent right-angled triangles and each triangle is equivalent to one quarter of a rectangle. So the hexagon's area is the same as five rectangles and then each rectangle must have an area of 12 cm^2. The given base of 6 cm tells us that the width of each rectangle is 2 cm, so each has a height of 6 cm. Finally, the hexagon's height is 1½ times that of a rectangle, and is therefore 9 cm.

Limerick

A dozen, a gross, and a score, plus three times the square root of four,
divided by seven, plus five times eleven, is nine squared (and nothing else more).

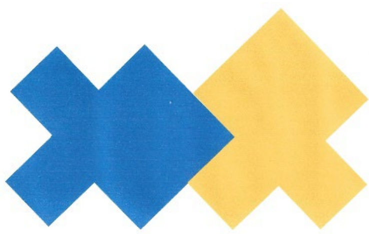
Consecucross

Two suggestions as to method: it is useful to list all the digits for each clue and strike them out as they are used, and the two-digit answers are the best ones to start with.

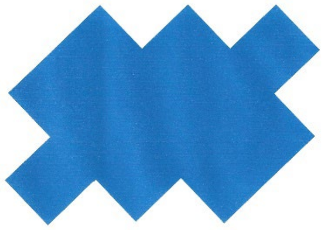


Congruent Halves

I think that only one solution exists:



. . . but you should be able to use symmetry to find an infinite number of solutions to the problem of dividing this into two congruent halves:



The Dartboard

17, 16, 25, 39, 24 gives 17 + 32 + 75 + 156 + 120 = 400

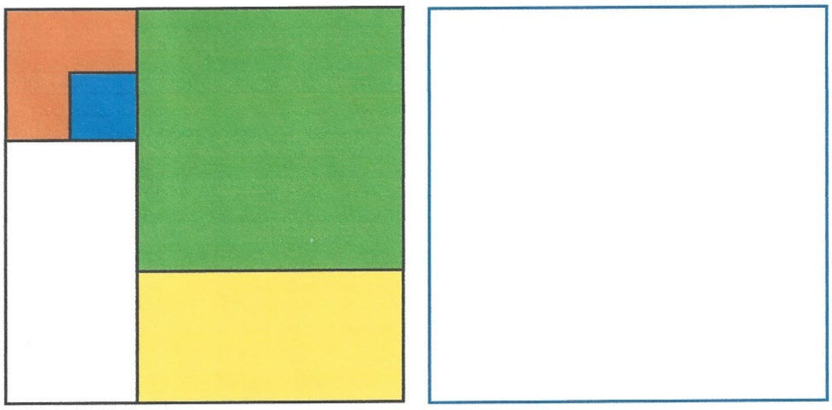
How quickly can you show that a total of 300 is not possible?

Square Integers that are Digitally Square

The highest possible digit sum for a three-digit number is 27. Since we can only use the digits 0,1,4 and 9 it is impossible to get a sum of 25 or 16, but we can get 1,4 or 9. So the SDS's are 100, 144, 400, 441 and 900.

Covering a Square

The colours would be seen as here -



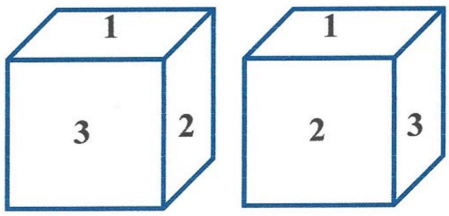
but what would the pattern be if the five squares were stuck together and then turned over (left to right?)

A Hexagon and Some Triangles

Here are two approaches: a) Cover the whole hexagon with half-size triangles. The coloured part makes up 24 of these while the complete hexagon is 54.
b) Slide the top 3 triangles to the right to form a coloured hexagon. Its width is two-thirds that of the large hexagon. The area fraction is four ninths.

Dice Markings

There are only two possibilities, usually referred to as 'clockwise' and 'anticlockwise':

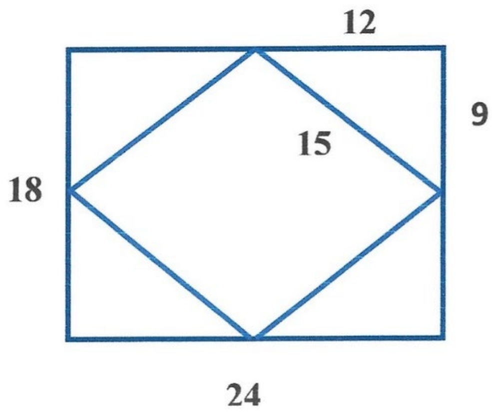


From *Puzzle Papers in Arithmetic* (adapted)

Simple 45° cuts at the top corners will do it, giving three trapeziums each with an area of 64 cm^2 (Since the sum of the parallel sides is the same in each case).

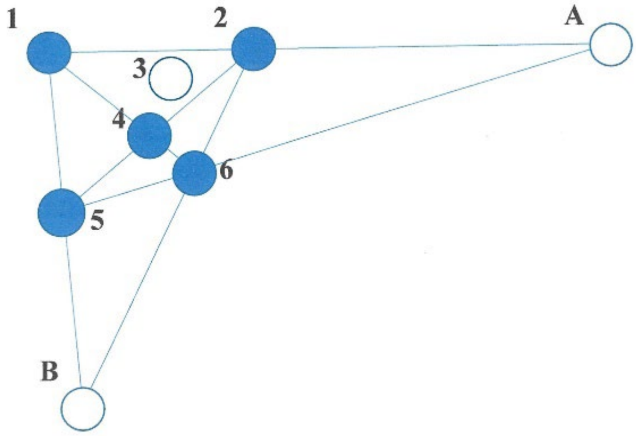
A Product $\quad 3 \times \frac{7}{3} \times \frac{13}{7} \times \frac{21}{13} \times \ldots \times \frac{421}{381} = 421$

Rhombus Revision



The clue is in the 'convenience' of 216. Why should it be so described? It is close to 15^2:
$216 = 15^2 - 9 = 15^2 - 3^2 = (15 + 3)(15 - 3) = 18 \times 12$.
So, it follows that a rhombus of this area could be formed by joining the mid-points of the sides of a rectangle of area 18 x 12 x 2. Although there are many such rectangles, fortunately the tempting 18 x 24 results in the corner triangles being three times a (3,4,5) triangle, giving the required edges of 15 for the rhombus.

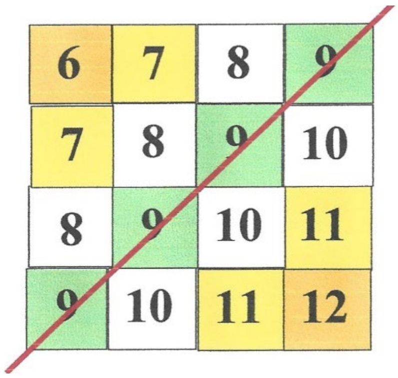
Moving a Circle



The circle labelled 3 could be moved to either of the points A or B shown here:

Add the Numbers

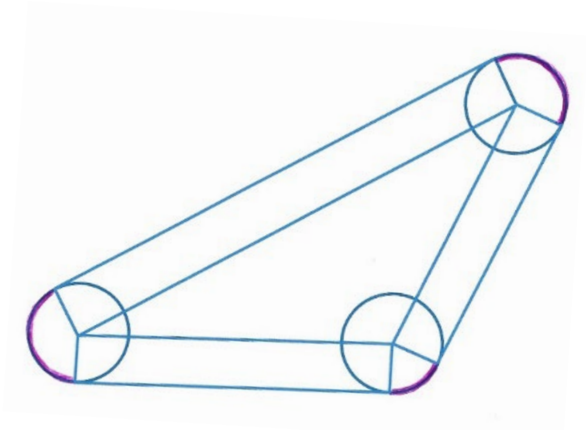
There are, of course, many methods available. I envisage this as a collection of unit cubes, with the number of cubes in each tower shown by the number given in that cell. My first thought was to use triangular numbers, since the first row is t(9) = 45, the second row t(9) + 9, the third t(9) + 9 + 9 and so on, giving a final total of
$9 \times t(9) + t(8) \times 9 = 9[t(9) + t(8)] = 9 \times 9^2 = 729$.



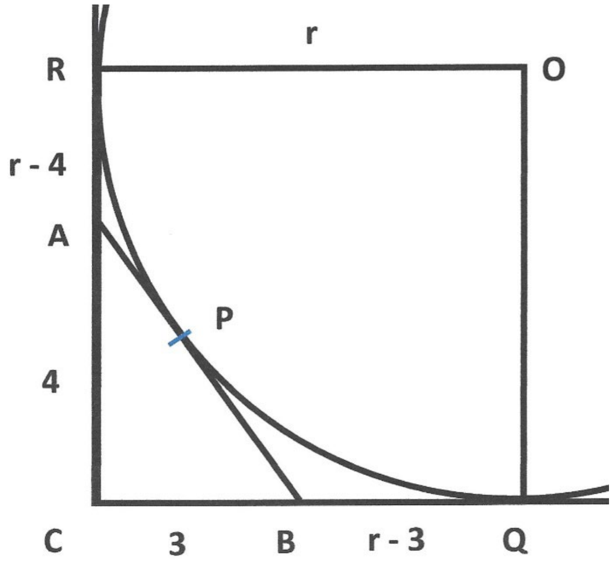
I think that the best insight is to realise that any pair of numbers that are exactly opposite each other – 'mirror images' in the line of reflection defined by the diagonal of nines – can be averaged and replaced by two nines. Which makes the total an obvious 81 x 9. Chopping off the top of my tower block and turning it over to insert it into the gap produces a cube.

The Belt

The portions of the belt in contact with the pulleys are equivalent to the circumference of one pulley so the length of the belt is
$3 + 5 + 6 + \pi = 14 + \pi$



A 3, 4, 5 Problem



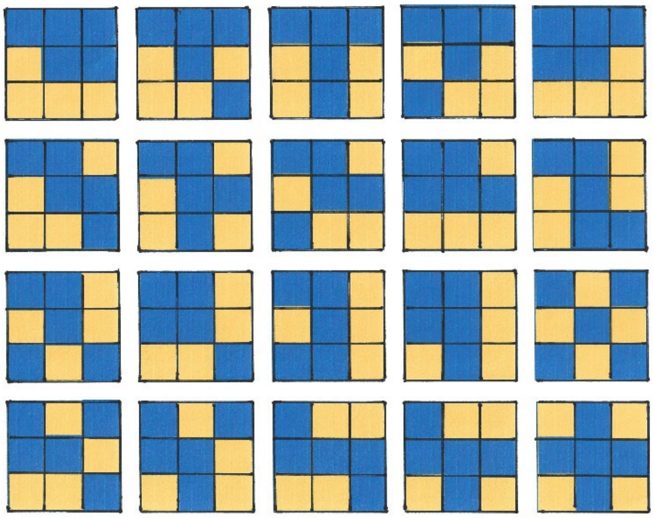
Things become clearer if we draw in the centre **O** and the radii **OR** and **OQ.** It is easy to show that **ROQC** is a square and **CR = CQ** (= **r**). Then we have **RA** = **r** – 4 and **BQ = r** – 3. Since they are tangents to the circle, **RA = AP** and **BQ** = **BP**. Then we have the easy equivalence

AP + PB = **AB,** giving

(**r** – 4) + (**r** – 3) = 5, and so **r** = 6 cm.

The Rubik's Cube Puzzle

There are 20 patterns: here they are in a logical order:



135

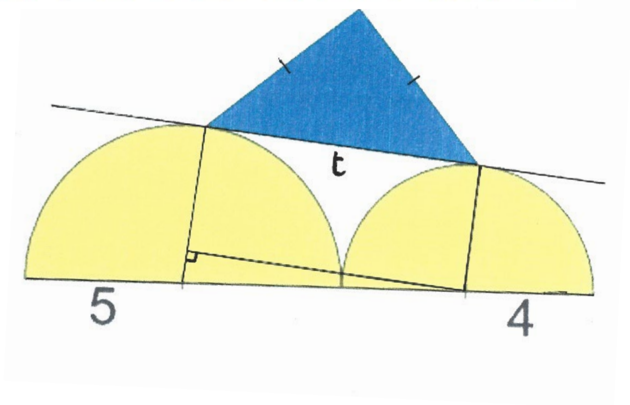
144 fits the first property:

(1 + 4 + 4) x (1 x 4 x 4)= 9 x 16 = 144.

175 fits the second:

$1^1 + 7^2 + 5^3 = 1 + 49 + 125 = 175$.

Balanced



Drawing the radii to the points of contact and an extra line as shown, we can use the right-angled triangle on the base line to calculate $\mathbf{t}^2 = (5 + 4)^2 – (5 – 4)^2 = 80$. The area of the given triangle could be thought of as half of a square with diagonal **t**. The area of any square is half of the product of its diagonals. So the area of the triangle is ½ x ½ x t^2, or one quarter of 80, i.e., 20 cm^2.