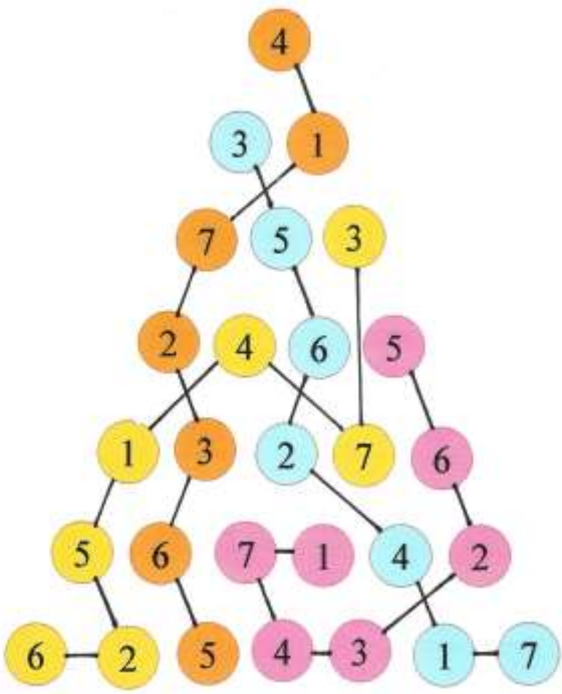
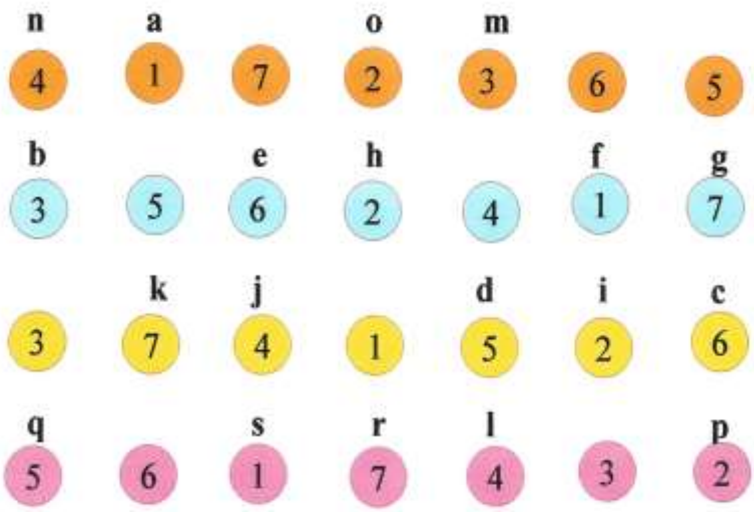
MAThematical PiE

Autumn 2021 **Notes for . . . . No 214**

Garlands on the Christmas Tree

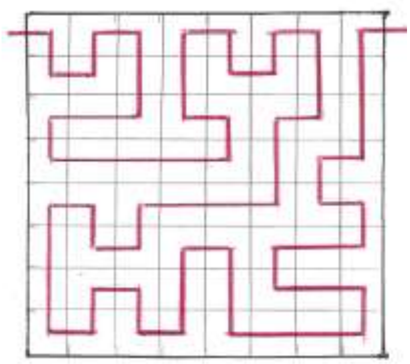
Scrambled Sudoku! Here is the tree with the lights on. I found it helpful to keep a record of the 'untangled' garlands as shown below. The letters **a, b, c,** etc. indicate the order in which I happened to find them, although there must be many paths to the final solution. The early ones are usually the only place on that garland that a particular digit can go.





Minimaze

The route through the square is shown here.



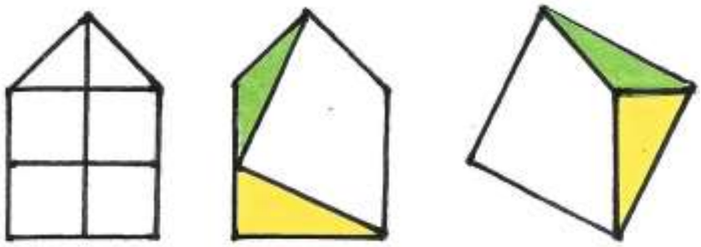
The correct order of the words is **G**radient, **A**rithmetic, **L**ongitude, **I**ntersect, **L**atitude, **E**quation, **O**ctagon, spelling the name **GALILEO.**

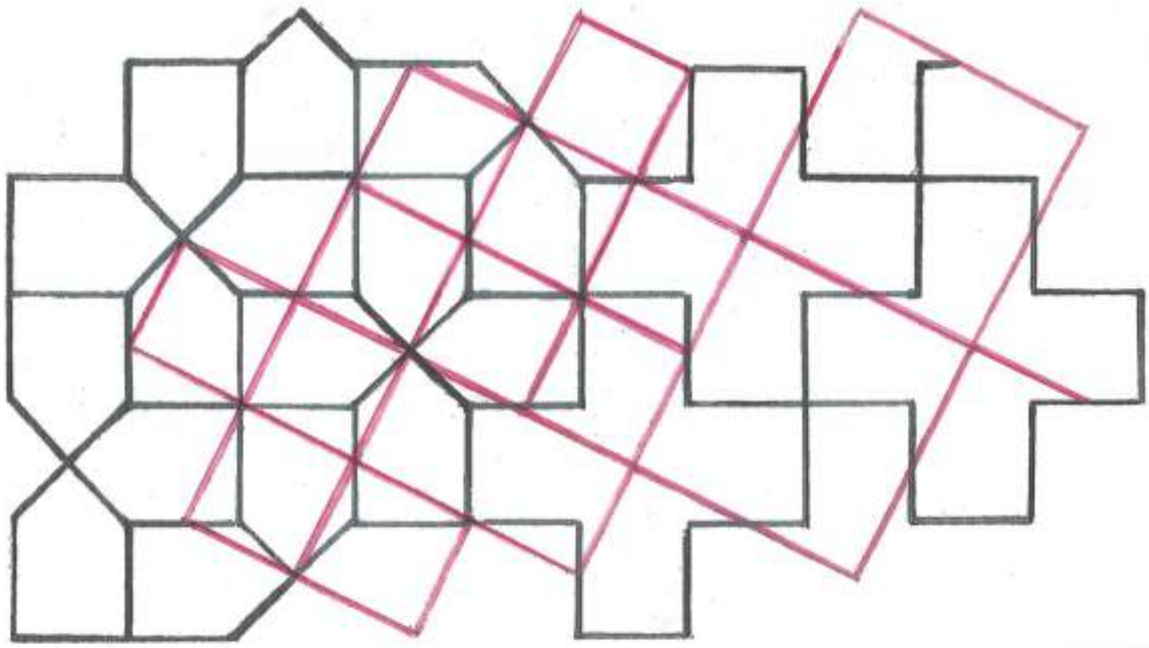
Two Lists

Factors are the key here. It turns out that list B consists of prime numbers and their powers and list A contains all the remaining (composite) integers. Put another way, the numbers in list B have only one prime among their factors while those in list A contain more than one prime factor. Since 729 is 3^6 it belongs in list B.

The Square Tabletop

There are several reasons why I particularly like dissection puzzles. The first is that dissecting a shape to rearrange the pieces into a square will inevitably require the application of Pythagoras' Theorem. For example in this case the pentagonal shape given can be regarded as having an area of five 'unit' squares, which indicates that the cuts that help to form the final square must go in directions whose vectors involve 1s and 2s since $1^2 + 2^2 = 5$. It is easy to demonstrate the change in a 'static' way as shown on the right - but I prefer to use actual cutting and physically moving the pieces which emphasises the transformations involved. In this case superimposing the two coloured diagrams above shows that the moves are 90° rotations. This leads to a second reason why such things are favourites of mine: the link with *tessellations.* Rotating the given pentagon produces the tessellation shown below. Drawing in the 'cuts' indicates an overlay of a square tessellation. Incidentally, this also uncovers the link with the more usual puzzle of dissecting a Greek Cross into four congruent pieces that will form a square.



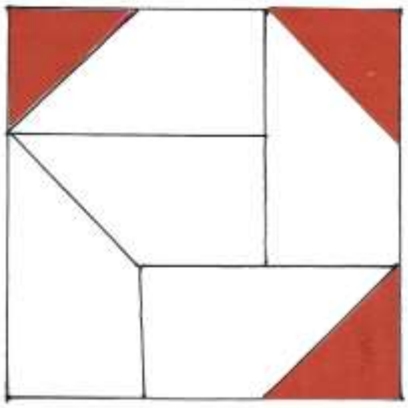


Primes in Time?

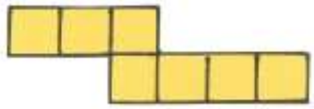
Of course we can rule out the even years and the one ending in 5. Digit sums also rule out 1617 and 1011 as multiples of 3. Of those remaining, we find 1819 is 17 x 107 and 2021 (if we do consider it) is 43 x 47. The only prime is 1213.

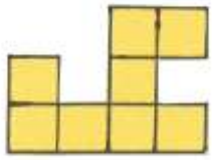
Quintessential

The heptagon has an area of 7.5 units and so one fifth has to be 1.5 units. The trapezia in the solution are examples of simple 'Rep-Tiles'. Such tiles are worth checking out on the internet or in chapter 19 of the fourth book in 'The New Martin Gardner Mathematical Library' (C.U.P)



The Square Perimeter Puzzle Here are two easy little subsidiary questions: (a) Configuration (a) has a perimeter of 16cm. How many squares can you add without increasing its perimeter? Configuration (b) has a perimeter of of 18cm. Can you add two squares that *reduce* its perimeter to14cm?
(b)





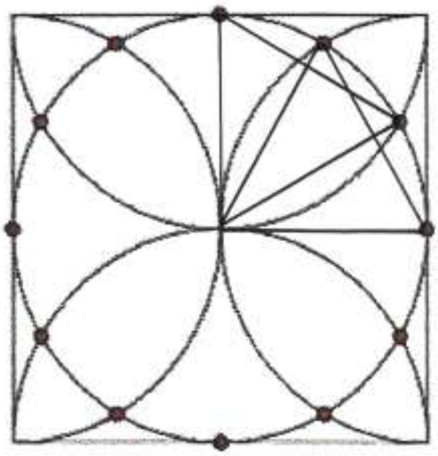
The maximum perimeter with 9 squares is 20cm. The most that any new square can add to the perimeter of an existing configuration is 2cm. (Four new edges added but two adjoining). So for ***n*** squares the maximum perimeter ***m*** is given by $m = 2n + 2$. Examples of configurations that achieve this are straight lines, crosses, or combinations of them such as (a) and (b)

Some More Light Algebra

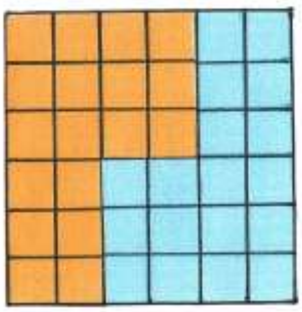
Or, of course, we could think about $a^2 = (a + b)(a - b) + b^2$

Jack and Jill Revisited

As in issue No. 212, Jill's right because the underlying squares and equilateral triangles result in angles of 60° and 30°.



Making the Cut



Four Dice

It is well-known that the opposite faces of a die add up to 7, so (listing the faces in the order front, back, left, right, top, bottom) the die on the top right is (5, 2, 1, 6, 4, 3). The 'touching faces' rule then fixes the two dice that touch it as (4, 3, 1, 6, 2, 5) and (1, 6, 2, 5, 4, 3). Then, working purely numerically, there are four possibilities for the final die: (6, 1, 2, 5, 3, 4), (6, 1, 2, 5, 4, 3), (6, 1, 3, 4, 2, 5) and (6, 1, 4, 3, 2, 5). However only the first and fourth of these are **identical** to the given dice. It's often forgotten that dice can exist in 'clockwise' or 'anticlockwise' forms. (When you look at the appropriate corner, one type reads 1, 2, 3 in a clockwise sense and the other type anticlockwise. These dice are of the **latter** type).

So the missing bottom face could be 4 or 5.

The Counting Triangles Puzzle

There are 16 'unit' triangles, (10 upright green and 6 hanging blue)

7 'four-unit', (6 upright with green corners and 1 hanging with blue corners)

3 'nine-unit' (all upright with green corners) and 1 'sixteen unit' So 27 in all.

TWINS

The identical two-digit twins are (22, 22).

The distinct ones are (12, 21)

In the set of three-digit numbers, there are five pairs of identical twins and five distinct:

(101, 101), (111, 111), (121, 121), (202, 202), (212, 212),

(102, 201), (103, 301), (112, 211), (113, 311), (122, 221).

There are ten sets of twins but only fifteen numbers are involved.

A Little Crossword

Here's 49:

			S	I	X
T	H	R	E	E	
E	F	O	V	I	
N	I	N	E	G	
	V	E	N	H	
	E			T	

Not a Word, Weather: no comment needed

Two Integers

Repeating digits often involve 37 x 3 = 111. Evidently we have here the product of the factors 7, 3, 37 (making 777) and 7, 11, 13 (making 1001). Commuting and associating in different ways can lead to several solutions, e.g.

(7 x 37) x (3 x 7 x 11 x 13) = 159 x 3003 or

(3 x 37 x 7 x 7) x (11 x 13) = 5439 x 143