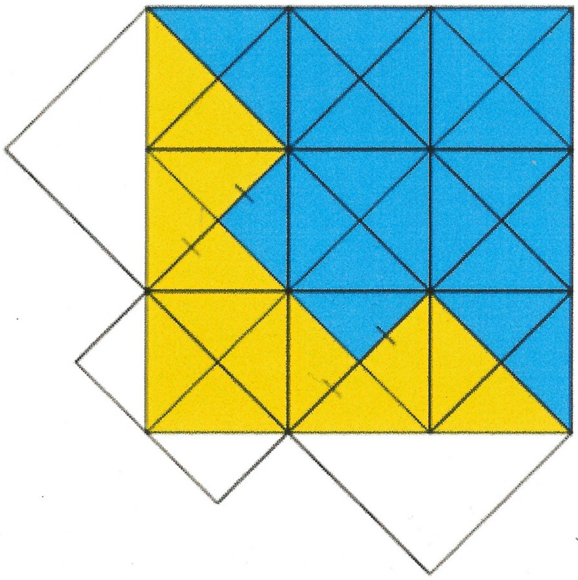
MAthematical PiE

Spring 2022 **Notes for . . . . No 215**

What's Left on the Right?
The lattice of triangles shows that the ratio of yellow to blue is 14:22 = 7:11 so the blue arrow has an area of 11 cm^2



Ages and Ages
Subtracting (10b + a) from (10a + b) shows that the difference between the two ages must be a multiple of 9. Biologically this probably becomes restricted to differences between 18 and 72. So, for example, if a parent is 18 when their child is born, the first 'palindrome' of ages will be 13 and 31. Adding one to each digit provides the next such occurrence, 24 and 42, and so on every 11 years. Which means that theoretically it might be possible for 5 generations of the same family to share this property.

The Descriptive Sequence Puzzle
Read each line from the second onwards in this way:

0	
1 0	"one zero"
1 1 1 0	"one one, one zero"
3 1 1 0	"three ones, one zero"

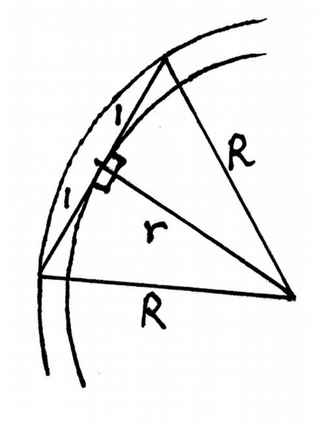
Each line describes the previous one. The seventh line is **311311222110**

Countup

Using all four digits each time makes a more challenging puzzle. With that proviso, I've found all but 16 and 30. Can you do better? Here is a tip on method: rather than looking for results in the order 1, 2, 3 . . . write down the simplest expressions first, e.g. 2 + 3 + 5 + 8 = 18. Then changing positives to negatives can provide simple expressions for several other results – in this case we can 'generate' 14, 12, 8, 2, 11, 7 and 5 from the expression above. Obviously using subsets of the four given digits makes things easier and then results like 16 = 2 x 8 or 30 = 2 x 3 x 5 are possible. 28 seems to be the most difficult one to get: 28 = (5 – 3/2) x 8 is not easy to spot!

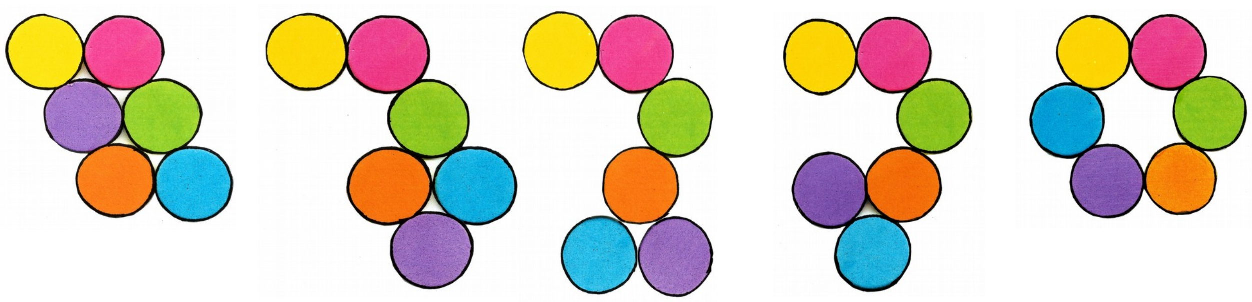
Packaged Polygons

The area of the annulus is $\pi R^2 - \pi r^2 = \pi(R^2 - r^2)$ and from the right angled triangles shown it is clear that $R^2 - r^2 = 1$ whatever other angles may be involved. So the area is always π.



Six Coins

Three coins will always fit snugly in an equilateral triangle:



Service Not Included

Two of the many solutions are shown below. In the first one there are several ways in which the **2, 1, 3** and **4** can be rearranged; and the first column could also be moved from the left (hundreds) to the right (units). I leave you to work out how many solutions this provides! The second solution does not generate as many – the **6** prevents some of the movements used above.

5 6 7	**4 9 5**
- 2 1 9	**- 1 6 7**
3 4 8	**3 2 8**

So far I have got 328 solutions

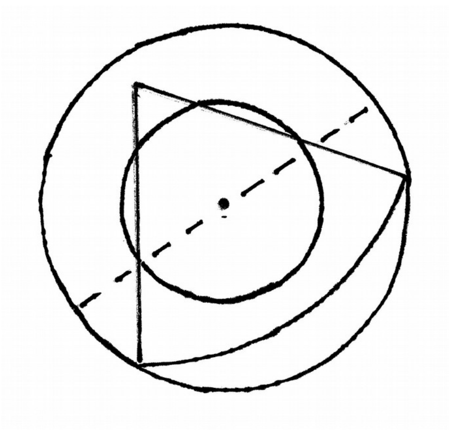
Again – can you do better?

Three-digit Squares 361, 529 and 784.

1	4	7	2	5	8	3	6	9
2	5	8	3	6	9	1	4	7
3	6	9	1	4	7	2	5	8
4	7	1	5	8	2	6	9	3
5	8	2	6	9	3	4	7	1
6	9	3	4	7	1	5	8	2
7	1	4	8	2	5	9	3	6
8	2	5	9	3	6	7	1	4
9	3	6	7	1	4	8	2	5

Sudoku+ In the given grid, 5,7 and 8 satisfy P9.

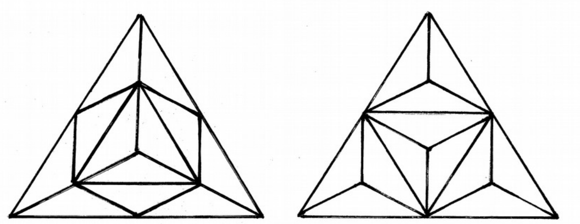
Visualising



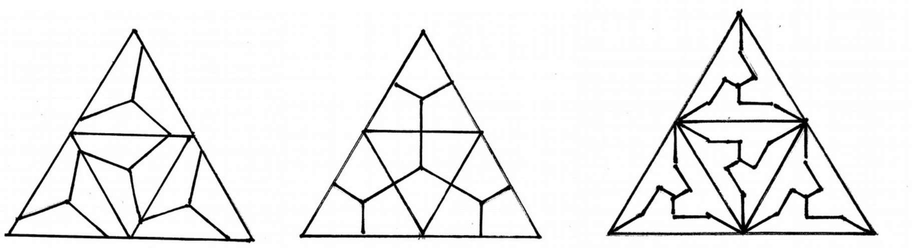
The answer should be 'The dish is fifteen feet away, in the opposite half of the circle to the dog'**.** The dog must be at least 5ft away from the tree: then there is always an arc 15ft away and the dish could be on any point of such an arc. The author's solution fails if the tree has a large trunk: then the dog cannot see the dish!

Congruent Twelfths

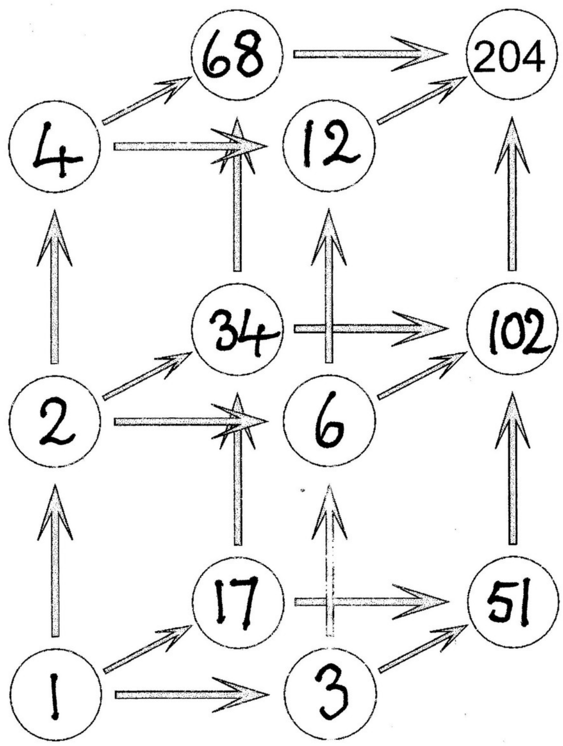
Here are two 'different ways' but they result in the *same* twelfth.



An infinite number of *different* congruent twelfths can be constructed by using the rotational symmetry of the small equilateral triangles in the second of the above diagrams:



Bubbles See right



The Surface Area Puzzle

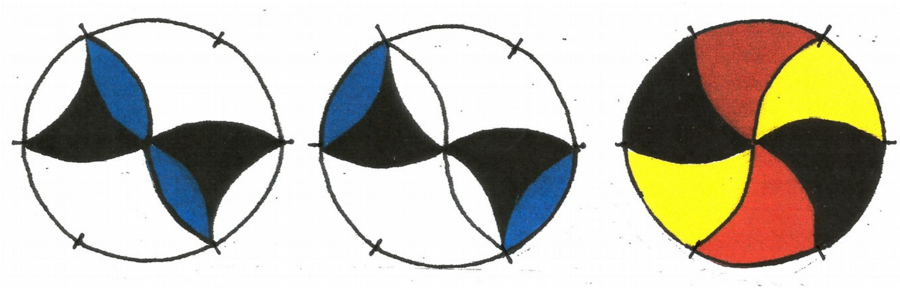
The dimensions are 4 cm, 6 cm and 8 cm so the volume is 192 cm^3.

Regular Polygons

Surprisingly many! Here is a list of (edges, angle) pairs: (3, 60), (4, 90), (5, 108), (6, 120), (8, 135), (9, 140), (10, 144), (12, 150), (15, 156), (18, 160), (20, 162), (24, 165), (30, 168), (36, 170), (40, 171), (45, 172), (60, 174), (72, 175), (90, 176), (120, 177), (180, 178), (360, 179).

Equal Arcs

One-third:



What Sort of Triangle?

The linear ratio of the triangles is 49:35:21 = 7:5:3 so AD = 25, DC = 15, CE = 15 and EB = 9. By subtraction, DE = 15 and so triangle CDE is equilateral.

The Yohaku Puzzle

The red box contains the number 2:

2	6	9	108
8	5	4	160
3	11	1	33
48	330	36	

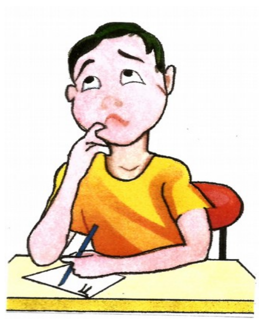
Geometry Problem

If we label the edges of the equilateral triangles ***e*** it is easy to see that the four green trangles can be rearranged into a square with edge ***e***. The smallest angle in a yellow triangle between the ***e*** edges is 30° so the area of each yellow triangle (half of absinC) is $(e^2 \sin 30^\circ)/2 = e^2/4$ and so the yellow area is also e^2.

On the Edge

vi) – i) gives **B** = 0 From vi), **RED** = 18 so in iv) 18 + **S** + **S** = 36 and **S** = 9, and from i) **A** + 18 = 23 and so **A**= 5. x) – i) gives **S** + **E** = 17 so **E** = 8. vii) – ix) gives **E** – **O** = 7 so **O** = 1.

Then substituting values of the above letters in iii) leads to **R** = 6, in ii) **N** = 3, in iv) **D** = 4, in vii) **U** = 2 and in viii) **I** = 7.
This makes 0123456789 **BOUNDARIES**.



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