Pack up a Penguin: Journeys into the Mathematics of Area
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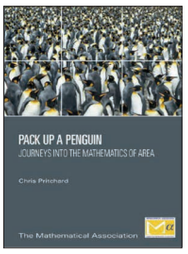
I was very much on the visual side … other people could do it but it wasn't their primary way of thinking, the variat-ion in the ways people thought about things was quite striking … I thought about the subject (relativity) very much in pictures … rather than equations – I mean I could do the equations when needed.

(Sir Roger Penrose, interview with Melvyn Bragg)



Chris Pritchard, in the final book of his trilogy on the "Mathematics of Area", takes us on an entertaining and stimulating journey of visualisation. Easy on the eye, with attractive photographs and helpful use of colour in the diagrams and proofs, this is a composed and welcoming book. Chris's straightforward but engaging prose takes the reader through a variety of topics on area at a comfortable pace. I found each chapter made a satisfying read in a sitting or two.

Each chapter is quite independent, but each an excursion on the journey into the mathematics of area. With an introductory fun discussion about the history of the "Wales" as a unit of measure for area (coincidentally, I came across the use of Luxembourg as a comparison of area for the very same iceberg, A68a, still traversing the Antarctic Ocean some three years from the first report) and a taste of geometric problems and styles of solution and proof, the author goes on to make the point that, rather than "tackling many problems in a single way", there are advantages in asking "for a single problem to be tackled in many ways". Comparison of methods helps form connections between apparently unrelated areas of mathematics, problem-solving skills are extended and practised, while simultaneously encouraging and building resilience.

First, penguins are packed up (in 2D) leading to optimal packing of circles in triangles and squares, using no more than Pythagoras and surds and some trigonometric identities. "Malfatti's marble problem", a problem of packing three circles in a triangle, is an instructive tale – a journey from accepted truth to gradual realisation that things were not quite what they seemed, unravelling over the 19th and 20th centuries.

There is a fascinating chapter exploring some of the geometry of Euclid and Archimedes and Heron. Interestingly, Chris makes the point that these mathematicians were at the helm of quite different mathematical traditions – Euclid, with a focus on logical structure, reasoning with rigour from simple to complex statements, using but not quantifying area; Archimedes, leading on the "metric tradition", building on contributions by such as Eudoxus, measuring space as area and volume; and a "utilitarian" tradition represented in the writings of Heron (a path of mathematics already explored to a degree by the Babylonians and Egyptians), measuring area against some standard unit, developing tools that were utilised in following centuries by craftsmen and architects, Byzantine constructions amongst those influenced. I found fascinating the section on Archimedes' method for finding the area of a segment in a parabola; illuminating to see how Archimedes approached such problems. Exciting, too, to read of relatively recent finds of long-lost writings such as Archimedes' *Method* and Heron's *Metrica*, separately rediscovered in Istanbul at the turn of the 19th/20th centuries.

A beautifully illustrated chapter follows, examining the surface areas of a variety of towers and buildings from a breadth of cultures – from Giotto's Bell Tower in Florence to the Minaret of Istanbul's Dolmabahçe Mosque, from a comparison of the ancient Bent and Red Pyramids in Egypt to the *exedra* of Cincinnati's Union Terminal, from South African Xhosa rondavels to Kent oast houses. We are guided through a variety of calculations as we journey through a range of 3D shapes. I was struck by how appropriate these examples might be for A-Level students, utilising skills in modelling as well as providing experience of elements of 3D geometry that they come across in both the pure and mechanics aspects of the courses – but here located in historically significant architectural contexts. The striking colour photographs make attractive entry points.

Further engaging material populates the three chapters on visualising algebra, trigonometry and series. The deft use of colour in the diagrams draws one in, easing the way through a variegated collation of examples, demonstrating identities and inequalities. I particularly liked the proofs by area of the addition formulae for sines and cosines – so much more intuitive than the ones I currently promote, using lengths, with my A Level students. Good to see one of my favourite demonstrations, illustrating how to sum an infinite geometric series – and exemplifying visually that clearly an infinite series can have a finite sum. I do think the visualisations of various series of powers of integers can be a balm to Further Maths students, when the busy-ness and notation of the algebra can, for some, obscure the beauty of the results. Perhaps viewing from a more holistic perspective – that visualisation provides – enables the results to be more easily appreciated.

Appropriately signing off these explorations with a Sierpinski-style fractal pattern from a 13th century tiling in the cathedral of Anagni, near Rome, the final chapter provides a wealth of further examples with discussion of solutions and engaging, clear diagrams. A helpful list of books and articles features in the References section.

I have enjoyed travelling this journey through area, engaging in the variety of mathematical thinking in new as well as familiar parts of the mathematical landscape, often placed in historical and cultural context. And it has given me ideas for new ways to approach topics with my A Level students. Very appropriate that the President of the MA should author a book on area in the 150th year of the association founded for the "Improvement of Geometrical Teaching". An enjoyable book with valuable resources for the teacher – "providing clarity to those for whom standard approaches have already failed and alternative ways of viewing … to those who have already grasped them" – this might well also be an attractive offering for an A-Level student's own reading. I now look forward to exploring the first two books in the trilogy!

Martin Jones