Primary Mathematics Challenge – February 2020

Answers and Notes

These notes provide a brief look at how the problems can be solved.

There are sometimes many ways of approaching problems, and not all can be given here.

Suggestions for further work based on some of these problems are also provided.

P1 **C** ($11^2 + 2 = 123$)   P2 **C** ($2020 \times 5\text{p} = 10\,100\text{p} = £101$)

1 **D** 11 p.m. A 15-hour flight that leaves Singapore at 3 p.m. will land in Manchester when it is 6 a.m. the following day in Singapore. This will be 11 p.m. in Manchester.

2 **E** 36% The area of toast remaining is $8 \times 8 = 64\ \text{cm}^2$, so the amount thrown away is $10 \times 10 - 64 = 36\,\text{cm}^2$, which is 36%.

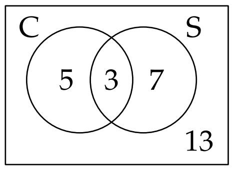
3 **C** 14 811 852 It should be noted that the units digit of the product of 3333 and 4444 will be the same as the units digit of the product of 3 and 4, namely 2: this succinctly identifies the correct option as 14 811 852. Alternatively, the calculation 3333×4444 will have an answer that is 3×2 times greater than 1111×2222. Now we can work out $2\,468\,642 \times 6$ exactly, which is indeed 14 811 852.

4 **D** 2550 Both Astrod and Steven are adding together 50 numbers, but each of the numbers in Steven's list is one greater than one of Astrod's. Hence Steven will have a total of $2500 + 50 = 2550$.

5 **E** $39\,\text{cm}^2$ The area of the square is $3 \times 3 = 9\,\text{cm}^2$; the area of each triangle is $3 \times 5 \div 2 = 7.5\,\text{cm}^2$. Therefore the area of the star is $9 + 7.5 \times 4 = 39\,\text{cm}^2$.

6 **E** more than 10 Each of the 40 *pairs* of socks will last 100 days, so that my teacher has $40 \times 100 = 4000$ days of sock-wearing. There being 365 (or 366) days in a year, 4000 days is more than 10 years.

7 **A** 3 We can illustrate the problem by means of the Venn diagram on the right. In the problem $8 + 10 + 13 = 31$ members of the class appear to be mentioned, but this is three greater than 28, the total number in the class. Therefore three members have been counted twice, namely those who like cabbage (C) *and* sprouts (S).



8 **D** 15 Referring to Dougie's dogs as P, Q, R, S, T and U, we have the following pairings: PQ, PR, PS, PT and PU; QR, QS, QT and QU; RS, RT and RU; ST and SU; and finally TU – that is, 15 pairings. The notes below suggest other approaches.

9 **A** 13 minutes The rider travels for 13 hours over 60 rides, which is an average time of $\frac{13}{60}$ of an hour per ride, hence 13 minutes.

10 **E** 135 cm We can see the combined height of the table and 3 bricks is 150 cm, whereas the combined height of one brick and the table is 140 cm. The extra two bricks have, therefore, a height of $150 - 140 = 10\,\text{cm}$ together, and so each brick must have a height of 5 cm. Hence, the table has a height of $140 - 5 = 135\,\text{cm}$.

11 **B** 26 We could try each of the options in turn: $15 \to 20 \to 40 \to 04$ does not work nor 37 ($\to 42 \to 84 \to 48$) nor 48 ($\to 53 \to 106 \to 601$) nor 59 ($\to 64 \to 128 \to 821$). Of the options only 26 works: $26 \to 31 \to 62 \to 26$. The notes below illustrate an algebraic approach.

12 **D** 6 Since the number of motorcycles is the same as the number of fire engines, we can count the points for these vehicles as equivalent to $9 + 2 = 11$ points for each fire engine. Then we can solve the problem by trial and improvement, working our way through the possible number of buses and noting that the number obtained by subtracting the total number of buses from 63 must therefore be a multiple of 11:

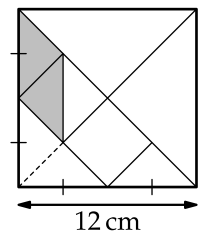
number of buses	points for buses	points for fire engines and motorcycles	
1	5	58	✗
2	10	53	✗
3	15	48	✗
4	20	43	✗
5	25	38	✗
6	30	33	✓
7	35	28	✗

Hence the number of buses is 6. The notes below show another approach to this.

13 **D** 17 Since the mean of Katherine's prime numbers is 8, their total is $8 \times 3 = 24$. The sum of three odd numbers is an odd number, so at least one of Katherine's numbers is even; however, the only even prime number is 2. Hence, the other two numbers have a total of 22. There are three possibilities where both numbers are prime: $3 + 19$, $5 + 17$ and $11 + 11$. The only possibility where the difference between the smallest two primes is itself prime is 2, 5 and 17.

14 **E** 70 Of the options, only 70 is 6 more than a square number (64) and also 11 less than another square number (81). In fact, 70 is the only possible number.

15 **B** 18 As can be seen in the diagram on the right, the shaded parallelogram is formed of two equal triangles, each of which is one quarter of a larger triangle, which is one quarter of the square. So the area of the shaded parallelogram is $2 \times \frac{1}{4} \times \frac{1}{4} \times 12 \times 12 = \frac{1}{8} \times 144 = 18\,\text{cm}^2$.

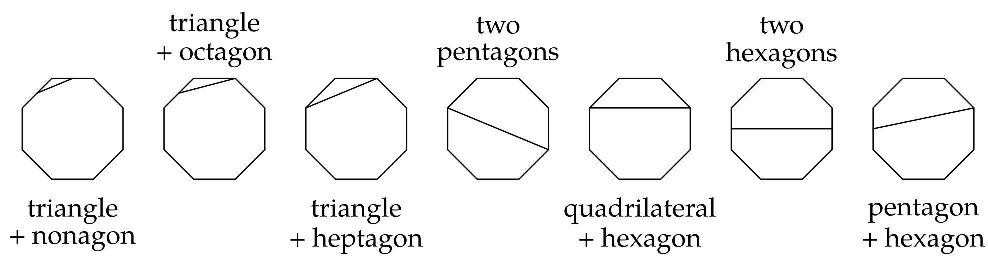


16 **B** 8 days If 4 machines take 6 days to harvest a crop, then 1 machine would take four times as long, that is 24 days. So 3 machines would take a third of this time, 8 days.

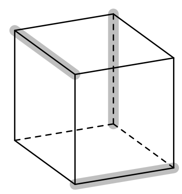
17 **C** 240 m The length of the line is $4000 \times 2 \times 3\,\text{cm} = 24\,000\,\text{cm} = 240\,\text{m}$.

18 **A** 75 421 We can find the correct option by scrutinising consecutive pairs of digits in each of the options, eliminating those which are not multiples of 3: B (637 542) has consecutive digits 37, C (721 543) has 43, D (751 263) has 26, while E (7 563 421) has 56 and 34. Option A (75 421) satisfies the conditions of the question, since 75, 54, 42 and 21 are each multiples of 3. The notes below explain why 75 421 is in fact the largest possible number to work here.

19 **A** 1 triangle and 1 hexagon The diagrams below show the three ways in which it is possible to cut off a triangle from the octagon (none of which leave a hexagon) and the ways in which to achieve the four other options.



20 **B** 3 Since there are 6 faces and each edge joins only 2 faces, Cassie must colour a minimum of 3 edges red. The diagram in the right shows that it is possible to choose 3 edges so that each face has a red edge.



21 **C** 3°

The exterior and interior angles of the three polygons are calculated as shown in the table below:

polygon	exterior angle	interior angle
pentagon	$360^\circ \div 5 = 72^\circ$	$(180 - 72)^\circ = 108^\circ$
hexagon	$360^\circ \div 6 = 60^\circ$	$(180 - 60)^\circ = 120^\circ$
octagon	$360^\circ \div 8 = 45^\circ$	$(180 - 45)^\circ = 135^\circ$

Therefore the angle of the overlap $= (108 + 120 + 135 - 360)^\circ = 3^\circ$.

22 **C** $15.75\,\text{cm}^2$

The rectangle in the bottom right with area of $5\,\text{cm}^2$ has an area $\frac{1}{4}$ times greater than the rectangle with area of $4\,\text{cm}^2$. They share the same base, and so the lower rectangle must have a height $\frac{1}{4}$ greater than the upper one. Now the same can be said for the rectangles on the left, and so the area of the unshaded rectangle is $\frac{1}{4}$ greater than $3\,\text{cm}^2$, that is $3 \times 1.25 = 3.75\,\text{cm}^2$. Hence the total area in square centimetres of the four rectangles is $3 + 4 + 5 + 3.75$, that is $15.75\,\text{cm}^2$.

23 **D** 3 : 2

Let the jars each initially contain, say, 100 ml of blackcurrant or orange juice.

jar X	jar Y	
100 ml of blackcurrant	100 ml of orange	at start
50 ml of blackcurrant	100 ml of orange + 50 ml of blackcurrant	half of X added to Y
50 ml of blackcurrant + 50 ml of orange + 25 ml of blackcurrant	50 ml of orange + 25 ml of blackcurrant	half of Y added to X

Now jar X holds 75 ml of blackcurrant + 50 ml of orange, a ratio of 75 : 50 = 3 : 2.

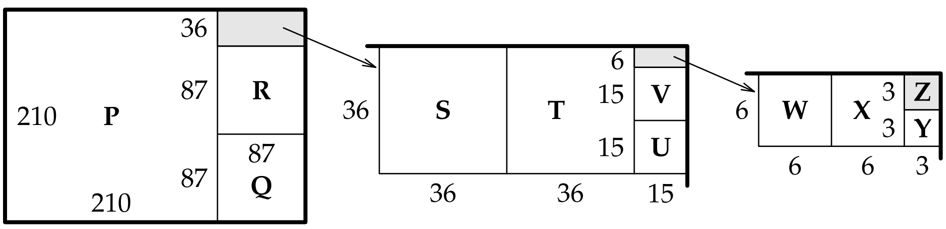
24 **B** 36

In order to contain 125 smaller cubes, the cube must be 5 small cubes across, back, and high. The cubes which do not touch the right, left, front, back sides or the base of the box form a cuboid which is 3 across by 3 back by 4 high; this contains $3 \times 3 \times 4 = 36$ small cubes.

25 **C** 3 mm

Below is a list of the squares and rectangles produced from the original 210×297 rectangle and a diagram of the dissection:

	dimensions of square (mm)	dimensions of rectangle (mm)
P	210×210	210×87
Q	87×87	123×87
R	87×87	87×36
S	36×36	51×36
T	36×36	36×15
U	15×15	21×15
V	15×15	15×6
W	6×6	9×6
X	6×6	6×3
Y & Z	3×3	3×3



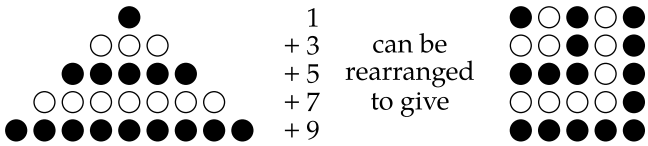
So the length of the side of each of these final two squares is 3 mm.

Some notes and possibilities for further problems

1 Ask pupils to research time differences between different cities and set each other problems based on these. Which cities have the greatest time differences? Which cities (from different countries) have the same time ... and why?

2 Pupils will almost certainly find several ways of arriving at the correct answer.

4 The diagram below indicates why consecutive odd numbers beginning with 1 add together to give a square number:



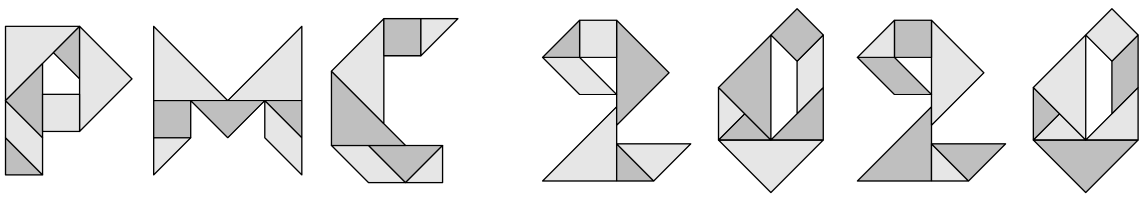
Astr**od** (a Welsh name meaning 'divinely beautiful') and St**even** ('crown') are British forenames. Can pupils think of any other names that might allude to common mathematical words?

8 It is clear that the number of pairs of dogs is the sum of consecutive numbers ($15 = 1 + 2 + 3 + 4 + 5$). With n dogs this is given by the formula $\frac{n\times(n-1)}{1\times2}$. If we were able to take 3 of the 6 dogs for a walk, then we could calculate the number of *combinations* as $\frac{6\times(6-1)\times(6-2)}{1\times2\times3}$, which gives the answer 20.

11 If we write the number we are seeking as 'ab'$= 10a + b$, then adding 5 gives $10a + b + 5$. Multiplying by 2, we have $2(10a + b + 5) = 20a + 2b + 10$. If this is the 'reverse' of the original 'ab', then we have $20a + 2b + 10 = 10b + a$. Rearranging we get $19a = 8b - 10$. Since $8b - 10$ is an even number, it must be that $19a$ is also even, and hence a itself is even. Taking $a = 2$, we have $b = 6$; since the remaining even values of $a < 10$ do not give whole number values for b, the only possible answer is 26.

The other observation that eagle-eyed pupils might make is that after adding 5 and then multiplying by 2, the result must be an even number. Hence the original number must have had an even tens-digit – this succinctly eliminates three of the five options.

15 Ask pupils to design their own tangram puzzles for each other. Using a standard tangram, arrange the pieces to make a shape, with no overlaps and no gaps. Draw round the outline and ask a friend to solve the puzzle. Perhaps they can make whole phrases:



17 Draw a line in the classroom or in the playground. Ask pupils to estimate how many 10 pence coins it would take to cover the line. Give them some resources, e.g. a short piece of string, a ruler, a 10 pence coin. How can they best check the accuracy of their estimates? Compare methods.

18 In order to find the largest number to satisfy the conditions of the problem, it is worth noting that the digits 3 and 6 can be combined only with each other in forming a two-digit multiple of 3. Therefore we can choose only from the five digits remaining, 1, 2, 4, 5 and 7. We should start by choosing 7 as the leading digit. Now the '7*' has to be a multiple of 3, so we should choose 75. Next, for '5*' to be a multiple of three, we choose '54', and then '42' (since the 5 has been used already) and then the only possibility left is '21'. Hence the largest answer is 75 421.

What would you have to change to design a question like this where each pair of consecutive digits has to form a multiple of 5?

19 Investigate shapes which could be made with two cuts. Or start with a different shape.

24 Can pupils generate a 'formula' (maybe in words) to solve this problem for different cuboids?

25 This question provides a way of visualising one method for finding the *highest common factor* of two numbers – it was first described by the ancient Greek mathematician, Euclid around 300 B.C. and is referred to as the *Euclidean algorithm*.