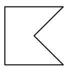
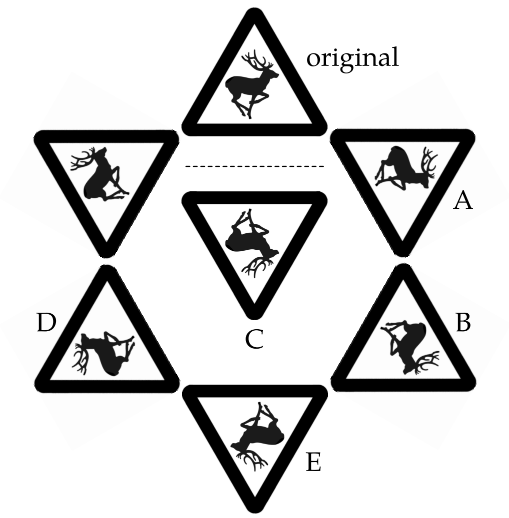
Primary Mathematics Challenge – November 2019

Answers and Notes

These notes provide a brief look at how the problems can be solved.
There are sometimes many ways of approaching problems, and not all can be given here.
Suggestions for further work based on some of these problems are also provided.

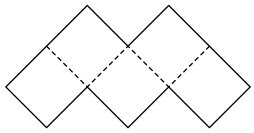
P1 **D** £10 P2 **E** 5 days

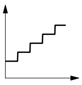
1	**B**	5 hours	17:34 is 5 hours later than 12:34.
2	**E**		Shape A is a square, B is a trapezium, C is a parallelogram and D is a rectangle – all of these are quadrilaterals. Shape E, however, has five sides, and is therefore a pentagon, not a quadrilateral.
3	**D**	518 m	The difference in the heights is $(828 - 310)$ m$=$ 518 m.
4	**B**	3	We can use one 50p coin, one 10p coin and one 1p coin to make a total of 61p.
5	**E**	Emperor	The average height of a eleven-year-old child is around 144 cm, and so it is the Emperor penguin which is closest to that height.
6	**C**	6 miles	Cycling at 12 mph, Speedy would go 12 miles in an hour. So in 30 minutes he cycles 12 miles $\div 2 = 6$ miles.
7	**B**	12	The graph first reaches a height of 20 mm after 12 weeks.
8	**E**	70	There are 6 starfish and 5 octopuses. So the number of boxes $= 6 \times 5 + 5 \times 8 = 70$.
9	**A**	70 minutes	Between 6 pm and 8 pm, Sam has 2 hours $=$ 120 minutes. The time he spends on sorting pencils, *Look-Time* and a break is $(10 + 25 + 15)$ minutes $=$ 50 minutes. So the time he spends on homework is 70 minutes.
10	**C**	4 years	Ellie is 6 years older than Lola, and Maddie is 2 years older than Lola. Therefore Ellie is 4 years older than Maddie.
11	**D**	12	Sam sings 2 songs in the time that Robyn swims 9 lengths. The time that she takes for 54 lengths is $54 \div 9 = 6$ times longer, so Sam will sing $2 \times 6 = 12$ songs.
12	**C**	4	There must be at least one boy and one girl in the family. But if there is just one boy or one girl, that boy will not have a brother and the girl will not have a sister. Therefore the smallest family will include two boys and two girls – four children.
13	**C**		The diagram below shows the five rotations of the original sign by multiples of 60°, including the options A, B, D and E. Also shown is the reflection, option C.



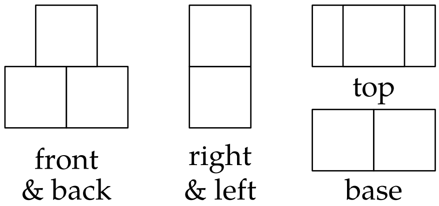
14	**D**	4	After removing 3 beads from the original 12, there are nine left, and thus three each of green, red and white. So there must have been 4 white beads originally.
15	**D**	39	The number of Salah's cards is 3 more than a multiple of 4, and 4 more than a multiple of 5. Of the two-digit options given, the only one that satisfies both criteria is 39 $(= 9 \times 4 + 3 = 7 \times 5 + 4)$.

16 **A** $125\,\mathrm{m}^2$ As the diagram on the right indicates, the M shape is made from 5 squares each of side-length 5 cm. Therefore its area is $5 \times (5 \times 5) = 125\,\mathrm{m}^2$.



17 **E**  As the crow drops each stone into the beaker, the level of the water will rise in separate 'jumps'. The only graph that shows this is graph E.

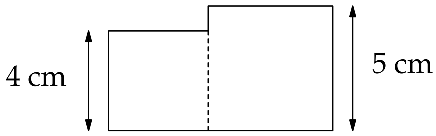
18 **A** $14\,\mathrm{cm}^2$ We will look at the shape from front and back, right and left sides, and from the top and underneath, as shown in the diagrams on the right. The surface area of the front (and of the back) is $3\,\mathrm{cm}^2$; the right and left, both $2\,\mathrm{cm}^2$; and the top and underneath both $2\,\mathrm{cm}^2$, giving a total surface area of $3+3+2+2+2+2 = 14\,\mathrm{cm}^2$.



19 **C** 2004 Given that Granny is 5 times older than each of the three triplets, we know that the age of each triplet is $120 \div (5+3) = 15$. So Cara, Cate and Chris were born in 2004.

20 **E** (16, 9) We can see that between points J and L there is a distance of $22 - 4 = 18$ units across, so the side of each square measures $18 \div 3 = 6$ units. Point K is two squares below point L, so the y-coordinate of point K will be $21 - 12 = 9$. Therefore the coordinates of point K are $(4 + 2 \times 6, 21 - 12) = (16, 9)$.

21 28 cm Separately the squares whose sides have lengths 4 cm and 5 cm have perimeters of 16 cm and 20 cm respectively. As shown in the diagram, when they are joined, the perimeter of the combined shape does not include the section of either where the two squares meet. Hence the perimeter has a length of $16 + 20 - 2 \times 4 = 28\,\mathrm{cm}$.



22 8 The largest number of ants will be less than $64 \div 6 = 10\frac{2}{3}$. If there were 10 ants, they would account for 60 legs, and so the remaining 4 legs would not suffice for a spider. Similarly, were there 9 ants, there would be $64 - 9 \times 6 = 10$ legs, enough for an 8-legged spider, but with 2 legs left over. However, with 8 ants, there would be $64 - 8 \times 6 = 16$ legs, enough for exactly 2 spiders.

23 10 Alternately subtracting 20 and 19 can be achieved more quickly by subtracting 39. We can see that $2019 \div 39$ is equivalent to $673 \div 13 \approx 51$. Since $39 \times 51 = 1989$, the sequence $2019, 1999, 1980, 1960, \ldots$ will eventually arrive at $2019 - 1989 = 30$. Subtracting a further 20 will get to 10, but a further 19 will reach $^{-}9$. Thus 10 is the last positive number in the sequence.

24 16 : 29 The four smaller squares on the left have the same combined height as the three slightly larger squares on the right. Since the answer is a ratio, we can choose, without loss of generality, to let the smaller squares have a side length of 3 units and the larger ones 4 units. The unshaded square has an area of $(4 \times 2)^2 = 64$ square units; the shaded area is $4 \times 3^2 + 5 \times 4^2 = 36 + 80 = 116$ square units. Therefore the ratio of the unshaded area to the shaded area $= 64 : 116 = 16 : 29$.

25 17 Let the values of the digits in the boxes be A, B, C, D and E, as shown on the right.

$$\boxed{A}\boxed{B}\boxed{C} \times \boxed{D}\boxed{E} = 11\,655$$

The question concerns the product of two numbers and it does not take so long to find that $11\,655 = 3 \times 3 \times 5 \times 7 \times 37$. The first thing we might consider is whether $D = E$. If so, then 'DE' would be a multiple of 11; however, 11 655 is not a multiple of 11, and so $D \neq E$. In consequence, the three digits of the first number are all equal, $A = B = C$.

$$\boxed{A}\boxed{A}\boxed{A} \times \boxed{D}\boxed{A}$$

$$\boxed{A}\boxed{A}\boxed{A} \times \boxed{A}\boxed{E}$$

Now we can assert that the form of the solution is either one of the two options shown on the right. If it is the first then we must have $A = 5$, since the last digit of 11 655 is 5. However, if that were the case then both AAA and DA would be multiples of 5, and so their product would be a multiple of 25, which 11 655 is not. Therefore we have the second option $AAA \times AE$. Again because 11 655 is a multiple of 5, either $A = 5$ or $E = 5$.

25 continued

If $A = 5$ then $AAA \times AE$ would be greater than $500 \times 50 = 25\,000$, which itself is already greater than 11 655. So it must be that $E = 5$, and by inspection 3 is the only value for A that gives an answer in the vicinity of 11 655. Verifying this, we have $333 \times 35 = (3 \times 111) \times (5 \times 7)$, and since $111 = 3 \times 37$ we have the product of prime factors obtained earlier. Thus the sum of the digits $= 3 + 3 + 3 + 3 + 5 = 17$.

Some notes and possibilities for further problems

P1 Mrs Truelove might have had more children. How much would each child receive if she had: 4 children; or 5 children; or 6 children or 7 children?

P2 Pupils may like to consider roughly how old they are in weeks – is there a quick way of calculating this? Is there a neat way of finding how many seconds a person has been alive?

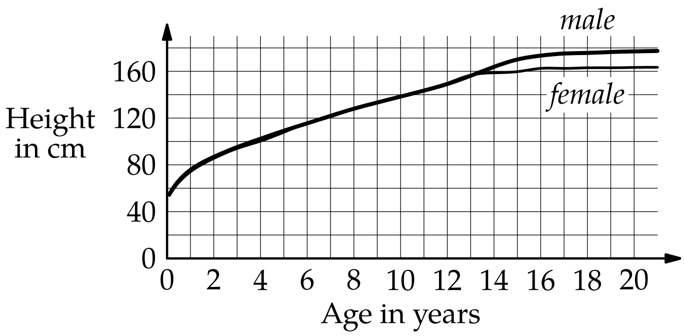
1 There are a number of countries where the time is not a whole number of hours different from Greenwich Mean Time: India and Sri Lanka, for example, are $5\frac{1}{2}$ hours ahead of London; Nepal is $5\frac{3}{4}$ hours ahead.

4 Pupils could draw up a table showing the least number of coins needed to make from 50p to 70p:

sum of money in pence	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	...
least number of coins	1	2	2	3	3	2	3	3	4	4	2	3	3	4	4	...

For which amount less than £1 is the least number of coins required the highest?

5 The graph below shows the average height of young people in the UK from birth to the age of 21.



6 How fast would Speedy be cycling if he takes an hour; or 30 minutes; or 10 minutes; or even 5 minutes?

7 One of the reasons that some people find spiders rather frightening is to do with their many legs and the way they move – very differently from any mammal. Each of a spider's legs has seven joints between where it is connected to the main head/body and the claws at the end. Moreover, they have no muscles; spiders 'operate' their legs using a system of hydraulics, pressurising the equivalent of blood.

8 Starfish are a rare example of a species with an odd number of limbs. Snails and slugs have only one 'foot'. Some might say that elephants and long-tailed monkeys have a trunk or a tail which serves as a extra limb. Other than these, the Japanese spider crab has eight legs (each of which can grow to a length of two metres), but it can survive losing up to three of them (which may in time grow back); eleven-armed sea stars can be found in the southern hemisphere.

11 In November 2018, the explorer Ross Edgeley became the first man to swim all around the British Isles, 1792 miles in 157 days. He claimed to have kept himself entertained by regularly singing to himself while swimming underwater.

17 The story relating a crow using this method for raising the level of water in order to drink is related in one of Æsop's Fables, "The Crow and the Pitcher", dating from the 7th century BC.

22 If we were to let a and s represent the number of ants and spiders respectively, then we could assert that $6a + 8s = 64$. Dividing both sides by 2 gives $3a + 4s = 32$. Since both 4 and 32 are multiples of 4, so must $3a$ be also; this will only be true if a is a multiple of 4. As $a < 10\frac{2}{3}$, we can take a as 8, whence $s = 2$.

23 The solution can be accelerated by subtracting multiples of 390: 2019, 1629, 1239, 849, 459, 69, ..., and then subtract 20 and 19 to reach 30; subtracting a further 20 leaves 10. Teachers might ask pupils to discuss and compare their methods and approaches: which do they think is most efficient?