Primary Mathematics Challenge – February 2019

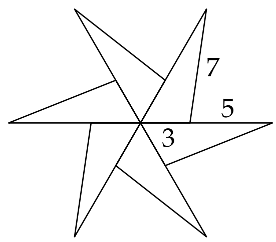
Answers and Notes

These notes provide a brief look at how the problems can be solved.

There are sometimes many ways of approaching problems, and not all can be given here.

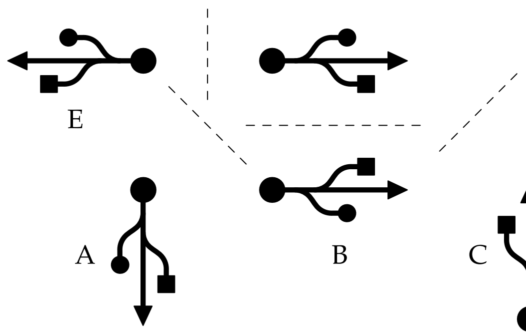
Suggestions for further work based on some of these problems are also provided.

P1 **E** ($12 = 3 \times 4$) P2 **D** (100 000 000)

1	**B**	1902	The options closest to 2019 are clearly B and C. Since $2190 - 2019 = 171$ whereas $2019 - 1902 = 117$, the closest of the options to 2019 is 1902.
2	**D**	75%	We can count that 12 of the 16 smaller triangles are shaded. Since $\frac{12}{16} = \frac{3}{4}$, as a percentage this is 75%.
3	**B**	after the 4	Paula can create these numbers: 54637, 45637, 46537, 46357, 46375. The smallest of these arises when the 5 is inserted after the 4.
4	**C**	20	There are 5 stations to start from and from each of them there are 4 other stations to travel to. Therefore the number of different journeys on this line is $5 \times 4 = 20$.
5	**D**	70	We can say that $10 \times 30 \times 50 \times 70 = 1 \times 3 \times 5 \times 7 \times 10\,000$ whereas $1 \times 30 \times 500 = 1 \times 3 \times 5 \times 1000$. So the number that should go into the box is $7 \times 10 = 70$.
6	**E**	$12\,\text{cm}^2$	The area of the outer square is $8 \times 8 = 64\,\text{cm}^2$ and that of the inner square is $4 \times 4 = 16\,\text{cm}^2$. Hence the area between the two squares is $64 - 16 = 48\,\text{cm}^2$. Since the two squares share the same centre, each trapezium has the same area, that is $48 \div 4 = 12\,\text{cm}^2$.
7	**D**	90 minutes	The number of Kit-Clats eaten every hour is $50 \times 60 \times 60$, so the number of minutes of production to make this number of Kit-Clats is $\frac{50\times60\times60}{2000} = \frac{5\times6\times6}{2} = \frac{180}{2} = 90$.
8	**C**	72 cm	The 12-sided star has six edges that are 7 cm long and six edges that are the difference between the 8 cm and the 3 cm sides, i.e. 5cm. So the perimeter of the star has a length of $6 \times (7 + 5)\,\text{cm} = 72\,\text{cm}$. 
9	**E**	2374	We could simply divide each of the five options by 7 to decide which of them is or is not a multiple of 7. However, having found that option A (2345) is a multiple of 7, and noticing that options B, C and D are 7, 14 and 21 larger than option A, and that the option E is 8 greater than option D, it is clear that option E (2374) is 1 greater than a multiple of 7.
10	**B**	FINLAND	The question is entirely about the repetition of letters in the seven-letter names of countries. In the name CRO**A**TI**A** there are two occurrences of the letter A and five other unrepeated letters – this pattern of repetition is the same as occurs in the name FI**N**LA**N**D, which has two Ns and five other letters.
11	**C**	40	Let Agnijo have n apps. Now Sam has $2n$, and Naomi $6n$. Therefore $n + 2n + 6n = 9n = 180$, and so $n = 180 \div 9 = 20$. Hence Sam has $2 \times 20 = 40$ apps.
12	**C**	700 g	Given that 10% of the weight of Erica's Camembert is 28 g, its total weight is $28 \times 10 = 280$ g. Half of this weight is $280 \div 2 = 140$ g, which is 20% or one fifth of the weight of Pete's Cheddar. Thus the weight of Cheddar is $140 \times 5 = 700$ g.

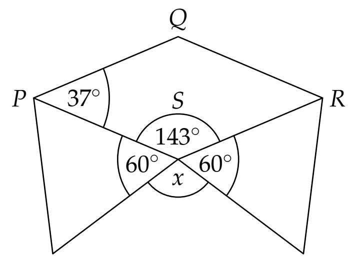
13 **D** 

Option B is a reflection of the original symbol in an axis parallel to its centre line; option E in an axis at right-angles to the centre line; options A and C are reflections in the two lines at 45° to the centre line. Option D cannot be a reflection because the circle and the square on the stems have been swapped without the stems being swapped.



14 **D** 97°

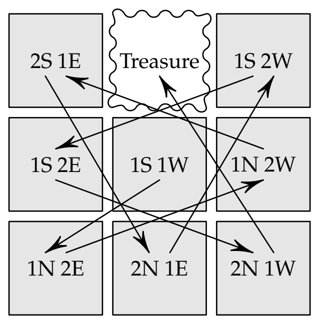
Because $PQRS$ is a rhombus, the lines PQ and SR are parallel, and so angle $PSR = 180° -$ angle $QPS = 180° - 37° = 143°$. Moreover, each of the angles in the two equilateral triangles is 60°.
Hence angle $x = 360° - 143° - 2 \times 60° = 97°$.



15 **E** 1S 1W

Since we know that Matthew must pass through all eight labelled squares, we can work out the combined direction after taking all eight instructions into account:

row	N	S	E	W
1		$2+1$	1	2
2	1	$1+1$	2	$1+2$
3	$1+2+2$		$2+1$	1
total	6	5	6	6



So the directions amount to 6 North, 5 South, 6 East and 6 West, or more simply 1 North. If Matthew is to reach the Treasure he must therefore start 1 South of the Treasure, that is, in the square marked 1S 1W.

16 **A** 7

If all the bunches are the same, then the number of bunches must be a common factor of 21, 35 and 28. The only common factors are 1 (in which case we have one very large bunch of 84 tulips) and 7 (which gives seven bunches, each of with 3 white, 5 yellow and 4 red tulips).

17 **B** 1.8 cm

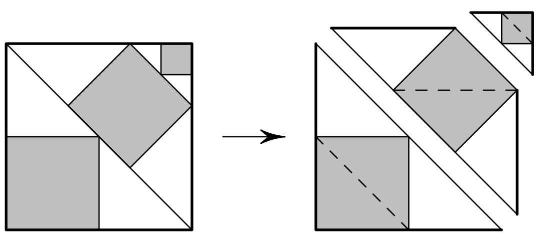
The volume of the brownie mixture in the rectangular tin is $15 \times 24 \times 2 = 720\,\text{cm}^3$. Given that Beattie puts this same volume in the square tin, the depth of the mixture will be $720 \div (20 \times 20) = 720 \div 400 = 7.2 \div 4 = 1.8\,\text{cm}$.

18 **B** 10 years

Panath and Ranesh share the 32 sweets in the ratio $20 : 12 = 5 : 3$. Therefore the sum of their ages a multiple of $(5+3)$, i.e. 8. However, the sum of their ages is also a multiple of 5, and so a multiple of 40. The only such multiple which is less than 50 is 40 itself, in which case their ages are 25 and 15, so the difference is 10 years.

19 **C** $\frac{1}{2}$

A simple way, but perhaps not the most obvious, is to regard the shaded areas as fractions of two triangles and a central strip, as shown below:

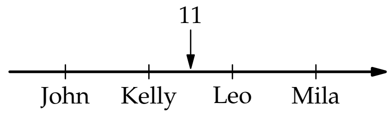


In each of the three parts, we can see that the shaded part is $\frac{2}{4}$ of the whole. Hence the total shaded area of the three smaller squares is $\frac{1}{2}$ of the large square.

20 **A** 000

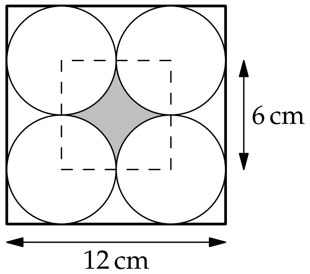
The product $123 \times 124 \times 125 \times 126 \times 127$ is a multiple of 125; moreover, it also has a factor of 2 three times, from 124 ($= 2 \times 2 \times 31$) and from 126 ($= 2 \times 63$). Therefore it is a multiple of $125 \times 2 \times 2 \times 2 = 1000$, and so it must end in 000. Alternatively, working from the options, it is easily seen that the product is a certainly an even multiple of 5 – so its unit digit is 0.

21 **E** 22



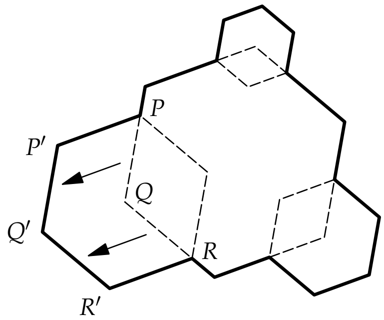
Since each of the averages are for two children at a time, we can place the children on a number line according to their ages, Kelly exactly halfway between John and Leo; Leo himself halfway between Kelly and Mila. Clearly the average of the ages of Kelly and Leo is also 11, and so the sum of their ages is 22.

22 **C** between $7\,\text{cm}^2$ and $9\,\text{cm}^2$



The area of the shaded region is equal to the area of the central square (shown dashed on the right) minus the area of the four quarter-circles inside it. The area of the central square is $6 \times 6 = 36\,\text{cm}^2$, and the area of the four quarter-circles is the area of one circle, which (we are told) is approximately $28.26\,\text{cm}^2$. Therefore the shaded area is roughly $36 - 28.26 = 7.74\,\text{cm}^2$.

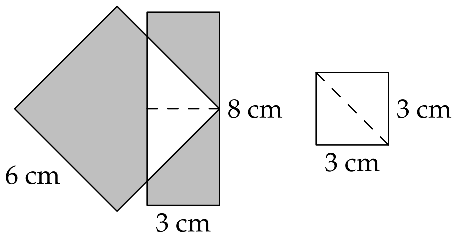
23 **B** 92 cm



In considering this problem, it is useful to see what has been subtracted from the perimeter of the large hexagon and what has been added to it. It can be seen that the section PQR of the large hexagon has the same length as the section $P'Q'R'$ of the smaller hexagon, so the section $P'Q'R'$ adds no extra length to the perimeter. However, the sections PP' and RR' do have the effect of increasing the perimeter, and their combined length is $\frac{2}{6} = \frac{1}{3}$ of the perimeter of that smaller hexagon.

Since a similar calculation can be applied to the two other smaller hexagons, the perimeter of the new shape is the perimeter of the large hexagon $+\frac{1}{3}$ of the total perimeter of the three smaller hexagons $= 60 + 96 \div 3 = 92\,\text{cm}$.

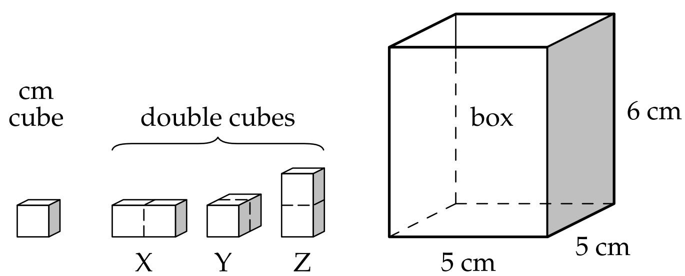
24 **A** $42\,\text{cm}^2$



The white triangle is right-angled and isosceles (since the diagonals of the square are parallel to the sides of the rectangle. So, cutting it in half into the right-angle gives two halves that may be formed into a square with sides of 3 cm (as shown on the left). Now the shaded area is the combined area of the square and rectangle minus twice the area of the white square $= 6 \times 6 + 3 \times 8 - 2 \times 3 \times 3 = 36 + 24 - 18 = 42\,\text{cm}^2$.

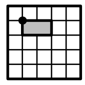
25 **B** 365

The illustration below shows a unit centimetre cube, the three possible ways of turning the double-cube around, and the box into which they will fit:

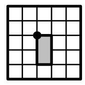


We shall consider how many positions there are for the double cube in the three ways in which it can be turned around: X, Y and Z.

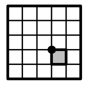
X Imagine that the double-cube X is lying on the bottom of the box. Looking down from the top, it might look like this diagram. From the possible positions of the dotted corner, it should be clear that there are $4 \times 5 = 20$ positions for X on the bottom of the box. However, there are 6 "layers" inside the box in which it could be placed in a similar position, and so altogether there are $20 \times 6 = 120$ positions when the double-cube is turned as X.



Y Now imagine that the double-cube Y is lying on the bottom of the box. As before, there are $5 \times 4 = 20$ positions for Y on the bottom of the box. Again, there are 6 "layers" inside the box in which it could be placed in a similar position, and so altogether there are $20 \times 6 = 120$ positions when the double-cube is turned as Y.



Z Finally, imagine that the double-cube Z is *standing* on the bottom of the box. There are $5 \times 5 = 25$ positions for Z on the bottom of the box. However, in any of those positions it could also be raised by 1 cm, 2 cm, 3 cm or 4 cm and still remain within the box. So, when the double-cube is turned as Z, there are altogether $25 \times 5 = 125$ positions.



So taking all three ways in which the double-cube might be turned, the total number of positions for the double-cube is $120 + 120 + 125 = 365$.

Some notes and possibilities for further problems

3 Where should the 5 be inserted to get the resulting number as large as possible? Use different numbers such as 11 111 or 98 765 or 50 505 and insert a 7 – what answers are there now?

5 What number will go in this box?

$$10 \times 30 \times 50 \times 70 \times 90 = 1 \times 30 \times 500 \times 7000 \times \boxed{}$$

8 If the lengths of the three sides of a triangle are a, b and c, and you find that $b^2 = a^2 + c^2 - a \times c$, then the triangle will have a 60° angle between the sides with length a and c. For the 8-7-5 triangle in the question, this is true because $7^2 = 8^2 + 5^2 - 8 \times 5$, and so the 60° angle is between the sides with lengths 8 cm and 5 cm. For this reason it is possible to fit six of these triangles around a point as shown with the star. Can you find other triangles with integer sides that also lead to a triangle with a 60° angle?

10 There are quite a few other countries with seven-letter names:

Albania, Algeria, Antigua, Armenia, Austria, Bahamas, Bahrain, Belarus, Belgium, Bermuda, Bolivia, Burundi, Croatia, Denmark, Ecuador, England, Eritrea, Estonia, Georgia, Germany, Grenada, Holland, Hungary, Iceland, Ireland, Jamaica, Lebanon, Lesotho, Moldova, Myanmar, Nigeria, Romania, Sarawak, Senegal, Somalia, St Kitts, St Lucia, Surinam, Tokelau, Tunisia, Ukraine and Vanuatu.

Do they all fit one of the five pie-charts in the question or do we need others?

What we know in English as a pie-chart has various different names (related to food) in other European countries: in Portuguese it is a *gráfico di pizza*, in Swedish a *tårtdiagram* and in French a *diagramme en Camembert* (named after the well-known cylindrical cheese).

11 Pupils may have used a variety of methods to solve this (and other questions). Ask them to discuss their methods and compare them.

19 Can pupils see why the shaded square in the bottom left-hand corner has the same area as the two smaller shaded squares combined?

21 Algebra can help one to arrive at a solution, though it also somehow hides the crux of the question.

Let j, k, l and l represent the ages of John, Kelly, Leo and Mila respectively. Then $k = \frac{j+l}{2}$, $l = \frac{k+m}{2}$ and $\frac{j+m}{2} = 11$, hence $2k + 2l = j + k + l + m = k + l + (j + m) = k + l + 22$, and so $k + l = 22$ and $\frac{k+l}{2} = 11$

25 The answer 365 is also the smallest number that can be written as a sum of consecutive square numbers in more than one way where the numbers squared are consecutive: $365 = 10^2 + 11^2 + 12^2 = 13^2 + 14^2$.

Not be outdone, 366 is the sum of four consecutive square numbers — which ones?

And finally: 364 is how many gifts are given altogether in the Christmas rhyme *On the first day of Christmas my true love sent to me*. Can pupils explain how to arrive at this total?