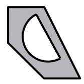
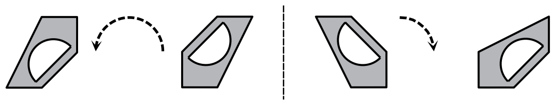
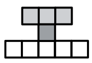
Primary Mathematics Challenge – February 2016

Answers and Notes

These notes provide a brief look at how the problems can be solved.
There are sometimes many ways of approaching problems, and not all can be given here.
Suggestions for further work based on some of these problems are also provided.

P1 **C** 327 P2 **C** 3

1	**D**	7	$2016 \times 1000 = 2\,016\,000$, which has 7 digits.
2	**D**	6	Given a mean of 1, there must be 20 brothers altogether. These are already accounted for by the 8 children with 1 brother and the 6 children with 2 brothers. So the remaining 6 children have no brothers.
3	**E**	$887 \div 8$	All the numbers to be divided are one less than a multiple of the number they are to be divided by. Hence the respective remainders are 3, 4, 5, 6 and 7.
4	**B**		The shape in B is the only one where the diameter of the semi-circle is parallel to the longest side of the pentagon. For the others:  E A C D. E is a 180° rotation of A; C is a vertical reflection of A; D is a 90° rotation of C.
5	**A**	25 000	A 90% decrease means 10% of 250 000 = 250 000 ÷ 10 = 25 000 of the lions remain.
6	**D**	D	If one of the statements is true, then the four remaining statements are false – this is precisely what statement D says.
7	**A**	130	The number of days = $16\,500 \div 125 = (16\,500 \div 1000) \times 8 = 132 \approx 130$
8	**E**		The shapes are all made from 9 squares, so the shape with a different perimeter from the others will be the one with a different number of internal joins between the squares: A, B, C and D have 10, while E has 8. Or we can just count the sides of the squares on the perimeter of each shape: A, B, C and D have 16, E has 20.
9	**D**	3	The value of the numerator is 1, while the denominator is $\frac{1}{3}$. Now $1 \div \frac{1}{3} = 3$.
10	**B**	90 cm^2	One can either work out the area of each shaded region, or think of the fraction of the whole square that is shaded. Given its four-fold rotational symmetry, we need only consider each smaller quarter square. Here the rectangle is half and the triangle a quarter of a half, so $\frac{5}{8}$ of each smaller square (and so the larger square) is shaded. The area of the large square is $(2 \times 6)^2 = 144\ \text{cm}^2$, and $\frac{5}{8}$ of $144 = 90\ \text{cm}^2$.
11	**B**	3	Dividing each of the numbers in the numerator and the denominator by 11 gives an equivalent fraction: $\frac{6+7+8+9}{1+2+3+4} = \frac{30}{10} = 3$
12	**B**	432 + 324	In options A and C, none of the numbers are multiples of 4; in option D, only one of the numbers (204) is a multiple of 4; in option E, both of the numbers are multiples of 4 (and of 8) and *do* have a total which is a multiple of 8. In option B, both numbers are multiples of 4, but their total 756 is not – because only one of them is itself a multiple of 8.
13	**B**	£9	If you choose one of every plate on the menu, you will get two of each item for £2.50 + £3 + £3.50 + £4 + £5 = £18. So one of each will cost £18 ÷ 2 = £9.

14 C 16

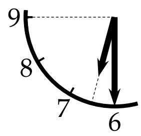
From the information we can compile and continue a table:

number of chameleons	number of flies eaten	time taken (min.)
6	8	4
6	4	2
6	12	6
2	4	6
8	16	6

so 8 chameleons will eat 16 flies in 6 minutes.

15 E 15°

At 6.30 pm, the minute hand points to the 6 and the hour hand to halfway between the 6 and the 7 (it is, after all, "half-past six"), as shown on the diagram on the right. The angle between each successive hour (here 6 and 7) is 360° ÷ 12 = 30°. So the angle between the two hands is 30° ÷ 2 = 15°.

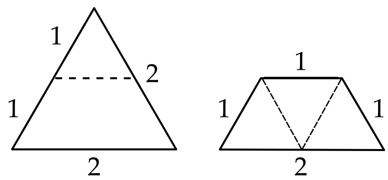


16 D 4

Since each human has two arms and two legs, the fact that in total there are more arms must be a consequence of the Martians each having more arms than legs. There are 91 – 47 = 44 more arms than legs, and since each Martian has 4 more arms than legs, there must be 44 ÷ 4 = 11 Martians. Since there are 47 legs and 33 of them are Martian, the other 14 legs must be human, and so there are 7 humans. Therefore there are 11 – 7 = 4 more Martians.

17 B 6 : 5

Let the sides of the equilateral triangle be 2 cm as shown on the right; so its perimeter is 6 cm. The trapezium can be split into three equilateral triangles, and so the length of the fold is 1 cm. The perimeter of the trapezium is therefore 5 cm.



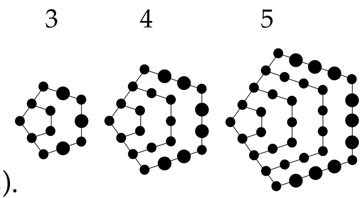
18 D 20%

The discount is £(10 800 – 8640) = £2160. As a percentage of the original price, this is

$$\frac{2160}{108\not{0}\not{0}} \times 1\not{0}\not{0}\% = \frac{2160}{108}\% = 20\%$$

19 C 117

It is evident from the sequence of numbers of mad teachers, (1, 5, 12, 22, 35, …), that the difference between each term increases by 3 each time, eg. 1 + **4** = 5, 5 + **7** = 12, 12 + **10** = 22, 22 + **13** = 35 and so on. Usually one should be reluctant simply to hope that a pattern continues, but here that hope is justified, as one can see comparing the diagrams for weeks 3, 4 and 5: each diagram grows by 4 extra 'corner' dots, but also on the 3 right-most edges the number of dots increases each week by an extra 1 dot on each edge (here shown as larger dots). Hence we can merely continue the sequence, 35 + **16** + **19** + **22** + **25** = 117. For a larger number of weeks (by which time *MTD* has become an epidemic), there is another method, and this is discussed in the Notes below.

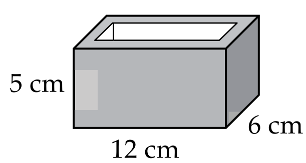


20 C 9

If Squirrel Nutcase had had one fewer nut when she divided them into piles of 2, 3 or 5 nuts, she would have no nuts left over, and so the number would have been a multiple of 2, 3 and 5. The number was therefore one greater than a multiple of 2, 3 and 5 (that is, one more than a multiple of 30) and also a multiple of 7. The only possibility less than 100 is 91, and so she must have eaten 9 nuts.

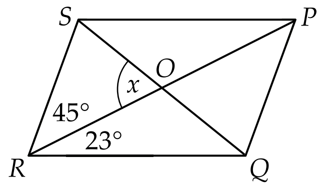
21 C 200 cm^3

The volume of the cuboid that forms the exterior of the box is 5 × 12 × 6 = 360 cm^3. Since the thickness of the plastic is 1 cm, the dimensions of the interior cuboid are 4 × 10 × 4, which has a volume of 160 cm^3. Therefore the volume of plastic is 360 – 160 = 200 cm^3.



22 **B** 3 We can find the last (units) digit by looking at the units digits of the four parts. We can quickly note that the units digits of 0^{2016} and 1^{2016} and 6^{2016} are 0, 1 and 6 respectively. Now then we have to look at powers of 2: $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$, and after this the units digits repeat ($2^5 = 3\underline{2}$, $2^6 = 6\underline{4}$, $2^7 = 12\underline{8}$, $2^8 = 25\underline{6}$, …). We notice that when the indices are multiples of 4 (eg. $2^4 = 16$, $2^8 = 256$, …) the units digit of the power of 2 is 6. Hence the units digit of $2^{2016} + 0^{2016} + 1^{2016} + 6^{2016}$ is the same as the units digit of 6 + 0 + 1 + 6, that is 3.

23 **A** 67° We shall refer to the point where the two diagonals cross as O. Given angle $SRQ = 45° + 23° = 68°$, and that RQS is isosceles (with $RQ = QS$), we have angle $RSQ = 68°$. So, considering the angles of triangle ROS, we find that $x = 180° - 68° - 45° = 67°$.



24 **A** 1 Two of the pieces of information relate to Chas, so we can try to relate the ages of the others to his age. Given Dave is 5 years older than Chas, but 3 years older than Benu, we can tell that Benu is 2 years older than Chas. Given also that Anil is 3 years younger than Chas, we can say that the sum of the ages of the four children is ($^-3 + 2 + 5 = 4$) more than four times Chas' age. So four times Chas' age plus four is twenty, and so Chas must be four. So Anil is just 1 year old.

25 **E** 3936 The first of the 41 numbers is either an even number or an odd one; we will look at each case separately.
If the first of the 41 numbers is even, there are 21 even and 20 odd numbers. Let the middle of these even numbers be n and then the even numbers are:

$$n - 20, n - 18, n - 16, \ldots, n - 2, n, n + 2, \ldots, n + 16, n + 18, n + 20.$$

Given that these have a sum of 2016, we have $21n = 2016$.
However, $2016 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 7$, and so $n = 2 \times 2 \times 2 \times 2 \times 2 \times 3 = 96$.
Oscar's list of 41 numbers will look like:

$$n - 20, n - 19, n - 18, \ldots, n - 1, n, n + 1, \ldots, n + 18, n + 19, n + 20, \qquad [*]$$

which has a sum of $41n = 41 \times 96 = 3936$.

Suppose now that there are 21 odd numbers and 20 even numbers. With the numbers labelled as in the starred list [*] above, the even numbers are

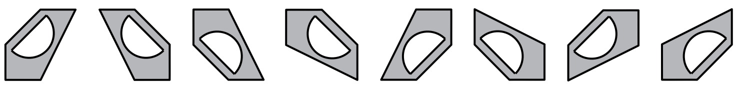
$$n - 19, n - 17, n - 15, \ldots, n - 1, n + 1, \ldots, n + 15, n + 17, n + 19,$$

and their sum is $20n$, which cannot equal 2016 since n is a whole number.

Thus the first of the 41 numbers must have been even, and the sum of all 41 numbers is 3936.

Some notes and possibilities for further problems

Q4 One could investigate how many shapes are reflections in vertical or horizontal axes, or rotations of multiples of 90° (or reflections of such rotations) of shape A? There are eight (including A itself):



Q7 Astonishing data on animals is available from many websites, including the Guinness Record site at www.guinessworldrecords.com. There we learn that

- the record time for a dog on a skateboard to cover 100 metres is 19.65 seconds,
- the loudest cat purr is 67.8 decibels, equivalent to the noise of a two-person conversation, and
- the record number of canned drinks opened in 1 minute by a parrot is 35.

In terms of sheer population, the United Nations publishes data on numbers of animals in the world, or country by country (http://faostat.fao.org): in 2013 there were (roughly)

- 1 335 312 000 ducks in the world,
- 395 000 horses in the UK, and
- 20 000 camels in Uzbekistan, as well as
- (at least) 10 000 000 000 000 000 dust-mites globally.

Q11 The simplification to get from

$$\frac{66+77+88+99}{11+22+33+44} \quad \text{to} \quad \frac{6+7+8+9}{1+2+3+4}$$

could look like a form of simplification by simply crossing off digits, a process which in general is unwise. Except for certain fractions, referred to as "anomalous cancellations", which do give a correct answer.

For instance, $\frac{19}{95} = \frac{1\not{9}}{\not{9}5} = \frac{1}{5}$ or $\frac{106}{265} = \frac{10\not{6}}{2\not{6}5} = \frac{10}{25} = \frac{2}{5}$ or $\frac{1019}{5095} = \frac{101\not{9}}{50\not{9}5} = \frac{101}{505} = \frac{1}{5}$

More of these can be found at http://mathworld.wolfram.com/AnomalousCancellation.html

Q12 Patrick's idea will never work in general. Perhaps pupils could put forward their own hypotheses about multiples and test them out on each other.

Q19 There is a formula to enable these 'pentagonal' numbers to be calculated without adding on from the previous one: the *n*th pentagonal number is given by the formula

$$p(n) = \frac{3n^2 - n}{2}$$

With around 450 000 teachers in the UK, it will take around 548 weeks (or 10.5 years) before all teachers are mad!

For other polygons, there are similar formulae to the one above:

for a polygon with s sides, the nth polygonal number is $p_s(n) = \frac{(s-2)n^2 - (s-4)n}{2}$

More details on these numbers can be found at
http://mathworld.wolfram.com/PolygonalNumber.html

Q22 We have seen that the units digits of consecutive powers of 2 follow the pattern 2, 4, 8, 6; similarly

- for powers of 3, the pattern is 3, 9, 7, 1.
- for powers of 4, the pattern is 4, 6.
- for powers of 5, the units digit is always 5, and similarly for 6.
- for powers of 7, the pattern is 7, 9, 3, 1.
- for powers of 8, the pattern is 8, 4, 2, 6.
- for powers of 9, the pattern is 9, 1.

Pupils could be encouraged to investigate the patterns from other powers – there are, in fact, patterns for the final *two* digits of powers – as there has to be, as there are only 100 possibilities (…00, …01, …, …98, …99) for these and so they must repeat at some stage. For example, the powers of 6 have the following final two digits: $06, \underline{36}, 2\underline{16}, 12\underline{96}, 77\underline{76}, 46\,6\underline{56}, 279\,9\underline{36}, 1\,679\,6\underline{16}, \ldots$

Related to these ideas is the instance of the awesomely amazing number 2 646 798
which is the sum of consecutive powers of its digits: $2^1 + 6^2 + 4^3 + 6^4 + 7^5 + 9^6 + 8^7 = 2\,646\,798$

The PMC is organised by The Mathematical Association

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