Primary Mathematics Challenge – February 2017

Answers and Notes

These notes provide a brief look at how the problems can be solved.
There are sometimes many ways of approaching problems, and not all can be given here.
Suggestions for further work based on some of these problems are also provided.

P1 **E** 20 P2 **B** 8.50 p.m.

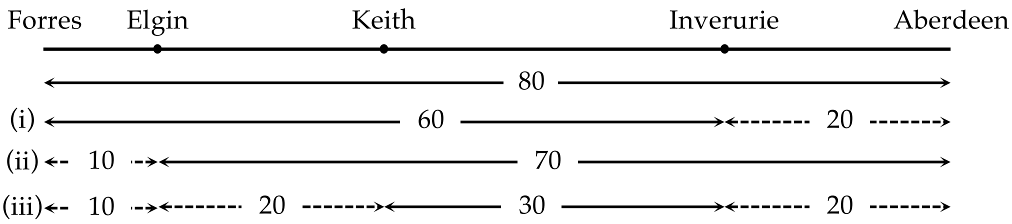
1 **B** 40 Estimating 8848 ÷ 224 as 8800 ÷ 220, we find Mount Everest is 40 times higher.

2 **C** 66 At a constant speed if one travels 11 miles in 10 minutes, one would travel 11 × 6 = 66 miles in an hour.

3 **C** 17 Buses leave the stop at 9.12 pm, 9.32 pm and 9.52 pm. Arriving at 9.35 pm, three minutes too late for the 9.32 pm, I would have to wait 17 minutes until 9.52 pm.

4 **C** 55 cm The rectangle is 9 radii long, and 2 radii high, hence 22.5 cm long by 5 cm high. Thus its perimeter is 2 × (22.5 + 5) = 2 × 27.5 = 55 cm.

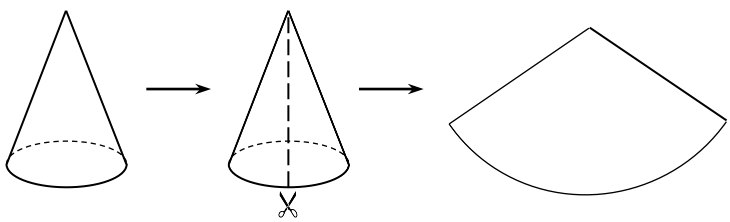
5 **B** 20 The diagram below illustrates the information we start with:



We can deduce as shown in stage (i) above that Inverurie to Aberdeen is 80 – 60 = 20 miles, and stage (ii) that Forres to Elgin is 80 – 70 = 10 miles. Hence, in stage (iii), Elgin to Keith is 80 – 10 – 30 – 20 = 20 miles.

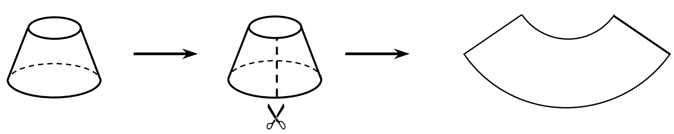
6 **D** £6 The initial ratio of the children’s money is 3 : 1 : 4. When Miguel shares half of his portion, the ratio becomes 3 + 1 : 1 + 1 : 4 – 2 = 4 : 2 : 2 = 2 : 1 : 1. Given that Clara now has £6 more than Miguel, Pablo now has £6.

7 **A**  One could consider the shape of the white *collar* around the cone by thinking first about a whole paper cone – here is an idealised, geometrical version:



We can cut from the circumference of the base to the highest point (*apex*) and flatten the resulting piece into a pie-shaped piece (a *sector*) – it has this shape as the distance from the apex to the base is the same for any point on the base.

Now we can think about the shape of the *collar* when laid out and flattened:



8 **D** 20 The length of the side of a shaded triangle is twice that of a white triangle. Therefore the area should be 4 times greater, hence $4 \times 5\ \text{cm}^2 = 20\ \text{cm}^2$.

9 **D** 36

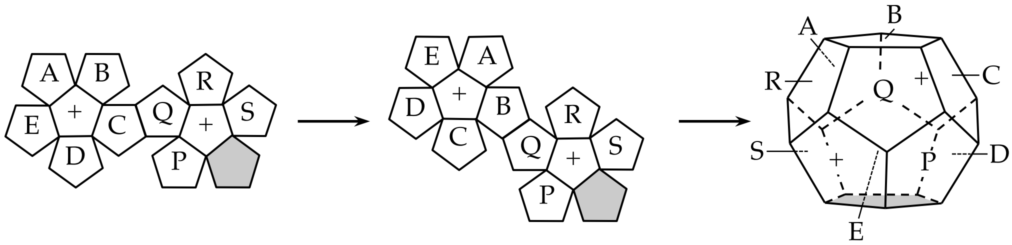
Let Herbie's age this year be h years. Then Gareth's age this year is $7h$ years.
In 6 years' time, their ages will be $h + 6$ years and $7h + 6$ years respectively.
We are told that $7h + 6 = 4(h + 6)$. This gives

$$7h + 6 = 4h + 24 \text{ so that } 3h = 18 \text{ and } h = 6.$$

So the present ages are 6 and $7 \times 6 = 42$, giving a difference of 36 years.

10 **B** B

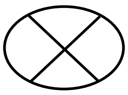
We label the two faces in the centre of the two rings of pentagons with a + sign and the remaining faces P, Q, R and S. It should be clear that the faces marked with a + will be opposite each other on the constructed dodecahedron. Since face B will join face Q also, we can redraw the net as shown below, leaving B on top:



11 **C** 35 million

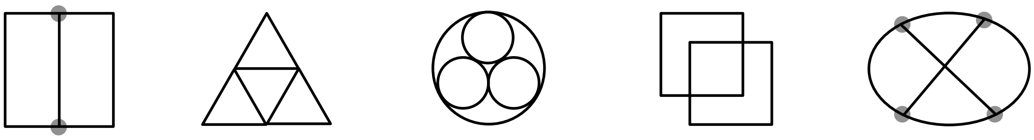
Given 4% of Canada's population is 1.4 million, we find that 1% is 1.4 million ÷ 4 = 350 000. Thus the population is 350 000 × 100 = 35 000 000.

12 **E**



It is likely that pupils will try each of the shapes before deciding that only E has no chance of being drawn without going over any line more than once or lifting the pencil off of the paper – in mathematical language, it is not *traversable*. However, it is possible to analyse the shapes to justify both why one always fails to draw E but also why one can succeed with the other shapes. In short, if it is possible to trace around a shape completely without going over any line more than once, we must look at where lines meet – *vertices* or *nodes*. If the shape has only two vertices where an odd number of lines meet, then it is traversable. This will be further explained in the Notes below.

When, for each of the graphs, we highlight the vertices which have an odd number of lines meeting – *odd vertices* – then we see that A to D have two odd vertices each whereas E has four:



Hence the only shape of the five that cannot be drawn (without going over a line twice or taking pen off the paper) is shape E.

13 **E** 36

18		j
	6	k
12		2

The product of each row, column and diagonal is 18 × 6 × 2 = 216. If we label two of the unfilled squares j and k, as shown on the left, we can work around the grid.

From the incomplete diagonal we have

$$12 \times 6 \times j = 216, \text{ so } j = 3.$$

From the rightmost column we have $3 \times k \times 2 = 216$, so $k = 36$.
The completed grid is shown here on the right.

18	4	3
1	6	36
12	9	2

14 **D** £67

Without the £10 fixed charge, the 2m radiator would have cost £38.
Hence the 3m radiator with the same height would cost £38 ÷ 2 × 3 = £57.
Adding the £10 fixed charge gives a total cost of £67.

15 **A** 1001

The prime factors of 209 are 11 and 19, so these must have been the reversed numbers that Miruna multiplied. The correct multiplication was 11 × 91 = 1001.

16 **C** $\frac{5}{24}$ The angles at the centre of the circle inside each of the two triangles are 45° and 60°. These leave a (180 – 45 – 60)° = 75° sector that is shaded. As a fraction of the whole circle 75° represents $\frac{75}{360} = \frac{5}{24}$.

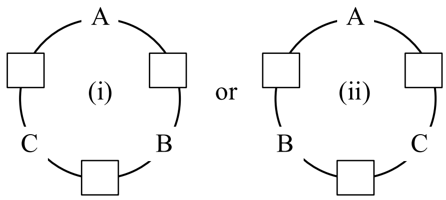
17 **E** 6 cm The area of a parallelogram is given by base × perpendicular height. Its base length is 14 cm and its area is 98 cm^2, its perpendicular height is 98 ÷ 14 = 7 cm. The area of a triangle is given by $\frac{1}{2}$ × base × perpendicular height, so the length of the base is 21 ÷ (7 ÷ 2) = 6 cm.

18 **D** 120 g Let the mass of one plum and the mass of one cherry be p g and c g respectively. Then $2p + c = 80$ and $2c + p = 70$. Hence $3p + 3c = 150$, and $p + c = 50$. But since $2p + c = 80$, we can deduce that $p = 80 – 50 = 30$, and so four plums weigh 120 g.

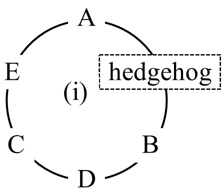
19 **A** Louisiana Looking at each state name for the number of vowels out of the total number of letters we have (with rounded percentages): Louisiana 6 out of 9 = two-thirds (67%); Georgia 4 out of 7 (57%, and more than half); California 5 out of 10 (50%); Oregon 3 out of 6 (50%) and Mississippi 4 out of 11 (36%). So Louisiana has the greatest proportion of vowels in its name, and so is the word from which one is most likely to pick a vowel at random.

20 **D** 8 Let the values of a silver spoon and a gold ring be £s and £r respectively. Then we know $12s = \frac{3}{4} \times 6r$, and dividing by 6, $2s = \frac{3}{4}r$, and so $8s = 3r$. So three gold rings are worth eight silver spoons.

21 **B** Bea If Alf and Bea, Bea and Cal, and Cal and Alf do not sit next to each other the three others must sit between them. So, looking down on the table we have two possible arrangements:

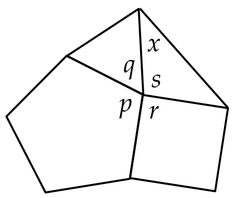


Given that Dev sits on Cal's right, he can either sit between Cal and Bea as in (i), or between Cal and Alf (ii). However, the girl with the hedgehog is on Alf's left, and so in (ii) Dev could not sit on Cal's right (without sitting on the hedgehog). So arrangement (i) is correct and Bea sits on the left of the girl with the hedgehog.



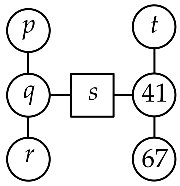
22 **B** 40 The ratio by weight of carrot : cabbage : yoghurt = 2 : 1 : 0.5 which we can double to give 4 : 2 : 1. We can see that here 2 represents the amount of cabbage and we want 2 kg of cabbage, so there will be 4 + 2 + 1 = 7 kg of coleslaw altogether. Therefore the number of pots is 7000 ÷ 175 = 40.

23 **A** 39° Labelling the interior angles of the regular pentagon, equilateral triangle and square as shown on the diagram on the right, we know $p = (180 – (360 ÷ 5))° = 108°$, $q = 180° ÷ 3 = 60°$ and $r = 90°$. This leaves s to be (360 – 108 – 60 – 90)° = 102°. The second triangle is isosceles as two of its side-lengths are the same as the side-lengths of the equilateral triangle and the square, and thus of the pentagon. Therefore $x = ((180 – 102) ÷ 2)° = 39°$.



24 **B** 90 There are two stages of the speed/time graph to interpret – the time between Kelly's start and 4 seconds later and the time from then until the end of the race 28 seconds later. In the first stage, her average speed is 1.5 m/s and therefore in 4 seconds she will cover a distance of 4 × 1.5 = 6 metres. In the remaining 28 seconds she runs at a constant 3 m/s, so for a distance of 28 × 3 = 84 metres. So altogether she runs 6 + 84 = 90 metres to finish at F.

25 **E** 61 Because the 7 consecutive prime numbers include 41 and 67, and as there are 5 prime numbers in between, the 7 prime numbers must be 41, **43**, **47**, **53**, **59**, **61** and 67. We shall label the circles and square as shown in the diagram and consider the total of the three lines.



We know that:

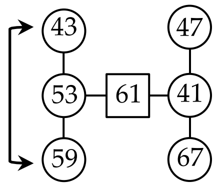
$$p + q + r = q + s + 41 = t + 41 + 67$$

and so $p + r = s + 41$ [1] and $q + s - t = 67$ [2].

From equation [1], if s is 43, 47 or 53, then the value of $p + r$ will be 84, 88 or 94 respectively – but no pair of the four primes left have these totals, so s is 59 or 61.

If $s = 59$, $p + r = 100$, a total which could arise only from 47 + 53. However, from equation [2], the difference of q and t is $67 - 59 = 8$, which is not possible with the remaining two primes 43 and 61. So s cannot be 59.

Given that $s = 61$ is the only option left, we have $p + r = 102$, a total which could arise only from 43 + 59. From equation [2], the difference of q and t is now $67 - 61 = 6$. Given the two primes left are 47 and 53, we can conclude that $q = 53$ and $t = 47$, and complete the diagram as shown on the right, where the arrows indicate that the positions of the 43 and 59 can be swapped around.



Some notes and possibilities for further problems

Q7 What shape would the white part of a square-based traffic 'cone' look like? Or even a hexagonal-based 'cone'? We could use the phrase 'traffic bollard' rather than cone for these other shapes!

Q12 A shape is *traversable* if the shape has only two *odd* vertices. This is because every time you go through a vertex you create two lines joined to it, and so (unless the point is the starting point or the end point) each point must have an even number of lines joined to it. Because you cannot take your pen off the paper, there can be at most two points that are starting or end points – in option A there are two, and in options B, C and D there are no odd vertices (so with these you can start wherever you wish and end at the same point). Ask pupils to create/design their own shapes which can and which cannot be drawn without lifting pen from paper or going over a line twice.

Q13

ab^2	1	a^2b
a^2	ab	b^2
b	a^2b^2	a

The square in the question has the smallest possible magic product and was published by the Frenchman G. Pfeffermann in 1893. You can make lots like this using the template on the left (just choose values for a and b). It is also possible to make squares where each row and column have the same total **and** the same product (though you have to forget about the diagonals) – the one on the right was made/discovered by Lee Morgenstern in 2007.

110	72	63	80
64	105	66	90
81	88	100	56
70	60	96	99

Q14 The idea of a fixed charge plus cost per item is a standard way for businesses to price their products. Imagine, for instance, a small cup-cake factory which costs £500 a week in machinery and wages to run. The ingredients for each cup-cake cost 5p. Each cup-cake is sold for 55p. How many must be made and sold to cover the cost?

Q18 It may be instructive for pupils to make up more problems like this. But they will have to be careful, because things can sometimes go wrong (as demonstrated in the following problems):

What is the weight of a plum if:

a) two plums and one cherry weigh 80 g, and four plums and two cherries weigh 160 g? or

b) two plums and one cherry weigh 80 g, and four plums and two cherries weigh 200 g?

The PMC is organised by The Mathematical Association

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