MAthematical PiE

Summer 2021 **Notes for . . . . No 213**

I'm honoured to have this opportunity to revisit a few old favourites!

Coded Magic Squares

These took me a long time to produce and I'm pleased with them. The first two squares are easy to solve since the complete diagonals given allow you to work out the 'magic total' and the rest follows quickly. In the third, the top row must total (26 + ***?***) which leads (6 + ***?***) in the bottom left cell. So the centre cell must be (26 + ***?***) - 11 – (6 + ***?***) = 9, and the bottom right cell (26 +***?***) - (6 + ***?***) - 11 = 3. Now we have a diagonal 15 + 9 + 3 = 27, making *?* = 1. The rest is easy:

4	9	2
3	5	7
8	1	6

2	13	9
15	8	1
7	3	14

15	1	11
5	9	13
7	17	3

In both of the coded cases the missing numbers are 1, 3, 7, 9, 13: ACGIM …...**MAGIC**!
In similar fashion, we have a complete column in the fourth square but in the final one a ***?*** is needed somewhere. I'd recommend placing it in the cell below the V(22)

26	3	2	23
6	19	20	9
18	7	8	21
4	25	24	1

2	21	9	22
23	8	20	3
24	7	19	4
5	18	6	25

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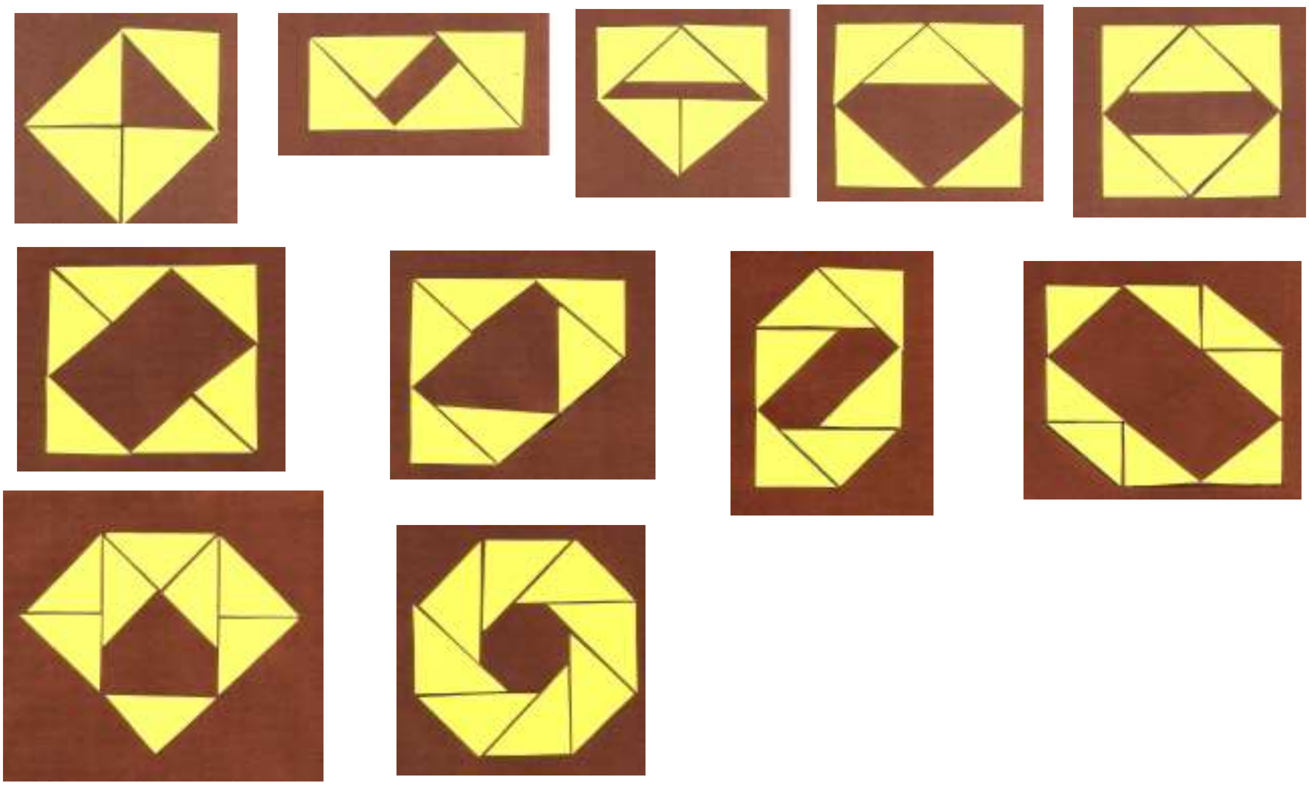
he final words are
WIZARDS and **WITCHES**

Total Squares and Utter Squares

A straightforward number search. Here are the three-digit total squares (and their square digit-totals):
100 (1), 121 (4), 144 (9), 169 (16), 196 (16), 225 (9), 324 (9), 400 (4), 441 (9), 484 (16, 529 (16) 900 (9), 961 (16).
The utter squares from this list (and their square roots with square digit totals) are
100 (10, 1), 169 (13, 4), 324 (18,9), 484 (22, 4), 961 (31, 4).

Shape and Space

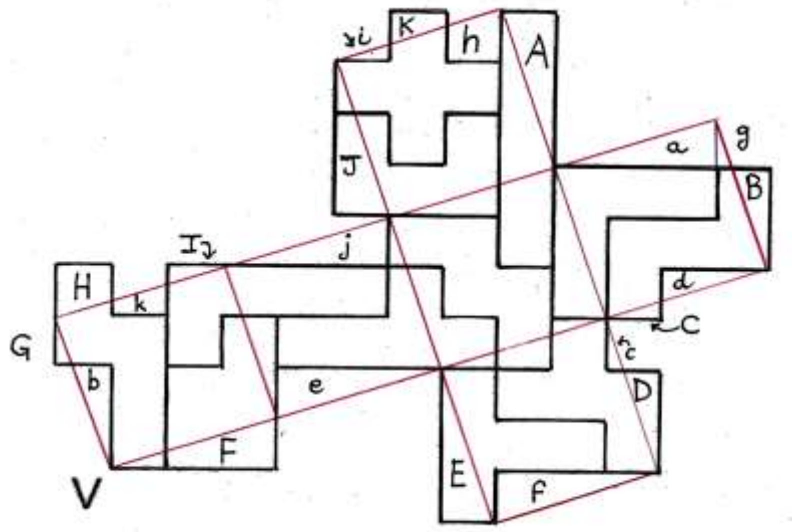
This simple exercise in the vocabulary of shape should be accessible to even our youngest readers. In the classroom I would very much recommend the use of a base card of contrasting colour to the triangles as shown here.



There will be many possible answers, these are just illustrations. Discussion of the relative merits of different answers is valuable.

Pythagoras Meets the Pentominoes?

This has strong links with dissections of shapes to rearrange into squares: the twelve pentominoes combine in an area of 12 x 5 = 60 squares , so each face of the resulting cube must have an area of 10 squares. This means that the edges of the cube must be the square root of 10, which suggests folds in the directions (3, 1) and (-1, 3) since $1^2 + 3^2 = 10$:



Strictly speaking the pieces 'added' to the pentomino outline are not 'flaps' since they do not need folding. All we need to do is cut around the combined outline, fold the red lines down and paste A to a, B to b, and so on..The photo shows 'one that the editor made earlier'.

ABBA

Trying a few palindromes should soon indicate that they are all multiples of 11. Personally, I would use this to introduce a proof by induction: it is easy to see that the As can be changed by adding 1001 to a palindrome and that the Bs can be changed by adding 110. Both these are multiples of 11. So if our first palindrome is a multiple of 11, so will the resulting two be. Then we can repeat the process as often as we like to get all the possible four-digit palindromes In other words, if we can find one 4-digit palindrome that is a multiple of 11, so will all all the others be We've already got a good low starter: 1001.

Algebraists might think it simpler to say ABBA = A x 1001 + B x 110 = 11(91A +10B).

Eight Tetrahedra

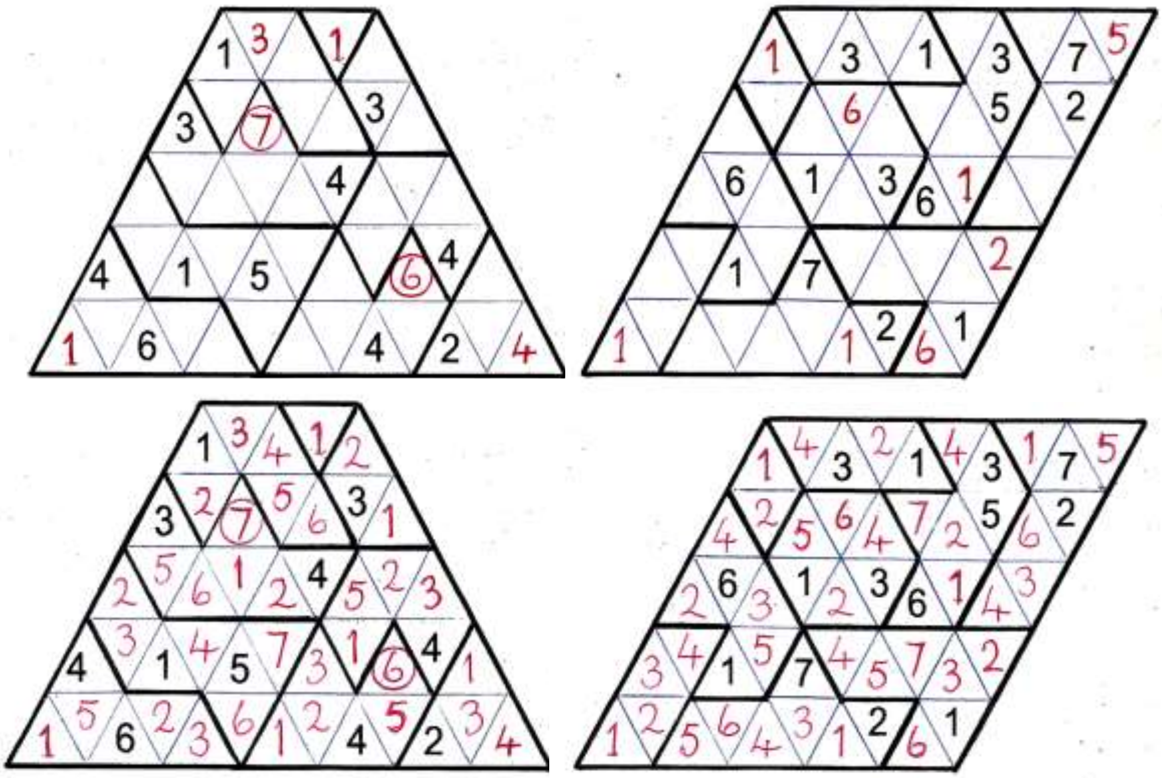
This unusual model-making is fiddly but the result is worth it. Accuracy of the triangles, folds, and the placement when glueing are all-important. My earlier models were fixed with a 'gluestick' but this can be difficult. For this one I used PVA and a brush:

Four Mathericks

These 'brief biographies" in limerick form have been a regular feature.

Isolation

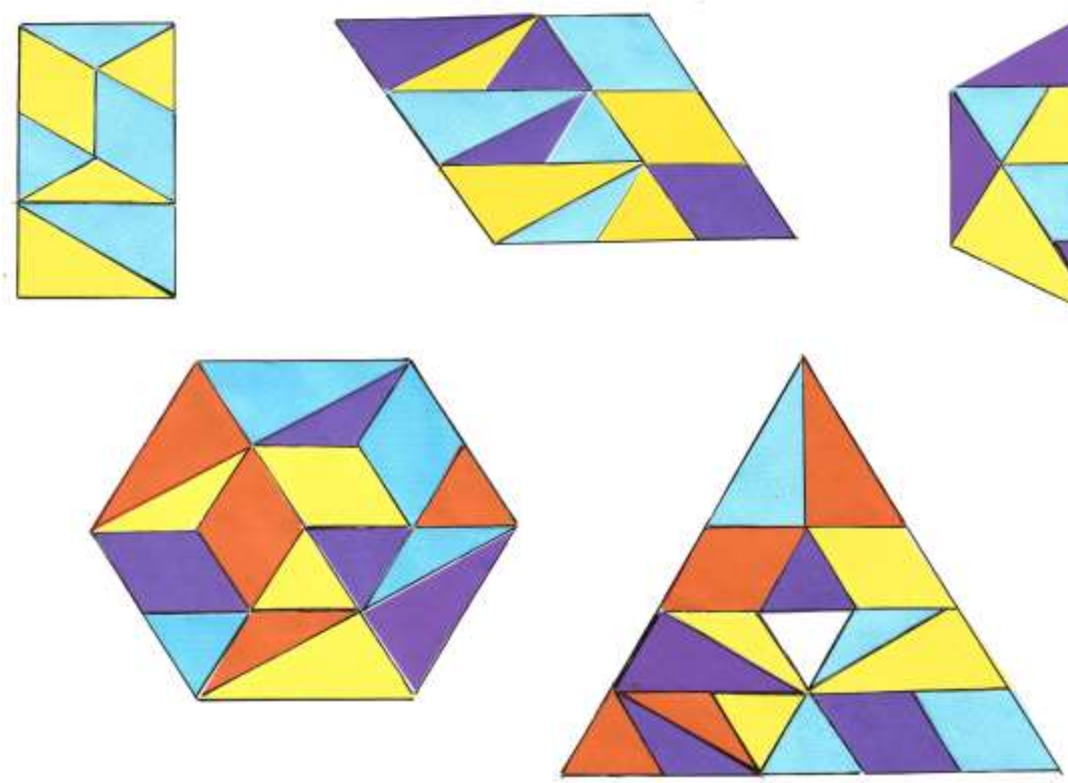
This is my 'isometric' version of *Suguru*, a type of puzzle that I prefer to *Sudoku*. With limited space for comment, the best help I can give is to point out that the numbers in red and the two in circles tn the upper diagrams should make a start.



Alphapower

There is only one number which when raised to 'the power of itself' gives a three-digit result: 4^4 =256, giving P = 2, I = 5 and E = 6 Then ii) gives L= 3. and iii) 2^Z =5M2 which must be 2^9 =512: Z = 9 and M = 1. Finally U = P + I = 2 + 5 = 7 so $E^U = 6^7$ = 279936 = PUZZLE.

Hexsectional



These are unlikely to be unique. In the later ones, some pieces have to be turned over.

Find the Sextuplets 213, 312, 324, 435, 543,and 768.

Literally Cyphered

We can see that O and G must be 2 and 3, although in which order is not certain. In either case T = 6, I = 5, and {M, S} = {8, 9}. Since M = I + O it must be 7 or 8, Hence it's 8 and S is 9. We end up with 0123456789 = ALGORITHM.

Reverse Gear

This messes with your mind! Call the number ***n*** and the product ***p***. The first digit of ***n*** must be less than 2, since otherwise ***p*** would be a six-digit number. So it is 1, meaning the first digit of ***p*** (and the last of ***n***) must be 9. The second digit of ***n*** must be less than 2. Trying 1 leads to inconsistencies but putting 0 here means the penultimate number of ***p*** must be 0, forcing the penultimate digit of **n** to be 8, then the centre digit 9 ;

p 9 __ __ __ 1 9 __ __ 0 1 9 8 __ 0 1

n 1 __ __ __ 9 1 0 __ 8 9 1 0 9 8 9

The final answers are 10989 and (using similar reasoning), 21978.

Can you use similar methods to find a six-digit number ABCDEF which is converted into BCDEFA when multiplied by 3?

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