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Diamonds are Forever: Tom O'Beirne's Polyiamonds

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Supporting mathematics in education

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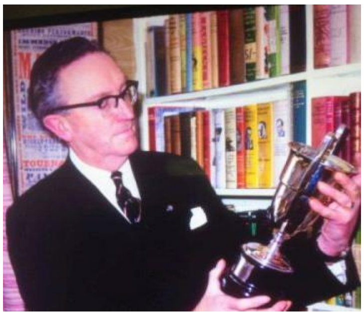
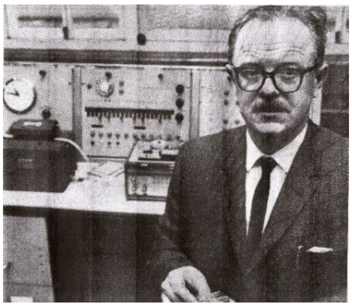
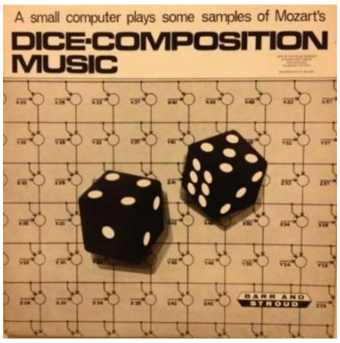
Sixty years ago, Thomas H. O'Beirne (1910–1975), known as Tom O'Beirne, was a mechanical engineer at Barr and Stroud Ltd., an optical engineering firm based in Glasgow, which produced binoculars and rangefinders for the Royal Navy and much else besides. He was also a leading authority on fairgrounds and circuses, later leaving a huge collection of photographs, cinefilm, posters and other memorabilia to the national archive for such material at the University of Sheffield. And if that were not enough, O'Beirne sported a substantial reputation as a recreational mathematician. To this day, that reputation rests in part on his invention of polyiamonds, one of the subjects of this essay.





Tom O'Beirne, with SOLIDAC in 1969



O'Beirne had an astonishing life and career. Born and raised in Glasgow, he attended his local university and graduated in mathematics and physics in 1938. During World War II, he was attached to the Royal Naval Scientific Service and when it was over joined a team at the Ordnance Survey, moving to Barr and Stroud in 1949 as chief mathematician.

During the early 1960s, Barr and Stroud attempted to diversify into electronics. Indeed, between 1958 and 1963, the company worked with Glasgow University to produce Scotland's first 'mini-computer' (the size of a desk). SOLIDAC (= **SOLID**-state **A**utomatic **C**omputer) was designed so that undergraduates could have first-hand experience of what was then the latest technology. O'Beirne was much taken with this machine and within a couple of years was using it to produce classical music with a stochastic twist.

Incidentally, the music is still available online at https://chipflip.wordpress.com/2020/02/25/a-small-computer-plays-some-samples-of-mozarts-dice-composition-music/
Are the sounds similar to that of a clarinet, as the blarb on the back cover suggests?

Bill Finlay, a former lecturer at Glasgow University, had an encounter with Tom O'Beirne at an impressionable age:

> 'In 1965 a public lecture was given by Tom (T.H.) O'Beirne, on the topic *Music, Numbers, and Computers* (the *Bull. Inst. Math. Appl.* later published a paper with the same title). Tom O'Beirne was a 'character' of the old school, such as one seldom encounters these days … Since music, numbers, and computers were three of my great interests, I attended his lecture. At the end of the talk Tom invited the audience to a demonstration. I took part, and was allowed to flick handswitches and enter numbers on the SOLIDAC console, which had a telephone dial for the easy input of decimal data. This was quite a thrill for a nerdy 17-year-old, half a century ago!

‘Thanks to Tom’s virtuoso programming in machine code, SOLIDAC was capable of impressive feats in the automated composition and performance of music. This resulted in the release of an LP album that was reviewed in *Gramophone*. In later years Tom joined the Department of Computing Science at Glasgow University, where he lavished care on SOLIDAC, keeping it operational well beyond its natural lifespan.’

I came across Tom O’Beirne’s name for the first time four years ago when I was looking into the founding of the Scottish Mathematical Council, a body that supports mathematics learning and teaching in Scotland. He had been one of the original members back in 1967 and when I was doing my research I was the Council’s Chair. I am grateful to Ian Anderson, another former SMC Chair, for letting me know that O’Beirne was also a prominent member of the Mathematical Association locally in Glasgow. He addressed the Glasgow Mathematical Association, or GMA (a sort of affiliate branch, no longer in existence) on at least three occasions, speaking on

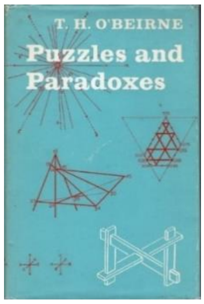
- ‘Some geometrical recreations’, 16 April 1953,
- ‘Practical mathematics’, 8 October 1959,
- ‘From Mozart to the bagpipes, with a small computer, 8 January 1970.

The subject of the first provides evidence that O’Beirne had been interested in recreational mathematics for a long time before he began writing for *New Scientist*. The second was his Presidential Address to the GMA and the third showcased SOLIDAC’s music-making capacity.

As a recreational mathematician, Tom O’Beirne left a legacy few could match. His book, *Puzzles and Paradoxes*, was published by Oxford University Press in 1965. It contains a ‘rearranged and amplified collection of some of the articles in a series … which appeared from January 1961 to February 1962 in the periodical *New Scientist*.’ With the articles and book came some small celebrity and he presented some of his puzzles on the BBC and on Dutch television.

I picked up a copy *Puzzles and Paradoxes* on the internet and was immediately taken by the comments on the nature of puzzles in the preface.

‘An arbitrary multiplication of complications is not the best recipe for a puzzle, as we see it: puzzles and their solutions should rather derive a maximum of effect from an economy of means. Both can then have an aesthetic element — charm, elegance, beauty — which we wish to make apparent; but some of this is absent when solutions give no suggestion of how they could originally have been obtained.’



Those readers who are aware of Martin Gardner’s books on recreational mathematics won’t be surprised by what’s in *Puzzles and Paradoxes*, but strangely it does not contain the three ideas for which O’Beirne is now, perhaps, best remembered:

- Algorithm for finding the date of Easter
- Polyiamonds
- O’Beirne’s Cube.

Easter Sunday is defined as the first Sunday after the first full moon after the vernal equinox. Perhaps it is no surprise therefore that numerous complicated procedures have been developed for finding its date, including the algorithm Gauss devised using modular arithmetic. A simpler algorithm, valid for just the 20th and 21st centuries, was published by Tom O’Beirne in 1966 (O’Beirne, 1966). According to Martin Gardner, ‘O’Beirne found he could memorize his procedure and as a party stunt give the date of Easter for any year during the relevant period by making all the calculations mentally’ (Gardner, 1997, p. 238).

O’Beirne’s algorithm has seven steps:

1. Call the year Y. Subtract 1900 from Y and call the difference N.
2. Divide N by 19. Call the remainder A.
3. Divide $(7A + 1)$ by 19. Ignore the remainder and call the quotient B.
4. Divide $(11A + 4 - B)$ by 29. Call the remainder M.
5. Divide N by 4. Ignore the remainder and call the quotient Q.
6. Divide $(N + Q + 31 - M)$ by 7. Call the remainder W.
7. The date of Easter is $25 - M - W$. If the result is positive, the month is April. If it is negative, the month is March, interpreting 0 as March 31, -1 as March 30, -2 as March 29 and so on to -9 for March 22.

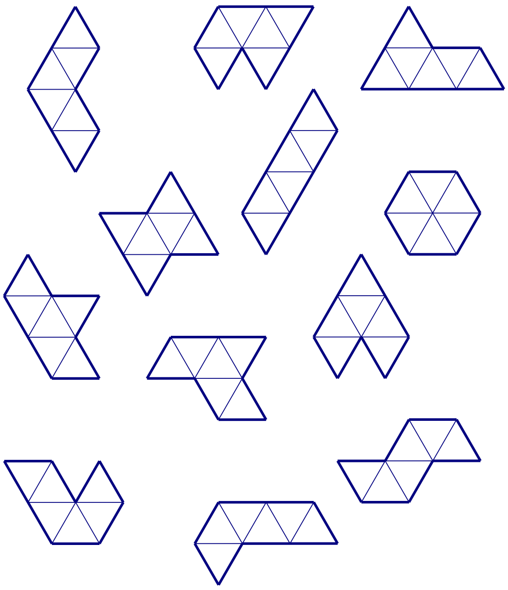
As an example, let’s see if we can find the date of Easter Sunday next year (2022)

1. $Y = 2022,\ N = 122$.
2. $122 \div 19 = 6$, remainder 8. $A = 8$.
3. $7A + 1 = 57$. $B = 3$.
4. $11A + 4 - B = 88 + 4 - 3 = 89$. $M = 2$.
5. $N \div 4 = 30$, remainder 2. $Q = 30$.
6. $N + Q + 31 - M$
   $= 122 + 30 + 31 - 2$
   $= 181$.
   $181 \div 7 = 25$, remainder 6. $W = 6$.
7. Easter Sunday has value $25 - 2 - 6 = 17$, and is on 17 April 2022.

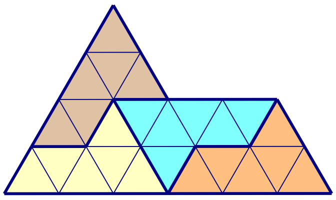
Question: What was the date of Good Friday 2019?

Polyiamonds are O'Beirne's response to Solomon Golomb's polyominoes. Sometime in 1959 he simply asked himself the question, "What if I were to replace squares with equilateral triangles?" O'Beirne was already in touch with Richard Guy, a British mathematician who went on to have a stellar career in Canada, collaborating with Erdös and John Conway (and dying in 2020 in his 104th year). Guy relates the story of O'Beirne's visit to the Guy family home in 1960 armed with the hexiamond puzzle. The gist of it is that "no one went to bed for about 48 hours." I presume that the puzzle to which Guy was referring is one which is typically fashioned in wood, and still selling well today.

The twelve possible hexiamonds are shown in more detail below.



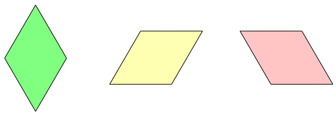
Do they all tessellate? One that does, in a rather nice way is the design in the top right corner, referred to as the sphinx. Four of them can be arranged so as to produce a larger version of itself. And of course, four of the larger design can also be so arranged, ad infinitum.



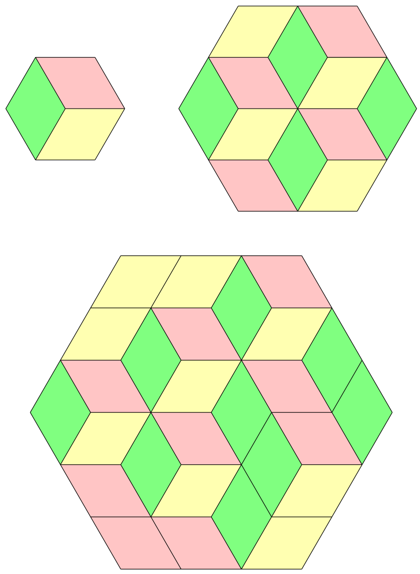
Task

Find all the pentiamonds and heptiamonds.

The polyiamond we know informally as a diamond is a rhombus made up of two equilateral triangles. In France the shape is known as a *calisson* because it somewhat resembles a confectionery of that name. Now there is a lovely result in elementary geometry about calissons. Imagine we are arranging calissons of three colours (green, yellow and pink, say) into a hexagonal design. We always place them in these orientations:

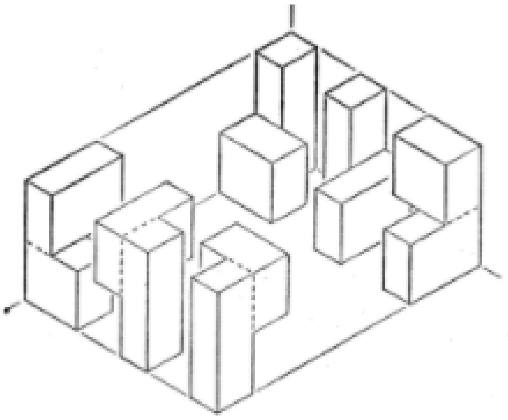


Here are some possibilities:



What do you notice about the number of green, yellow and pink calissons? Now explore different arrangements of calissons and different sizes of hexagon. Does what you've discovered still hold? Further information can be found in David & Tomei (1989).

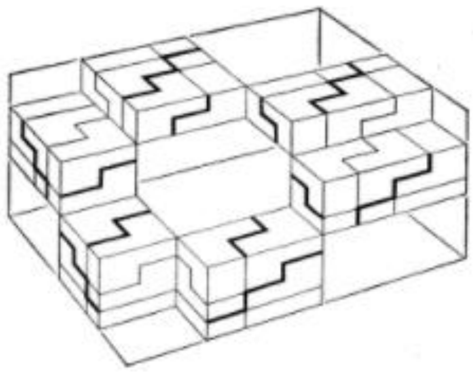
Finally, we come to what is probably O'Beirne's finest invention, a cube which splits into components in numerous ways and then reassembled into other cuboids. Apparently, it was invented while considering the best design for standard boxes to fit optimally in a delivery lorry when bricks. Perhaps it is only eclipsed by the invention of the Soma Cube by Piet Hein, Pentominoes by Golomb and Rubik's Cube. O'Beirne's units are $6 \times 4 \times 3$ bricks. It's rather obvious that $2 \times 3 \times 4 = 24$ of them would fit inside a cube of side 12. But what O'Beirne did was to stick the bricks together in fours. This gives the six components shown below, taken from the original paper (O'Beirne, 1961).



These components can be translated (i.e. moved without rotation) to make a closed cycle of six different cuboids as shown below, each move having two steps:

$$12 \times 12 \times 12 \to 8 \times 24 \times 9 \to 18 \times 8 \times 12 \to 16 \times 18 \times 6 \to 16 \times 12 \times 9 \to 12 \times 24 \times 6,$$

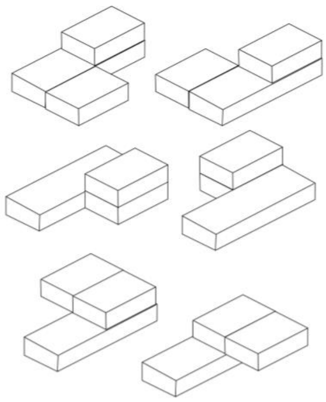
each move consisting of two steps.



There are video clips of the moves at two sites: www.youtube.com/watch?v=fMIpy5-oMfo , www.puzzlemuseum.com/month/picm12/2012-02-obeirnes.htm

At the AIMS Center for Maths and Science Education at Fresno Pacific University in California, students are asked by Dr Richard Thiessen to explore the potential of the 'O'Beirne Cube' for use with Fifth, Sixth and Seventh Grades (10-13 year olds). There are six pieces, each piece consisting of one long and two short (half-length) pieces stuck together. So there is a nice question about the particular pieces, followed by the task of fitting them together to make a cube.

AIMS math & science sandbox



By today's standards, Tom O'Beirne's life was not a long one and so it's not unreasonable to wonder about those projects that remained incomplete at his death. Two that we know of are books that were never published, the manuscripts held in the Mitchell Library in Glasgow. The first is on Hengler's Circus, a hugely popular entertainment in Glasgow from the 1860s to the 1920s. It is remembered in the name of a Wetherspoon's pub in Sauchiehall St. The subject of the second is the panopticon, a building designed to *see all around* from a central point, conceived by Jeremy Bentham for prisons and other institutions. (For aficionados of the Walter Presents foreign language thrillers on Channel 4, Professor T was incarcerated in one.)

References

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Answers

- In 2019, Easter Sunday was on 21 April, so Good Friday was on 19 April.
- There are 4 different pentiamonds and 24 different heptiamonds. Here are the heptiamonds.

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