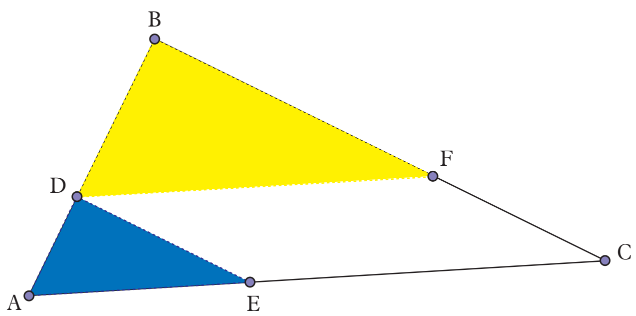
Shifting the Paradigm?

Beautifully moving

by **Ben Sparks**

When I was training as a maths teacher I remember being slightly shocked by some glowing feedback I received from my tutor, visiting from the university. The lesson was simple (an introduction to triangles and related vocabulary), the end goals not ambitious (understand and use the words ‘congruent’ and ‘similar’), and my classroom management had not been particularly praiseworthy (no blood loss, but it was a near thing). The part that had garnered such positive feedback (and which clearly had rescued an otherwise stultifying lesson) was a simple dynamic geometry file projected on the board.



A triangle had a (general) point marked on one side, from which lines parallel to the other two sides were drawn, until they met these other two sides. At this point the old adage a “picture is worth a thousand words” should help me stay under my word limit (see diagram!). **http://www.geogebratube.org/material/show/id/65084**

So far so good, I’m sure you will all notice the blue and yellow triangles have to be similar (convince me?). So why the praise from my tutor? This is where the adage breaks down – and this picture, despite its good ‘worth to word-count’ ratio, does not do full justice. My picture was *moving*. The general point on the left side was sliding up and down, and the rest of the construction (the parallel lines) moving with it.

I am not trying to claim my dynamic geometry file was awe-inspiring, ground-breaking or even particularly stimulating, but it was *moving* (in the non-emotional sense). The effect it had on the class, while not quite miraculous, was nevertheless profoundly gratifying to a young trainee teacher. They watched in semi-rapt quasi-silence, as these pair of triangles waxed and waned on repeat. It also led to some inevitable questions – “What is changing? What is staying the same?”. The lesson progressed with some relatively respectable discussion of the properties of the two triangles: the angles (staying the same) and the sides (changing, in proportion), and some less respectable hilarity caused by a volunteer’s inability to stop the animation precisely when the blue and yellow triangles were *congruent*. But the point had been made, similarity and congruency had been discussed, and we were all the richer for it. The final clincher came when, with the general point in the middle (so blue and yellow were congruent), I moved one of the original points of the triangle, and the whole construction changed – but not the congruency. The generality of the triangles, and our observations, now crystallized, and the lesson moved on to some discussion of the properties of similar triangles.

What am I trying to say here? I think something profound happened (not necessarily intentionally!) when I introduced something *moving* into my lesson. The idea that the initial point on the triangle (D) was a general point did not need any explanation because it was manifestly not a ‘particular’ one – it was *moving* all over the place. Now John Mason claims that “a lesson without the opportunity for learners to generalize is not a mathematics lesson.” Does something *moving* not provide an immediate jumping-off point for generalization?

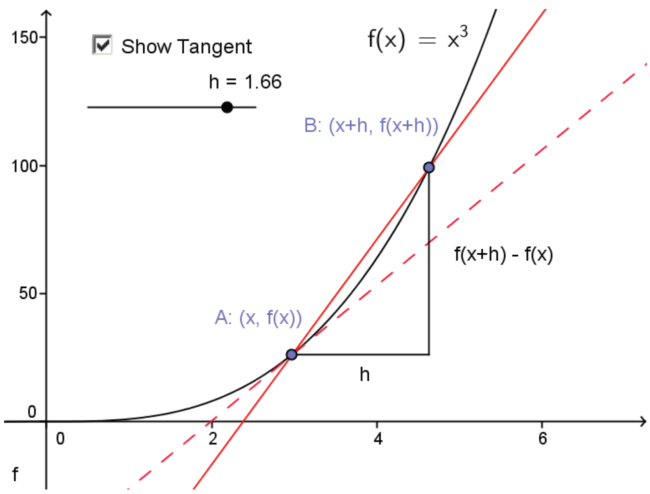
Dynamic geometry software has changed things here. The Greeks worked quite happily on their geometry without it, but when you can drag a point yourself, and see a diagram change (and some things *not* change) you immediately begin to understand the generality contained therein.

When you *move* a point on a semicircle around with your mouse, and see the right angle (subtended by diameter) stay unchanged; when you *move* the corner of a triangle and see its incircle move with it; when you *move* (change, gradually) a coefficient of x^2 in a quadratic from positive to negative and you see the movement created; all of these situations give insight to students (and teachers) which is hard to achieve from static diagrams. Not only that, they seem to have a magical (if temporary) effect on an audience – moving images undeniably attract and hold the attention better than static ones.

I do realize the irony of writing a printed, *static* article about the importance of dynamic, *non-static* images. Perhaps this can best be addressed by some examples, at the end of this article, in the form of links to *GeoGebra Tube*. The examples are not meant to be highly polished, finished products – but if they inspire any further exploration of how to make movement part of mathematics they will serve their purpose.

The ease of access to dynamic geometry software is improving all the time. *Cabri, Geometer's Sketchpad* and *Autograph* (among others) pioneered the revolution, and the baton has been picked up by open-source software like *Desmos, GeoGebra* and many more. The free and near universal access (for students and teachers alike) is making these software packages increasingly useful for quick access to visualizations on demand, and also for more carefully prepared resources.

If we, as teachers of mathematics, ignore the power of visual generalization that comes free with *moving* images, then I feel we are missing a vital trick in our quest to communicate the wonder and usefulness of mathematics. Technology can often appear helpful, and yet in the end add little extra value (or even remove it). In the case of dynamic geometry, if it is paired with teaching expertise, I believe it could cause a profound and useful paradigm shift in teaching and learning mathematics.

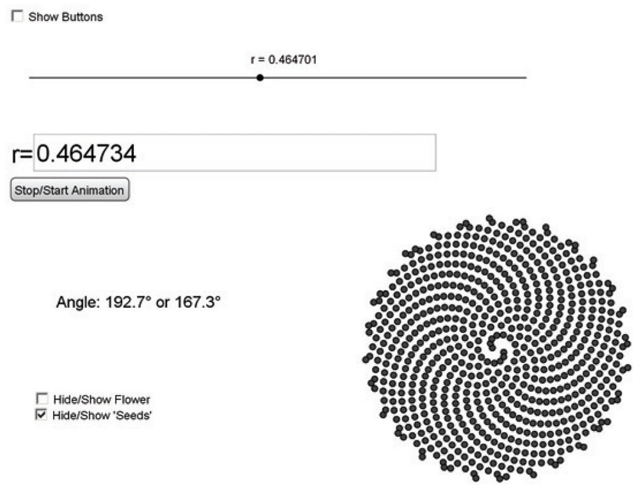


Some Examples

Calculus

http://www.geogebratube.org/material/show/id/65262

Differentiation from first principles is included in some A level syllabi (and frankly it's a shame if it's left out completely even when not examinable!). Here's a simple animation which captures the flavour of the chord AB approaching the true tangent to a curve at A.



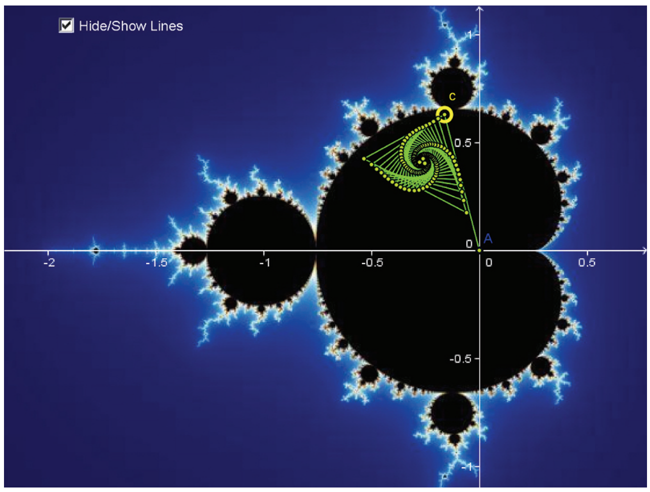
Sunflower

http://www.geogebratube.org/material/show/id/1725

Most mathematicians, many teachers, and some students will have heard of the golden ratio. There are so many myths (that need debunking, e.g. the most aesthetic ratio? proportions in the human body?), and yet so many valid and intriguing mathematical properties (that need celebrating – e.g. the most irrational number? Fibonacci sequences?). Here is one animation that is beautiful because it's moving, but has some deep maths of irrational numbers buried behind it.

Mandelbrot

http://www.geogebratube.org/material/show/id/32947



The Mandelbrot set is a well known piece of beautiful mathematics. How can it move? The iteration that creates it ($z_{n+1} = z_n^{\,2} + c$) can be easily plotted on the spreadsheet function of *Geogebra*:

1. Create a point 'c'.
2. Change it to a 'Complex Number' instead of a 'Cartesian Coordinate' in its properties (Algebra tab).
3. Type "=0" in the cell A1 of the spreadsheet.

4. Type "= A1 ^ 2 + c" into A2, and copy it down for 100 or so cells.

5. (If these new points aren't visible simply select the column A, right click for a menu, and click Show Object).

Now dragging the point 'c' around will create fabulous orbits on the complex plane. Stable orbits come from points in the Mandelbrot set, unstable ones from points outside. Seeing this *move* was the first time I understood the simplicity of the Mandelbrot set – it has stayed with me ever since.

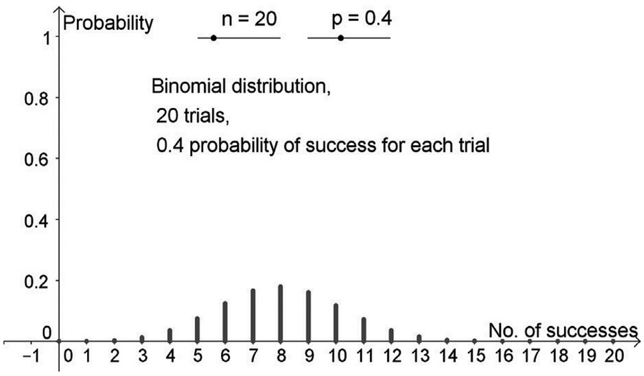
Statistics – Binomial Distribution

http://www.geogebratube.org/material/show/id/69985

Here the binomial distribution is initially shown with its 'p' parameter animating. It is a pleasing motion, and often helps fuel a discussion on what a 'distribution' actually *is*. The graph is shown with 'discrete' bars, unlike the default picture produced with the BinomialDist command.

A link to my *Geogebra Tube* account, where all these examples are stored is here:

http://www.geogebratube.org/user/profile/id/870



Keywords: Dynamic geometry software; *Geogebra*.

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