Task

For the following numbers add a square to make another square.
Example 20 + 16 = 36

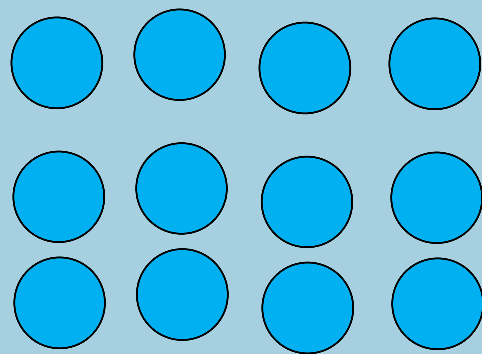
27

29

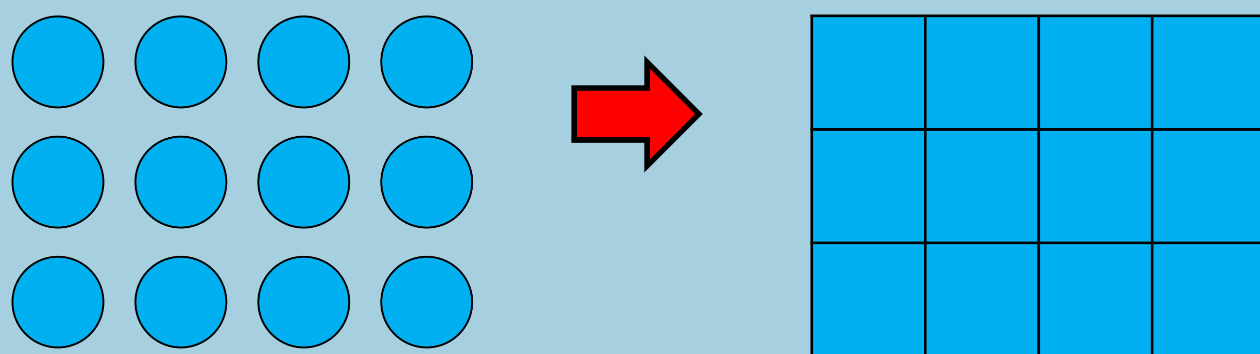
28

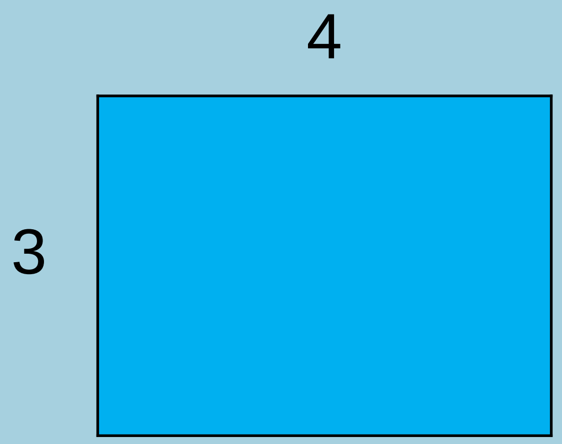
31

Multiplication



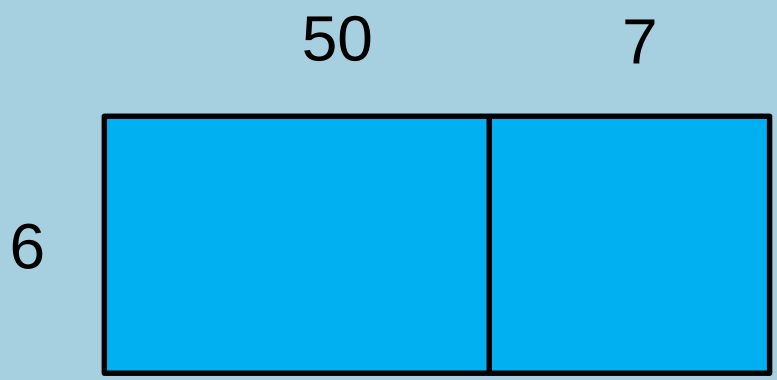
This is three four times
or 3 multiplied by 4
or 3 x 4



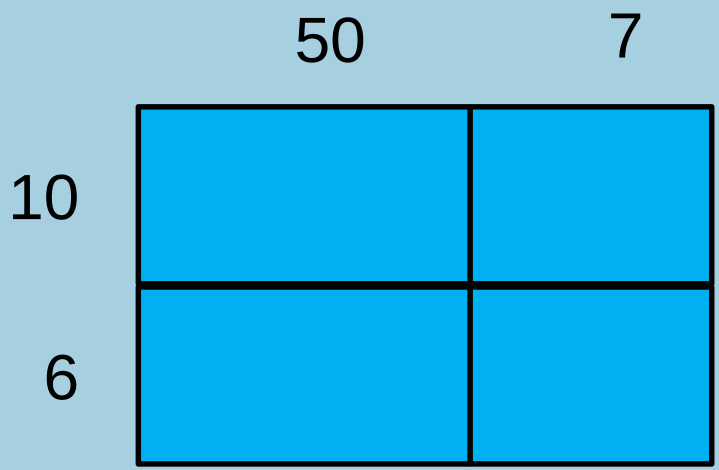


This means that multiplication can be represented by rectangular areas.

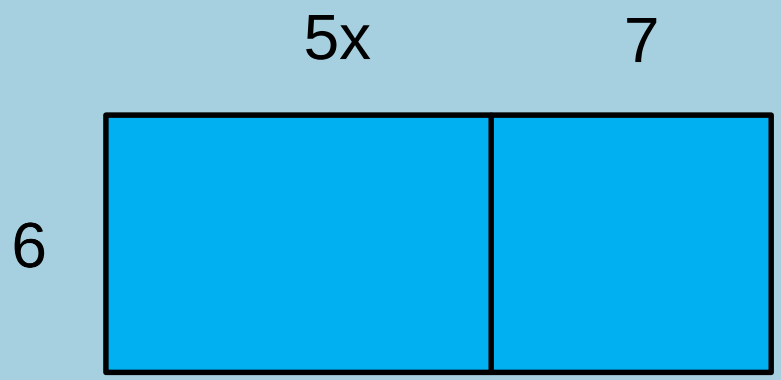
6 x 57 can be represented by



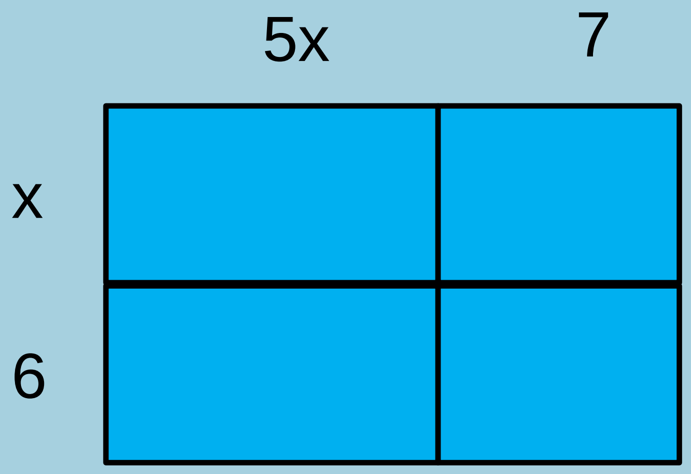
16 x 57 can be represented by



6(5x + 7) can be represented by

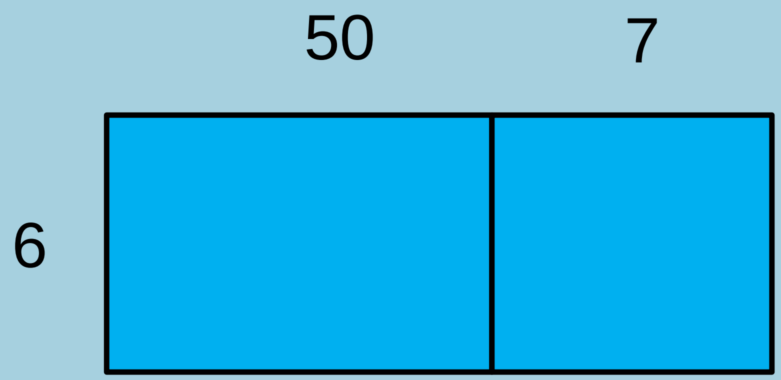


(x + 6)(5x + 7) can be represented by



Expanding brackets

6 x 57 can be thought of as 6(50 + 7)

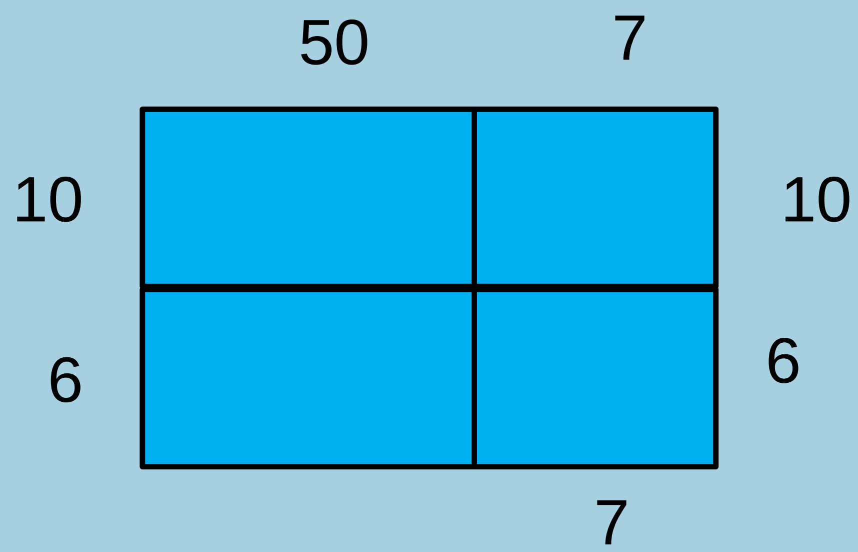


Factorising can be seen as the inverse of expansion

On a basic level we look for a common factor and ‘put’ the expression into brackets

Visually we construct one large rectangle from two smaller ones

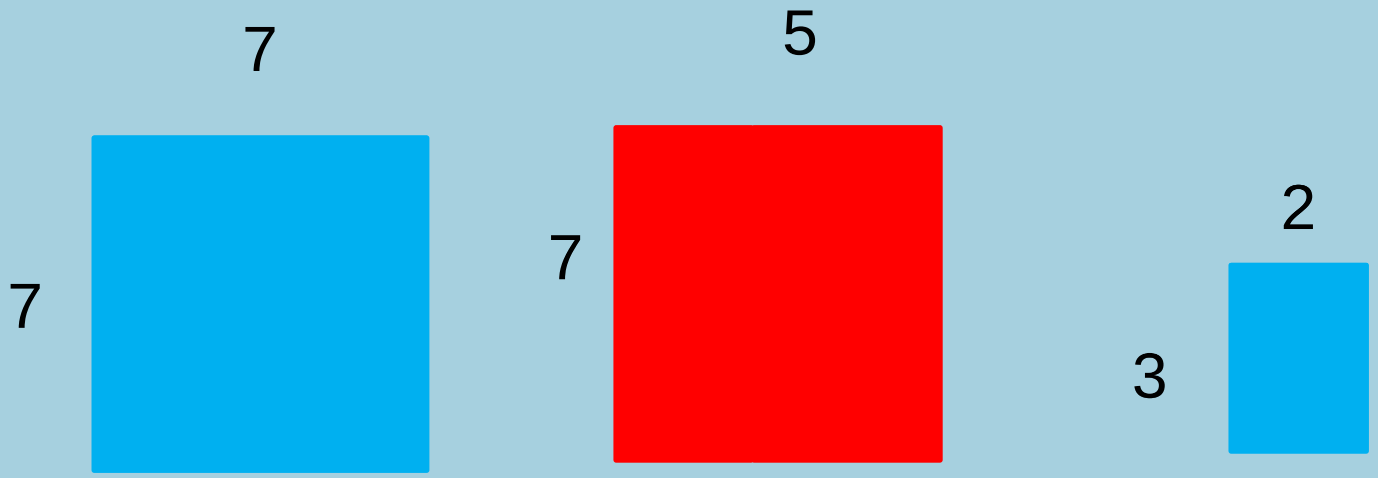
Expanding two brackets is expanding a single bracket twice



Factorising is just(!) the reverse

Consider the following task:

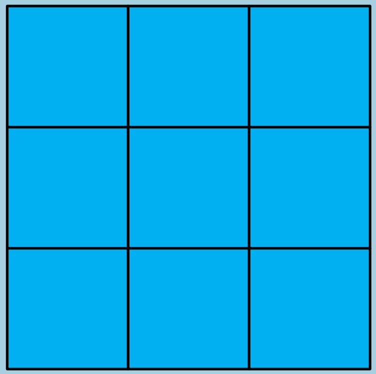
By making a horizontal or vertical cut in the red rectangle, arrange the now four pieces into one rectangle



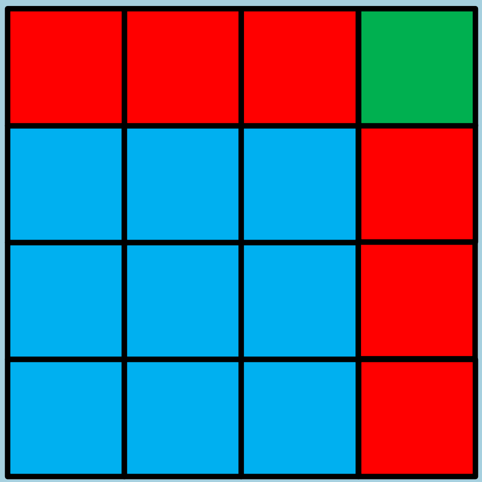
click

Square numbers

Think about the name, we can produce diagrams to illustrate.



Arithmetically the difference between successive square numbers is always an odd number.
Can we show this?



Starting with 3^2 we add a row of 3, a column of 3 and 1 more to fill the gap.

We can generalise….

Using an example look at $19^2 = 361$

To get to the next square number we add 19, 19 and 1

$361 + 19 + 19 + 1 = 400 = 20^2$

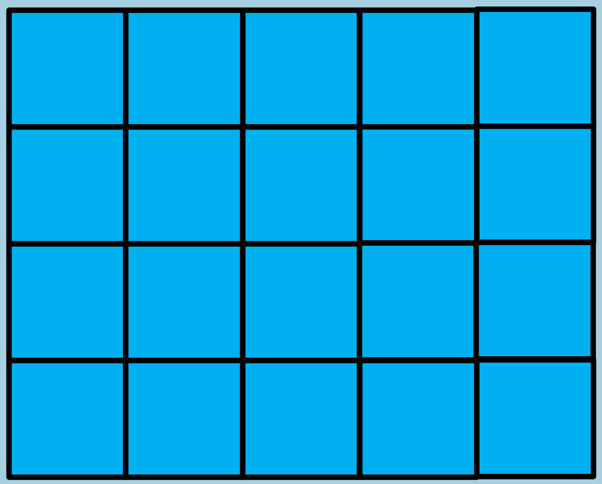
Similarly in reverse …

$21^2 = 441$

If we now subtract 1, 20 and 20 we get

$441 - 1 – 20 – 20 = 400 = 20^2$

Start with a square.



Add an extra column

Here we have 4 x 4 becoming 4 x 5 which is 20

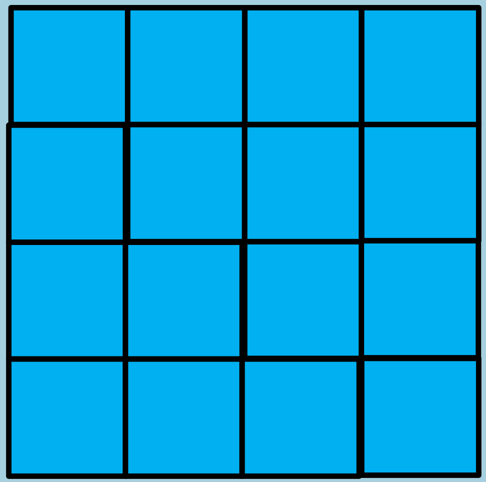
This is a Pronic Number because it can be written as n(n + 1)

Start with a square

Remove the main diagonal

Slide to make a rectangle

Pronic numbers are created by adding or subtracting rows / columns from squares

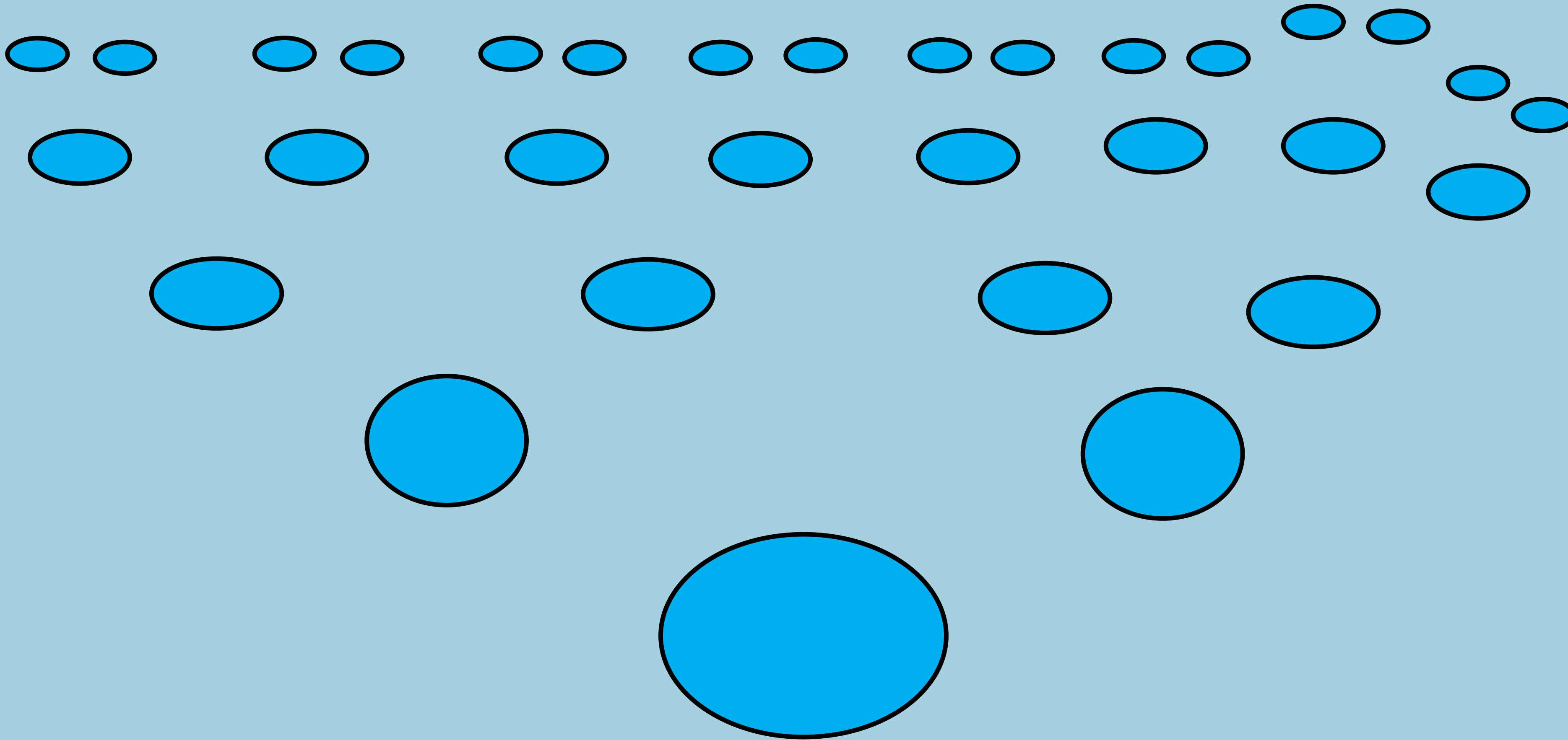


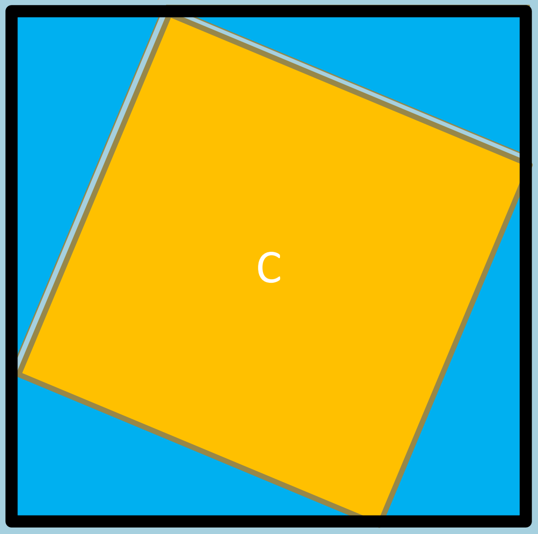
Remove the main diagonal

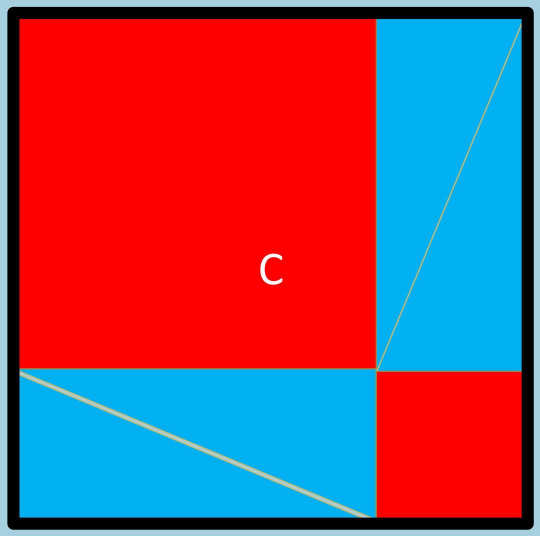
This leaves two identical triangular designs

Triangle numbers appear in several situations including: Handshakes problem, Mystic Rose and number of games in a league.

Amoebas are the simplest animals.
They have one cell and reproduce by separating into two







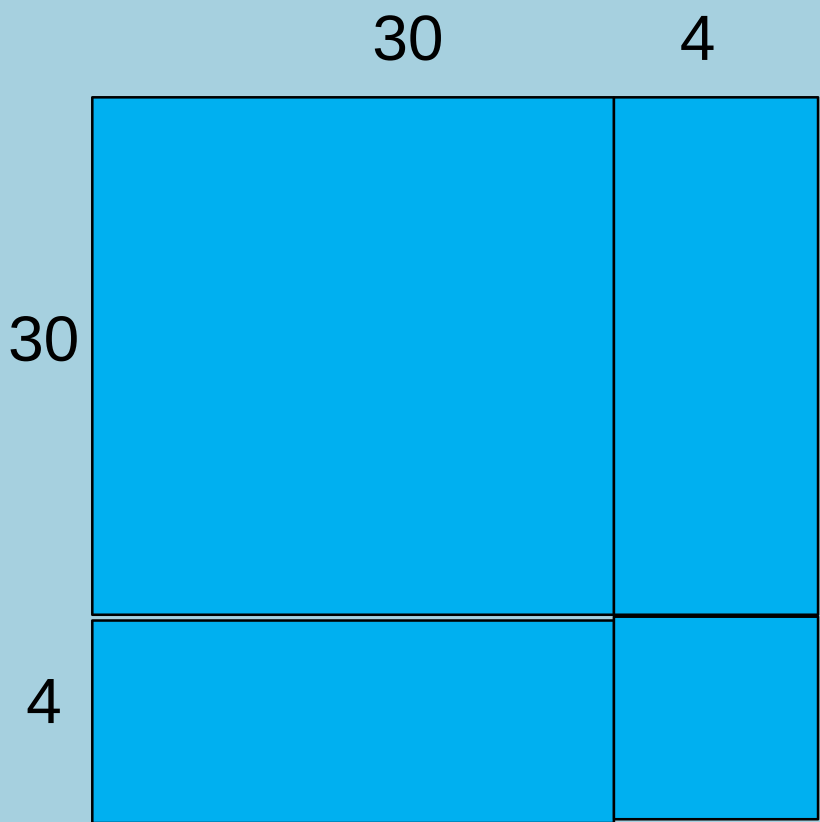
The yellow shape is a square.
How do you know?

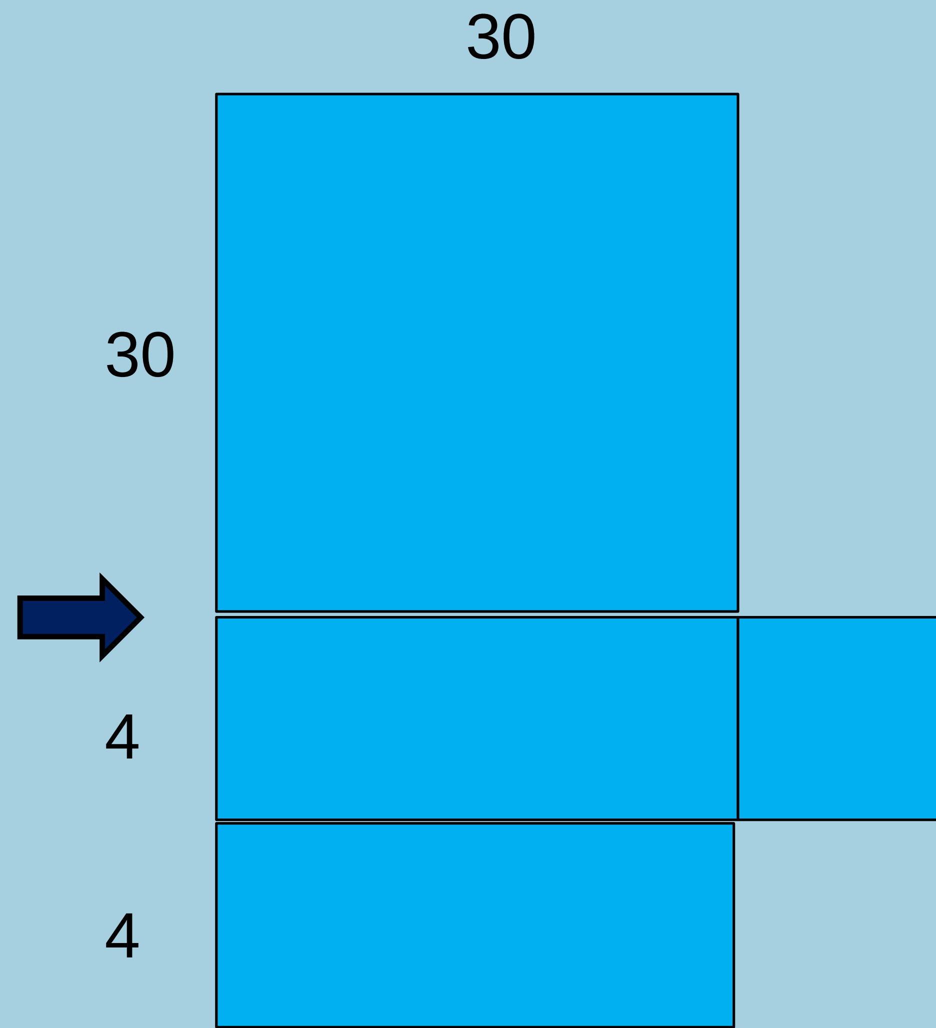
The red shapes are squares.
How do you know?

The two black outline shapes are identical squares.
How do you know?

The area of the yellow square is the sum of the areas of the red squares.

Consider 34^2





$34^2 - 4^2 = 30 \times 38 = (34 - 4)(34 + 4)$

Completing the Square

This tells us something very important:

Completing the square is part of the same piece of mathematics as the difference of two squares,

They can both be done arithmetically and visually

Returning to the original questions:

27 is 9 x 3. Half the difference is 3 The add on is $3^2 = 9$ and we get 36

29 is 1 x 29. Half the difference is 14. The add on is $14^2 = 196$ and we get 225

28 is 14 x 2. Half the difference is 6. The add on is $6^2 = 36$ and we get 64

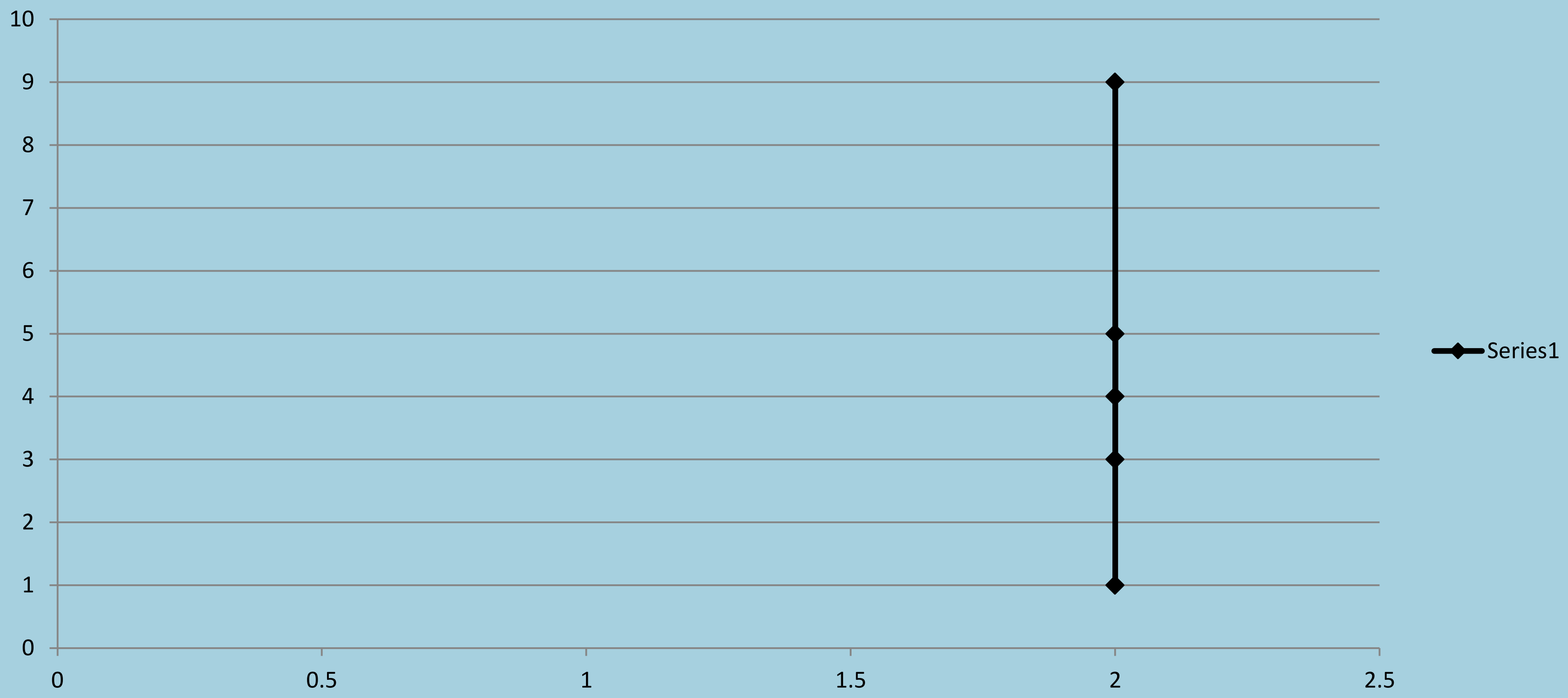
31 is 1 x 31. half the difference is 15. The add on is $15^2 = 225$ and we get 256

Look at the following pairs of coordinates

(2, 3, (2, 9), 2, 4), (2, 5), (2, 1)

Predict what you might see.

Now plot them and see if you are correct.



Find more pairs of coordinates that will ‘fit’

Describe in words what they have in common

Can you give coordinates for a horizontal line?

Predict what the outcome might be if you plotted the following:

(1, 2), (3, 6), (5, 10), (2, 4)

(2, 6), (3, 9), (5, 15), (1, 3)

(3, 12), (1, 4), (2, 8), (5, 20)

Sequences

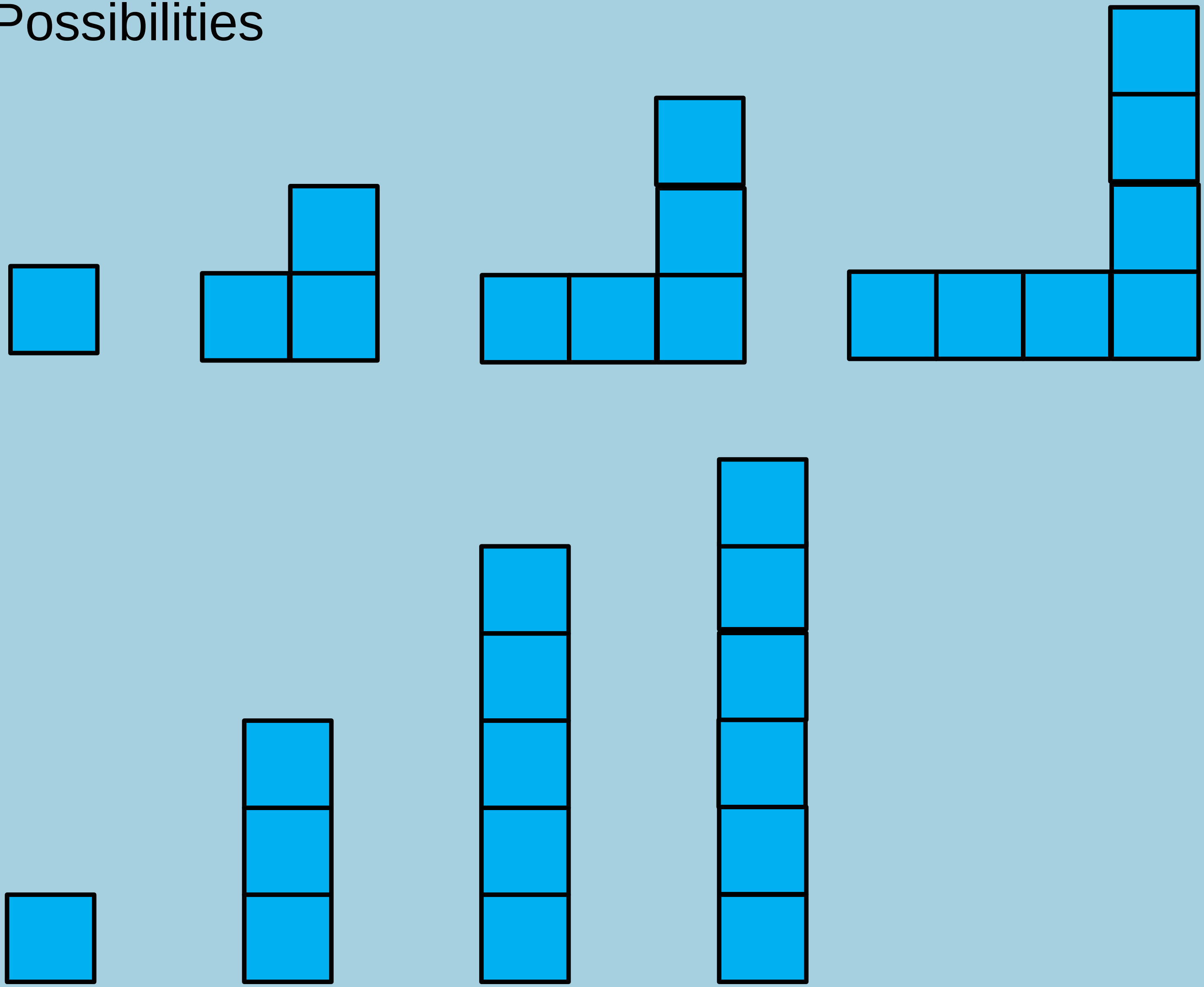
Last year I described the use of diagrams to help children understand algebraic notation.

I would also suggest being creative and construct diagrams from given sequences.

Look at the sequence generated by $2n - 1$.

Construct a set of diagrams to illustrate it

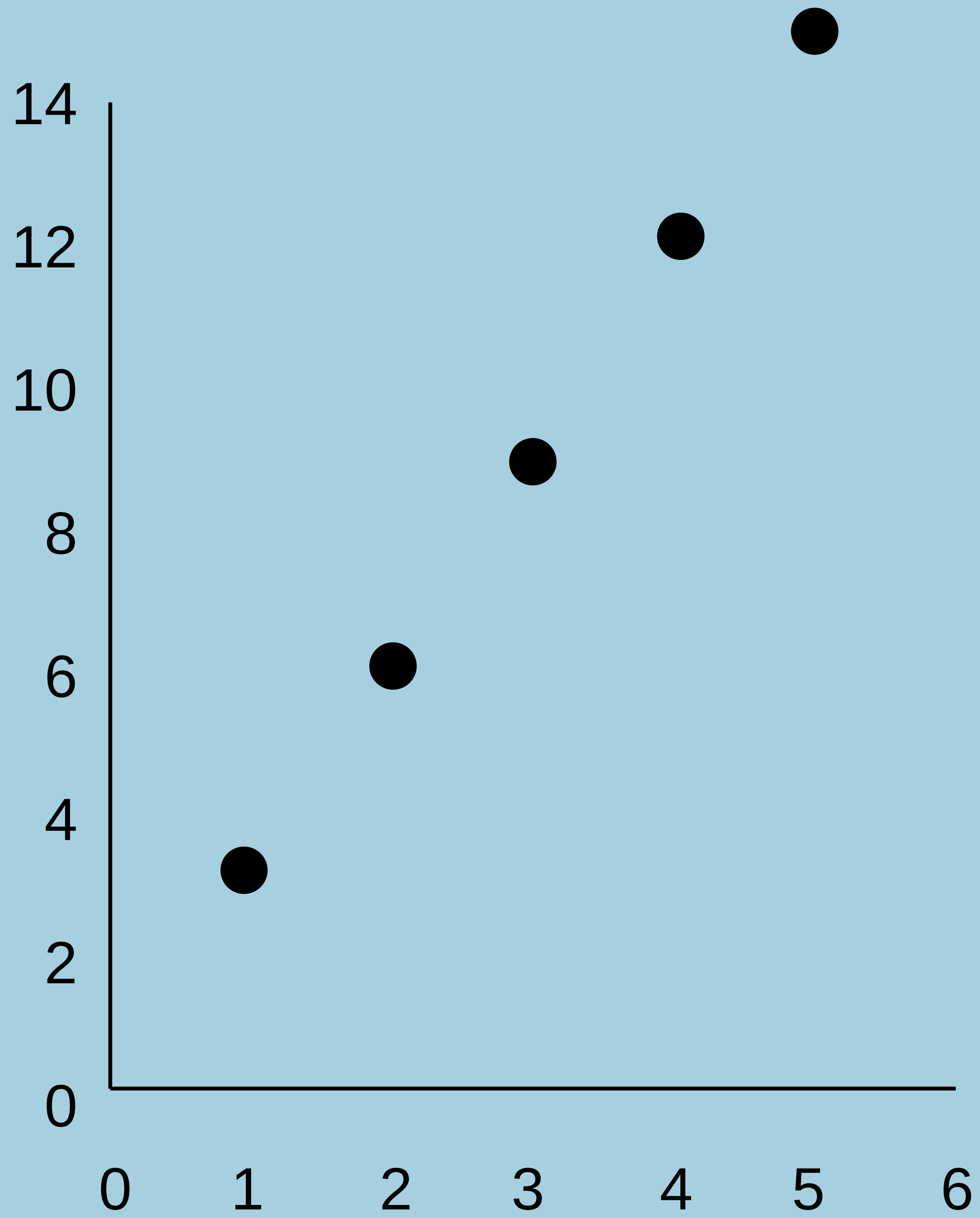
Possibilities



Consider the sequence 3, 6, 9, 12, 15 ….

It would be reasonable to suppose the next term would be 18.
Gaps between successive terms are always 3.

We can also show this on a graph by plotting:
(1, 3), (2, 6), 3, 9), (4, 12), (5, 15).
The x-coordinate represents the position of the term.


14
12
10
8
6
4
2
0
0
1
2
3
4
5
6

Try this with other sequences such as
4, 8, 12, 16, 20, …….

2, 4, 6, 8, 10, 12, …..

What would be the rule for these sequences?

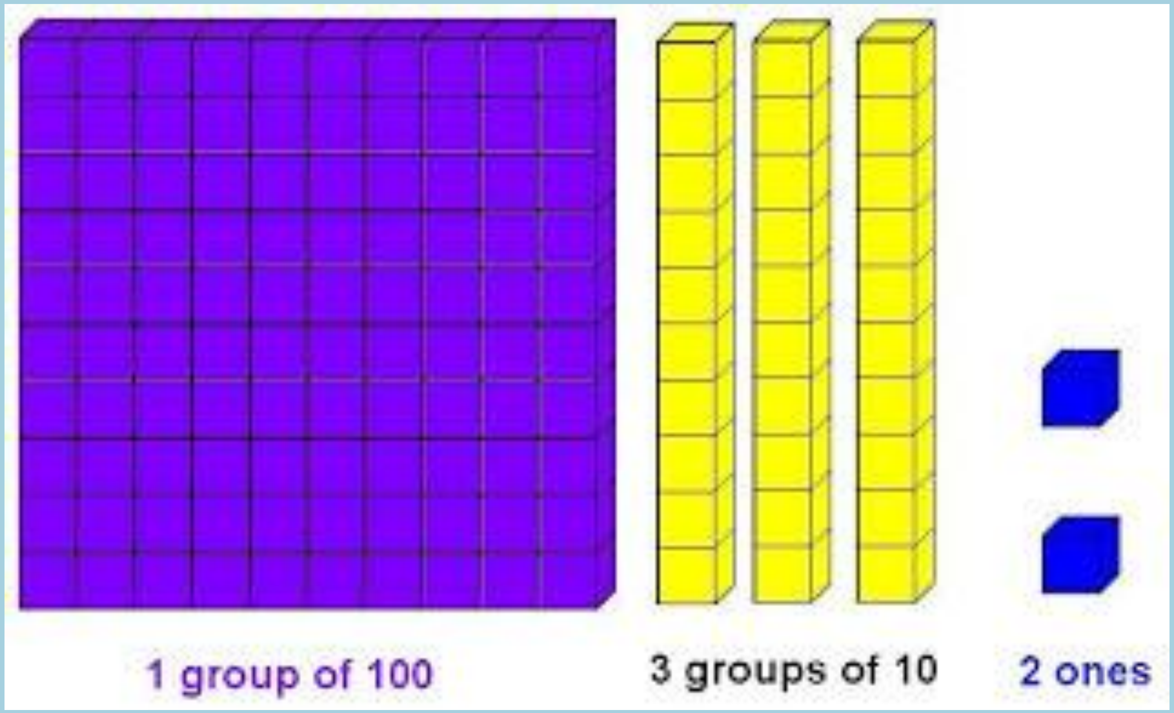
Note the larger the multiplier the steeper the line.
Questions to be asked:
What a rule of n give as a graph?
How could you get a line less steep than this?
Can you get a line that goes the other way?

Compare sequences where the rules are say 3n, 3n+1, 3n + 2 ..

Things that arise here include parallel lines, gradient and Intercepts.

Additional

Dienes Blocks



Place value Counters

1232+3114

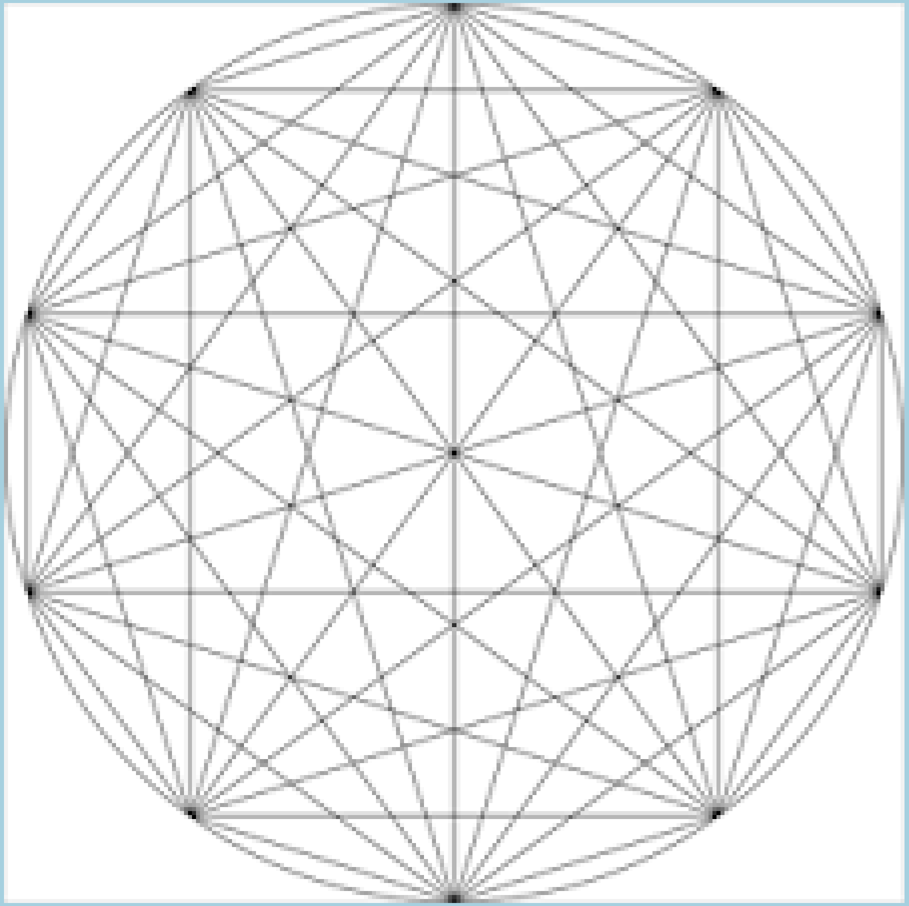
	TH	H	T	O
	1000	100 100	10 10 10	1 1
+	1000 1000 1000	100	10	1 1 1 1

Handshakes

How many handshakes would there be if everyone in the group shook hands with everyone else once?

click

Mystic Rose:
How many lines are needed to draw these diagrams



click

A tournament involving 6 people is played.
Everyone plays all the other players once.
How many games will there be?

	A	**B**	**C**	**D**	**E**	**F**
A	X	√	√	√	√	√
B	√	X	√	√	√	√
C	√	√	X	√	√	√
D	√	√	√	X	√	√
E	√	√	√	√	X	√
F	√	√	√	√	√	X

click