Responding to the 2021 Mathematics Ofsted Research Review

A practical guide for the classroom practitioner

The Joint ATM and MA Primary Group

The Joint Primary ATM and MA Group, established in 1999, is an active group that is drawn from two subject associations (the Association of Teachers of Mathematics and the Mathematical Association). Both associations are known for their strong and enthusiastic commitment to supporting teachers and educators in all phases and ages of mathematics. The Joint Primary Group has over a hundred members who are academics, teachers and consultants who have specialist understanding of primary mathematics and belong to one or both associations. We work collectively to deepen and widen our knowledge of mathematics education with the intent of supporting practitioners in the teaching and learning of mathematics. This document is a product of that collaboration and is written for anyone with an interest in mathematics education.

Two similar aims crossing both organisations are to *'encourage increased understanding and enjoyment of mathematics'* (ATM) and to *'support and promote confidence and enjoyment in mathematics for all'* (MA). Both associations *'actively interact with teachers and others'* (MA), to *'share and evaluate teaching and learning strategies and practices'* (ATM).

Mathematics is a '*creative and highly interconnected discipline*' (DfE, 2013, p.3) which is often misinterpreted as having a narrow, arithmetic focus. Both associations uphold the current National Curriculum (NC) aims of fluency (conceptual understanding), reasoning (following a line of enquiry, conjecturing relationships and generalisations, developing an argument, justification of proof using mathematical language) and problem solving (non-routine problems). One of the purposes of this document is to remind practitioners of the importance of these aims in interpreting the Ofsted Research Review. After all, these aims support a '*high-quality mathematics education [which] provides a foundation for understanding the world, the ability to reason mathematically, an appreciation of the beauty and power of mathematics, and a sense of enjoyment and curiosity about the subject*' (DfE, 2013, p.3).

Contents

Aim of this document

This document is intended to support on-the-ground interpretations of the Ofsted research review for mathematics, hereafter referred to as 'the review', published in May 2021. In writing this response, we have engaged with the review and considered how the recommendations might translate into positive mathematical experiences for early-years and primary-aged children. Our hope is that this document stimulates thinking by educators and leaders about the nature of mathematics learning and teaching in their settings, and can act as a springboard for further practitioner research.

Further Reading

We also encourage you to read other responses to the review from other mathematics educators and associations. At the time of publication, the following were available:

Anne Watson: http://www.pmtheta.com/uploads/4/7/7/8/47787337/conceptual_learning.m4a and http://www.pmtheta.com

Association of Mathematics Education Teachers (AMET): https://www.ametonline.org.uk/app/download/12837138/AMET+Ofsted+complaint.pdf

Cambridge Mathematics: https://www.cambridgemaths.org/blogs/review-noun-a-critical-appraisal/

Christian Bokhove: https://threadreaderapp.com/thread/1409107127512219651.html

Early Childhood Mathematics Group (ECMG): https://earlymaths.org/response-to-the-ofsted-mathematics-review-of-research/

Mathematics Mastery: https://www.arkcurriculumplus.org.uk/news-events/ofsteds-research-review-standing-on-the-shoulders-of-maths-giants

These **statutory documents,** which relate to the 4-11 age range of the review, are important to note:

- EYFS Framework
- National Curriculum for Mathematics for KS1 and KS2
- Ofsted School Inspection Handbook

Thank you for taking the time to read our response.

Please note: all members of the Primary Group contribute as individuals and as representatives, and views expressed by the Group are not necessarily those of organisations or individual members represented.

How to use this document

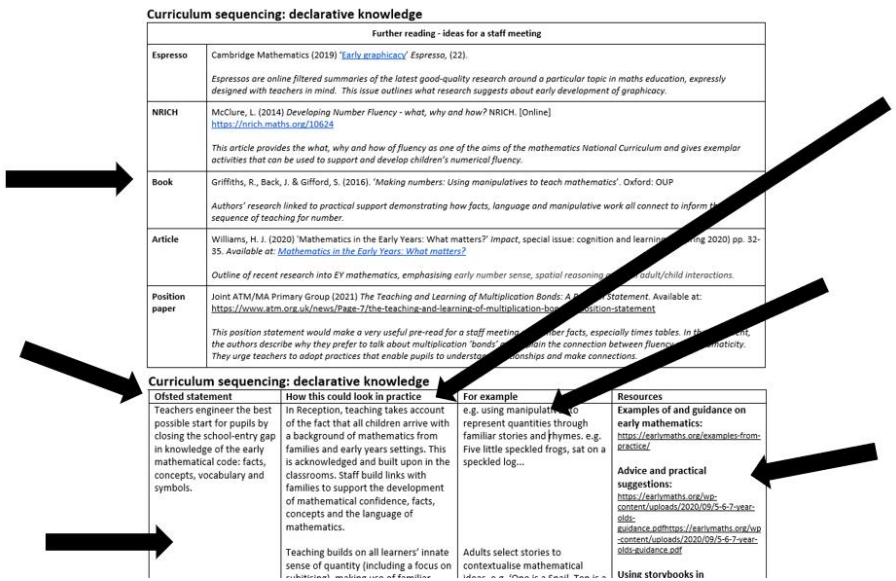
In the review, each section concludes with a set of bullet points under the heading 'based on the above, high-quality maths education *may* have the following features'. This document is framed using these bullet points, and will support teachers and leaders to confirm evidence of good practice in their schools aligned with the Ofsted proposals.

In writing this document, we have followed the sections used in the review.

The first box contains a list of research and key texts on this topic which have been chosen because they reflect the principles of both associations, and they are easily accessible to all users. Schools may wish to use these texts as they research and develop aspects of their practice.

In all but the first section (where Ofsted do not provide a summary of bullet points) there is then a table with four columns.

The first column contains the bullet points from the research review.



Curriculum sequencing: declarative knowledge

Further reading - ideas for a staff meeting	
Espresso	Cambridge Mathematics (2019) 'Early graphicacy' *Espresso*, (22). *Espressos are online filtered summaries of the latest good-quality research around a particular topic in maths education, expressly designed with teachers in mind. This issue outlines what research suggests about early development of graphicacy.*
NRICH	McClure, L. (2014) *Developing Number Fluency - what, why and how?* NRICH. [Online] https://nrich.maths.org/10624 *This article provides the what, why and how of fluency as one of the aims of the mathematics National Curriculum and gives exemplar activities that can be used to support and develop children's numerical fluency.*
Book	Griffiths, R., Back, J. & Gifford, S. (2016). *'Making numbers: Using manipulatives to teach mathematics'*. Oxford: OUP *Authors' research linked to practical support demonstrating how facts, language and manipulative work all connect to inform th… sequence of teaching for number.*
Article	Williams, H. J. (2020) 'Mathematics in the Early Years: What matters?' *Impact*, special issue: cognition and learning… ing 2020) pp. 32-35. *Available at: Mathematics in the Early Years: What matters?* *Outline of recent research into EY mathematics, emphasising early number sense, spatial reasoning … adult/child interactions.*
Position paper	Joint ATM/MA Primary Group (2021) *The Teaching and Learning of Multiplication Bonds: A … Statement*. Available at: https://www.atm.org.uk/news/Page-7/the-teaching-and-learning-of-multiplication-bo… osition-statement *This position statement would make a very useful pre-read for a staff meeting … ber facts, especially times tables. In th… ent, the authors describe why they prefer to talk about multiplication 'bonds' … ain the connection between fluenc… matticity. They urge teachers to adopt practices that enable pupils to understa… ionships and make connections.*

Curriculum sequencing: declarative knowledge

Ofsted statement	How this could look in practice	For example	Resources
Teachers engineer the best possible start for pupils by closing the school-entry gap in knowledge of the early mathematical code: facts, concepts, vocabulary and symbols.	In Reception, teaching takes account of the fact that all children arrive with a background of mathematics from families and early years settings. This is acknowledged and built upon in the classrooms. Staff build links with families to support the development of mathematical confidence, facts, concepts and the language of mathematics. Teaching builds on all learners' innate sense of quantity (including a focus on …	e.g. using manipulat… to represent quantities through familiar stories and rhymes. e.g. Five little speckled frogs, sat on a speckled log… Adults select stories to contextualise mathematical …	**Examples of and guidance on early mathematics:** https://earlymaths.org/examples-from-practice/ **Advice and practical suggestions:** https://earlymaths.org/wp-content/uploads/2020/09/5-6-7-year-olds-guidance.pdfhttps://earlymaths.org/wp-content/uploads/2020/09/5-6-7-year-olds-guidance.pdf **Using storybooks in**

The second column – '**How this could look in practice' –** makes suggestions of what leaders might see in classrooms as evidence of good practice.

The third column provides some examples from classrooms.

In the final column there are links to resources that may support this aspect of practice. They are resources that are in keeping with the principles of the associations, and in many cases are authored and used by members of the Primary Group. Many are repeated as they match the ideas in a number of sections.

The suggestions are appropriate for Reception, Key Stage 1 and Key Stage 2 unless stated otherwise.

Where a (£) is used, this indicates a paid for resource.

In what follows, where the Ofsted suggestions are duplicated, we refer back to our previous suggestions.

Ambition for all

Further reading – ideas for a staff meeting or INSET	
Espresso	Cambridge Mathematics (2017) 'Maths anxiety' Espresso, (6). *Espressos are online summaries of the latest good-quality research around a particular topic in maths education, expressly designed with teachers in mind. This issue outlines what research suggests about how maths anxiety affects mathematics learning.*
NRICH	Gifford, S. (2015) *Early Years Mathematics: How to Create a Nation of Mathematics Lovers?* NRICH. *What research suggests about how to create more positive attitudes to and higher achievement in mathematics, beginning in the Early Years.*
Book (£)	Boaler, J. (2016) *Mathematical Mindsets.* San Francisco, CA: Jossey-Bass. *Mathematical Mindsets is an accessible research-informed collection of practical classroom activities that support teachers to ensure that all children believe that they can achieve in mathematics. The chapters sequentially address changes in practice that lead to a growth mindset.*
Article	Dowker, A., Cheriton, O., Horton, R. and Mark, W. (2019) 'Relationships Between Attitudes and Performance in Children's Mathematics', *Educational Studies in Mathematics,* 100, p. 211. *The authors' research into links between success in, and attitude to, mathematics of English and Chinese children in their first school year.*
Article	Marks, R. (2013) '"The Blue Table Means You Don't Have a Clue": the persistence of fixed-ability thinking and practices in primary mathematics in English schools.' FORUM, 55(1) pp. 31–44. *This article is an account of how fixed-ability thinking prevails among teachers and pupils. Marks describes how pupils are aware of their maths 'status' within the class and also that teachers interact differently with pupils from different groups. This would make a useful reading to provoke teacher reflections on their own practices.*
Article	Willingham, D. T. (2009) 'Is it true that some people just can't do math?' *American Educator*, Winter 2009–2010 pp. 14–19, 39. *In this article, Willingham argues that everyone can learn mathematics. He describes the importance of persistence and hard work on the part of the learner, and sets out some approaches that teachers might take that can lead to success for all.*
Books (£)	Drury, H. (2014) *Mastering mathematics: teaching to transform achievement.* Oxford: OUP. Jain, P. and Hyde, R. (eds.) (2020) *Myths and legends of mastery in the mathematics curriculum.* London: Learning Matters. *These two books offer a guide as to how you might support a mastery approach in schools.*

Curriculum progression: the planned and purposeful journey to expertise

Further reading – ideas for a staff meeting or INSET	
Book (£)	Clements, D. H. and Sarama, J. (2021) *Learning and Teaching Early Math: the Learning Trajectories Approach* (3rd Edition). Routledge. *Comprehensive volume on the authors' extensive research into developmental learning trajectories and how they can be used to plan effective mathematical journeys for young children's mathematical learning.*
Article	Ofsted (2009) Mathematics: Understanding the score. Improving practice in mathematics teaching at primary level. 080283, London: Crown Copyright. *This booklet was drawn from the findings of 84 Ofsted inspections in primary schools. From the evidence gathered they drew out features of good mathematics teaching and provided examples of what this looks like in practice, including the need to carefully sequence lessons and make links to previous learning.*
Espresso	Cambridge Mathematics (2018) 'Introducing early algebraic thinking' Espresso, (12). *Espressos are online summaries of the latest good-quality research around a particular topic in maths education, expressly designed with teachers in mind. This issue outlines what research suggests about how early algebraic thinking can be introduced.*
NRICH	Pennant, J. (2014) *Developing Excellence in Problem Solving with Young Learners*. NRICH. *A clear presentation of how a focus on the choice of task (between the five types of problems outlined in the 2004 Primary National Strategy) and the different stages of problem solving, can develop confident and competent problem solvers.*
Book	Kilpatrick, J., Swafford, J. and Findell, B. (2001) Adding it up: helping children learn mathematics. Washington, D.C. Hadleigh: National Academy Press. *This book includes the 'strands of mathematical proficiency' model. The five strands (adaptive reasoning, strategic competence, conceptual understanding, productive disposition and procedural fluency) are presented as combining to make a rope, this being a metaphor for the way in which these strands should combine equally to form a balanced curriculum. It is available as a free pdf download.*

Ofsted statement	How this could look in practice	For example	Resources for the classroom
Successful curriculum progression is planned from the beginning of a pupil's education through focusing on core content, to develop pupils' motivation and to allow more breadth and depth later.	Teaching is underpinned by knowledge of developmental progressions which supports identification of what needs to be learned next as well as the prerequisite knowledge needed (to inform assessment). Consistent and developmental use of core manipulatives and representations. All staff have a deep understanding of the role of representations in the development of mathematical understanding. Children are empowered to 'own' their learning through planned problem-solving approaches and investigative tasks, building on their keen and natural ability to problem solve. Teachers are confident in pedagogical subject knowledge in all strands of mathematical learning. Consistent teaching approaches are used throughout the school, including the curriculum within a year – carefully planned sequences of lessons are modified for the specific group.	Using developmental learning trajectories on which to base a sequence of teaching learning experiences and to inform interactions, e.g. (i) Calculation methods are taught across the school by linking manipulatives with formal and informal methods, e.g. purposeful use of base-ten blocks, leading to formal addition and subtraction, and number tracks and lines leading to mental methods. (ii) Using the NCETM progression list when planning a sequence of lessons. (iii) Planning the use of low-threshold high-ceiling open tasks to build children's understanding, confidence and motivation.	**Learning Trajectories** US site detailing developmental learning trajectories for birth to Y4 in all areas of mathematics, based on the work of Doug Clements and Julie Sarama. See the Making Numbers book recommended in the **curriculum sequencing: declarative knowledge** section. **NCETM early years support** A bank of resources to support EYs practitioners, including the Number Blocks materials. **DfE non-statutory guidance** This document illustrates progression in key concepts from Y1–Y6 and includes a set of assessment tasks for each of the ready-to-progress criteria. **NCETM PD materials** These resources set out a teaching sequence and provide detailed pedagogical support for teachers in three curriculum strands: (i) number, addition and subtraction, (ii) multiplication and division, and (iii) fractions. **National Curriculum resource tool** From NCETM. Making connections, articles, activities, exemplification

Ofsted statement	How this could look in practice	For example	Resources for the classroom
			and videos available for each aspect of the curriculum Y1–Y6. **NZ Maths** A comprehensive bank of resources from New Zealand. **Hooked on Mathematics** (£) A programme of study for 7–11 year olds based on freely available resources. From MA. The **teacher resource book** from any published scheme will guide this.
The planned curriculum details the core facts, concepts, methods and strategies that give pupils the best chance of developing proficiency in the subject.	Across all year groups there is a clear programme of learning embedded in practice and continually reviewed and improved through lesson study and reference to a wide range of research.	All staff work together to understand how they support all children's mathematical development across the range of ages in their setting.	The **teacher resource book** from any published scheme will guide this. **DfE non-statutory guidance** Described above. **NCETM PD materials** Described above. **National Curriculum resource tool** Described above. **NCETM early years support** Described above. **Developing spatial reasoning** An in-depth review of the evidence on the development of spatial reasoning in young learners, and practical ideas for the classroom.

Ofsted statement	How this could look in practice	For example	Resources for the classroom
The teaching of linked facts and methods is sequenced to take advantage of the way that knowing facts helps pupils to learn methods and vice versa.	Children use intelligent practice exercises to use the relationship between facts to enable appropriate recall. There is commitment from all staff to the consistent and developmental use of core manipulatives. All staff have a deep understanding of their role in the development of mathematical understanding. Evidence of a coherent school curriculum (core content, NC) that has been planned to enable pupils to make connections across and within different areas of mathematics, e.g. teaching addition and subtraction in parallel. Use variation strategies by planning strings of related questions/ problems to delve into children's developing understanding.	If I know $3 \times 4 = 12$, what else do I know? Or sequenced questions so that pupils are guided to build on what they know. $3 \times 4 =$ $3 \times 40 =$ $3 \times 400 =$ $1200 \div 3 =$ $120 \div 3 =$ $12 \div 3 =$ Calculation methods are taught across the school by linking manipulatives with formal and informal methods, e.g. purposeful use of base ten blocks leading to formal addition and subtraction, and number tracks and lines leading to mental methods. The pattern of number relationships is emphasised in number songs and rhymes (e.g. 1 less in 'ten in the bed') using manipulatives to show the addition bonds.	**Young children's understanding of number patterns** An article discussing children's understanding, based on practical classroom activities that could be replicated. **Ideas for developing fluency** A short article setting out how promoting fluency doesn't mean a return to 'drill and practice'. **Variation Theory** This free website includes banks of high-quality, sequences of questions and examples using key principles from Variation Theory. The examples are particularly for KS2.
Sequences of learning allow pupils to access their familiarity with the facts and methods they need in order to learn strategies for solving problem types.	**Please see subsequent section on Curriculum sequencing** and **Pedagogy: consolidation of learning.**	Please see subsequent section on **Curriculum sequencing** and **Pedagogy: consolidation of learning.**	Please see subsequent section on **Curriculum sequencing** and **Pedagogy: consolidation of learning.**

Curriculum sequencing: declarative knowledge

Further reading – ideas for a staff meeting or INSET	
Book (£)	Griffiths, R., Back, J. and Gifford, S. (2016). '*Making Numbers: Using Manipulatives to Teach Mathematics*'. Oxford: OUP *This book is one output of a Nuffield funded research project into how facts, language and manipulative work all connect to inform the sequence of teaching for number. The book contains wonderful images of objects and resources being used to support understanding of numbers to 200 and beyond. Further research reports from this project are available on the Nuffield website.*
NRICH	McClure, L. (2014) *Developing Number Fluency - what, why and how?* NRICH. *This article provides the what, why and how of fluency as one of the aims of the mathematics National Curriculum and gives exemplar activities that can be used to support and develop children's numerical fluency.*
Position paper	Joint ATM & MA Primary Group (2021) *The Teaching and Learning of Multiplication Bonds: a Position Statement.* *This position statement would make a very useful pre-read for a staff meeting on number facts, especially times tables. In the document, the authors describe why they prefer to talk about multiplication 'bonds' and explain the connection between fluency and automaticity. They urge teachers to adopt practices that enable pupils to understand relationships and make connections.*
Article	Williams, H. J. (2020) 'Mathematics in the Early Years: What matters?' *Impact*, special issue: cognition and learning (8, Spring 2020) pp. 32–35. *Outline of recent research into EY mathematics, emphasising early number sense, spatial reasoning and rich adult/child interactions.*
Espresso	Cambridge Mathematics (2019) 'Early graphicacy' *Espresso,* (22). *Espressos are online summaries of the latest good-quality research around a particular topic in maths education, expressly designed with teachers in mind. This issue outlines what research suggests about early development of graphicacy.*

Ofsted statement	How this could look in practice	For example	Resources for the classroom
Teachers engineer the best possible start for pupils by closing the school-entry gap in knowledge of the early mathematical code: facts, concepts, vocabulary and symbols.	In Reception teaching takes account of the fact that all children arrive with a background of mathematics from families and Early Years settings. This is acknowledged and built upon in the classrooms. Staff build links with families to support the development of mathematical confidence, facts, concepts and the language of mathematics. Teaching builds on all learners' innate sense of quantity (including a focus on subitising), making use of familiar contexts (including stories), concrete experiences and images. Teaching enables learners to make links between quantity, abstract numerals and abstract number names and includes developing children's understanding of measures, shape, space and pattern. Rich contexts provide opportunities for mathematical talk which is more than vocabulary acquisition. Discussion provides opportunities for learners to hear and use new language, to reason and predict. Vocabulary is supplemented with gesture and manipulatives (including fingers) to support understanding. There is a focus on developing meaningful understanding of concepts, mathematical symbols	Using manipulatives to represent quantities through familiar stories and rhymes, e.g. 'Five little speckled frogs, sat on a speckled log...' Adults select stories to contextualise mathematical ideas, e.g. 'One is a Snail, Ten is a Crab' can be used to teach the order of operations (Y6). Teaching builds on pupils' concrete understanding. Children engage in 'maths chats' with adults. They are enabled and encouraged to talk about their mathematical activity and introduced to relevant mathematical language in context. Numerals are introduced alongside quantities, e.g. in simple collecting games. Mathematical symbolisation is linked to practical tasks such as scoring in games.	**Building firm foundations in mathematics with 5, 6 and 7 year olds** Advice and practical suggestions from the Early Childhood Maths Group. **Meeting ELGS in Reception** This is a blog post for BSRLM which includes advice for interpreting the new Early Learning Goals. **Maths through stories** **Book ideas for early math learning** This website provides ideas and resources for incorporating story or picture books into mathematics lessons. **Data talks** Guidance and activity ideas to engage pupils in interpreting and analysing through classroom talk. **Images showing different numbers of items** A bank of images to support subitising, estimating and counting. **Mathematical moments** The ECMG mathematical moments is a growing collection of cameos which provide situated examples of mathematics pedagogy. They demonstrate the importance of exploring the mathematics in

Ofsted statement	How this could look in practice	For example	Resources for the classroom
	and vocabulary based on prior experience and giving pupils opportunities to make conceptual links through reasoning using different representations – including the use of manipulatives – of the same concept. There is a strong focus on thinking mathematically, using symbols to assist with this as appropriate, rather than on the symbols themselves.		children's everyday experiences at home and in educational settings.
Pupils are taught core facts, formulae and concepts that are useful now and in the next stage of education.	Young learners develop an understanding of the number system through a balance of teaching and exploration. They develop knowledge of the composition of numbers based on perceptual and conceptual subitising. Learners start to develop an early practical understanding of how the part-whole relationships in composition link to addition and subtraction. Children conceptually subitise in order to support their understanding of composition. Conceptual understanding is prioritised, all staff appreciate how important this is for later success (e.g. in primary, place value and equivalence).	A wide range of manipulatives, both structured and natural, and images are used to embed young children's understanding of the number system. These include number tracks, games, books, number lines, as well as natural resources such as pinecones and shells, and structured resources such as Numicon and Cuisenaire. Exploring the composition of number, e.g. filling and discussing the smaller numbers contained within 5-frames and, later, 10-frames. Structural apparatus and representations which emphasise place value structures are used regularly to support secure conceptual understanding (e.g. base-ten materials, arrow cards, Gattegno charts).	**Developing additive fluency video** This video from NCETM features Rosie who is using a rekenrek. **Stick and Split** This app supports children to learn multiplication facts and understand the links between them. **Getting to grips with multiplication facts** A short article by Mike Askew with practical ideas for the classroom. **Videos from NCETM** Four lesson videos showing pupils learning number bonds and multiplication facts through reasoning and making connections. **Fluency without fear** In this article, Jo Boaler highlights the issue of valuing memorisation of facts over number sense.

Ofsted statement	How this could look in practice	For example	Resources for the classroom
Teachers help pupils develop their automatic recall of core declarative knowledge, rather than rely on derivation, guesswork or casting around for clues.	Learners start learning number bonds as the composition of small quantities and use this knowledge to derive further facts. Research seems to suggest that learners develop automaticity in number bonds **through derivation strategies**, rather than despite them. The knowledge develops (later to automaticity) through the process of reasoning about the derivation.) Early algebraic thinking is based in the exploration and recognition of pattern, e.g. gradually being able to recognise the unit of repeat within a repeating pattern. Generalising arises from anticipating and predicting patterns, whether visual or in games. Where direct instruction is used to 'point out' fundamental features of mathematics, assumptions are not made that learners now 'know' this. Instruction should be balanced with opportunities for sense-making and the joy of discovery, problem solving, independent and collaborative work.	In Reception using manipulatives to represent that when one speckled frog has jumped off the log... how many are left on there? how many are off? In key stage 2 roll a dice and use the numbers to generate a proper fraction, how many fractions greater than less than or equal to ½ are possible. Playing a simple game repeatedly leads to discussion about strategy and 'what might happen if...', e.g. starting with a dish of 11 items, each player removes either 1 or 2 on their turn, the winner is the player who removes the last item. Or, rolling a dotty dice to fill 10-frames, first to exactly 20 wins. Opportunities are provided for children's independent exploration of taught key ideas, in every year group, through exploratory talk and/or structured independent problem solving.	See blog post by **Cartwright** on using games in the classroom in the **pedagogy: consolidation of learning** section. **Games from Michael Minas** A set of 78 videos showing Michael playing mathematics games with his child. All games are adaptable and require only simple resources. **Stick and Split** This app supports children to learn multiplication facts and understand the links between them. **Counting collections** Article on using the counting of collections to develop early number learning. **The importance of pattern** Article about the importance of pattern understanding in early mathematics.

Curriculum sequencing: procedural knowledge

Further reading – ideas for a staff meeting or INSET	
Article	Skemp, R. R. (1976) 'Relational understanding and instrumental understanding.' *Mathematics Teaching*, (77) pp. 20–26. *Although originally published in 1976, Skemp's findings are as relevant today as they were then. He discusses two ways of learning mathematics: relational understanding, 'knowing what and why'; and instrumental understanding, 'rules without reasons'.*
Espresso	Cambridge Mathematics (2020) 'Developing concepts of time' *Espresso,* (32). *Espressos are online summaries of the latest good-quality research around a particular topic in maths education, expressly designed with teachers in mind. This issue outlines what research suggests about developing concepts of time.*
NRICH	Back, J. (2013) *Manipulatives in the Primary Classroom.* NRICH. *This article responds to Ofsted's 2012 Made to Measure report, which suggested that manipulatives were not used as widely or as effectively as they might be in primary classrooms. With a focus firmly on manipulatives as a tool for all, rather than a crutch for some, this article supports practice by providing examples of activities and the manipulatives that best support them.*
Article	Kuchemann, D. and Hodgen, J. (2017) 'Models of multiplication: Unlock the power.' *Primary Mathematics*, 21(2) pp. 18–21. *In this article, the authors explore different models to explore the structure of multiplication. They begin with the question,* ***which is larger, 22 × 53 or 21 × 54.*** *Models shared include stories and diagrams, and the authors conclude that these give meaning to the concept of multiplication.*
Books (£)	Fosnot, C. and Dolk, M. Series of four books: *Young Mathematicians at Work: Addition & subtraction (4–8 years), Algebra (4–14 years), Multiplication and Division (7–10 years), Fractions, Decimals and Percents (10–14 years).* *Four titles resulting from the authors' research developing the computational knowledge of learners between 4–14 years of age, using a sequence of interconnected lessons, investigations and games.*

Ofsted statement	How this could look in practice	For example	Resources for the classroom
Teachers teach younger pupils non-distracting and accurate mathematical methods that encourage them to use recall over derivation.	Children are encouraged to learn about number bonds by composing and decomposing small quantities using real objects. This developing knowledge is used to encourage them to derive further facts. Automaticity is gradually acquired over time through the process of reasoning about the derivation. Adults discuss in a rich variety of contexts how children 'see' amounts, building on conceptual subitising to perceptually subitise.	5-frames, egg-boxes and later, 10-frames, are used to reveal the connections between small quantities; all the small numbers 'hidden within' the larger numbers (e.g. when you are looking at 7 counters, can you spot a group of 4 and a group of 3?) A wide range of items of all sizes are used to make and discuss quantities with children: 'what do you see?' 'what do you notice? Structured manipulatives are selected carefully to represent numbers in order to expose clear connections between them, e.g. Numicon and Cuisenaire.	**Guidance on early mathematics** Advice and support from the Early Childhood Maths Group. **Subitising: What is it? Why teach it?** An informative article – with plenty of suggestions for teachers – that describes this important innate ability and why it is crucial to future mathematical development. **Images showing different numbers of items** A bank of images to support subitising, estimating and counting. See the Making Numbers book recommended in the **curriculum sequencing: declarative knowledge** section.
Teachers plan to teach older pupils efficient, systematic and accurate mathematical methods that they can use for more complex calculations and in their next stage of learning.	As learners develop their understanding of larger numbers, they learn how the properties of operations and the base ten number system allow calculations to be broken down and completed in different ways. They apply this deep understanding of the structure of the number system and, therefore, are not tied to any specific procedures or methods. Instead they systematically and accurately select the most appropriate and efficient method for the calculation, and being able make sensible decisions about	Teaching focuses on developing understanding of core mathematical structures such as the additive and multiplicative relationships, identifying the links between addition and subtraction, and between multiplication and division, and the important differences between reasoning additively and multiplicatively. Similarly, learners develop understanding of the properties of the operations (commutativity, associativity, distributivity) to understand, simplify and derive	**National Curriculum resource tool** From NCETM. Making connections, articles, activities, exemplification and videos available for each aspect of the curriculum Y1–Y6. **NCETM PD materials** These resources set out a teaching sequence and provide detailed pedagogical support for teachers in three curriculum strands: (i) number, addition and subtraction, (ii) multiplication and division, and (iii) fractions. **DfE non-statutory guidance**

Ofsted statement	How this could look in practice	For example	Resources for the classroom
	when and how to calculate mentally, and when it is more appropriate to use a calculator. Carefully chosen representations including manipulatives are used to support the development of conceptual understanding throughout the Primary Years, which can be used to reveal structure. E.g. showing the decimal number system using base-ten blocks. A planned and consistent progression of manipulatives and other representations (including language) ensures that prior learning is activated efficiently. Learners are active users of representations (including manipulatives) both to make sense of mathematical situations and to convince others that their explanation is correct. Teachers facilitate rich discussions that enable learners to understand and make connections to each other's strategies (e.g. through use of manipulatives or number lines) and build their understanding of the number system.	calculations, rather than relying on procedures. Developing understanding of formal subtraction methods through using base-ten blocks, but later using the base-ten blocks to convince someone else (and themselves) that their 'answer' is correct. Children can prove the (length times breadth) divided by two formula for finding areas of triangles using Geostrips. Using Cuisenaire rods to generate an image which represents 2 × 4 to demonstrate how many 2s are in 8  How would you do each of these calculations? (Without paying attention to the answers.) 121 – 78 121 – 59 121 – 4 121 – 20	This document illustrates progression in key concepts from Y1–Y6 and includes a set of assessment tasks for each of the ready-to-progress criteria. **Models in mind** Mike Askew article explaining how models and representations are used as tools for thinking by pupils, e.g. using an array to represent multiplication with integers and fractions. **Making use of manipulatives** Guidance with supporting evidence for the use of manipulatives in KS2 and KS3. **A deep sense of number starting with dots** An ATM publication of over 50 activities, games and ideas to guide children in developing number sense. **Cuisenaire – from Early Years to adult** Suitable for teachers in all phases of education, this book supports teachers to use Cuisenaire rods to support learners' conceptual understanding. From ATM.
Teachers help pupils to use these methods to see new connections of number, geometry and time.	Children are given problems to solve which make use of their developing number knowledge.	Can these two teddies have an equal number of biscuits each?	**NRICH curriculum mapping** Activities are mapped against curriculum statements from EYFS to

Ofsted statement	How this could look in practice	For example	Resources for the classroom
	Clear links are made between operating on numbers and practical applications of calculations, such as the timings required to bake a cake, or the measurements required to construct a scale model.	How many pebbles balance the pineapple? What if I try corks? What equal groups can I make with all 12 buttons? I know how to mentally add 531 + 263 because I recognise that this is a calculation which requires no exchange. I start with 531 and adding 200, then 60 then 3. I can use the same strategy to calculate what time to take the cake out of the oven if it takes 1 hour and 30 minutes to cook, and I put it in the oven at 4.20, i.e. I add 1 hour and then 30 minutes.	Y6. All activities have teacher notes and require and promote pupil reasoning. **National Curriculum resource tool** From NCETM. Making connections, articles, activities, exemplification and videos available for each aspect of the curriculum Y1–Y6. **Developing spatial reasoning** An in-depth review of the evidence on the development of spatial reasoning in young learners, and practical ideas for the classroom.
Teachers encourage pupils to use core mathematical methods rather than resort to guesswork, cast around for clues or use unstructured trial and error.	Please see the previous section **Curriculum sequencing: declarative knowledge**	Please see the previous section **Curriculum sequencing: declarative knowledge**	Please see the previous section **Curriculum sequencing: declarative knowledge**

Curriculum sequencing: conditional knowledge

Further reading – ideas for a staff meeting or INSET	
Espresso	Cambridge Mathematics (2019) 'EAL students in mathematics classrooms' *Espresso,* (26). *Espressos are online summaries of the latest good-quality research around a particular topic in maths education, expressly designed with teachers in mind. This issue outlines what research suggests about supporting EAL students in mathematical classrooms.*
Talk	Kate Palmer at Babcock Mathematics Research Talks #7: Supporting Reading Comprehension for Maths Word Problems. *This 28-minute research talk is by Kate Palmer, a speech and language therapist. In this video, she shares her thinking about why some pupils struggle with interpreting and solving word problems and shares strategies for teaching pupils to tackle word problems. More information, an article and sample resources can be found on her website.*
Videos	Building discussion in mathematics learning through the use of number talks – a collection of videos *The video by Sherry Parrish is one of a set of short 'number talk' videos demonstrating children encouraged to explain their thinking, what they know and to justify their answers to a calculation.*
NRICH	NRICH team (2017) *Creating a Low Threshold High Ceiling Classroom*. NRICH. *The NRICH project defines a low-threshold high-ceiling task as one which offers everyone the chance to start and everyone the chance to get stuck. However, it is acknowledged that tasks alone are not enough and this article discusses the key features of a low-threshold high-ceiling classroom and how this environment can nurture learners.*
Book (£)	Mason J., Burton, L. and Stacey, K. (2010) *Thinking Mathematically* (2nd Edition). London: Pearson. *This book discusses ways of developing your own mathematical thinking and that of others. It guides the reader to develop their ability to tackle problems through questioning, conjecturing, getting stuck and revising ideas.*
Article	Cuoco, A., Paulgoldenberg, E. and Mark, J. (1996) 'Habits of Mind: An Organizing Principle for Mathematics Curricula', *Journal of Mathematical Behaviour*, 15, pp. 375–402. *This article discusses the fact that as technology changes, so does mathematics and we therefore need to focus on the 'habits of mind' that can empower our children for the future by developing their mathematical thinking.*

Ofsted statement	How this could look in practice	For example	Resources for the classroom
Teachers teach useful, topic-specific strategies to pupils, as well as how to match them to types of problem.	Children are aware that there are a range of different approaches to solving mathematical problems. In his influential book, *How to Solve it*, Polya (1945) identified 10 key problem solving strategies: 1. Guess and check 2. Make a table or chart 3. Draw a picture or diagram 4. Act out the problem 5. Find a pattern or use a rule 6. Check for relevant or irrelevant information 7. Find smaller parts of a large problem 8. Make an organised list 9. Solve a simpler problem 10. Work backwards. Children become aware of these and recognise when and how to use them, and learn that some problems can be solved using more than one strategy. To meet the demands of the National Curriculum aims and the EYFS, children are engaged in a problem-solving approach to learning, by solving their own problems as well as those posed by others. Both staff and children know that the *whole point* of learning mathematics is to be able to solve problems.	Sort a set of problems (with answers) into those that can be solved by making a table, and those which can be solved by guess and check strategies. Compare strategies and discuss efficiency.  Which strategy would be helpful for this problem? Is there a rule? Will an organised list help (NRICH 2019).	**Mathematical Problem Solving in the Early Years** Advice on developing problem solving in the Early Years with suggested activities. **Primary Padlock Challenges** (£) A set of logic puzzles from MA. **Problem solving in Primary classrooms** Feature including articles, advice and problem-solving activities to support learners in developing understanding of mathematics and developing problem solving skills. **Data talks** Guidance and activity ideas to engage pupils in interpreting and analysing through classroom talk. **The Aliens have Landed** (£) A resource from MA. 175 illustrated maths problems suitable for KS2.
Pupils are confident using linked facts and methods that are the	Teachers familiarise themselves with developmental learning trajectories	Children are already familiar with right angles and triangles, and they	**Learning Trajectories**

Ofsted statement	How this could look in practice	For example	Resources for the classroom
building blocks of strategies, before strategies are taught.	in order to support the building blocks of the key ideas in mathematics. This enables children to use what they already know and apply this in different contexts and to solve new problems. Teachers may choose to give children a task which enables them to pre-assess children's prior knowledge, and maybe supplement, before embarking on next steps.	explore what happens when they 'cut off' the three angles of a triangle and lay them together, to make a straight line, which is equivalent to two right angles. They can already represent 53, they explore how to show 530 and 5300 using base ten blocks and compare 'what is the same/what is different?' Children are already familiar with a set of building blocks and are encouraged to use a small selection of these to play a 'make the same as mine' game, in pairs, or to recreate a model from a diagram /photograph and describe the blocks they have used.	US site detailing developmental learning trajectories for birth to Y4 in all areas of mathematics, based on the work of Doug Clements and Julie Sarama. **DfE non-statutory guidance** This document illustrates progression in key concepts from Y1–Y6 and includes a set of assessment tasks for each of the ready-to-progress criteria. **NCETM PD materials** These resources set out a teaching sequence and provide detailed pedagogical support for teachers in three curriculum strands: (i) number, addition and subtraction, (ii) multiplication and division, and (iii) fractions. **National Curriculum resource tool** From NCETM. Making connections, articles, activities, exemplification and videos available for each aspect of the curriculum Y1–Y6.
Teachers encourage pupils to use core, systematic strategies rather than resorting to guesswork or unstructured trial and error.	Teaching explicitly models a 'habits of mind' approach to tackle non-routine problems (problems that do not have an immediately apparent solution, which require thought and may result in becoming 'stuck'). This includes modelling of and encouragement for useful learning dispositions, including e.g. curiosity,	Specific strategies: Imagining and expressing Specialising and generalising Conjecturing and convincing Classifying and characterising Organising, reflecting and extending. Example questions: What do I notice? What if…?	**John Mason blog** In this blog post, John writes about establishing conjecturing atmospheres. The ideas would particularly suitable for KS2 classrooms. **Graham Fletcher's website** Maths progressions and problem-based lessons.

Ofsted statement	How this could look in practice	For example	Resources for the classroom
	perseverance, collaborating with others etc. Teaching also models strategies, questions and actions, encouraging learners to adopt these for themselves in order to break into and break down problems by revealing mathematical structures that they recognise. They can then use reasoning to identify known facts and procedures that may find a solution.	How can I show this? How else? What's the same? What's different? What am I given? What do I need to find out? What examples can I give? And another and another? Have I seen something like this before? What could be going on here? Why? Is there a relationship? Can I describe this in general terms? Example actions: Visualising, predicting, describing, comparing, seeking patterns, manipulating, representing, recording, purposeful trialling, changing as little as possible, generating and articulating hypotheses, convincing, proving, being systematic, reviewing, evaluating, investigating further.	**Marilyn Burns' website** Rich task resource for teachers EY–Y6. **Thinking Mathematically Everyday Series Y1–Y6** (£) Series of books from ATM containing rich tasks with teacher guidance, resources and suggestions for questioning and extension. **Reasoning progression maps from NCETM** Reasoning questions mapped to National Curriculum statements.

Curriculum sequencing: meeting pupils' needs

Further reading – ideas for a staff meeting or INSET	
Espresso	Cambridge Mathematics (2017) 'Working memory for mathematics learning' Espresso, (10). *Espressos are online summaries of the latest good-quality research around a particular topic in maths education, expressly designed with teachers in mind. This issue outlines what research suggests about why working memory is important for mathematics learning.*
Article	Ollerton, M., Stratton, J. and Watson, A. (2020) 'Inquisitive about inquiry? Loaded with cognitive load?' *Mathematics Teaching,* (270) pp. 32–36. *The authors outline compatibilities between theories of teaching and learning from cognitive science and inquiry approaches, and what is important to consider when meeting pupils' mathematical needs.*
NRICH	NRICH team (2017) *Nurturing Successful Mathematicians*. NRICH. *This article shares a model of what it means to be a successful mathematician, couched in child-friendly language. The five key ingredients it outlines, can be seen modelled and linked to particular tasks in this webinar.*
Book (£)	Askew, M. (2015) *Transforming Primary Mathematics: Understanding Classroom Tasks, Tools and Talk.* London: Routledge. *Updated to reflect the current National Curriculum, this book encourages teachers to carefully choose the tasks they use to support mathematics learning, consider the tools that extend thinking and also to pay attention to classroom talk. One chapter looks specifically at 'variation'. There is a companion publication, 'A Practical Guide to Transforming Primary Mathematics'.*
Research report	Trundley, R., Wreghitt, C., Edginton, H., Eversett, H. and Burke, S. (2017) *Supporting children to be active and influential participants in mathematics lessons through effective use of assigning competence and pre-teaching, Final report*. July 2017. Babcock LDP, Devon County Council, Jurassic Maths Hub, Cornwall and West Devon Maths Hub. *This fascinating and accessible report is the output of a year-long action research project exploring strategies for supporting children to access age-appropriate mathematics and increase in-class participation. The report provides many useful examples and recommendations which will be of interest to teachers wanting to explore these approaches in their own classrooms. This project is also the topic of a 48 minute research talk.*

Ofsted statement	How this could look in practice	For example	Resources for the classroom
New content draws on and makes links with the content that pupils have previously acquired.	Teachers anticipate pupil responses and plan for how they can be used to support the learning of all pupils. Teachers' intentions regarding what they want their children to think about during mathematical interactions are based on what the children know and can do, and the children's developmental stage. New content is applied and consolidated. This might be through exploratory play, tasks and engaging problems.	See Section on **Curriculum sequencing: conditional knowledge** At snack time, adults encourage children to discuss how many pieces of fruit each child has and how to share the snacks fairly, building on their previous understanding of cardinality and comparing quantities.	**The Lesson Study Group (US)** US site with discussion, resources and videos of children learning through problem solving approaches. **Collaborative Lesson Research (UK)** UK site with resources to support lesson research. See suggestions in earlier sections related to **curriculum sequencing**.
Curriculum progression is by intelligent design rather than by choice or chance.	The curriculum and teachers' preparation for teaching mathematics displays their underlying knowledge of both the children and of mathematical structures and connections. There is consistent and developmental use of core manipulatives and representations. All staff have a deep understanding of their role in the development of mathematical understanding. Children are empowered to 'own' the learning through problem solving and investigative tasks, building on their keen ability to problem solve. Teachers are confident in pedagogical subject knowledge in all strands of mathematical learning.	Children have access to a range of structured manipulatives to explore connections between numbers, which will continue to be used as they progress through the school. Calculation methods are taught across the school by linking manipulatives with formal and informal methods, e.g. purposeful use of base-ten blocks leading to formal addition and subtraction, and number tracks and lines leading to mental methods. Manipulatives are selected carefully for children to explore mathematical ideas. Manipulatives are planned for and their use develops over time. NCETM progression documentation is used to inform lesson preparation and planning.	**Birth to 5 Matters** **Development Matters** Non-statutory guidance for the EYFS including Reception. **DfE non-statutory guidance** This document illustrates progression in key concepts from Y1–Y6 and includes a set of assessment tasks for each of the ready-to-progress criteria. **NCETM PD materials** These resources set out a teaching sequence and provide detailed pedagogical support for teachers in three curriculum strands: (i) number, addition and subtraction, (ii) multiplication and division, and (iii) fractions. Also see section on **curriculum sequencing.**

Ofsted statement	How this could look in practice	For example	Resources for the classroom
			The Teachers' Guide to any published scheme will support this.
Rehearsal sequences align with curriculum sequences.	Pupils are given time to consolidate what they have been taught. NRICH tasks are used related to current learning to assess and develop mathematical thinking.	Continuous provision and independent tasks are linked to what children have been taught including, e.g. providing 1–4 spotty dice and pens and paper alongside loose parts to practice subitising and recording small amounts. In Key Stage 2, children might learn about dividing a two-digit number by a single-digit number, and then explore how to arrange the digits 345 into [ ][ ] ÷ [ ] = to get answers with/without remainders, and as many different solutions as they can; this may lead to generalisations about when there will be a remainder.	**Mastery assessment materials** Guidance for assessing mastery and assessment tasks for Y1–Y6. From NCETM. **NRICH curriculum mapping** Activities are mapped against curriculum statements from EYFS to Y6. All activities have teacher notes and require and promote pupil reasoning. **National Curriculum resource tool** From NCETM. Making connections, articles, activities, exemplification and videos available for each aspect of the curriculum Y1–Y6.
Pupils who are more likely to struggle or who are at risk of falling behind are given more time to complete tasks, rather than different tasks or curriculums, so that they can commit core facts and methods to long-term memory.	Teachers plan for low threshold, high ceiling tasks that are accessible to all. This allows every child to work on a similar area of mathematics and gives teachers the opportunity to support those that need more support where necessary. Pre-teaching a few identified children for even a few minutes prior to a lesson has been shown to greatly benefit those children that might not have been able to access that content. This has been found to effectively assign competence to those children who might have struggled otherwise.	Before using dice in a number game, the teacher revisits subitising tasks children are familiar with, to make links with dots on the dice. Children are learning about formal methods for subtraction; the teacher takes 10 minutes in advance/or part way through the lesson to enable children to revisit the use of base-ten blocks for addition and makes connections with subtraction as inverse.	**Low-Threshold High-Ceiling – An Introduction** Information on low threshold/high ceiling tasks, and links to tasks to use in the classroom. LTHC tasks are those that all pupils can access and some can take further. See research by **Trundley et al. on assigning competence and pre-teaching** as detailed in the 'further reading' table at the start of this section.

Pedagogy and new learning

Further reading – ideas for a staff meeting or INSET	
Books (£)	Haylock, D. and Manning, R. (2019) *Mathematics Explained for Primary Teachers* (6th edition). London: SAGE. Haylock, D. and Cockburn, A. (2017) *Understanding Mathematics for Young Children* (5th edition). London: SAGE. *These books by Derek Haylock and his colleagues, written for trainee teachers, set out how to teach all aspects of mathematics. The books describe and exemplify the 'connections model' which recommends linking concrete experiences, pictorial representations and symbols with language. They would be a useful addition to a PPA room.*
Guidance reports	EEF. (2020) *Improving mathematics in the Early Years and Key Stage One – guidance report*. Education Endowment Foundation. EEF. (2017) *Improving mathematics in Key Stages Two and Three – guidance report*. Education Endowment Foundation. *Two key EEF reports for two age bands with recommendations from research for schools to put into practice that make a significant difference to pupils' learning.*
Espresso	Cambridge Mathematics (2019) 'Variation in mathematics education' Espresso, (20). *Espressos are online summaries of the latest good-quality research around a particular topic in maths education, expressly designed with teachers in mind. This issue outlines what research suggests about how variation theory might affect mathematics teaching and learning.*
Article	Russell, S. J. (2000) 'Developing Computational Fluency with Whole Numbers in the elementary grades.' *Teaching Children Mathematics*, 7(3) pp. 154–58. *The author discusses the need to balance skills and understanding – she highlights the fact that 'mathematical memory' is developed not by memorising but through the linking of mathematical ideas and relationships.*
NRICH	Woodham, L. (2018) *Using NRICH solutions as a resource*. NRICH. *As well as being a source of mathematical tasks, the NRICH website also publishes children's solutions to these tasks. This article outlines some ways in which these solutions can be used to support the teaching and learning of mathematics (for audiences of both pupils and teachers) by exemplifying particular tasks and their accompanying children's solutions.*

Ofsted statement	How this could look in practice	For example	Resources for the classroom
Teachers remember that it is not possible for pupils to develop proficiency by emulating expertise, but by emulating the journey to expertise.	Mathematical proficiency is fostered by adults deliberately working to develop confidence and a positive 'can do' attitude. Learning avoids being about 'answer getting' and supports learners to grapple with concepts by reasoning and by engaging with non-routine problems to encourage them to make sense of ideas in terms of what they already know.	In Reception to emulate the journey to being a mathematician, adults integrate aspects of the statutory 'Characteristics of Effective Teaching and Learning' (DfE 2020) into all their mathematical provision: ● playing and exploring ● active learning ● creating and thinking critically. In Primary, sequences of learning aim to develop fluent understanding, reasoning and problem-solving proficiency alongside social and emotional learning skills (e.g. mathematical resilience) so that learners can experience struggle and make their own decisions.	**Characteristics of effective learning in EYs** Defining and exemplifying the characteristics of effective learning. **Nancy Stewart interview** Background on the Characteristics of Effective Learning from a key member of the team advising on the EYFS 2012 review. **Maths with meaning** A short article about the connection between learning skills and solving problems, and the importance of challenge. **First Maths Challenge** **Primary Mathematics Challenge** These are annual events for pupils in Y3-4 and Y5-6. Past papers are available to download.
Systematic instructional approaches to engineer success in learning are incorporated into all stages and phases.	Adults are aware of developmental learning trajectories and use these to plan their mathematics teaching and children's experiences, building on what children know and can do. Embedded formative assessment practices allow for adaptations of learning both in-the-moment and in subsequent plans so that learning is tailored to reflect the cohort.	Use of dialogic pedagogies (see EEF report for an explanation) to encourage learners to share their understanding, use of mini-whiteboards to share responses and well-designed hinge questions that reveal understanding and common misconceptions.	**Dialogic Teaching** EEF report on the use of dialogic teaching pedagogies to develop learning including mathematics. **Early mathematics pedagogy** Description of exploration, apprenticeship and making sense. From ECMG. **Learning Trajectories** US site detailing developmental learning trajectories for birth to Y4 in all areas of mathematics, based

Ofsted statement	How this could look in practice	For example	Resources for the classroom
			on the work of Doug Clements and Julie Sarama.
Teachers aim to impart core content in alignment with the detail and sequence of the planned curriculum.	Adults are aware of developmental learning trajectories and use these to plan their mathematics teaching and children's experiences, building on what children know and can do. See previous notes connected to **curriculum progression**.	Children are introduced to 5-frames before 10-frames and encouraged to subitise small quantities and to use this knowledge to 'see' larger quantities. Children learn that angle is a measure of turn before they learn about the angles inside a triangle. See previous notes connected to **curriculum progression**.	**High Impact Teaching** Australian recommendations for effective teaching. **Metacognition** EEF recommendations on the impact of metacognition and self-regulated learning. See previous notes connected to **curriculum progression**.
Teachers help pupils to avoid relying on guesswork or unstructured trial and error.	In Reception adults create enabling environments and positive relationships that support the unique child to follow their interests and use their innate learning abilities through play and exploration. By first standing back and observing and then playing alongside and asking questions, adults understand and support learning that is child-led. Adults have a deep understanding of developmental trajectories and how children learn, supporting children through continuous provision, by constructing opportunities for sustained shared thinking and, when appropriate, leading group activities that stimulate interest and/or introduce new language or concepts.	Reception learners are developing their understanding of the composition of numbers to five. With the whole group, the teacher has shared some images of different arrangements of five objects and asked children 'What do you notice?' to share different ways that five can be composed of two parts. Continuous provision includes different collections of loose parts and when children are playing a game, skilled adults play alongside and respond to children's spontaneous noticing of parts within a whole. They ask follow-up questions and facilitate shared thinking about quantities. In a Year 6 lesson on the distributive property of multiplication, the teacher provides learners with a pair of calculations: **10 × 9 + 3 × 9**	See the **EEF guidance reports** detailed in the 'further reading' table at the start of this section.

Ofsted statement	How this could look in practice	For example	Resources for the classroom
	In Primary, teachers respond to formative assessment information to scaffold learning appropriately throughout whole class teaching and in paired, small group and independent work. Consistent use of familiar language and representations enables learners to make links to prior learning. Common misconceptions and difficulties are anticipated and explicitly incorporated through questioning and activities so that learners develop conceptual understanding. Pupils are consistently asked to justify their thinking.	and **6 × 9 + 7 × 9** and asks them to consider 'What is the same? What is different?' Children go on to generate other similar calculations and through reasoning and making use of pictorial representations of the area model for multiplication, reach a verbal generalisation of distributivity and to link this to formal methods for multiplication.	

Pedagogy: consolidation of learning

Further reading – ideas for a staff meeting or INSET	
Espresso	Cambridge Mathematics (2016) 'Confidence assessments in mathematics learning' Espresso, (2). *Espressos are online summaries of the latest good-quality research around a particular topic in maths education, expressly designed with teachers in mind. This issue outlines what research suggests about how assessing confidence affects learning and testing in mathematics.*
Research reports	Nunes, T., Bryant, P. and Watson, A. (2007) Outputs from the research project: 'Key understandings in mathematics learning'. *Of the eight papers in the review, papers 2–5 are primary-focused. The authors connect three key questions in understanding mathematics: what insight is needed, how informal mathematics knowledge relates to school learning, and what understanding is needed to build new mathematical ideas.*
Blog	Cartwright, K. (2021) *Games: Tools for mathematical learning*. Primary Learning. *The author discusses the role of games in consolidating primary pupils' learning, referencing Russo, Russo and Bragg's (2021) research into the use of games in mathematics classrooms and describes the mathematical value of a few favourite games.*
NRICH	NRICH team (2014) *Reasoning: the Journey from Novice to Expert*. NRICH. *This article describes a progression in mathematics reasoning which is divided into 5 stages (ending in proof). It clearly defines each stage, invites us to consider these definitions alongside examples of children's work and thinking, and is complemented by a second article called Reasoning: Identifying Opportunities, which provides guidance about further embedding the tasks and pedagogy into classroom practice.*
Book (£)	Borthwick, A. and Cross, A. (2018) *Reasons to Reason in Primary Maths and Science.* London: SAGE. *The authors recommend that reasoning is a part of mathematics education because it is an essential skill for the future. This book offers a framework of ideas, models, scaffolds, skills and practical ideas to develop mathematical reasoning in the primary classroom.*

Ofsted statement	How this could look in practice	For example	Resources for the classroom
Educators plan to give pupils opportunities to consolidate learning that go beyond immediately answering questions correctly.	Adults encourage children to reason about their answers.	By including questions such as: How do you know? Are you sure? Can you convince me? Is this always true?	**NRICH curriculum mapping** Activities are mapped against curriculum statements from EYFS to Y6. All activities have teacher notes and require and promote pupil reasoning. **Geometry Juniors** (£) From MA. This is an exploration of shape and space enabling children to test their knowledge and think mathematically. **Primary Questions and Prompts** (£) This ATM publication will support teachers to plan their teaching interactions, broaden their repertoire of questions and find out more about how their pupils develop the ability to think mathematically.
Educators plan to give pupils opportunities to consolidate learning that involve overlearning.	Children are given plenty of opportunities to expand their knowledge in a wide range of familiar and unfamiliar contexts, and in a range of routine and non-routine problems.	Time is provided for consolidation, discussion and sense-making which enables teachers to assess children's next steps.	See previous sections on **Pedagogy and new learning**
Educators plan to give pupils opportunities to consolidate learning that align with the detail and sequence of the curriculum.	See previous sections on **Curriculum sequencing**	See previous sections on **Curriculum sequencing**	See previous sections on **Curriculum sequencing**

Ofsted statement	How this could look in practice	For example	Resources for the classroom
Educators plan to give pupils opportunities to consolidate learning that are free of distraction and disruption.	The National Curriculum promotes dialogic learning and this is seen in talk-rich classrooms. Children are enabled to engage in purposeful activity and dialogue, teachers listen carefully to children's dialogue, and where appropriate they scaffold their discussions and use children's ideas to further the learning of others. In Reception, adults draw on the statutory Characteristics of Effective Teaching and Learning when planning mathematics interactions.	Children are given time to think about a question by themselves before using 'talk pairs' to discuss what they are thinking. Children are encouraged to be 'Active learners' of mathematics by supporting them to concentrate, to keep trying and to celebrate what they achieve. Adult-child discussions continue beyond simple question and answer, to include speculation and encouragement to 'tell me more…'.	Talk Maths Website focussing on the value of talk in learning. The site includes lots of free resources for teachers and parents. Talking Maths (£) ATM publication with activities to promote talk in KS2 and KS3 mathematics classrooms. Sustained shared thinking Article on this key Early Years pedagogy developing early mathematical talk.
Educators plan to give pupils opportunities to consolidate learning that strike a balance between type 1 and type 2 practices. There are 2 'types' of practice: – 'type 1' involves the rehearsal of core facts, methods and strategies that can be used to complete exercises and solve problems now and in the next stage of education – 'type 2' includes explaining, justifying and proving concepts using informal and diagrammatic methods, parsing and derivation of number.	Mathematics is integrated throughout the day as well as in dedicated time. To embed and further develop number facts or written algorithms, teachers may pose a question or present an investigation that engages and motivates learners, whilst providing opportunities for practice and developing mathematical thinking. Children have opportunities to embed new ideas through interesting activities, and to use newly developed understanding in a range of contexts, both mathematical and non-mathematical.	At the start of the school day, pupils self-register by moving their name onto a large pictogram. Pupils work out how long it is until lunch break. In Reception, children compare lengths of cloth to make a superhero cape for a teddy. To embed addition and subtraction facts: 'What happens when you add three consecutive (next door) numbers?' or 'I add three one-digit numbers together; my total is 14. What numbers could I have added together?' To embed written multiplication methods: 'What is the closest product to 1028 that I can make with these four digits?'	See previous sections on **Pedagogy and new learning.** Creative Star Learning A website packed with ideas to make learning of mathematics active and outdoors. ATM maths snacks A collection of short videos introducing intriguing problems. Love Maths games Short videos demonstrating simple but engaging games using cards and other easily available resources. Pick and Mix (£) This e-book from ATM provides tasks that develop fluency with number, algebra and geometry

Ofsted statement	How this could look in practice	For example	Resources for the classroom
		Children use recently learned place value understanding, to discuss the proximity of planets in our solar system.	together with the independent thinking required for problem solving.
Educators plan to give pupils opportunities to consolidate learning that avoid creating a reliance on outsourced memory aids or physical resources.	Adults plan learning paying attention to the effective use of a range of manipulatives and other representations, integrating this with mental work, fingers and informal written recording; helping children understand the links between all the representations. Teachers are aware of the value of interweaving concrete, pictorial, and abstract in mathematics lessons.	Adults model a hiding game with 6 objects, and encourage children to explain what they see and how they know, to draw solutions and answer related questions such as: 'how many are in the box if 3 are outside?' Teachers encourage pupils to visualise the manipulatives and representations, using the same language, so pupils make connections.	See previous sections on **Pedagogy and new learning.**
Educators plan to give pupils opportunities to consolidate learning that help pupils to avoid relying on guesswork or unstructured trial and error.	See previous sections on **Ambition for All,** and **Pedagogy and New Learning.**	See previous sections on **Ambition for All,** and **Pedagogy and New Learning.**	See previous sections on **Ambition for All,** and **Pedagogy and New Learning.**

Assessment

Further reading – ideas for a staff meeting or INSET	
Espresso	Cambridge Mathematics (2017) 'Effective feedback to mathematics students' Espresso, (9). *Espressos are online summaries of the latest good-quality research around a particular topic in maths education, expressly designed with teachers in mind. This issue outlines what research suggests about the characteristics of effective feedback to mathematics students.*
NRICH	Hatch, G. (2005) *Using Games in the Classroom*. NRICH. *This article reports first-hand classroom experience of using games as an effective medium for learning mathematics. Separate consideration is given to the learning that took place, the ways of working that were observed and the different experiences encountered over time by pupils who were offered this means of engagement with the subject.*
Book (£)	Hansen, A. (2020) *Children's Errors in Mathematics.* (5th edition). London: Learning Matters. *This book supports teachers in understanding the common misconceptions in primary mathematics learning, enabling teachers to assess children's understanding and plan to expose potential barriers to understanding.*
Article	Boaler, J. (2014) 'Research suggests timed tests cause math anxiety.' *Teaching Children Mathematics*, 20(8) pp. 469–474. *In this article, Boaler explores the link between timed tests of mathematics facts and maths anxiety. She describes the connection between experiencing stress and struggling to access the working memory, leading to underachievement in timed tests. She suggests alternative approaches.*
Blog	Cox, S. (2021) *EEF Blog: Integrating evidence into mathematics teaching - addressing assessment*. EEF. *Blog from EEF describing the multiple purposes assessment serves: tracking pupil progress for school purposes; identification of ideas and concepts requiring re-teaching; and the highlighting of misconceptions for pupil support.*
Book (£)	Hodgen, J. and Wiliam, D. (2006) *Mathematics inside the black box.* London: GL Assessment. *A booklet providing an excellent clear introduction to the ideas of AFL offering advice and guidance on how to develop formative assessment in mathematics. The book is organised around three themes: classroom dialogue; feedback and marking; and peer and self-assessment.*

Ofsted statement	How this could look in practice	For example	Resources for the classroom
Pupils are well prepared for assessments through having learned all the facts, methods and strategies that are likely to be tested.	Children have the opportunity to explore and apply the use of recently acquired understanding in new and varied contexts.	When I can count to 7, I can explore different ways of showing 7 on 2 hands. How can this be recorded? What about 8? or 5? is there a pattern? Child rolls 5 dice and makes a 3-digit number and a 2-digit number. They explore the highest and lowest products possible.	**Mastery assessment materials** Guidance for assessing mastery and assessment tasks for Y1–Y6. From NCETM. **DfE non-statutory guidance** This document illustrates progression in key concepts from Y1–Y6 and includes a set of assessment tasks for each of the ready-to-progress criteria. **The dyscalculia assessment** (£) This resource includes activities to assess pupils' understanding.
Teachers plan frequent, low-stakes testing to help pupils to remember content.	Children respond to up to five key questions about the big ideas at the end of a sequence of work. This may be practical or written, and will not form part of data collection, but is used to inform the teacher. In Reception teachers build in short reflection time after some experiences or after some teacher-led activities	In the last 10 minutes of a lesson children may respond to three carefully chosen questions about the content of the lesson. Children write two quiz questions about their learning at the end of a lesson. I took a picture of you while you were building your tower, what can you tell me about what you were doing? The class are shown four different solutions to a relevant problem and discuss the efficiency of the methods and unpick any errors.	**See resources listed for previous statement** **AfL in mathematics** (£) This resource describes a variety of AfL strategies, with advice for the teacher. Activity materials are included. From MA. **Diagnostic Questions** A bank of questions with 1x right answer and 3x wrong answers to reveal common misconceptions. **Concept Cartoons in mathematics education** (£) This set of concept cartoons is organised according to topic.
Lessons incorporate timed testing to help pupils learn maths facts to automaticity.	See previous section on **Ambition for All** and **Consolidation of Learning**	See previous section on **Ambition for All** and **Consolidation of Learning**	See previous section on **Ambition for All** and **Consolidation of Learning**

Systems at the school level

Further reading – ideas for a staff meeting or INSET	
NRICH	Carruthers, E. and Worthington, M. (2010) *Children's mathematical graphics: Understanding the key concept*. NRICH. NRICH team (2013) *Primary Children's Mathematical Recording*. NRICH. *These articles consider the place and value of meaningful mathematical recording. The first focuses on the mark-making of children in EYs/KS1 and highlights the differences and connections between recording and representing mathematics. The second – looking at KS1 and KS2 – suggests that there are three particular contexts in which recording might take place. The implications for classroom practice are considered and tasks are signposted that offer pupils opportunities to capture their thinking in different ways.*
Blogs	Burns, M. and Sibley, R. (no date) *Maths journals boost real learning*. Scholastic. Douglas, H. (2019) *5 types of maths journals and how to use them*. Maths No Problem! The Lesson Study Group (no date) *Teaching through problem solving*. The Lesson Study Group. *If you are considering how and what pupils record in their mathematics books, then these three short articles may be useful. They describe some benefits of journaling, examples from real classrooms, and also include useful ideas for introducing these with learners including the use of scaffolds to support pupils' journaling.*
Espresso	Cambridge Mathematics (2019) 'Effective continuing professional development' Espresso, (11). *Espressos are online summaries of the latest good-quality research around a particular topic in maths education, expressly designed with teachers in mind. This issue outlines what research suggests about what makes for effective continuing professional development in mathematics teaching.*
Book (£)	Rowland, T., Turner, F., Thwaites, A. and Huckstep, P. (2009) *Developing Primary Mathematics Teaching: Reflecting on Practice with the Knowledge Quartet.* Los Angeles, CA: SAGE. *The book is a collection of papers and a CD of the accompanying lessons, which enable the reader to understand the knowledge required to teach mathematics. The quartet provides a framework for lesson observation that focuses on the teacher's choices to support learning.*
Article	Archer, R. (2016) Lesson Study: A trip to Japan. *Mathematics Teaching* (250) pp. 36-40. *The author describes her research into using Lesson Study – a collaborative professional development tool that originated in Japan – in a UK context.*

Ofsted statement	How this could look in practice	For example	Resources
School-wide approaches to calculation and presentation in pupils' books. [**Ofsted Tweet 8/6/21** '*We don't require pupil work in a specific format in our inspections – or for it to be kept for us*'].	Young children are supported to represent their thinking informally in a range of contexts. In the same way as children learn to record in writing and comprehension tasks in English (both for personal note-taking and to communicate to others), they also have opportunities to learn how to record mathematics in a meaningful way, both for personal use (to help them do the mathematics) but also to communicate to others. They learn that some mathematics is recorded symbolically, and in universally understood formats (e.g. it is helpful if some 3-digit by 2-digit multiplications are recorded in a formal algorithm, or that an addition can be recorded in an equation such as 7 = 3 + 4).	Children know that mark-making not only has meaning in writing but also in mathematics. Children use journals to record their thinking in mathematics, and to record in ways that enable them to rehearse, practice and communicate their thinking, strategies and methods.	See the **blog posts** and **NRICH articles** above for ideas for using journals and encouraging pupils' own recording.
School-wide approaches to providing time and resources for teachers to develop subject knowledge and to learn valuable ways of teaching from each other.	Staff have opportunities to collaborate to plan effective lessons and to learn from each other. All forms of CPD are highly valued in school, including that which is led internally, externally and through lesson study.	Teachers across EYFS, KS1 and KS2 are given regular time to discuss developmental learning trajectories together and to observe in each other's classrooms, jointly developing an understanding of what they mean by 'manipulative use', or 'developing reasoning'.	**The Lesson Study Group (US)** US site with discussion, resources and videos of children learning through problem solving approaches. The lessons use journaling and promote deep mathematical understanding. **Collaborative Lesson Research (UK)** UK site with resources to support lesson research.

https://www.atm.org.uk

https://www.m-a.org.uk

MA/ATM primary group alisonborthwick@me.com (Chair)