Primary Mathematics Challenge – February 2018

Answers and Notes

These notes provide a brief look at how the problems can be solved.
There are sometimes many ways of approaching problems, and not all can be given here.
Suggestions for further work based on some of these problems are also provided.

P1 **C** (30 cm ÷ 5 = 6 cm) P2 **D** $(12 \div 2) \times 3 = 18$

1 **C** 25

We can simply calculate £50 ÷ £2 = 25.

2 **A** $2 \times (0+1+8)$

Evaluating each option:
for A: $2 \times (0+1+8) = 2 \times 9 = 18$;
for B: $2 - 0 - 1 - 8 = -7$;
for C: $2 \times 0 \times 1 \times 8 = 0$;
for D: $2 + 0 + 1 + 8 = 11$;
for E: $2 + (0 \times 1 \times 8) = 2 + 0 = 2$.

3 **E** 228

Having read three-quarters of the book, I read a further $22 + 35 = 57$ pages, the remaining quarter. Hence the book has $57 \times 4 = 228$ pages.

4 **C** 9 geese and 5 sheep

All five options will account for a total of 14 heads, so we must find which option gives a total of 38 legs:

geese		sheep		
number	legs	number	legs	total number of legs
8	16	6	24	40
6	12	8	32	44
9	18	5	20	38
5	10	9	36	46
10	20	4	16	36

So Jake will see 9 geese and 5 sheep.

5 **A** $\frac{15}{24} = \frac{1\times5}{2\times4}$

Taking each option we have $\frac{1\times5}{2\times4} = \frac{5}{8} = \frac{15}{24}$ so option A is true. For the other options,

B: $\frac{2\times4}{3\times5} = \frac{8}{15} = \frac{24}{45} < \frac{24}{35}$

C: $\frac{3\times3}{4\times6} = \frac{9}{24} = \frac{3}{8} < \frac{33}{46}$ since $\frac{3}{8}$ is less than $\frac{1}{2}$

D: $\frac{4\times2}{5\times7} = \frac{8}{35} < \frac{42}{57}$ since $\frac{8}{35}$ is much less than $\frac{1}{2}$

E: $\frac{5\times1}{6\times8} = \frac{5}{48} < \frac{51}{68}$ since $\frac{5}{48}$ is much less than $\frac{1}{2}$.

6 **B** 12

In the first 10 minutes Dr Jabbemall will have 'jabbed' 60 patients, and so has 240 patients to jab in the next 20 minutes. This amounts to $240 \div 20 = 12$ jabs per minute.

7 **E** 401 to 460

The first multiples of 91 are: 91, 182, 273, 364, 455 — the last of these is the first to be found in the ranges given.

8 **D** 13

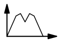
Gethin's best choice is to get as many songs as he can at the reduced 5 for £3.50 rate and then see how many he can get with the amount of money he has left. Since 3 × £3.50 = £10.50, he can only make use of the reduced rate twice, buying $2 \times 5 = 10$ songs. With £$(10 - 2 \times 3.50) =$ £3 left, it can be calculated that £3 ÷ 79p is 3 with $(300 - 3 \times 79)$ = 63p change. Hence he can buy $10 + 3 = 13$ songs. In fact with these numbers, Gethin could get 13 downloads with just one group of five and the rest in singles, though he would get less change.

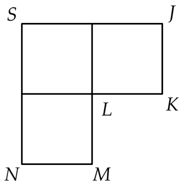
9 **C** 155

The most efficient way to calculate this is to notice that each of the next ten numbers (11, 12, 13, …, 20) is 10 more than the corresponding number in the original ten numbers (1, 2, 3, …, 10). So their sum is $10 \times 10 = 100$ greater, ie. $55 + 100 = 155$.

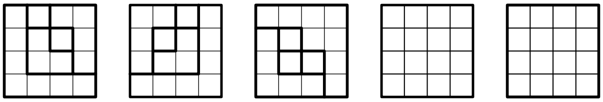
10 **D** £1 750 000 Audrey weighs 50 000 grams, so her value could be calculated as $50\,000 \times £35 =$ £1 750 000. Of course, to her father, Audrey is priceless.

11 **B** Beans The first and third lines of the menu reveal that beans are 40p more expensive than egg. Now since the most expensive of the five items is twice as dear as the others, beans must cost 80p and the other items 40p.

12 **A**  Asmita's run can be split into six stages: from S to J, J to K, K to L, L to M, M to N, and N to S. Between S and J, her distance from S increases, and continues between J and K but not quite as quickly. But between K and L, Asmita gets closer to S. After L the graph of her distance as she returns to S will be the reflection of the graph up to L. Therefore the only graph that represents this is graph **A**.



13 **E** 80 The number of diamonds is equal to the number of faces of the golden cubes visible from any viewpoint apart from the base. So looking from each of these five directions, we see:

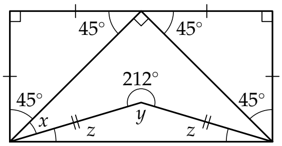


front left | front right | top | back left | back right

It should now be clear that we need $4 \times 4 \times 5 = 80$ diamonds.

14 **D** 9 The factors of 20 are 1, 2, 4, 5, 10 and 20, so $\boxed{\mathbf{20}} = 6$. Similarly the factors of 18 are 1, 2, 3, 6, 9 and 18, so $\boxed{\mathbf{18}} = 6$ and so $\boxed{\mathbf{20}} \times \boxed{\mathbf{18}} = 6 \times 6 = 36$. Now the factors of 36 itself are 1, 2, 3, 4, 6, 9, 12, 18 and 36, so that $\boxed{\boxed{\mathbf{20}} \times \boxed{\mathbf{18}}} = 9$.

15 **E** 29° Two of the triangles are both right-angled and isosceles, and so have angles of 45°, 45° and 90°. Labelling the remaining angles y and z, we have the following:



There are now two approaches to finding angle x:

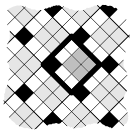
(i) Angle $y = (360 - 212)° = 148°$, and angle $z = (180 - 148)° \div 2 = 16°$. Hence $x = (90 - 45 - 16)° = 29°$

(ii) Considering the interior angles of the "arrowhead", $360° = (90 + 212 + 2x)°$, and so $x = 29°$.

16 **B** 02:22 on Tuesday We know that 1800 minutes is $1800 \div 60 = 30$ hours, 180 minutes is 3 hours, so that 2018 minutes is 33 hours with another 38 minutes remaining. So 2018 minutes is equivalent to 1 day, 9 hours and 38 minutes. Counting backwards from Wednesday midday gets to Tuesday at 02:22.

17 **B** 1 : 4 : 4

If you look carefully, you can see that a 3 by 3 pattern repeats throughout the tiling, as highlighted on the right. This means that the ratio of black to grey to white tiles for a large enough wall will be the same as the ratio for the 3 by 3 pattern, that is 1 : 4 : 4. For further information about analysing tessellations, see the work of Elizabeth Williams and Hilary Shuard on tessellations and lattice points, in Williams, E. & Shuard, H. (1970) *Primary Mathematics Today*, Longman, London (ISBN 9780582360044).



18 **E** 500 018

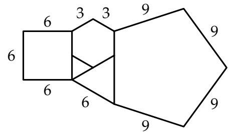
Given that 100 001 is 1 less than a multiple of 7, we know that 100 002 is a multiple of 7. So the numbers 200 004, 300 006, 400 008 and 500 010 are multiples of 7, as they are multiples of 100 002. Hence the numbers 100 003, 200 005, 300 007, 400 009 and 500 011 are one more than a multiple of 7. Of the options, only 500 018 is a multiple of 7 away from these, and so is one more than a multiple of 7.

19 **A** 1 : 2 500 000

The scale of the map is 18 cm : 450 km; converting the actual distance to centimetres, this is equivalent to $18 : 450 \times 1000 \times 100 = 18 : 45\,000\,000$. Simplifying gives a scale of 1 : 2 500 000.

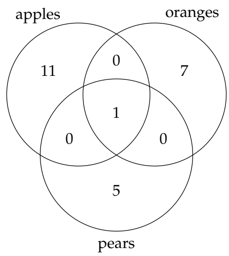
20 **E** 66 cm

Given the perimeter of the square is 24 cm, its side-length is $24 \text{ cm} \div 4 = 6$ cm. The smaller equilateral triangle and the hexagon have the same side-length and this must equal half of the side-length of the square, that is 3 cm. So the side-length of the larger triangle must be 6 cm and hence the side-length of the pentagon is $(6 + 3)$ cm = 9 cm. The perimeter of the combined shape consists of three sides of the square, two sides of the hexagon, four sides of the pentagon and one side of the larger triangle $= 3 \times 6 + 2 \times 3 + 4 \times 9 + 6 = 18 + 6 + 36 + 6 = 66$ cm.



21 **D** 23

In the class of 24, 12 children eat oranges, 8 eat apples and 6 eat pears; it is, therefore, not possible for all the children to eat only one kind of fruit, as $12 + 8 + 6$ is greater than the total number of children. The diagram on the right shows how it is possible for 23 children to eat only one kind of fruit.

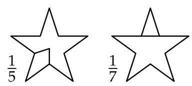


22 **C** 5396

When put in order, the numbers are: 3569, 3596, 3659, 3695, 3956, 3965, 5369, 5396, 5639, 5693, . . .

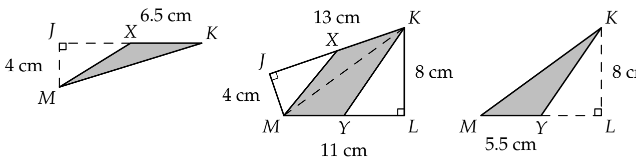
23 **D** $\frac{19}{35}$

Since the fraction shaded in shape P is $\frac{4}{5}$, the fraction that is unshaded, which is a kite, is $\frac{1}{5}$. Likewise the fraction unshaded in shape Q is $\frac{2}{7}$, and so the area of each of the triangles is $\frac{1}{7}$ of the total area — as shown on the right. The shaded area in shape R is made up of two such kites and one such triangle. So the fraction shaded in shape R is $2 \times \frac{1}{5} + \frac{1}{7} = \frac{14}{35} + \frac{5}{35} = \frac{19}{35}$.



24 **C** 35 cm^2

To find the shaded area, we will split it along the line MK as shown:



The area of triangle $KXM = \frac{1}{2} \times 6.5 \times 4 = 13 \text{ cm}^2$; the area of triangle $KYM = \frac{1}{2} \times 5.5 \times 8 = 22 \text{ cm}^2$. So the area shaded = $(13 + 22) \text{ cm}^2 = 35 \text{ cm}^2$.

25 **A** 48 First we must work out the number of tickets of each colour. Let the numbers of red, white and blue tickets be r, w and b respectively; we have $r : w = 5 : 4$ and $w : b = 8 : 7$. Writing $r : w$ as $10 : 8$, we can combine the ratios to find $r : w : b = 10 : 8 : 7$. Hence for a total of 100 tickets, there must be 40 red, 32 white and 28 blue tickets. Now the largest number of red tickets one can select *without* having half of them is 19, and likewise 15 and 13 for white and blue tickets. So it would be possible to select $19 + 15 + 13 = 47$ tickets and still not have half or more of any particular colour of ticket. However, the very next ticket, the 48th, would have to be a red, a white or a blue one, and so one of the colours would now have at least half of its number selected.

Some notes and possibilities for further problems

5 There are many other examples of what are referred to as *anomalous calculations*, where you can leave out the same digit (or digits) on both the top and the bottom of a fraction and create an equivalent fraction – here are a few:

$$\tfrac{26}{65} = \tfrac{2}{5} \qquad \tfrac{116}{464} = \tfrac{11}{44} \qquad \tfrac{1016}{4064} = \tfrac{101}{404}.$$

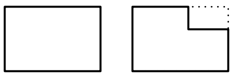
Equally as strange as these are *Printer's errors* where a multiplication sign is carelessly left out and powers become ordinary digits: $2^5 \times 9^2 = 2592$ and $3^4 \times 425 = 34\,425$.

8 In the question there is more than one way to get 13 songs for £10. Pupils might invent their own special offers and investigate rates to explore the best buys.

10 Audrey's father can now measure amounts of money in "Audreys", using the conversion 1 Audrey = £1 750 000. In this way a *Limited Edition Bugatti Chiron* will cost him 1.43 Audreys (= £2.5 million), and a Mars bar a mere 0.000 000 3 Audreys (= 60p) – perhaps the Audrey will not catch on. It is not unusual for unconventional units of measurements to be used; for instance, when a large iceberg broke away from the Larsen C ice-sheet in Antarctica in July 2017, it was described as being "a quarter of the size of Wales" – this being much easier to grasp than $5000\,\text{km}^2$ or even half a million football pitches. Of course, the *mile* had its origin in the distance a Roman soldier could cover in a thousand paces, *mille passus*. Perhaps your pupils can think of their own "useful" units of measurement!

12 Pupils could investigate the graphs for routes around other polygons – squares, rectangles, hexagons, dodecagons, circles. Some of them are more complicated than pupils might think!

13 There is a similarity between the situation here (removing parts of the cube, keeping new faces parallel to those of the original cube, but not changing the total surface area), with the 2-D equivalent whereby the perimeter of a rectangle is unchanged when a smaller rectangle is removed from its corner as shown on the right. Could this principle apply to other shapes too?



14 It should be noted that 36 has an odd number of factors. This is true of all square numbers, as with any square number n we can pair off all but one of its factors so that each pair has a product of n itself – but there is always one factor left on its own, namely the square root of n.

18 There is a little-known method for deciding whether a number is divisible by 7, without having to divide. The method involves crossing off the units digit, but before you forget it completely multiply the digit by 2 and subtract it from the remaining number – continue repeating this process until you recognise a multiple of 7 (including 0, of course) in which case your original number is a multiple of 7, or not, in which case the original number was not. For example, to test the number 3465 (which is $5 \times 7 \times 9 \times 11$) we do: $3465 \to 346 - 2 \times 5 = 336 \to 33 - 2 \times 6 = 21$ (a multiple of 7).

21 What is the smallest possible number who could eat only one kind of these fruits? What is the largest number that could eat exactly two fruits? There are many questions that arise from this information – some of them have answers, and some of them may not!