Equals

Realising potential in mathematics for all

for ages 3 to 18+

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ANXIETY AND RECALL

Realising
potential in mathematics
for all

Letters and other material for the attention of the Editorial Team to be sent by email to: edmiston01@btinternet.com

Advertising enquiries: Janet Powell
e-mail address:
jcpadvertising@yahoo.co.uk
Tel: 0034 952664993

Published by the Mathematical Association, 259 London Road, Leicester LE2 3BE
Tel: 0116 221 0013
Fax: 0116 212 2835
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Designed by Nicole Lane

Editor's Page

This edition of *Equals* is packed with a range of articles under the umbrella of anxiety – its causes and some possible solutions.

You will notice we have several new contributors. I happened to meet Les Staves in June and as a result he was able to offer us a piece on the issues surrounding counting and the development of mathematical thinking. Pete Jarrett reflects upon the impact that Laurie Buxton has had upon his thinking and in tribute we include the piece he refers to, one that was originally published in *Struggle*.

In a similar vein Clare Lee, Sue Johnston-Wilder and Lucy Rycroft-Smith all reflect upon the roots of mathematical anxiety. Mark Pepper tracks the changes to the assessment of mathematics which, inadvertently, be causing a rise in anxiety in the classroom. Finally Professor Steve Chinn shares some of the causes of anxiety and suggests way that we can all, with reference to what we actually know about the development of mathematical thinking, take effective steps to support all of our learners.

A quick glance at the contributors to this edition serves to highlight a change that is taking place in *Equals*. More and more of you are getting in touch to use *Equals* as a forum to share your thoughts about how you are helping those who struggle to cope in the mathematics classroom. The focus for this edition is anxiety and I hope you find the articles helpful as you seek to support the needs of all your learners. As I shared in a previous edition anxiety is something that can affect all learners regardless of their age or ability. I still have not really come to terms with my own daughters shift in attitude towards a subject she always excelled in at age 17.

I would like to build upon this shift in focus and open the next edition to all readers. Please feel free to send in your hints and tips concerning activities that you always do as you feel they reach out and open the eyes of those who do not view themselves as mathematically able. It could be something you have read or a lesson, activity or resource you know will always work to engage all pupils.

Personally I would not be without Rummikubs in my classroom. It is a game we love as a family yet it is one that I used to break down some significant barriers with a group of internally excluded pupils in Key Stages 3 and 4 a few years ago.

Coming from another perspective I personally feel the teaching of the measurement system is neglected in many schools and is drowned out by the dominant focus upon counting. Counting is not easy as Les Staves so eloquently highlights in his article and so it may that complementing this focus with an exploration of the measurement system (so laying the foundations for both ratio, fractions and scale) could bear much fruit. I have a range of activities based around measurement that I use across Year 1 – 9 which I will share in the next edition.

Anxiety and recall

Stimulated by a prescient article written by Laurie Buxton many years ago Pete Jarrett reflects upon the lot of special needs pupils in the face of the current drive towards a more formal mathematics curriculum.

Mark Pepper brought an old copy of *Struggle* to an Editorial meeting a few months ago, and in that edition there was an article by Laurie Buxton entitled 'maths phobia' that has been re-printed in this edition. The article is as current now as it was in 1992 when it was published. However, in the last 25 years our awareness of maths anxiety has increased significantly, as is shown by the contributions in this current edition of *Equals*. Maths anxiety is complex and the contributing factors are many, but often resulting in a very real fear of doing mathematics.

there is a problem with negative attitudes to mathematics in society

On page 81 of Adrian Smith's *Review of Post-16 mathematics* (Smith, 2017) there is a section on culture and appreciation of mathematics in society. The section draws attention to the 'deep seated and enduring' issues with mathematics, the acceptability of 'I can't do maths', the gender bias in the uptake of mathematics courses, and the poor understanding of the role and function of mathematics in a numerate society. At the bottom of this article is a link to a short '*Pisa in Focus*' paper (OECD, 2016), 'Is memorisation a good strategy for learning mathematics?'. Note to Mr Gibb – It doesn't seem to be especially helpful, certainly not used by a high percentage of learners in the countries that we admire so much such as Shanghai-China and Singapore. Around the same time as Professor Smith's report came out, another report '*The Wellbeing of Secondary Pupils with Special Educational Needs*' was published (Barnes and Harrison, 2017). Pupils with SEN have poorer wellbeing than non-SEN pupils around school and school work.

To summarise, there is a problem with negative attitudes to mathematics in society, remembering stuff isn't as good as understanding why to do stuff, and SEN pupils are unhappier and more likely to suffer from mental illness due to school and school work. Many readers of *Equals Online* will now be gently banging their heads on the desk, mumbling, "I know, I know, I know, please offer some suggestions on how to help, I am sooo tired and it's not *****mas yet!"

Well, not quite yet my friends – I have more moaning to do! Over the summer, two esteemed Professors of my acquaintance had their attention drawn, by a third party, to a request to respond to some questions about a new Year 4 times tables test. Both Professors have an outstanding international reputation in maths learning and cognitive neuroscience, are largely retired after very successful careers, and pretty much know what they are talking about – they may be considered experts. One of these Professors was perplexed, perhaps in all his career he had missed something about the absolute importance of knowing your times tables in the development of mathematicians, and politely asked if the nice man from the DfE could help him fill in the gaps. He got no reply. The other Professor was very angry, and thought that many students would be harshly judged due to poor working memory, slow speed of processing and weak numerosity. As this Professor knows quite a lot about maths anxiety, he was also rather concerned that 9-year olds may be subjected to a testing regime that is going to deepen any anxieties. Neither of these Professors is what is known as a snowflake in modern parlance, their concerns are very real, formally testing at this age is going to disadvantage many and probably offer no extra meaningful information. Neither was suggesting that recall of knowledge was not useful, it can make maths so much more efficient if you can recall these facts, rather, that not all learners are able to access facts at the required level of efficiency and that recall is not the only ability required in maths learning.

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In a bid to aid public understanding of times table knowledge recall and multiplicative reasoning I found a couple of random 8 to 9-year olds and asked them a few closed times table questions, testing recall and perception of the commutative law, and a couple of open questions about how they worked their answers out – a small amount of recall, some finger counting, some use of the distributive law. It took around 2 minutes and I am sure that it provided as much information about the learner's mathematical abilities and understanding as a formalised test. I believe it used to be called teaching. I then undertook the same task with some adults – a small amount of recall, some finger counting, less confidence in the distributive law.

As I was now fully engaged in unreliable and invalid research methodologies I thought I would delve deeper in to some of the traditional negative attitudes about mathematics to see if I could draw out anything that may offer some insight to the birth of these attitudes. Luckily, I have access to an interesting sample group, 70 odd online marketers, all self-employed and requiring a reasonable amount of maths knowledge, all embracing a new way of doing business, all women, from a range of backgrounds, mostly aged between 25 and 55, many being parents, and, most importantly, always willing to fill in a short online questionnaire of dubious quality (during the process of this research

I also discovered that my Unicorn name is Blissful Fuddle-Bucket and my star sign suggests that my perfect career would be interior designer).

When asked how happy they were at school, on a 5-point Likert scale, with 1 being ‘very unhappy’ and 5 being ‘very happy’, the mean response was 3.33 and the modal response was a 4.

When asked how happy they were in maths lessons, on the same 5-point scale, the mean response was 2.88 and the modal response, with 29% of respondents was 1.

When asked were they good at maths, 32% said no and 43% said they knew enough to get by.

When asked ‘Is it OK to be bad at maths?’ only 13% said it’s not that important

When asked if doing maths makes them anxious 45% said yes. Below are some of their comments when asked about the kind of situations that make them anxious:

> *If I have to do it in front of people or it’s for something important, I’m scared of getting it wrong.*
>
> *When I need to work out the answer quickly in my head.*
>
> *In particular when still using my fingers to count in front of others and also being expected to quickly come up with an answer. I tend to take a while to come up with an answer (if I’m able to come up with one).*
>
> *I hate maths to the point if I know I have something that needs maths I can have panic attacks.*

There were some common themes, mental arithmetic, having to do maths in front of people, answering quickly. These themes are consistent with the research that I have published previously in *Equals Online* (Jarrett, 2015). Often the causes of anxiety are built around how maths is done rather than the maths itself, speed of processing, recall of maths facts and the allied use of alternative strategies and having to do maths in public, feeling slow and worrying about appearing to be thick.

The premise behind formal testing of maths facts is that fluency in recall both aids understanding and reduces anxiety, which is true, but only for some. There is no doubt that good recall leads to efficient calculation, but not everyone is able to achieve efficient recall, and there are a multitude of reasons why not, some cognitive, some due to teaching and some due to societal influence. Many of the reasons that people struggle to work efficiently will fall in to the range of SEN, meaning that it is these SEN learners that will become unhappy with their difficulties in keeping up, working in the way they have been asked to do, and feeling part of an inclusive maths learning community.

When asked ‘Is it OK to be bad at maths?’ only 13% said it’s not that important, which I thought was comforting. Interestingly though, several respondents did say that ‘basic maths’, by which they mean functional numeracy, is important but more complex maths is not.

The most interesting responses came to the question 'what made maths at school a good or bad experience?':

> *A good teacher, a non-disruptive classroom environment.*
>
> *Consistent teaching*
>
> *My teachers were fun and loved maths*
>
> *Great teachers, lots of opportunities to learn in different ways*
>
> *I had a patient teacher who was able to explain concepts in a number of ways and give me time and opportunity to practise*
>
> *My teacher made it a fun experience. And even though I wasn't good enough he pushed me to do more and higher.*

That is the answer to what works. But 30% of respondents said that their teacher didn't like them, so perhaps not everyone has got the 'consistent, engaging and passionate' memo. Here are some of the more negative responses:

> *I just felt I could never get it right and was too scared and embarrassed to ask for help for fear of being called 'thick'*
>
> *Timed tests and being put on the spot to answer questions in front of the class*
>
> *Indifferent bad tempered teacher unwilling to accept that different teaching methods would probably suit different students.*
>
> *I was in set 1 for Maths in my 1st year at Secondary school. By the time I left school I had 13 O Level A Grades and a Grade c in GCSE Elementary Maths! I didn't understand some things in that first year and fell further and further behind. It ended up that I couldn't do it at all and became anxious at even having to think about it. That has carried on though my entire life*
>
> *My teacher made it bad by saying he didn't see the point in teaching maths to girls!*

Teaching maths isn't just about the maths, it's about the script that we speak from, the environment we create, and the learners we expect our students to become. It's about fostering positive learning relationships, grit, resilience, mindset and blackbox thinking. It is as much about the psychology of the learning brain as it is about the confusion of fractions to the untrained.

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Maths Phobia

Laurie Buxton

I expect that you teach with great clarity and a caring regard for your pupils' sensibilities, but might it just be that some have problems left by other teachers less perceptive than you? You remember the Cockroft Report? One group of researchers approached adults at random to ask them their views about mathematics. Though they were firmly assured that they were not going to be asked to do any, about half of them flatly refused to have anything to do with the survey. I wonder if the same will be true if this is tried again in, say, ten years time.

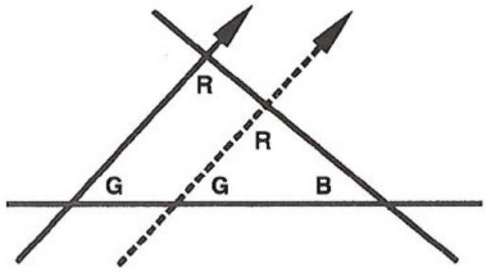
From the work that has gone into presentation we might believe that once we find how to explain things properly, they will all understand. It is not so. That is not to decry the valiant efforts made, the devising of new schemes and the invention of concrete materials of great subtlety. They help. But the root of many problems is emotional.

The nature of mathematics does not help. The content is not value-laden, and it is emotion-free. By that I do not mean that one can not respond to it in an emotional way, but that the content is not so charged. Once we enter natural language emotion is constantly with us. One effect of this is that it feels unnatural to many teachers of mathematics to discuss how a person feels as s/he actually does maths. They may sympathise with someone who is so bold as to say that s/he does not like maths, but mostly do not empathise.

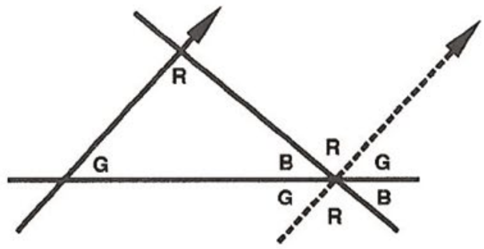
Firstly, a tale of Elaine. Elaine was my favourite case of maths anxiety, and I worked with her over many hours. Then I made an ILEA television programme. In it I taught her that the angle sum of a triangle was 180 degrees. There was nothing clever about the presentation as far as the maths was concerned, but what she said was very revealing.

We looked at the triangle, the angles coloured red, green and blue, and I asked her if it looked as if the angles added up to 180 degrees. She said "Generations of maths teachers have told us it was so, but I do not really believe it." This form of "belief" is at an emotional, not rational level. As the Buddha so aptly put it "Believe nothing on the mere authority of a teacher, myself or any other". Look at the triangle and see if you really believe it.

I then started a hackneyed proof where I first moved one side down parallel to itself, thus:-



She agreed that I had held it at the same angles with the other two sides, for "That is something I might do in dressmaking" but she felt that the three angles should now add up to less than before. When a little later I commented that she had thought the three angles now added up to less, she firmly corrected me "I did not *think* they were less, I *felt* they were less. What I think and what I feel are not the same thing".



When I drew the line down to the vertex and she saw them clustered there, she was delighted.

This form of blockage has its roots in intuitions about how things should be. The prime example is the product of two negative numbers. People will not believe that "minus times a minus is a plus' as we sometimes crudely put it. These blockages are cleared by means that are part cognitive, part affective. It is no use at all responding with a solid proof of something that is not believed. The beliefs existing in the learner's mind must be examined and discussed. Leaving them unresolved sets up irritants that can lead to distaste for the whole of maths.

Young children often form the (quite sensible) view that "multiplying makes it bigger". And so it does in the only numbers they know at this stage. Taking it a stage further, ask someone to give you a number and they will not reply "seven and five-eighths". There is a lot of rubbish to clear before we agree that a quarter is a number and that multiplying by it will make the other number smaller. Part of this lies in the language we use. If we said "two lots of three" rather than "two times three" we might more easily accept "one and a quarter lots of three" or "a quarter lot of three".

That is the first aspect. Many of the people I have dealt with have had far more traumatic and undermining experiences. They range from brutal beatings to mild "tut, tuts" to children who feel dreadfully diminished by someone they respect and love.

The classroom is heavy with authority. The young teacher struggling for control may not find this easy to believe, but had they not the structural authority inherent in their post there would be no rebellion against it. Equally, the experienced teacher with excellent relations may believe that the pupils regard them as friends. Maybe... but boss friends!

In this authority lies judgement, and in judgement lies the power to diminish. This is not, of course, confined to the maths classroom, but there are features there that render the situation more fraught than elsewhere. Let us list a few:-

1. Maths is perceived as being right or wrong, with the unfortunate social implications of these words.
2. Ability at maths is equated with intelligence, and this with moral worth.
3. Emotion is not a suitable topic for discussion in a maths lesson.

These are three major heresies!

"The teachers believed in caning at primary school and that put the fear of God in me and I was never at school."

"I've always associated maths with fear and trembling. I managed to escape with a beating once a term."

These are quotes from two people with very different backgrounds. The second went on to say "It was the whole school ethos. You got your figures in nice lines, and you had vertical lines in the maths books (which my handwriting didn't 'fit' anyway), and if you didn't, it was a mark of insolence and deviation and all sorts of things". Do I detect sighs of relief? Things are not like that now, of course.

But listen again. "They were kind, in their way, and would jolly me along, saying 'Of course you can do it', when of course I couldn't, and the panic would rise because there was something wrong with my powers of communication, that I could not make them understand." Does that strike nearer home? It does for me. I can hear myself saying that sort of thing, and cringe now when I think of it.

For those who wish to learn, who regard you well, and wish to please you, the merest indication, of dismay at their inability, will strike as hard as the gross injustice of the beatings that were once the rule.

An institution such as a school has an aim and intent which it and its employees seek to fulfil. The aim is that the pupils learn; a simple aim. The problem is that some will always learn more easily than others. Those that do are likely to attract praise for helping the institution to reach its aim, so clever people are 'good' at their work and we have accepted heresies 1. and 2. Beware! Do you praise a pupil when they get the question 'right'? Of course you do. Should you? Think!

At this stage I could launch into my 'Denigration of Praise', but it takes a whole chapter. Read the second edition of my 'panic' book entitled *Math Panic*. You may need to translate from the American.

The whole area of teaching maths is a minefield, but it can and must be done. What is interesting in the classroom is how children think and feel, so examine it. It is only when the thinking is awry that there is interest for the teacher... certainly not crossness at their failure. Enjoy finding out how they thought about the question, do not seek pride in the fact that you have succeeded in getting them to achieve some correct answers. Let them tell you how they feel as well as how they think. Remember the words of Elaine. Help them enjoy their thought processes. If they learn to enjoy learning, your task is really done and you are superfluous, and that is the aim of the operation.

Swaffham, Norfolk

Building mathematical resilience

Clare Lee (Open University) and **Sue Johnston-Wilder** (Warwick University) share their thoughts on the type of environment that either breeds anxiety or fosters resilience. They provide some helpful guidance on those who wish to develop a growth mindset among their students.

Introduction

Mathematical resilience is a positive stance towards mathematics, which can be developed by certain ways of teaching mathematics. When a student has begun to develop Mathematical Resilience they know they can grow their capacity to tackle problems in mathematics, they feel included and supported within a community of mathematics learners, they understand that learning mathematics is not a simplistic task, but one that requires struggle and perseverance, but not too much, and they know how to recruit appropriate support to enable them to succeed. In other words, developing mathematical resilience means that students become able to overcome the barriers and impediments that are often part of learning mathematics and which, without an environment that supports the development of mathematical resilience, can mean that anxiety grows and impedes learning.

Experiencing difficulties in understanding mathematics is widespread and those difficulties can and often do cause anxiety

Experiencing difficulties in understanding mathematics is widespread and those difficulties can and often do cause anxiety (Ashcraft & Krause, 2007, Devine, Fawcett, Szűcs & Dowker, 2012, Foley, Herts, Borgonovi, Guerriero, Levine & Beilock, 2017). This article will explain how anxiety may be inevitable for many people, possibly due to the way that mathematics is currently experienced in many classrooms, but it will focus more on teaching *for* mathematical resilience that is, acting within the classroom in ways that build students' ability to persevere and succeed despite the difficulties inherent in learning mathematics.

Mathematical resilience is not something that the student does or does not have, it can be grown with help from those charged with helping those who are learning mathematics. Mathematical resilience can be built by students in classrooms that maintain a strategic and explicit focus on creating a supportive and inclusive culture that stresses the belief that everyone can grow their mathematical learning. Five characteristics of a learning environment that is known to build mathematical resilience are:

1. helping students know that brain capacity can be grown, ability in mathematics is not fixed, everyone can get better with good teaching and support;

2. enabling everyone to feel included in and

supported by the community of people that are learning mathematics;

3. helping students to see mathematics as relevant in the world in which they live and of personal value to them;

4. asking students to struggle, but not too much, helping them to understand that growing mathematical capability requires perseverance but that they do not have to struggle alone unless they want to;

5. modelling ways to work at mathematics, showing how to seek and get support as well as how to give effective support.

Such an environment usually, but not always, asks students to work collaboratively as this requires communication, which in turn builds mathematical thinking, but always works in ways that enable every student to feel positive, comfortable and supported as they endeavour to move their learning forward in mathematics.

Mathematics Anxiety and Avoidance

We know that currently numeracy continues to decrease in the adult population (see for example, https://www.nationalnumeracy.org.uk/what-issue). Only 24% of 16-24 year olds who achieved an A*-C grade at GCSE also reached the equivalent level in the Skills for Life assessment, and therefore even those achieving what is considered to be a successful grade in school can often be functionally innumerate (Department for Business Innovation and Skills, 2012). Most people also know from experience about the high proportion of the adult population who react negatively to any mention of mathematics. The current result of many years compulsorily spent learning mathematics is:

- many disempowered individuals, as an unwillingness to use mathematical ideas means people can misunderstand much about what is happening in their personal lives and are unable to make good decisions;

- a disempowered society, with generations of school leavers avoiding any courses that appear to demand mathematics (Brown, Brown & Bibby 2008, Johnston-Wilder, Brindley, & Dent, 2014), causing a skills shortage in important professions.

So what is causing this problem? Many authors (e.g. Hernandez-Martinez & Williams, 2013; Hembree, 1990) suggest that it is highly likely to be mathematics anxiety which causes avoidance behaviour. Mathematics anxiety and avoidance is acquired when people are repeatedly put in situations which either push them beyond their current capability, even with support, or which cause them humiliation or exclusion. For example, when someone is shouted at for not knowing an answer this causes humiliation. The anxiety in this case may also be exacerbated as it is likely that they are already beyond their current capability, evidenced by not being able to produce the required answer. Mathematics anxiety also seems to be engendered by many common school practices, which are only used because teachers are led to consider that their use will be beneficial to their students in the long run. These practices include demanding instant public recall of answers and asking students to learn many discrete processes for getting the “right” answer, rather than enabling students to make connections with what they already know. Mathematics anxiety can also be learned from a mathematics anxious adult, be it a parent or teacher. The parent or other adult that says “I could never do maths” is setting up the conditions for the learner to begin to avoid mathematics themselves.

Mathematics anxiety is, in effect, an emotional handbrake, because students who become anxious when they are faced with mathematics do not want to, or are unable to, fully engage and therefore do not learn the ideas as fully as they might. Imagine if a child were short-sighted but it was not diagnosed, it would prevent them engaging fully and therefore it is likely they will underachieve. Once treated, and with a fine new pair of glasses, they can engage fully and learn well. Similarly, historically, children with undiagnosed dyslexia and dyspraxia have underachieved and failed to thrive in academic situations unless someone, often an eagle-eyed SEN teacher, picked out the specific difficulties. Mathematics anxiety also causes reduced progress across time and is often the cause of students’ continued low attainment in mathematics. Mathematics anxiety, whilst it is acquired and disabling, is also treatable. Once the conversation about how anxiety can be overcome is started, we have found that more sufferers will come forward and more people will be able to fully engage in learning mathematics without the disabling effects of anxiety.

many students live in fear of being separated from their friends by being ‘sent’ into a lower set if they do not get good marks

Diagnosing mathematics anxiety (MA) as a problem is not always straightforward, as the anxiety usually causes avoidance strategies to be deployed. Avoidance can be seen as a strategy for safeguarding yourself in the face of a perceived threat, and mathematics lessons are perceived by many as a threat, often a social threat, where there is a high potential for humiliation or exclusion. Many people feel threatened by mathematics because of remarks that have made them feel stupid, for example, exclusion (‘you are not clever enough to do maths’), or humiliation (‘why don’t you understand – it is simple’). These may not even be verbal threats, many students live in fear of being separated from their friends by being ‘sent’ into a lower set if they do not get good marks. If you are panicking or experience threat, the ancient part of your brain (the amygdala) will take over, triggering a fight/flight/freeze response. This response makes cognitive thought relatively inaccessible; a student experiencing this response will be overwhelmed with the desire to run away (storm out of the classroom) or fight (swear at the teacher and/or throw a book) or freeze (mutter incoherently and/or burst into tears). Thus MA and avoidance is disabling and causes underachievement because anxiety affects cognition, it interferes with thinking and it also interferes with short term memory (Shi, & Liu, 2016). Teachers and other adults who work with those learning mathematics may find it hard to identify the problem as MA, as it may manifest as avoidance, anger, tears or statements such as ‘I can’t do maths’. If you look for MA, and especially if you enable students to have the vocabulary to discuss and describe their feelings towards mathematics, MA is often there, stopping learners from learning, enjoying and using mathematics.

Lyon and Beilock (2012) worked with highly mathematics anxious individuals (HMAs), using digital imagery and they found:

> *"best educational practices for enhancing math competency in HMAs is not to generate costly math courses specifically for the HMAs (Gresham 2007) nor is the best method likely to be one that focuses solely on eliminating one's initial anxiety response (for a review of these and other approaches, see especially Hembree 1990). Instead, classroom practices that help students learn how to marshal cognitive control resources and effectively check one's math-related anxiety response once it occurs—but before it has a chance to reduce actual math performance—will likely be the most successful avenue for reducing anxiety-related math deficits."*
> *(Lyons and Beilock 2012 p. 2109)*

So at this stage we have established that mathematics anxiety is a widespread problem that often causes individuals to avoid mathematics, which in turn reduces the supply of students on courses that are likely to expose those individuals to mathematical work, which is costly to society. We have also established that mathematical anxiety and avoidance can be costly for individuals themselves, often leading to a lower income and a lack of willingness to check things like energy bills, salary slips and quotes for work, thereby disempowering these individuals. So the first idea that we would like to present is that of the growth zone model as this model is designed to help learners of mathematics 'marshal' their cognitive resources and recognise and 'check their mathematics related anxiety' when it occurs. After a discussion of the growth zone model, we will discuss mathematically resilient approaches that can be instigated in order to attempt to prevent mathematics anxiety developing or to reframe the learners' approach to mathematics so that they are empowered know that mathematics is not a threat to them and therefore to be able to use and control mathematics in any way that they want to. It is important to note that what follows may be applied to maths anxious or avoidant staff as much as students.

Learning Mathematical Resilience

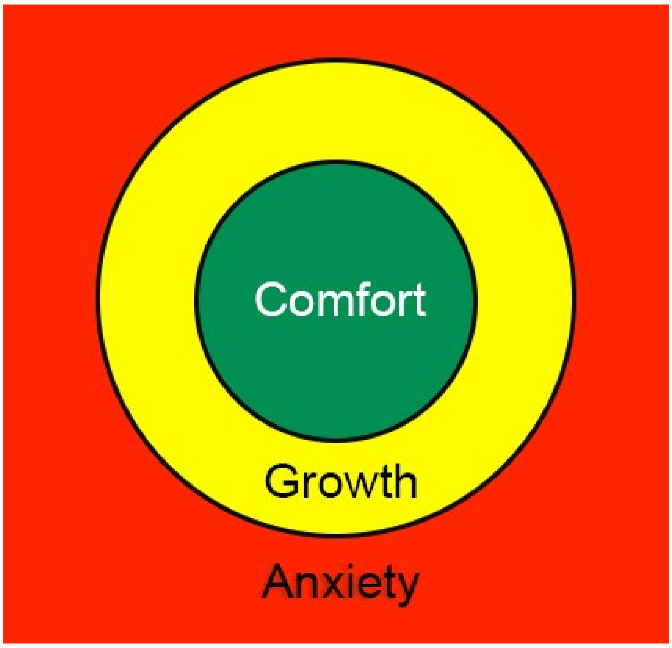


Figure 1: The Growth Zone Model

The Growth Zone Model

The growth zone model is used to help students understand and articulate the feelings they may experience when they are learning mathematics. The growth zone can be thought of as sitting between a comfort zone and an anxiety or danger zone, see Figure 1.

In the comfort zone, students will feel comfortable with the mathematical ideas on which they are working. They will often be accomplishing useful work, they may be consolidating or gaining fluency with ideas that have already been learned. It is often a good place to be. However if students stay too long in the comfort zone, they will not be learning anything new. Some people say that mathematics is tedious (Nardi & Stewart 2003) and one reason for this may be that there is a lack of the exciting feelings of challenge and risk which can lead to successful learning, which students find in the growth zone.

In the **growth zone**, students will be learning and as a consequence they will feel challenged and that they have to struggle to understand. It is likely that they will feel pressure as they encounter and overcome barriers. Learning mathematics is not easy for anyone, even accomplished mathematicians, and students will make mistakes in the growth zone and have to persevere, just as they do, for example, when learning a new move on a skateboard. Making some mistakes and getting stuck shows that the challenge is at the appropriate level; if they are not making mistakes then they are still in the comfort zone and are not learning. Some learners will feel that they are in a risky place, so they need to be re-assured that risk and uncomfortable feelings are all part of learning and growing. Support will be necessary in the growth zone, as the feelings of risk and uncertainty must not become overwhelming as then learners will move into the red zone and that feels very unsafe.

Learning mathematics is not easy for anyone, even accomplished mathematicians

Avoiding the feelings that are associated with the **red zone** are the reason that many people avoid mathematics. Underperformance is not caused by mathematics being the sort of subject that causes its learners to encounter barriers and feel some anxiety about how to overcome the barriers in order to succeed. It is caused by learners not knowing that they should expect feelings of risk and uncertainty and mistakes when in their growth zone and that they have to learn how to handle these feelings. If a learner does not expect these feelings or know how to handle them, then such feelings could easily be interpreted as not being able to “do” mathematics. A supportive, collaborative environment is vital when venturing into the growth zone, as well as an understanding of what to expect.

If a student moves into the red zone, they will experience panic, it is likely that the freeze, flight or fight reflex will be triggered, where higher level thinking becomes difficult or impossible, as the brain moves into survival mode. Unfortunately many traditional ways of teaching mathematics have not helped students recognise that learning can be stressful, risky and uncertain and this has led to many students going straight into the red zone when faced with any mathematical problem. The learner needs to know in advance that being in the growth zone may trigger productive levels of adrenalin, not too much, but just right. The physical reaction to challenge is a supporting influence on potential cognitive development, provided the learner feels any risks are under their control.

Using the language of the growth zone model, learners can become aware of moving into the red zone and therefore develop strategies for moving back into the growth or comfort zones. Any incipient anxiety can be recognised and articulated using the language associated with the growth zone model, giving opportunities to manage those feelings and return to learning. Anxiety management approaches include breathing exercises, especially slightly longer out breaths or breathing with the diaphragm, physical exercise, talking to someone and learning to step away until calm. Such approaches focus on ways in which the sufferer can take back control. In the growth zone, discussion of self-safeguarding, peer support and that errors are a sign of learning, are ways in which support can be offered and issues which can arise in the growth zone overtly addressed.

Mathematically resilient students know that they can make progress if they struggle a little and persevere

Sometimes sufferers of mathematics anxiety have to learn to 'cope ahead'. This involves thinking through what options there may be if a sufferer becomes so anxious they cannot continue to learn. Considering solutions to this problem beforehand, with a clear, calm mind, allows a learner to come up with a way they can cope with the situation and gain the space to think or the support they need. The sufferer will then feel more confident when learning mathematics and equipped to deal with stressful situations that arise during that time.

Teaching for Mathematical Resilience

Those students with mathematical resilience will encounter barriers when they are in their growth zone. They will understand the feelings of risk and uncertainty that are associated with being challenged but instead of framing this experience as "I can't do maths" they will gain control over these feelings by marshalling the resources they need. They may talk about the problem with a peer or teacher, quietly think things through, doodle something that is useful for them, use their highlighter pens to sort out the important information or look up something pertinent in a textbook or on the internet. Mathematically resilient students know that they can make progress if they struggle a little and persevere and they know that they may need support and will know how to get the support they need.

Teaching for mathematical resilience is about enabling students to become convinced that they can do mathematics. There are myths about mathematics which seem to say that mathematics requires you to have a phenomenal memory, in order to remember the 'right way' to do lots of discrete processes, which are all straightforward and easy ways to get an answer. However real mathematics is an interconnected subject (Askew, Brown, Rhodes, Johnson, & Wiliam, 1997) where the solution for problems can be found by considering what you already know that might help. There are efficient ways to achieve solutions but it is more likely that those efficient processes are remembered if they have been worked out by connecting with and building on current knowledge. The path to solutions will not be smooth and straightforward at first, barriers will be encountered, but the barriers just mean that mathematics is being learned.

So teaching for mathematical resilience involves learners developing: a growth mindset, a willingness to struggle, a knowledge of how to work at mathematics and an understanding of the meaning, value and purpose there is in mathematics. These ideas will now be developed further:

learning can always be grown and it is never too late to come back to learning mathematics

A growth mindset. Our culture holds a relatively fixed mindset when it comes to mathematics. One example is that schools 'set' pupils according to some notion of ability which reinforces the fixed mindset idea that 'some people can do mathematics and some people can't'. Another is that examinations are taken at a given age e.g. the GCSE is taken at 16yrs, if society acted as though abilities can be grown, then examinations would be taken when ready, as they are in music. Recent neuroscience has shown that new connections can be made throughout the lifespan (Siegel, 2010). Therefore learning can always be grown and it is never too late to come back to learning mathematics. Helping students know that they can always learn more in mathematics can seem to be a mammoth effort as society seems to continually undermine a teachers efforts. But it is worth it, as accepting that "I can't do it **yet**!" is the first step towards "I can do it!"

A curriculum that offers challenge so that learners can learn that mathematics requires struggle, it is not just a case of applying learned algorithms. Overcoming challenges will require support, but it has to be the right kind of support. Some learners like to collaborate from the start, others like to think on their own, hence the ethos of the mathematical learning environment must be that students accept challenges, understand the need to struggle and also articulate the support they need, and receive it, in order to meet those challenges that learning mathematics supplies.

Know how to work at mathematics: According to Williams (2014), mathematical resilience is made up of confidence, persistence and perseverance. Williams makes a distinction between *persistence*, which means to keep on trying, but trying in the same way even though it has proved unproductive so far and perseverance which means to keep on trying by recruiting support or finding an alternative approach. Learning to persevere is what it takes to stay longer, safe but challenged, in the growth zone. It means learning to try hard but not to just try one way, but look for what else you know; if the hammer does not work, the screwdriver might, try another tool in your toolbox.

accepting that "I can't do it yet!" is the first step towards "I can do it!"

Knowing how to work at mathematics can be hard, as another one of those mathematics myths is that if you cannot immediately see what steps to follow, the teacher, and only the teacher, will have to explain those steps again. The idea that the path to a mathematical solution is not a smooth one can involve students "unlearning" accepted wisdom and re-learning the idea that there will be bumps in the road and barriers to overcome and these ***do not*** mean that you "can't do maths".

Sometimes the new work that students have to do so that they can become mathematical problem solvers will mean they get stuck. Students will need approaches to use when they get stuck, as there is only one teacher and lots of learners. Some teachers have developed a 'stuck poster' which contain the students' ideas of what to do when they get stuck. Such posters can be displayed every lesson and added to whenever a student finds a good way to help "unstick" themselves. Learning agency is an important part of developing mathematical resilience; only the student is in a position to learn the mathematics they need, hence within a supportive environment as they become more and more able to take steps to continue to work at and learn about mathematics, they become more agentic and alongside becoming more resilient.

they are working as mathematicians, and are doing real mathematics and starting to see value and purpose in it

Understand the meaning, value and purpose there is in mathematics: environments that help learners see how mathematics is related to and valuable in the real world are important as learners often ask, what use is this to me? However, this is not just about financial matters or measuring walls to see how much paint will be needed (when everyone knows you just buy a big tin and go back and get another if you absolutely have to). Mathematics can be real if it is seen as part of your world, so when learners ask their own mathematics questions about space flight or dinosaurs or the World Cup or whatever is in the news at the moment, and then work together to answer the questions they have posed, then they are working as mathematicians, and are doing real mathematics and starting to see value and purpose in it. Similarly, if they are cooking a meal or scheduling a journey.

In Conclusion

Mathematics anxiety is a widespread problem in many countries across the world. It seems to affect really quite young children, and persists into adulthood where it affects study and career choices and can disempower adults in their daily lives. Mathematical resilience, as a construct, stands in opposition to anxiety. Working for mathematical resilience in learning environments mitigates anxiety once it has developed, but may also prevent disabling anxiety developing. Mathematical resilience requires a growth mindset, everyone needs to think in growth mindset terms, if the learner is to be convinced that there is no ceiling on what they can do with effort and the right support. 'Everyone' is different in different situations but in school this will mean the students, the teachers, the teaching assistants, the office staff and cleaners and most especially the parents who will need support themselves to really think and speak growth mindset. The parents can be helped to see their role as listeners who take an interest but do not take over, and do not know the answer but get involved with the process. Parents can encourage their child and help them think of connections or formulate questions, and they can encourage their child to contact their peers or look for help on the internet. Parents who are introduced to the language of the growth zone model can find they then have the language to help their child.

Developing mathematical resilience means being open and honest about the fact that mathematical learning is sometimes difficult and usually requires learners to struggle. When working on mathematics, students will get stuck and make mistakes, this is the expected state of a learner who is being challenged. The teachers' role is not to smooth the path but to equip the student with the tools they need to overcome any barriers they encounter. Students will need help to know how to collaborate and help one another in ways that maximise learning; it is tempting to tell someone the right answer but that will not help that learner learn useful processes and experience future success. The student will need to know how to work at mathematics, how to persevere not just persist, what approaches to use if they get stuck and how to recruit the help and support they need. Overcoming anxiety and becoming mathematically resilient can be difficult, but it has great and empowering rewards; feelings of barriers successfully overcome and a hard challenge met are rewards themselves, without the improved examination passes that we know are also attainable.

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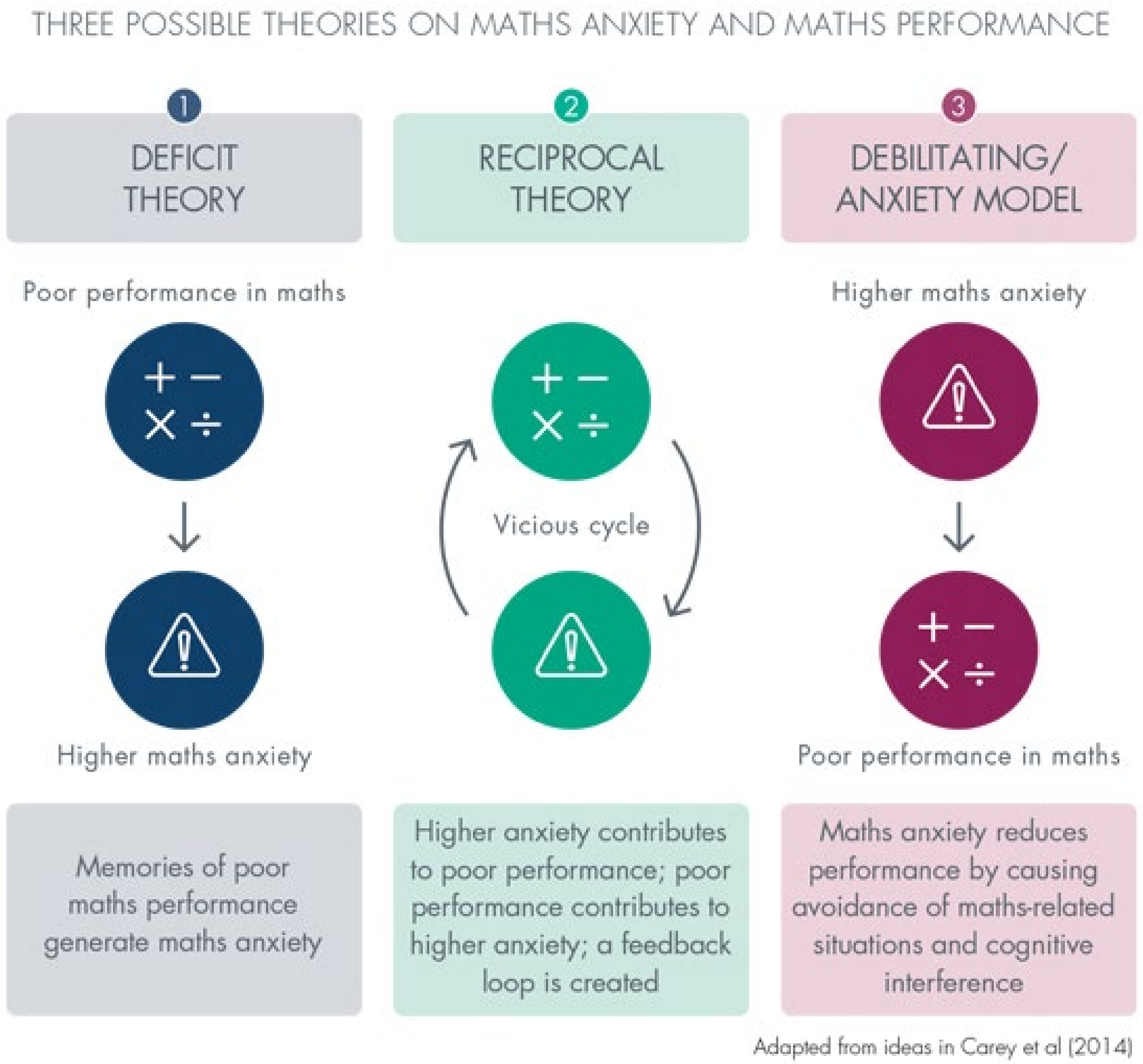
Conversations with my daughter

For Lucy Rycroft-Smith a chance conversation with her daughter lead to some soul searching regarding the nature of anxiety. This piece mirrors very closely that written by Clare Lee and Sue Johnston-Wilder and so the two should be seem as complementary.



Last month my eight-year old daughter rushed to find me, excited by a mathematical ‘trick’ she had found on YouTube. “Ok mum,” she said. “You take your birth year...’

I worked hard to hide familiarity with the simple underlying algebra, excited to be able to explain why it worked afterwards.

“Now you need to multiply by 20 and add 1015’ she said. ‘Do you need a calculator? Oh of course not, ‘she smiled, a gloriously proud look in her little face that said *my mum’s a mathematician*.

Then; my heart felt like it stopped. I grinned back at her just a little too wide. Never mind blank - my mind was a blizzard, an untuned TV set, an eddy of liquid coiling into a creeping vortex that I couldn't control. I tried to line the numbers up in my mind, like soldiers, but they lost all meaning and became scribbles of colour, dissolving into one another. I felt hot. Prickles of uncomfortable embarrassment spidered up my back. I choked.

In ten years of maths teaching, I have never experienced this strong a physical reaction – although I've had plenty of hairy mathematical moments, made hundreds of mistakes, and had to perform many a calculation in front of an audience. Was what I experienced maths anxiety, and is that something different to just feeling that something mathematical is a challenge, or that others may be judging you on your maths ability?

As part of my work at Cambridge Mathematics, I am lucky enough to be able to write monthly Espressos: filtered maths education research documents for teachers with hyperlinked references and classroom implications. In June, I looked at the issue of maths anxiety and the current research around it.

Although this is quite a recent field of study in maths education, there is clear evidence that maths anxiety cannot be reduced to either general anxiety or test anxiety – and is not simply a proxy for low mathematics 'ability'. (Mahoney & Beilock, 2012). Researchers do not currently agree as to which theoretical model might explain the link between maths anxiety and maths performance (see image) – whether the link is one-way, or a cycle (Carey et al, 2014). Pupils with maths anxiety tend to avoid maths-related tasks and there is a disruptive effect on their working memory, both of which contribute to a negative correlation with mathematical 'performance' (Ashcraft & Krause,2007). Brain imaging shows that maths anxiety has a distinct pattern, showing as decreased activity in regions associated with working memory and numerical processing (Lyons & Beilock, 2011).

There is also evidence that maths anxiety can be transmitted from teacher to student (Haciomeroglu, 2013); teachers who are anxious or negative about mathematics can instil the same attitudes in their students (Mahoney & Beilock, 2012). There are obviously implications here for teacher training and CPD – confident maths teachers are an incredibly important and valuable resource for creating confident maths learners. Maths anxiety is linked with intense feelings of shame or guilt (Haciomeroglu, 2013), which maths teachers may recognise as something that all too often prompts students to count on fingers under the table in a panic, or flush and go blank when asked for answers in class.

It is hard to measure exactly how many people suffer from maths anxiety, but research suggests it could be anywhere from 25% to 60% (Beilock & Willingham, 2014; Perez-Tyteca et al, 2009). The evidence suggests there is a stronger effect dependent on gender, too – girls report maths anxiety more than boys (Devine et al, 2012) and female teachers' anxieties seem to transfer more to female pupils, negatively affecting both their achievement and attitudes to mathematics (Beilock et al. 2010).

So what can be done? One study showed that students who reported maths anxiety were able to perform nearly as well on a difficult maths task if clear help was given to eliminate the negative emotions (Lyons & Beilock, 2011). It would also appear that, as students with early difficulties in numerical and spatial skills are more likely to develop maths anxiety, interventions to help support these skills may help to prevent it developing (Mahoney & Beilock, 2012). Students who believe they can improve with practice are much less prone to maths anxiety than those with more fixed beliefs (Dweck, 2008). Johnston-Wilder et al (2014) have also developed the Growth Zone model to help people characterise and deal with maths anxiety.

So, how did I deal with it, having read all this research? I looked my daughter in the eye, and told her my mind had gone blank for a moment. 'Don't worry Mummy, ' she said, airily.' It happens to everyone.'

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Counting – a deceptively simple skill

In a piece that needs reading by all those who are serious about supporting their children Les Staves shares his thoughts on why mathematics starts long before the words one, two, and three pass a child's lips.

Many people consider counting to be a simple skill, but that leads us to be deceived – we often hear young children recite a string of words and assume they can count.

Sharing attention to counting

The fact that we might not be focusing on the same things when we count together with children is supported by research into children's beliefs about counting. Research suggests that, despite being able to perform quite well, a high proportion of children do not understand the adult's purposes of counting before they start school (Munn, 1997)[ii].

Research showed that pre-school children had ideas about counting as a social experience, they thought it was for pleasure, or to please others, but it was rare for such young children to explain that it was 'to know how many'. Their impressions were that it was a social or playful activity, but they did not connect it with quantification. Penny Munn could see a combination of things that alongside the pre school child's lack of appreciation of other people's mental activities might contribute to lacking understanding about *why* they were counting with an adult.

She observed:

- during joint counting activities between very young children and adults the process of saying words seems important, but the actual aim of finding quantity is often not emphasised
- the strength of children's own natural concentration on the physical aspects of counting activities, touching and handling, often obscures the intended mental function of finding quantity
- children are dependent on adults to provide for them, they have no urgent real reason to check and tally, so counting is usually a game
- count words often occur as parts of games that are not to do with quantity, e.g. One, two, three – go.

If such typical pre-school children have not yet seen the meaning beyond the social facade of the counting they do with adults. How do children with learning difficulties fare in the process? Is there any wonder they sometimes watch our faces, to check if they are doing the right thing, rather than look at the objects they are counting. If we could read the thoughts behind their eyes what would be the question in their mind. 'Does he want me to touch them? Shall I make that *'Won too three noise?'*

Penny Munn has a number of suggestions that might help us make our scaffolding of early counting and modelling activities more effective:

1. We need to establish *what* children believe we are doing together when we count.
2. Counting has many sub skills and we need to

be able to see which of these are the things that they think we want them to do. *e.g. itemising, ordering, naming*.

3. Though we need to be aware when their counting is only a form of recitation, we also need to take their non-numerical counting seriously.
    a. *Though it may not yet be mature and related to quantities their recitation of number words is important practise in establishing word order.*
    b. *This recitation of number words is also an important point of social contact we can share with them and from which we can help them establish connections with quantity.*
4. We need to make the purpose of counting explicit. She suggests that as we are involving them in the use of counting words we could also muse aloud providing feedback about aspects of quantity that are involved. Children respond well to this kind of incidental information, and even as adults we use forms of thinking aloud to direct our actions. (*See private speech in chapter on communication.*)
5. We need to encourage children to use numerical goals, making use of counting for tallying, checking and comparing items and events.

Typical children often have both:

1. a practical knowledge of small quantities, and
2. are able to produce a series counting words.

When adults hear children counting there is a tendency for them to think that they have integrated these two aspects of counting. But this is not always the case, and these connections must be made if children are to *count meaningfully* or to understand the role and purpose of counting in practical calculations.

The process of counting

In order to count a group of objects children must be able to itemise them and tag each with a number name. Through imitating others counting children learn to connect experiences of

- looking at and touching objects

With

- The verbal symbols of the count words.
- Understanding the meaning and consequences of the connections.

So co-ordinating visual and tactile activities with verbal labelling are important parts of counting *but* meaningful counting requires an understanding of the quantities involved and an understanding of the purpose of counting.

The Principles of counting

To count accurately and with meaning there are five principles that must be understood and applied (Gelman and Gallistel, 1978)[ii]

Principles about how to count

- The one to one principle
- The stable order principle
- The cardinal

Principles about what can be counted

- The abstraction principle
- The order irrelevance principle

About the 'How to count principles'

The one to one principle

Understanding and ensuring each item receives one and only one tag, which requires:

- *Tagging*. Summoning up and applying distinct names one at a time.
- Physically keeping track, or mentally *partitioning*. Which items have already been counted and which remain.

It is necessary to realise that the name tags are specially for counting with, they are nothing to do with other characteristics of the items being counted.

Teaching related to the one to one principle

In the early stages of learning to count children may be vague or imprecise about their pointing, they wave their fingers in the general direction, but often let the rhythm of the oral counting sequence dominate the speed at which they count, and consequently lose correspondence. Teaching needs to make them more aware of the importance of coordinating the itemising and tagging. Promoting strategies that will help them to notice if they have double counted or missed items e.g.

- practising arm gestures and pointing at large objects
- controlling clapping and counting events
- controlling sequential touching
- moving items – to keep track on which have been counted and which are left
- placing counters or other tallies on or next to the objects being counted
- using number lines or tracks as templates to place items on as they are counted
- making marks next to objects being counted
- using the index finger to touch point (*declarative point*)
- *using the index finger to point without touching*
- developing eye pointing and nodding.

There are many times when working or playing together we can make sure good itemising or marking takes place. There are many different levels of itemising, vocalising, and naming, all too often in games we simply expect or prompt the child to count successfully when it would be more fruitful to have a less ambitious objective. This is a prime area for breaking skills down; games provide motivating opportunities to practise the parts of counting.

The stable order principle

The name tags must always be used in a stable order.

Using number names to provide ordinal names for things being counted presents the child with the problem of remembering a long list. (Miller, 1956)[iii]. This is the role of the traditional nursery rhyme – developing the Childs ability to reflect upon the sound patterns of words, building up the sequence of number words as sound patterns. They will be learned and extended in short chunks[1] and we need to be aware of the current extent of a pupil's performance so that in our modelling and during peer interaction we can emphasise the next steps[2].

Teaching related to the stable order principle

We need to recognise that many of our pupils have grown past the age at which nursery rhymes are appropriate – but still need this phonological development and practice. In our teaching we need to recognise the valuable role of the way we use our own voices when we model and teach. We must be aware of the power of intonation and rhythm, in offering connections and prompts that make it easier to create memorable 'chunks' or connections that help learning a sequence of words.

Some suggestions that might form the basis of, or be included in, learning activities are listed below:

- Sound and word play to promote discrimination.
  - *emphasising key sounds within word, identifying word from first sound or sound profile, creating strings of sound*.
- Linguistic rhyme and rhythm games incorporating number words
  - *Raps, chants, catch phrases, modified songs – downloaded video clips e.g. YouTube – Sesame St count it higher - Sesame St Feist 1234.*
- Games to highlight the importance of coordinating words with items
  - *including fast counting, slow counting, pauses, rhythmic counting and counting without rhythm.*
- Intonation emphasising important aspects of the sequence
  - *punctuating the sequence well and emphasising the last item counted to emphasise the cardinal number – see below.*
- Using gestures and movement along with sounds, and visual representations e.g. number lines or rows of items or people.
  - *hand and arm gestures, pointing, clapping, stamping, nodding.*

Though the emphasis within the suggestions above is on learning the string of number words we should also bear in mind

- the importance of fingers movements and gestures accompanying speech
- the importance of coordinating the speech and movement with visual arrays such as number lines, rows of objects or people.

Martin Hughes (1986)[iv] notes how children still resort to pointing and tapping to assist their counting even when objects are out of sight and a critical factor for building a number system is connecting the spatial/perceptual representations we have in number sense with learning the count sequence.

The cardinal principle

The final number represents the size of the set.

When the child understands this principle they recognise that earlier numbers were temporary steps towards the last number tag, which is special, because it is the **cardinal number and represents 'how many' items have been counted**.

Appreciating the importance of *cardinality* is an important milestone in a child's mathematical development; it is a keynote in understanding that the process of counting is not just sound making or a game but actually has meaningful and useful purposes – *related to finding out how many*. It is a principle that a child must understand before they can begin to carry out **addition by 'counting on'**.

It is necessary for children to understand this principle before they can use the technique of 'counting on' to carry out addition or to determine or compare the equivalence of two groups.

Fully grasping the cardinal principle depends on understanding the previous two principles, it therefore matures after them.

There are four phases in its development (Fuson and Hall 1983)[v]. Reciting the last number with no clear idea that it relates to quantity – *probably because they think it is the response the adult expects*.

- Understanding that the last number of the count relates to the quantity. In which case they may be able to understand that they can respond to a request like give me three – by counting.
- Understanding the progressive nature of cardinality i.e. if they are stopped in he middle of a count they can say how many they have counted so far, and then carry on. Being able to compare size represented by numbers, or understand that the next number in a sequence represents a larger quantity

Teaching related to the cardinal principle

It is useful to observe if the child has an appreciation of the purposes of counting and if so at which level they apply the cardinal principle, so that we can arrange appropriate individual modelling, and also be aware of what to emphasise as we work with them in practical activities or games with peers.

- To understand the last number of the count relates to the quantity.
- When the child's counting is recitation without meaning our teaching should include modelling counting things for a practical purpose. We can emphasise the special importance of the last item in a count by intonation, and use checking to reinforce purposefulness.

- Making and naming groups *e.g. counting the row of objects – and gathering them together as the count finishes – or restating the cardinal as they are gathered.*

- To understand the progressive nature of cardinality.
- Provide models of pausing and continuing in practical counting activities.
- Demonstrate adding one more model and involve pupil in checking result.
- Model and involve pupil in solving practical addition problems by counting on.
- Give pupil opportunities to use apparatus or objects in play and exploration that illustrate progression of number, experiencing staircase image or nesting.
- It may be necessary to be aware of pitching the level of counting within the zone of the child's established number words and accurate perception of groups.

- Being able to compare size represented by numbers.
- Use counting to compare groups that have been named – and associated to a numeral, checking answers by counting again.
- Using templates or number lines to make visual comparisons of groups that have been named.
- Compare photos or pictures of groups – compare and count.
- Model and involve pupil in adding or finding

one more to groups and checking.

- Provide models and involve pupils in practical activities of carrying out addition by 'counting on'.

The principles about what can be counted

The abstraction principle

Counting can be applied to any collection – real or imagined.

Adults realise that they may count physical or non-physical entities, similar and dissimilar things, objects that are not present or even ideas. Young children on the other hand count physically present items and they group things in accordance with how they see immediate relationships. Variations in material properties or position may affect their view as to whether an item can or should be included in a count.

When we are working with or assessing pupils at early stages of development, who may have difficulty with abstract thinking we need to be aware of what they conceive as allowable within a counting sequence.

- What they might think about including or leaving out of the count on grounds of physical properties, position and so on.
- Are they able to understand they can count objects they cannot see?
- Can they count events as they happen, and events that occur elsewhere?
- Can they count ideas?

Teaching related to the abstraction principle

- Sorting by different criteria – discussing different reasons for groupings.
- Model counting groups of things that are nor identical.
- Counting items before hiding, or storing in drawers or cupboards, bag, thinking about them and counting them when they are not visible.
- Using photographs or video of items to count them when they are not there.
- Using narratives and remembering events, using fingers tallying, mark making and counting to remember and record things that happened, use photographs or collected objects to check.
- Use narratives and discussion about the future – counting things that might happen or things we think about – e.g. How many of us will come to school tomorrow?

The order irrelevance principle

The order in which items are counted is irrelevant; the same cardinal value will be reached.

This principle requires knowledge about the previous four principles. Grasping it entails understanding that each counted item is still an individual thing, not a 'one' or 'two' etc. because number name tags are temporarily given for the purpose of counting, not renaming things and they do not necessarily adhere to the objects once the counting is finished.

Whatever order the objects are counted in the same cardinal result occurs. It is necessary to grasp this principle in order to be able to generalise the use of counting flexibly as a tool without having to put things in a line. It helps us confirm the consistency of the quantity of a group and it is confidence in that consistency that enables us to be sure about making comparisons. Such confidence helps us to override the messages of perception that may confuse us when spatial changes make things appear bigger, and it may therefore underlie our ability to recognise the conservation of number.

Teaching related to the order irrelevance principle

- Developing pointing strategies
- Counting linear ordered groups in different order and forward and back
- Making linear groups and recounting them
- Rearranging linear groups to random and re-counting
- Counting and rearranging random groups
- Counting groups rearranging spatial distributions
- Discriminating and counting specific items from pictures or photographs of mixed groups.
- Pointing at and counting items that are distributed about the room
- Keeping tally of I spy type games

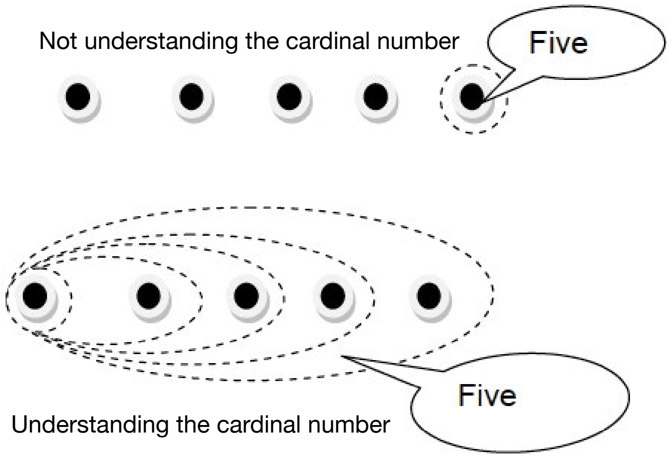
Some confusion about language when counting

- When we are counting we use the number words in different ways:
  1. as ordinals – to keep order as we tag each item
  2. as a cardinal when we name the size of the group.

Children may not always appreciate these distinctions for example:

If we ask a typical four year old to count a row of five objects they may count them correctly and declare that there are 'five'. But If we then ask them – 'so show me five' they will often point to the fifth object – saying 'that one'.

- This suggests that the child is interpreting the words one, two, three etc. as names for the individual elements in the count *like Monday, Tuesday, Wednesday are names of the sequence of elements of the week*. So for that child on that occasion the word '*five*' stands for the last object in the group – and not for the entire group. See Fig 1 below.



The lower diagram also illustrates 'hierarchical inclusion' - the idea that number is progressive and bigger numbers contain all the smaller numbers before them.

Fig 1

To carry out the process of counting and naming the child must understand

1. sometimes we use number words temporarily as 'ordinals '– to keep track of the order
2. then we can use them as 'cardinals' – to name the size of the group.

In order to understand that you can describe the 'quantity' in the group you have to appreciate that a larger number includes all the smaller numbers. This is another of the 'big ideas' – *hierarchical inclusion*.[3]

When we are using or modelling the use of number words with children for practical purposes it is important that we try to make it clear when we are using them as tags to show or keep order – and when we are using the as cardinals to name the quantity.

Just to add a little more confusion sometimes number names are used just as names (nominals) *e.g. the number 4 bus.*

More big ideas

When children have become fluent in applying the principles of counting there are still a number of 'big ideas' – concepts – they need to understand to for their counting to be a useful part of their mathematical thinking. These include

- Hierarchical inclusion - *understanding that numbers build by exactly one each time they progress – and that they nest within each other by this amount.*
- Compensation – which entails:
  - Appreciating the balance of increase and decrease.
  - Applying understanding of whole and part relations within numbers.
  - Understanding that different combinations can make up the same number.

Readers interested in the big ideas will find them discussed in more detail in The Equals Guide to Mathematicsvi and in 'Young mathematicians at work' Fosnot and Dolk 2001vii

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1 P scales recognise progression in the extent of a child's rote sequence – and that a differential will exist between the rote sequence and accurate counting.

2 The central role of learning from others is highlighted by Vygotsky's 'zone of proximal development', which indicates how much further a child can go when learning with the support of a teacher, parent, carer or peer.

3 The development of the cardinal principal and understanding hierarchical inclusion are interrelated – each is helped by the other.

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Pre-empting maths anxiety

Professor Steve Chin, backed by over forty years of research, has a track record in supporting those who struggle with mathematics. In this piece he gives some very wise advice on overcoming the barriers that exist in many classrooms.

'Anxiety occurs when an anticipated event is expected to make demands for which the person is unprepared…. Not only is the anxious person not able to simply dismiss the anxiety-producing possibility, but he can become obsessed with it.' (Costello, 1976)

I want to use this, relatively old, definition of anxiety as the basis for my article. I am not actually apologising for it being relatively old, because in terms of maths anxiety it is relatively young. I want to focus on '*when an anticipated event is expected to make demands for which the person is unprepared*' (and when they are fully aware that they are unprepared).

I could re-word this from a learner perspective: 'Experience tells me that I always fail at this task. When I am told to do more of this task, do not be surprised if I show signs of anxiety.'

We should, by now, have some ideas as to the situations or topics in maths for which learners are unprepared. In my survey of maths anxiety for 11 to 16 year-old students in mainstream and in specialist schools for dyslexics (Chinn, 2009), my data ranked some of the main demands and situations that created high anxiety, for example, 'doing long division without a calculator' and 'having your results read out loud in class.'

Then, from an informal survey of teachers over the past two decades, a sample now running into thousands, I know that a noticeable number of pupils are giving up (avoidance and withdrawal) on maths at age 7 years. I know from this survey that, for example, division and fractions are internationally disliked by many students.

I know that a noticeable number of pupils are giving up (avoidance and withdrawal) on maths at age 7 years

So, we can focus on two classroom based examples of anxious situations; what children try to learn before the age of 7 years old that creates anxiety to a level where they withdraw from maths and what topics and situations create anxiety in older pupils. Some issues are not age specific.

As a teacher for some 40 years, I continue to ask myself how we might teach maths to alleviate as many of these anxiety-creating issues as possible.

Learning from the outliers.

John Holt, in his classic book 'How Children Fail' (1965) said that 'everything I learn about teaching I learn from the bad students.' My own experience of some 17 years of mainstream teaching followed by 24 years of teaching students with specific learning difficulties would reflect Holt's experiences.

More recently, Murray et al (2015) explained, 'In essence students with learning disabilities are representative of a set of outliers on a continuum where all students experience difficulty in mathematics to some degree. While many students' struggles with mathematics can be attributed to dyscalculia, which appears to impact basic number sense and the learning and remembering of arithmetic facts and procedures, there are other reasons why students have difficulty with mathematics, such as difficulty understanding and using representations, poor executive function, and affective issues. Students identified with mathematics disability represent one extreme of students who struggle with mathematics, and we believe these students can inform the ways in which approaches to mathematics education should be designed for all.'

Students identified with mathematics disability represent one extreme of students who struggle with mathematics

There is much in this quote on which to build this article. My key hypothesis is that, for many, 'struggles' can create anxiety and withdrawal. So, how do we adjust teaching and the classroom ethos so that they do not generate anxiety?

Teaching

What are the potential barriers to learning? What do we know about learner characteristics? What can we do in the classroom to reduce resistance to learning and any sense of inadequacy and lack of resilience?

Children and adults will not learn if the communication that promotes the learning is not efficacious and there are many facets to this communication.

Communication

Students need to be able to receive information, visually and orally, effectively and efficiently. Their ability to receive information will be affected by a number of basic factors such as hearing and the frequency range available to them (for example, distinguishing between 'hundred' and 'hundredth'), seeing, including the use of colour (which can also support short term memory), design and clutter (minimising distraction whilst maintaining interest).

The amount of information given orally should be cognisant of those pupils with poor short-term memories. The way mental arithmetic tasks are designed should be to challenge maths abilities, not working memory capacity. I do like to reflect on just what it is I am testing, for example, applying a maths concept or working memory capacity. One of these can be trained, the other cannot.

In the USA, students can request a 'vanilla environment' for examinations, an environment free from any distractions. Busy classrooms are great for some students, but not all. Focusing on the key task is not an automatic behaviour and anxiety will not help that ability to focus.

Receiving information can also mean copying, from a board, a screen or paper. This can be a challenge for speed of writing and for short-term memory. It is a potential source of frustration and then anxiety as the pupil predicts another failure in comparison to his peers.

Feedback

One of the key factors in creating anxiety is a fear of negative evaluation. This is probably at its worst with spelling and with arithmetic as answers are so absolute. This fear is often made worse by the attitude of those who can be accurate and, maybe, a tad arrogant about that. I have seen too many examples where appraisal of creative writing becomes negative marking for inaccurate spelling.

And we should avoid fake praise!

So, 7 + 8 = 14 is wrong. For 2y + 5 = 31, y = 6 is wrong (but explainable). We cannot change that, but how work is marked and how results are presented to pupils are within our control. 'Waiting to hear your score on a maths test' ranked highly in the 20 items used in my anxiety questionnaire (Chinn, 2016).

In my days as a school Principal I made use of Seligman's attributional style as a key way to address situations that had the potential to create anxiety and de-motivation. I certainly did not want to generate learned helplessness.

Expectations

Expectations, not surprisingly, permeate education. Sometimes these are renamed objectives or targets. I was often battling the low expectations that surrounded many of my dyslexic students, a situation by no means confined to that learning problem. But getting each objective, each expectation just right for that individual in that subject is a very challenging task. At its simplest, set it too high and failure is likely, too low and motivation disappears and maybe self-esteem, too. Teachers and parents can extrapolate erroneously expressing views such as, 'She is so good at writing, why can't she add?' or (worse) 'His brother was brilliant at maths!' Failing to meet expectations can create anxiety and de-motivation and this will be exacerbated by saying, 'Never mind, you did your best.' Pupils who did their best don't want that to result in 'Never mind.'

And we should avoid fake praise!

Speed of working

It would be an unrealistic expectation to expect all pupils to finish the same amount of work in the same time. It would be equally unrealistic to expect all children to answer mental arithmetic questions in the same time, even if this is at the recall of basic facts level, for example, 12 x 12, as the UK Government currently expects.

It would be equally unrealistic to expect all children to answer mental arithmetic questions in the same time

Speed of processing is likely to lie on a normal distribution curve, so we should not be surprised at a range of times for responses. Maybe students could be considered as individuals and be given fewer examples (and maybe some get more than the average). A reflective question is, 'How much evidence do I need to know that this student has a reasonable grasp of this topic?' I'm not a great fan of busy work for any group of learners.

Making sure that worksheets and tasks in general are differentiated to meet this issue would make a good Maths Department exercise, creating many opportunities for discussion about topics and students. Another task for busy teachers, but one side benefit is that they would be building up a bank of resources.

It is not always possible to appreciate the time a student took to do a homework task

It is not always possible to appreciate the time a student took to do a homework task. The written evidence may not paint an accurate picture.

I am not advocating wrapping students in cotton wool. I am advocating setting tasks that achieve sensible and realistic goals for as many students as possible … and thus helping to maintain motivation and reduce anxiety.

I think questions such as 'Can't you go any faster?' are somewhat simplistic in learning situations and are very likely to raise anxiety levels.

Early maths

So, what are key topics in early maths and what can we do to make them successful and unanxious?

Butterworth (2010) considers subitising, approximate number sense and numerical stroop to be the foundations for early numeracy. Subitising is the ability to accurately and quickly quantify a number of randomly organised small circles. Adults should be able to do this for 7 dots. A child needs to be able to make this judgment consistently if the connection between quantity and the number symbol is to set the foundations for maths. Numerical stroop is also about creating a secure interpretation of number symbols. Skemp (1986) observed that, 'It is largely by the use of symbols that we achieve voluntary control over our thoughts.' Yet one definition of anxiety (Datta and Scarfpin, 1983) states, 'mental block anxiety may be initiated, for example, by a mathematics symbol or a concept that creates a barrier for the person learning mathematics.'

I like using patterns to develop number sense, patterns that are similar to those found on playing cards, with the exception of the two rows of three representing six. I prefer a five pattern plus 1. In my two decades of teaching children with severe difficulties in maths I have found that 1, 2, 5 and 10 are the familiar (and comfortable) numbers. The spectrum of difficulties in maths suggests that this will be true for many children. Breaking down numbers into 'chunks' that are automatically evaluated is one way of achieving this goal. One of three key findings from a major study in the USA on how children learn (Bransford et al, 2000) sets the mantra for dealing with learning facts:

To develop competence in an area of enquiry, students must:

(a) have a deep foundation of factual knowledge,

(b) understand facts and ideas in the context of a conceptual framework, and

(c) organise knowledge in ways that facilitate retrieval and application.

It is (c) that tells us that there is an alternative to an over-emphasis on rote learning. A recent PISA survey (2016) found that students in the UK relied heavily on memory for maths, which is not the best approach for understanding concepts, but has the, often erroneous, appeal of easily attainable accuracy. I believe that teaching children how to use maths strategies to work out the facts and procedures they do not know or cannot remember. For example, collecting like terms in algebra links back to understanding that multiplication facts are based on repeated addition of the same number. An example is that 7 x 8 (possibly the hardest fact for children to commit to memory) is 8 + 8 + 8 + 8 + 8 + 8 + 8 which is 5 x 8 plus 2 x 8 and thus 56. So, y + y + y + y + y + y + y is 7y (or 5y + 2y). Desoete and Stock (2011) emphasises the importance of classification and seriation as foundation skills.

Maths is a very developmental subject and a consequence of this is the need to know and understand the basics, the foundations and the skills. Young children need that, not just recall, to build confidence in their own efficacy to address learning problems.

Conclusion

There are cognitive barriers that handicap the learning of maths. Addressing these may reduce the affective barriers.

There are cognitive barriers that handicap the learning of maths. Addressing these may reduce the affective barriers.

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The progression of the assessment of KS2 mathematics from Sats to NTs to NCTs

Mark Pepper has taken the time to track the changes to the way maths is assessed and has unearthed some very interesting and disturbing trends! This article is a must for all who wish to take charge of the way we 'test' pupils from the very earliest age.

Standardised Assessment Tasks (SATs) were introduced in 1991 and their purpose was to assess the progress made by pupils within the core subjects of the National Curriculum in English, mathematics and science. The science component was later withdrawn and the acronym SATs also had to be relinquished due to objections from an American company who used this title prior to its introduction in the UK. As a consequence of this they were then re-named National Tests (NTs). Despite this longstanding change many commentators and journalists continue inexplicably to refer to these assessments as SATs. Ironically the change in name was most apt as there was a significant change in the assessment procedure as the practical tasks of the SATs era became transformed into tests within the NTs. Hence instead of performing tasks pupils were required to predominantly answer closed questions with the use of pencil and paper (though one of the three papers was exclusively devoted to mental maths). There was a further change to the title in 2016 when the tests became known as National Curriculum Tests (NCTs).The reason given for this change was a requirement that the tests should become even further aligned to the N.C.

Standardised Assessment Tasks

As previously mentioned the original SATs covered English, maths and science and consisted of the performance of tasks.

An episode that took place shortly after the introduction of SATs illustrates some of the practical difficulties in administering the tasks which arose from time to time.

I was actively involved in a science SAT in an inner London primary school. My role was to assist a Year 2 class teacher in the administration of the test. She had been thorough in her preparation which had included the acquisition of all of the materials required for the task. One of these consisted of a carrot. The teacher effectively prepared the children with a clear explanation of what would be required of them. All appeared to be running smoothly until I noticed that the teacher was becoming increasingly agitated and started anxiously staring at the table. She also looked under the table. I discovered the source of her anxiety when she whispered to me

I noticed that the teacher was becoming increasingly agitated and started anxiously staring at the table

"The carrot has disappeared!"

One of the boys overheard this and shamelessly confessed "I ate it, Miss."

The task had to be postponed to a later date when the teacher ensured that the carrot did not leave her possession until the task was completed!

National Tests

In the initial years of National Tests the format consisted of three papers. One of these was exclusively devoted to mental maths whilst the other two papers consisted mainly of closed questions that required written answers. In one of these papers candidates were permitted to make use of a calculator. Hence the candidates could concentrate on the problem solving aspects of the questions without the additional burden of carrying out tortuous written calculations.

The inclusion of the mental maths paper was doubtless influenced by the format of the Daily Mathematics Lesson (DML) which was a central component of the National Numeracy Strategy. The first fifteen minutes of the DML consisted of a mental maths starter and the mental maths paper of the early NTs provided a means of assessing progress made within mental maths.

National Tests 2015

A significant change took place in 2015 when permission to use a calculator in one of the papers was withdrawn. This was a surprising development in view of the fact that calculators have been permitted within one of the GCSE papers from its inception to the present time. In addition to the general advantages of pupils using a calculator in the primary sector it is essential that they develop practical calculator skills both at face value and in preparation for their involvement within the GCSE.

National Curriculum Tests 2016/7

There were major changes in the format of the NCTs in comparison to the NTs. The tests still consisted of three papers but the mental maths paper was replaced by a paper entitled *Arithmetic*. The other two papers which had been entitled Paper A and Paper B now bore the title of Reasoning. The Arithmetic paper consisted in its entirety of rote learning in the form of questions involving the recall of number facts or the application of taught algorithms. The prominence of a rote learning approach within this paper is further evidenced by the dramatic increase in the number of questions involving long multiplication and long division. In the 2013, 2014 and 2015 tests within all three papers there was just one question that involved long multiplication or long division. In the Arithmetic papers of both 2016 and 2017 there were two long multiplication questions and two long division questions. Furthermore in 2016/7 pressure was applied to use the "formal method" to answer long multiplication and long division questions. This pressure consisted of the offer of an additional mark if no more than one error was made and the candidate used the formal method in answering the

In addition to the general advantages of pupils using a calculator in the primary sector it is essential that they develop practical calculator skills

question. In long division questions this meant using the “bus stop” method. The offer of an additional mark is explicit both in the advice to candidates in the test papers and within the marking scheme. Hence there is pressure on both teaching staff and candidates to not only apply taught algorithms to long multiplication and division questions but to use a single method, the formal method.

The use of the title of reasoning for the other two papers is highly misleading. The title reasoning implies questions that require problem solving and creative skills. Yet a comparison of the type of questions contained in Papers A and B and those questions in the Reasoning papers reveals that there is virtually no difference between them with regard to the degree of creative thinking required in answering the questions.

Conclusion

It can be seen that the progression of the assessment procedure from the early SATS to the current NCTs have been characterised by a continuous movement from a fairly progressive approach to one that has become increasingly dependent upon rote learning in the form of the reproduction of number facts and the application of taught algorithms. This is also evident in the withdrawal of a mental maths component of the test and the ban on the use of calculators throughout all three papers of the test.