The Die

If 3 is scrubbed from the **first** die, then of the possible outcomes three will give a total of four: **1 + 3, 2 + 2** and **0 + 4**; and three will give a total of nine: **4 + 5, 5 + 4** and **6 + 3.**

Tabular Triangles

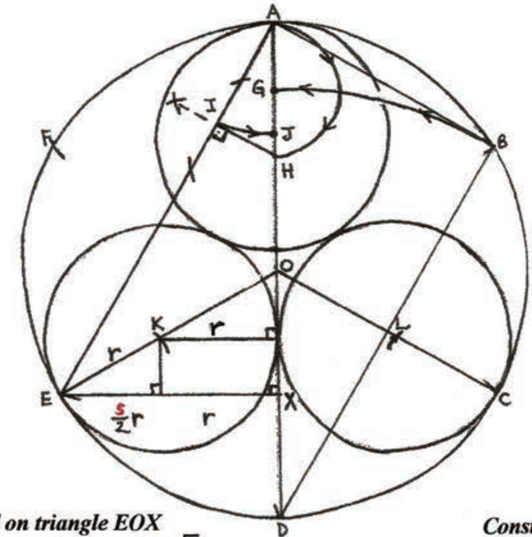
The third table is a simple multiplication table. Using **t(n) = n(n + 1)/2,** we can show

$t(a + b) = (a + b)(a + b + 1)/2 = (a^2 + ab + a + ab + b^2 + b)/2 = (a^2 + a)/2 + (b^2 + b)/2 + ab$

$= a(a + 1)/2 + b(b + 1)/2 + ab = t(a) + t(b) + ab.$

Three Poached Eggs

I've managed this solution but it does seem complex! Is there an easier way?



Working based on triangle EOX

(radius of pan = ***R***, of eggs = ***r***, $s = \sqrt{3}$

EX = $sr/2 + r = sR/2$ so $sr + 2r = sR$

$r(2 + s) = sR$

Rationalise the L.H.S:

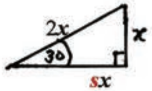
$r(2 + s)(2 - s) = sR(2 - s)$

$r(4 - 3) = r = sR(2 - s)$

Working based on triangle DAB

AD = 2***R*** and AB = ***R***. ABD is a right angle.

By Pythagoras', DB = sR



(The only way is sx)?

Construction

Draw a circle centre O radius ***R***.

With radius ***R***, step off points ABCDEF.

Draw lines AD, AE, OE, OC, DB.

Centre D, radius DB, mark G on DA.

Since DB = sR, AG = $2R - sR = R(2 - s)$.

Centre G, radius GA, mark H on AD.

'Drop' a perpendicular from H to AE at I.

AI is now $sR(2 - s)$, the required ***r***.

Centre A, radius AI mark J on AD.

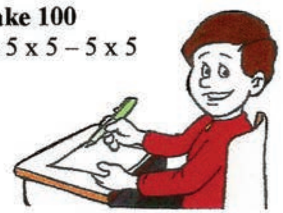
This is the centre for our first egg.

Radius AJ, centres E and C, mark the other two centres at K and L.

Draw the three eggs!

Make 100

5 x 5 x 5 – 5 x 5

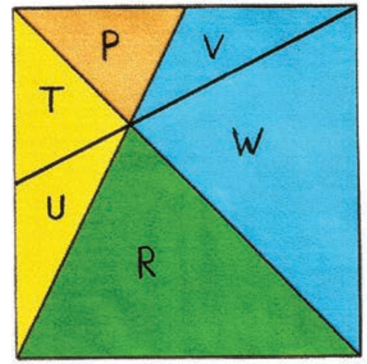


MAthematical PiE

Summer 2020 **Notes for . . . . . . No. 210**

Four Areas

Complete the diagonal symmetry by inserting one more line:



Then **P** and **V** are congruent to **T** and **U** respectively, and **R** is congruent to **W.** Since **P** and V have equal 'bases' (on the top line) and the same 'height', they have the same area and so **P = T = V = U**. Clearly **U + T + P** form a triangle whose area is one-quarter of the complete square, which means each is one-twelfth, as is **V**, so **U + T + P + V** = 4/12 or 1/3 of the square. Subtracting these from the whole square leaves **R + W** = 2/3 making each of **R** and **W** 1/3, equivalent to 4**P** .

Returning to the original diagram with **P** = 1, we have -

Q = U + T = 2**P** = 2, **R** = 4**P** = 4, **S = W + V** = 4**P** + **P** = 5

The Longest Palindrome

Find the frequency of each digit: 0 1 2 3 4 5 6 7 8 9

1 3 5 7 4 4 3 3 5 5 . . (check the total is 40)!

In a palindrome, only the central digit of an odd-length number can appear an odd number of times. So we can only keep one odd frequency (say that of 0) but must reduce all the other odd frequencies by one, leaving us with 1 + 2 + 4 + 6 + 4 + 4 +2 + 2 + 4 + 4, a total of 33 digits.

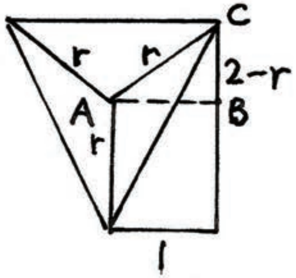
Numbers in Reverse

i) For a number with digits ***ab*** to be increased by 75% when reversed, we need $7(10a + b) = 4(10b + a)$, from which $66a = 33b$ and $b = 2a$. Solutions: 12, 24, 36, 48.

ii) The 24-hr clock places several restrictions on the possible numbers. I think that 00.32 and 23.00 give the largest difference of 22hr 28m.

A Square and a Circle

The circle circumscribes an isosceles triangle. If its radius is r then in the triangle ABC shown, AB = 1, BC = $2 - r$ and AC = r. Using Pythagoras', we obtain $r = 1.25$.



8 ÷ 2(2 + 2)

No further comment needed

From *Puzzle Papers in Arithmetic* (adapted)

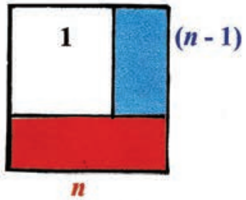
607 = 6 x 100 + 7 = 6 x 102 - 5

Chocolates

As a minimum, the first person would take one chocolate and each subsequent person just one more than the last, making the total a triangular number. Since the 13th triangular number is 91 but the 14th is 105, there can have been no more than 13 people.

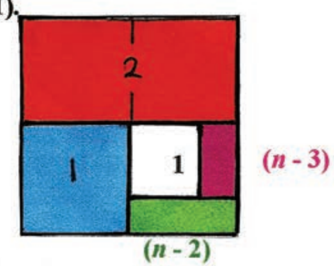
Squares within Squares

Most even numbers are possible: to divide a square into **2*n*** others, first draw a row of ***n*** squares along one edge, then a column of **(n – 1)** squares down an adjacent edge. The remaining square brings the total to $n + (n - 1) + 1 = 2n$.



The least value of ***n*** required to form a gnomon is 2. dividing the square into 4.

Most odd numbers are possible: to divide a square into **($2n$ – 1)** squares: start by drawing a row of **two squares** along one edge. Complete a gnomon by drawing **one square** down an adjacent edge. This leaves a congruent square which we need to divide into **($2n$ - 4)** squares. Since this is even, we continue in a similar way to the above - in the remaining square, draw a row of **(n – 2)** squares, and complete a gnomon by drawing a column of **(n – 3)** squares down an adjacent edge. The remaining square brings the total to $2 + 1 + (n - 2) + (n - 3) + 1 = (2n - 1)$.



The smallest possible value of ***n*** to allow the final gnomon is 4, so 7 is the smallest number of squares that we can form by this method. The impossible numbers are 2, 3 and 5.

The Path

Squares and Primes

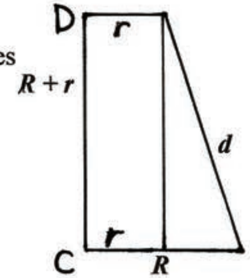
121, 289, 529, 625, 961

Round a Trapezium

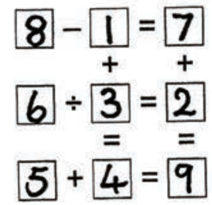
If CX = R and XD = r, the diameter d of the semicircles is given by $d^2 = (R + r)^2 + (R - r)^2 = 2(R^2 + r^2)$

The area of the two semicircles is then Pi times $d^2/4$ = half of $Pi(R^2 + r^2)$,

but the area of the two circles is $Pi(R^2 + r^2)$

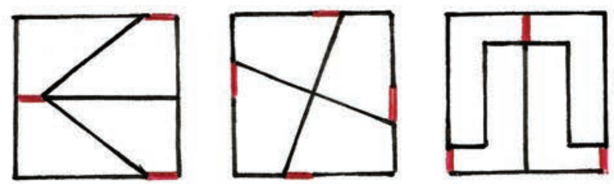


1 to 9



Congruent Quarters

Each of the given dissections can generate an infinite number of others by varying the coloured lines below (but keeping them equal in each case).



Five Numbers

Three ways: 12 + 12 + 12 + 32 + 32, 12 + 12 + 14 + 14 + 48. 14 + 18 + 18 + 18 + 32.

Icon Counting

viii)– v) = SMART – MAST = R + 33 – 27 = 6

viii) – x) = SMART – STAR = M = 33 – 25 = 8

x) – ix) + STAR – TAPS = R – P = 25 -19 = 6 and so P =0

Substituting values in vii) gives I = 1, in iv) gives G = 5 and in vi) gives S = 9

i) + ii) – ix) leads to O = 4. Further substitution in ii), iv) and v) give T=3, C= 2, A= 7:

0	1	2	3	4	5	6	7	8	9
P	I	C	T	O	G	R	A	M	S