The Hot Grapes

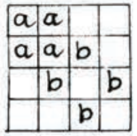
The grapes contain 10g of solids and 990g of water. After evaporation, the new mass m is made up of the same 10g of solids with (98%m)g of water so

$$m = 10 + 0.98m$$
$$100m = 1000 + 98m$$
$$2m = 1000$$

The grapes now weigh only 500g - a beautifully surprising result!

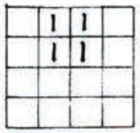
Squaring Up

The four moves in the sample game could be played in any order, so choosing any of the squares 1, 2, 3 or 4 will lead to a four-move game with 'evens' winning. Starting with either of the first two squares illustrated leads to immediate defeat for 'odds' in a two-move game:

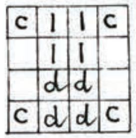


If a = 1 then b = 2 and vice-versa.

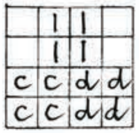
The winning first move is



. . which can only proceed



or

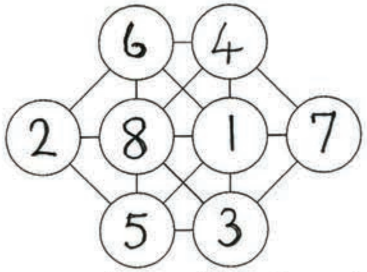


If c = 2 then d = 3 & vice-versa.

Two Special Triangles

It's relatively easy to spot (6, 8, 10) with area and perimeter 24, but one can also use the equations $ab = 2(a + b + c)$ and $c^2 = a^2 + b^2$ to find that $a = 4 + 8/(b - 4)$. For integer values, $(b - 4)$ can only be 1, 2, 4 or 8. I leave it to you to calculate the values of a and b that arise!

1 to 8



Twisted Logic

I had to be told the answer: starting with the 1 near the centre, the squares spiral clockwise in the order 1, 2, 4, 8, 1, 6, 3, 2, 6, 4, 1, 2, 8 . . since these are the digits of the powers of 2, the next three digits are 2, 5, 6, placing 6 in the empty square.

Mathematical PiE Notes 209

MAthematical PiE

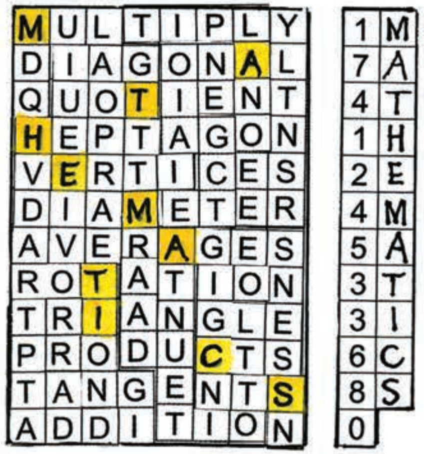
Spring 2020 **Notes for . . . . . . No. 209**

Borromean Rings

Very attractive. Removal of the central ring frees the rest. One crucial difference is that the removal of *any* ring undoes the linkage in the traditional three-ring verson. I don't think that this is possible with any more than three rings. Search the internet to see interesting *African Borromean rings* carved from wood.

Jigword

I have done a physical cut-and-paste to demonstrate:



Mathematical Link-Words

Number, triangle, coordinate, set, vector

Sumo Wrestling

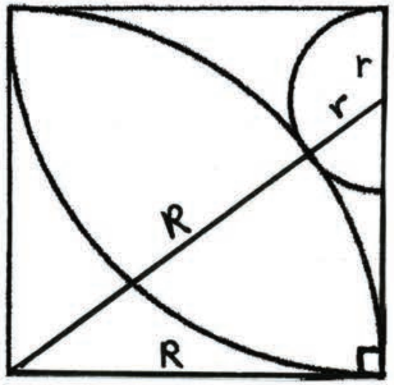
In the quarter-diagram shown,

$(R + r)^2 = R^2 + (R - r)^2$, so

$(R + r)^2 - (R - r)^2 = R^2$ and

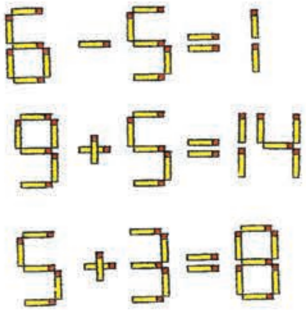
$2R.2r = R^2$ or $4r = R$

In the original diagram, the centre of the small circle is situated one-eighth of the way down the central line.



Five Numbers
We need the primes to be close to 60: 47, 53, 61, 67, 72

Just as Easy as 1, 2, 3!
Of course it is relatively easy to make the equations 'true' by using one match to cross out the equals sign - but apart from such sneaky dodges, there are solutions such as these -



Easy Multiplication
The essential pattern is the multiplication of numbers with the same tens digit ***a*** and 'complementary' units digits ***b*** and ***b'*** *where* ***b' = 10 - b.*** We then have

$$(10a + b)(10a + b') = 100a^2 + 10ab' + 10ab + bb'$$
$$= 100a^2 + 10a(10 - b) + 10ab + b(10 - b)$$
$$= 100a^2 - 100a - 10ab + 10ab + 10b - b^2$$
$$= 100a(a + 1) + b(10 - b)$$
$$= 100a(a + 1) + bb'$$

So 11 x 19 = 209, 12 x 18 = 216, 13 x 17 = 221 etc.,
21 x 29 = 609, 22 x 28 = 616, 23 x 27 = 621 etc.,
31 x 39 = 1209, 32 x 38 = 1216, . . .
Which gives 45 such examples in total (discounting commutative results).

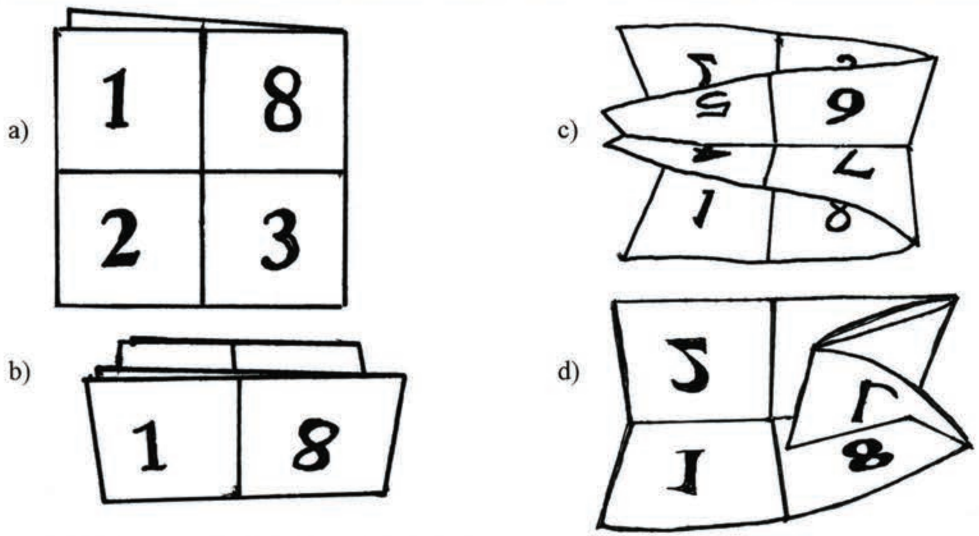
Multiply and Add
To maximise the number of 9's in the total we need to place 9 centrally and use the next four largest digits (8, 7, 6, 5) in some order round it. Suppose we place 8 at 'N'
To maximise the number of 8's we must multiply it by the next largest available digits (4, 3), so place these 'NE' and 'NW'. Then we would rather have 4 x 7 than 4 x 6, and prefer 3 x 6 to 3 x 5 so this fixes the positions of 7, 6, 5. Finally 7 x 2 is preferable to 6 x 2, fixing 2 and 1.
The total is then: 9(8 + 7 + 6 + 5) + 8(4 + 3) + 7 (4 + 2) + 6(3 + 1) + 5(2 + 1) = 371

3		8		4
	■		■	
6		9		7
	■		■	
1		5		2

3	24	8	32	4
18	■	72	■	28
6	54	9	63	7
6	■	45	■	14
1	5	5	10	2

What is the lowest total?

A Mad Map
The process is not easy to describe: I hope that the drawings below help to convey the process! I used a 10cm x 20cm piece of tracing paper so the numbering showed on both sides:



a) Fold the square 7, 4, 5, 6 under 1, 8, 3, 2.
b) Then fold 2,3 under 1, 8.
c) Looking 'inside' the package it can be seen that 6, 7, 8 are in order but 1, 2 are separated by the central 4, 5: so pinch the 4,5 section and tuck it behind the 6.
d) Flatten the packet back down to look similar to b) and fold the left 1, 2 under the rest.

From *Puzzle Papers in Arithmetic* (adapted)
Jack is 16 and Jill is 25, since 625 = 25^2 and 256 = 16^2

Trios, not Triples
Once a few of the empty cells have been filled using the 'not triples' rule, the 'once in a row/column' sudoku style leads to quick completion.
For example, the second cell in the top row of the first grid i) cannot be 5 or 6 since these are already in that row, ii) cannot be 1 or 3 - already in that column, iii) cannot be 4 since the trio would sum to 5 + 4 + 6 = 15 = 5 x 3. So this cell contains 2.
The second column can be completed quickly since we cannot place 6 between 5 and 1 because 5 + 6 + 1 = 12 = 4 x 3.

5	2	6	3	1	4
2	5	3	6	4	1
1	4	2	5	6	3
4	1	5	2	3	6
3	6	4	1	5	2
6	3	1	4	2	5

2	1	4	6	3	5
4	3	6	5	2	1
1	6	3	2	5	4
3	2	5	4	1	6
6	5	2	1	4	3
5	4	1	3	6	2

Abundant, but Weird Nevertheless
70: 1 + 2 + 5 + 7 + 10 + 14 + 35 = 74