Equals

Realising potential in mathematics for all

for ages 3 to 18+

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Anniversary Edition

Save the albatross

MATHEMATICAL ASSOCIATION
supporting mathematics in education

Equals

Realising potential in mathematics for all

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Editors' Page

A spacecraft could not have been landed on an unbelievably distant comet without the application of a great deal of mathematics. All children deserve to be shown something of the power that knowledge of mathematics can give them over their environment. Not a day goes by without all of us applying copious mathematics even if it is only making simple estimates of times, directions, and distances.

In order to set the scene for this anniversary edition of *Equals*, which has been published since the 1970s, we have included the editorial to the first edition of *Struggle*. This was the title given to the early editions of what later was renamed *Equals* to indicate the challenges met by those pupils who struggle to achieve any qualifications in mathematics and persistently remain lower attaining pupils in mathematics.

The journal, under the name of *Struggle*, was the creation of Peter Kaner, then an inspector for the Inner London Education Authority. Peter was determined that the whole population should be given some facility with mathematics. Knowing that many strugglers in mathematics were taught by non-mathematicians - often fearful of the subject themselves - *Struggle* was founded to support these teachers.   Because Peter left the ILEA just before the first issue was printed Rachel Gibbons took over the publication of that issue and has kept it going ever since as the lead to the editorial team. Links were made with leading organisations in the field of special education, such as NASEN (formerly National Association for Special Educational Needs) and SENJIT (Special Educational Needs Joint Initiative for Training, Institute of Education, University of London). When the ILEA was closed and education in London taken over by the individual boroughs, the Mathematical Association took on *Equals* as part of their suite of journals and has continued to provide this support for the publication.

My time writing for *Equals* over the past 10 years has helped me clarify my own thinking regarding the development of mathematical understanding in the pupils I have had the pleasure of teaching. On reflection I think I have always been aware of the danger of presenting mathematics to pupils in a way that divorces it from the fact that being mathematical is essentially a human endeavour. Many concepts we take for granted (the idea of zero, the column method etc.) have emerged from a struggle to make sense of the world around us. More now than ever many very young students are presented with these difficult concepts as a body of facts that needs to be mastered and learned. The downward creep of "facts" in primary school is a very damaging example of this. Without acknowledging such struggle we mistakenly assume that children will just pick it up because we give them the algorithm. Listening to my own children has made me realise that this just isn't so - children need to time and space to collect their thoughts in order to develop their understanding.

Equals to me has always been about this – providing practical ways for teachers to support their pupils to develop their own understanding. This understanding is not linear and will never proceed at 2 sub-levels per term or whatever measure is used to measure schools, departments and teachers. Our model of development now assumes all children will develop at a uniform rate, in line with expectations, and teachers will suffer if they do not! Life is full of beauty with much to cause us to wonder about the mathematics that can help us to appreciate this. *Equals* has shown this to me over the years and with your help it will continue to do so for many more to come.

First edition of *Struggle* – Editorial

Dear Readers,

Sorne of you will be aware of the time taken to get this first issue underway because it is some months since we canvassed support for the idea. Well, we didn't have much sympathy with 'The Times' and its industrial problems until we started on the endeavour of producing STRUGGLE!

But now that we've got underway we intend to produce issues once a term.

A group of us in London felt it was time to try to come to terms with some of the prevalent ideas about the pupils who finally become those resolutely non-CSE, non O-level, fail-everything-else at 16+ characters who sit day after day in mathematical lessons in our schools and whose performance (they tell us) on leaving has led to hands upraised in horror and disillusionment with the educational process.

If one inspects the final examination of school arithmetic courses one will see essentially the four rules (including long ones) on whole numbers, fractions and decimals. Now, unlike so much of the school curriculurn, a value can be put on this knowledge simply because there are calculating machines that can do the job.

Peter Kaner, the initiator of STRUGGLE and until recently an inspector of mathematics in the ILEA, gives a rough impression of the cost effectiveness of many arithmetic courses in secondary schools by comparing them with modern electric calculators. '.... if you could add, subtract, multlply and divide all sorts of numbers to an accuracy of 8 digits (including getting the decimal point in the right place), never get a wrong answer (except when you applied the wrong question) your skill would be worth around £5.

'We are all familiar with the pupil who will only achieve 30% of the efficiency of the machine. This puts a value of £1.50 on his skill. What about the cost of obtaining this skill? In Secondary School alone our candidate has experienced 4 hours a week of maths for 5 years. This comes to 800 hours of time spent on the part of the pupil, or nearly 1 complete year at school. School cost, we are told, is approximately £1000 per year per Secondary pupil (including teachers' salaries, etc) so that the COST must be taken to be something over £900 to finally achieve calculating power worth approximately £1.50.'

If we are still failing to teach arithmetic satisfactorily in our secondary courses for low achievers what should we do? Even if we do decide that the purchase of a calculator for each pupil is part of the answer, the problem will still not disappear. In an attempt to answer this question our magazine has been conceived, and while we are answering this one, why not share knowledge on all topics which connect with the problem? How should we organise special help, how do we identify particular problems, how can we extend the range of mathernatics, how can they learn to solve problems, how can we help them to deal with numerical aspects of living, where can information and teaching materials be found etc, etc?

'ANSWERS TO MANY OF THESE QUESTIONS HAVE BEEN FOUND BY INDIVIDUAL TEACHERS and it is hoped that we can, through the issues of this magazine reach their ideas and solutions and offer them to a wider public.

You may be one such teacher and we hope you will write in a contribution or at least a letter. We hope to interest teachers in special schools as well as those in Remedial Departments of Comprehensive Schools. Also, many of the ideas should be relevant to those working on Appendix ll work in colleges of Further Education (the part of their work that helps youngsters qualify in basic skills before they are able to embark on technical courses) but most of all we aim to distinguish mythology and fact in a very controversial area of education.' *

We shall include in each issue an example of a workcard which has proved successful in the classroom. This issue contains 'Old Oak' and 'Dealing the Cards'. Here again the editors would welcome contributions ranging from successful teachlng games to your own brilliant method of doing long division (even if some people think you shouldn't). Everything is airned to be down to earth and fitting the school system as it is and not necessarily as it will be after we have achieved earthly paradise. Don't expect to agree with everything that is written here as we deliberately want to foster alternative points of view, but if you do disagree strongly, please write in and (space permitting) you will find your own point of view is pubiished and that not everyone agrees with you either!

* Peter Kaner

Attitudes to learning: working with a group of lower achieving Year 5 children.

Emma Billington and **Jennie Pennant** look at strategies to develop a positive learning disposition in maths lessons for this group.

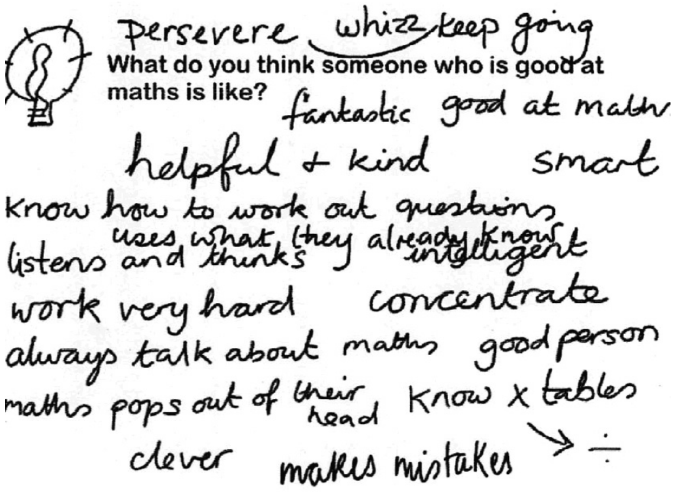
This collaboration began in the autumn of 2006 when Emma invited Jennie to look with her at the challenges the 17 children in her group faced with mathematics. Conversations with, and observations of, the group – a mixture of boys and girls – revealed that they had begun Year 5 with the feeling that they were poor at mathematics and felt they were likely to fail and get it wrong. As a result the group were unwilling to offer their ideas and emergent thoughts about the mathematical concepts they were exploring.

Emma and Jennie spent a session with children looking at three key questions with them

- What do you think someone who is good at maths is like?
- What skills do you need to be good at maths?
- How can we help each other to get better at maths?

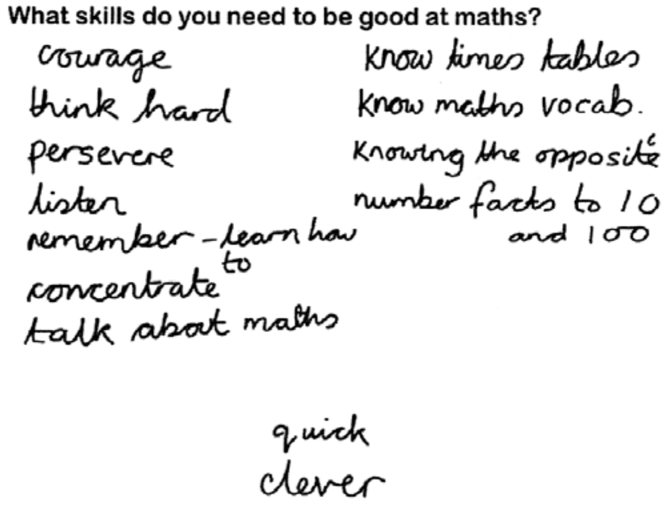
The children worked on their answers individually on paper at first, then shared their ideas with a partner and finally Emma took feedback from the whole class.

Their responses were recorded on the electronic board and are shown below.

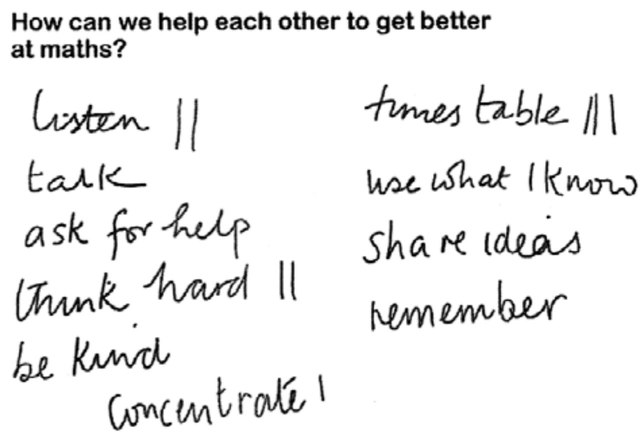


The comment 'maths pops out their head' is rather fascinating. It suggests that to be good at maths requires very little effort. This could be very offputting for these children who may well find that maths does not come easily.

The idea of needing courage to be good at maths is an interesting one. Sometimes it can be hard for children to offer their ideas and thoughts in the classroom if they are not at all sure about them.



It needs a very safe and supportive classroom environment to make this a possible and comfortable activity for children.



The tally marks show where more than one child in the group offered that particular idea. Emma felt that this feedback from the children was very useful and that, as a group, they could work on this in each lesson.

She decided to keep the electronic board notebook so that they could refer back to it in each lesson. The children could find examples for themselves of times when they were managing to carry out their ideas, such as managing to listen, and also discuss ways of supporting each other in developing these ideas. This enabled the children to have a high degree of ownership of the process in the lesson and also meant that progress could be clearly tracked by teacher and children alike.

In December, Jennie and Emma reviewed the progress of this strategy with the children. Emma felt that the explicit focus on the 'attitudes to learning' through the electronic board notebook was helping and that the children were gaining confidence in contributing their thoughts and ideas to the lesson. Jennie and Emma discussed the idea of having a celebration with the children when there was learning as a result of a mistake, so that mistakes became more friendly and positive.

Emma and Jennie also talked about de-cluttering the curriculum and Emma tried to focus on a few significant key skills such as learning multiplication tables and number bonds to 10 and 100. As these key skills developed, Emma then made links between these and other skills, gently providing steps to support the children in progressing. For example, building on number bonds to 100 she took the children into looking at totalling decimals to make 10. From here it then worked well to look at the effect of dividing a number by 10. Emma identified fundamental concepts such as place value that underpin the number system, and worked in depth with these to give the children a confident base from which to explore further mathematical concepts.

By the summer term, the strategy for using the 'attitudes to learning' notebook had developed into focussing on one point per week. At the start of every lesson, Emma looked at the attitude with the children - e.g. being kind to one another, listening, concentrating and together they listed all the things they could do in the lesson to demonstrate that. Emma tried to match the attitude chosen to the maths focus for the week. For example, during the highly practical capacity maths focus she chose the attitude of working together – a development of the children's original list. Emma reported that they have been able to pause mid-lesson and add ideas, or reflect on how well they are doing with the chosen attitude. Very exciting!

In order to develop the listening and sharing of ideas that the children put on their original list Emma worked hard to encourage the children to explain their thinking.

Emma reflects:

> 'If a child tells that 67 rounds to 70 as the nearest ten, instead of saying "yes" I say "why?". It has helped me to identify where learning is going right/wrong, and more importantly has shown the children the need to express themselves clearly. Prior to the children explaining, I will often give them the opportunity for a one minute chat with a partner to sort their thinking, before verbalising in front of the class. This has helped them feel more confident in sharing their thinking.'

Reflecting on the project, towards the end of the summer term, Emma is aware of how much more willing the children are to participate than they were last September and that they are now keen and confident to volunteer their ideas. However, her overriding reflection is that, in her experience, success in maths for those who are struggling depends very much on relationships - with the teacher and between the children in the group.

Emma reflects:

> 'Real progress has only taken place with my group as we have got to know each other, and as trust has developed in each other and in me.'

The question at the beginning of the year – 'how can we help each other to get better at maths?' – has at its heart that need to build trusting classroom relationships so that risk-taking is possible. Without risk–taking how can there be any meaningful learning?

Pinkwell School, Hillingdon and BEAM Education

Successive reflections for pupils and teachers.

To Mundher Adhami it seems possible to think of learning in a mathematics lesson, or in a professional development session, as a sequence of reflections. In such a sequence the subject matter for each reflection emerges as an abstraction or a generalisation from the previous one.

Here is an example from a recent CAME[1] course for experienced teachers. (see note 1) The course is based on collective design, classroom trial and writing of guidance for new Thinking Maths lessons. A further outcome is to describe for ourselves principles for the design of new thinking lessons.

Classroom reflection

The latest suggested lesson we worked on was on **common errors in subtraction** and the logical reasons for them. It was called 'Wrong Answers'. The pupils would first explore errors in subtraction calculations and describe them in their own words. In a second episode pupils should explore ways of checking for ***sensible*** as well as ***accurate*** answers based on a sense of the size of numbers. All of this would fall under the label metacognition, i.e. being conscious of your own and other people's knowledge, ways of working, feeling etc. We started using the label 'reflection' to focus on conceptual knowledge.

Before we went into the classroom we worked on the task ourselves. Three questions with potential difficulties were offered: 205 - 78, 312 – 65, and 1204 – 359. We ended up using only the first one, since the possible outcomes were unexpectedly rich and should be fully explored.

In the first draft offered to this planning stage, the common erroneous answers for the subtraction 205 - 78 were expected to be 13, 37, 77, 103, 107, 137, and 207. These were expected as results of the partial use of the rules:

- You cannot take a number from a smaller number. So you take the smaller.
- You cannot subtract from 0 so you go to the next number to the left

- You borrow from the next number from the left then pay back
- You cannot borrow from 0 so you borrow from the number on the left.

However the work by teachers at this stage produced five more erroneous answers with 'partial logic': 123, 133, 273, 283, and 585.

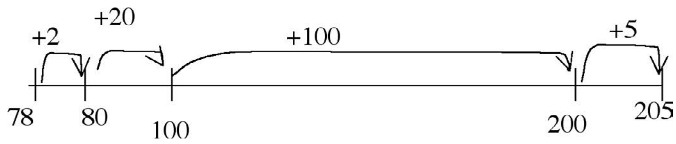
We changed the first draft, and tried the newly sketched lesson in two Y6 classes for about one hour next morning. The start-up story was the same: 'a trainee teacher friend of mine is puzzled and upset that one subtraction question in homework was found very difficult by his Y4 class. Can you help him see why?' Six different subtraction questions were used in each class and the lesson flow was different in details, but with similar levels of engagement and fruitful reflection by the pupils.

In both classes pupils' first thoughts were to blame the trainee teachers : 'not up to it' or 'taught them wrong'.

One class worked out the right answers first and shared their methods. There was the number-line add-on method, the standard columns method, and one which is 'working in the head' method with simpler numbers first than adjusting. They called this the 'normal method' and the 'made-up' method.

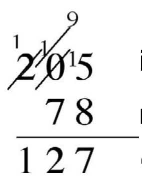
A long discussion followed on what is common between the three methods, ending in them calling them all 'complicated' and recognising that in each case there are many mini-steps, and that each of the mini-steps allows for errors.

A:



They then added 100+20+5+2 in order to get the right answer 127.
The steps were in drawing 5 lines or line markers, and writing 14 numbers on the page. But everything is visible and you do things in order.

B:



The steps include three 'crossings', including a double crossing, and placing numbers in the right places in columns. 'You get lost in the small 1s'.

C:
'We take 70 from 200 then take eight more then add 5.' The two pupils who suggested this method couldn't really explain their method well, but were convinced of it. The teacher came to the rescue, and the class realised that despite the seeming simplicity they can get lost in what to add or take away at the end.

That class went on then to find reasons for errors in six answers given, some finding other errors. The pair that used the number line thought that younger pupils worked well on adding then suddenly realised they should be doing subtraction so they subtracted the last number. The class did focus on the fact that the 'small number has to come out of the bigger number', and about difficulties with zero, the 'carrying' or 'borrowing' but suggested other attention errors which were aired in discussion, and which the teachers had not thought of.

At the end of the lesson one of the three teachers who had taken a turn in running this one hour trial asked the class if we had helped them, and there were ideas like: ‘we worked it out ourselves’, and ‘we taught each other’ and ‘everyone who came up to the board helped.’

The second Y6 class first worked on the simpler question 255-38, and found the error in the unit place, then the error of adding instead of subtraction and not keeping to the columns. Then the original subtraction question was given with an A3 sheet with 6 different wrong answers, to choose one to work on. They talked about the same problems and confusion, but added others. For 37 the working was

$$\begin{array}{r} \overset{\cancel{1}}{\cancel{2}}\overset{1}{0}\overset{1}{5} \\ 78 \\ \hline 37 \end{array}$$

Discussions included what to call the ‘crossing’ and reducing the numbers. ‘Borrowing’ was called ‘decomposition’ by a pupil, a word which was not accepted by his partner. ‘Borrowing without showing’ was called ‘stealing’, and the taking of the small number from the bigger was called a ‘swap’.

When the pupils at end of the lesson were asked what they’d found useful in it, one pupils said ‘I didn’t know there are so many mistakes possible’ and another ‘I didn’t know it makes such a difference which way round you do it. I will not do it again’.

We call the questions suggested for end of lessons that elicit views of the learning process in the classroom ‘reflective questions’ since they do allow pupils to have an overview and see patterns they may not notice when they work on details. The Thinking lesson therefore can be understood as a sequence of steps, some to engage with the task, others to sit back and reflect on the work.

Back at the education centre the teachers agreed that the single subtraction question opened a ‘can of worms’ and allowed some deep reflection in pupils. They had to focus on things they had missed before and describe them, and they therefore had to handle new challenges, each at their level. The pupils were comforted by the fact that errors still have logical reasons, and that they can learn from looking at these. This is a ‘culture of learning issue’ which is best to emerge in reflection slots in the classroom.

Teacher’s reflection on specific shared classroom practice.

This kind of follow-up discussion amongst teachers was reflection on the teaching, and is similar to the standard CAME PD practice. This is based on teachers going on cycles of:

- discussing lessons that have already proved fruitful in the hands of other teachers, based on published guidance that includes key questions, typical pupils’ responses and examples of interactions.
- planning their own first trial of the activity in the classroom, and conducting or observing lessons with peers.
- reflecting as a team on the immediate or recent shared practice.

In the Extension course this process starts at an earlier stage, with the groping for ideas and logical steps within a mathematics topic with difficulties and richness, and with planning a new lesson from scratch. Also, the reflection after the first trial involves identifying the salient fruitful responses of the pupils and what the teachers did to promote them. All that goes into possible modifications to the first draft and towards a second one. However, the reflection remained focused on a single lesson and on shared work in planning and implementation.

the common erroneous answers for the subtraction 205 - 78 were expected to be 13, 37, 77, 103, 107, 137, and 207.

In our discussion after the trials on the Wrong Answers lesson, the teachers discussed:

- the 'hook'[2], or the story line, at the start of the lesson. (In this case it was "a student teacher asking us for help to understand why this question was so difficult for his Y4 class so that answers such as 33, 133, 585, 107, 207 and 238 were found.)
- whether an easier question should be given first, without the zero, e.g. 255 -27
- should a question on 'which of these 6 answers are obviously wrong without calculations?' be early or late in the lesson?
- whether or not it is necessary first for youngsters to try to find the right answer and list the methods before tackling reasons for mistakes
- if the right answer was deliberately sought, how far to go with comparing the different methods (number line, standard and mental informal) and understanding what makes a problem more difficult or prone to errors.
- what language to use for the confusing word 'borrowing', and the alien word 'decomposition'. The words 'exchange' and 'breaking up' seem promising, linked with mentioning the 10's or 100's place.

Thinking about the range of levels possible, it seems clear that the access level to this lesson is familiarity with three digit numbers, subtraction and some number bonds, i.e. good level 3 in NC terms. The difficulties however are in coordinating several rules and keeping track of a sequence of steps. These could be above level 5 when there are about 10 mini-steps such as in this example. More than half the primary school population would seem to benefit from such a lesson.

Similar kinds of discussion occurred on each of the previous new Thinking Maths lessons, often repeatedly as we changed lessons and retried them. It is this process spanning several lessons that is now made the subject of reflection.

Cognitive Acceleration Associates

1. CAME Cognitive Acceleration in Mathematics Education
2. Mark Dawes had introduced the term 'hook' in the previous round in this course, as part of a presentation for a Master-level module, and it became currency in the group, at times replacing the notion of the starting 'story', which Alan Edmiston had earlier suggested as a key pedagogic feature. It is often additional to the skeletal planned lesson, and depends on the teacher and the particular class and time.

If you can't hear how can you do as you are told?

Amy[1] tells Rachel Gibbons how her deafness held back her education.

Amy is a carer and a good one at that. She notices what needs doing without having to be told. She is concerned for her patients' well-being, gently jokes them out of their worries and is deft at making them as comfortable as possible. Whenever she has a spare moment she has a book in her hand and is lost in it. Yet until she was seven years old nobody noticed Amy's hearing difficulties and by that time she had learnt to make up her own mind about what she should do in lessons. Much of it differed from the teacher's plans for the class but that did not worry Amy. In response to being ignored she went her own way. Gradually she had persuaded her mother to buy her 5 tape recorders In class she could hide the earphones under her long hair, thread the leads down inside her sweater to the tape recorder in her pocket and listen to music. When her tape recorder was confiscated in one lesson she had another ready for the next. At the end of the day she went round to her various teachers collecting all her equipment ready for the next day. The intelligence shown in all these ploys was clearly not minimal, indeed I would argue that Amy was giving herself practical lessons in logic when planning all these strategies to get her own way.

Eventually when her special needs were recognised Amy was offered a support assistant but refused because she did not want to stick out as the one person in the class who was different and who needed extra help.

By the time she was 17 and in college she still could not read. She had got by so far, making her way in the world by asking passers by what words meant when it was essential to know. Often Amy says that the people she asked were unable to answer her questions either. How many of them, she wonders, could not read?

Often Amy says that the people she asked were unable to answer her questions either. How many of them, she wonders, could not read?

Amy was popular at college and she was invited to go on holiday with some of her new friends. They told her to be sure to bring a book. Of course not being able to read, she did not possess any books. But, having seen his film, she did know something about Stephen King and she could recognise his name, indeed she had probably picked up more words than she was aware of, and she bought all King's books she could find and took them with her. On that holiday she started to read for enjoyment and enlightenment and has never looked back.

Now, at the age of 29, Amy is an avid reader and wherever she goes she is seldom without a book in case she has time for a few minutes' delight between jobs.

Rachel Gibbons was formerly an inspector for mathematics in the Inner London Education Authority

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1. Name changed to preserve anonymity

Mathematics in unusual places 1 – Manhole covers

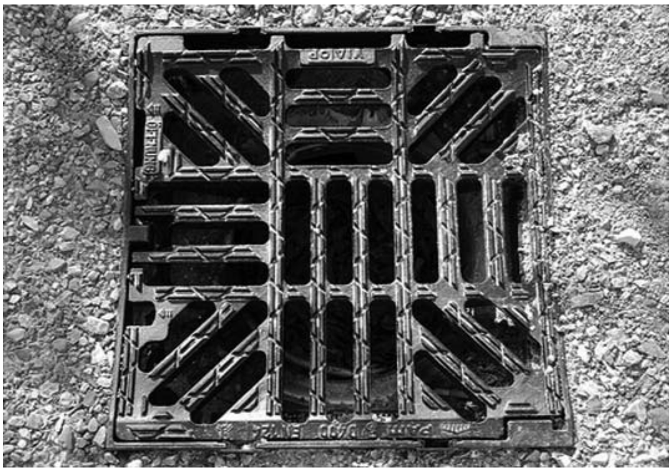
Manhole covers are more than just heavy pieces of metal that stop people from falling into holes in the ground Matthew Reames explains. They come in a variety of shapes: circular, square, rectangular, hexagonal and more - and can be an exciting way of exploring mathematics.

The designs on the surfaces of the covers range from simple circles or squares to groups of parallel lines and some rather complex shapes. Some covers have patterns involving translations or reflections while others have line symmetry or rotational symmetry. A quick look at the collection of photographs of manhole covers at http://tinyurl.com/manholecovers will show some of the huge variety of shapes, sizes and patterns throughout Europe. (As an aside to purists, the collection includes photos of both manhole covers and drain gratings.)

Using different manhole covers is a real-life way of incorporating maths skills into your lessons. What is the area of each cover? The perimeter? Can you find a pair of manhole covers that have the same perimeter but cover different areas? Do rounded corners make any difference to the calculations? Do some covers have corners that are more rounded than others? The area calculations are fairly straightforward with rectangular covers but become slightly more difficult for round covers. Other shapes such as hexagons, ovals, and triangles each have their own challenges.

Children who are studying circles and Pi may benefit from the opportunity to use what they have learnt to measure and calculate the area and perimeter of round manhole covers. They can have a very good discussion about the best way to measure the circumference of a round cover, something that is not nearly as simple as measuring the perimeter of a rectangular cover. Investigating round covers becomes even more interesting if you have round covers of different sizes. Compare the diameter, circumference and area of each circle. Can you find rectangular manhole covers with the same area or perimeter as the round covers?

Looking more closely at the covers, have the children examine the surface patterns. Is there line symmetry? Rotational symmetry? Can they find examples of translations, reflections or enlargements? Some surface designs are intricate tessellations while others are complex swirls. Are any of the designs completely random? Do any



covers have a completely blank surface? Why do you think the designers created the patterns that they used?

A note on photographs

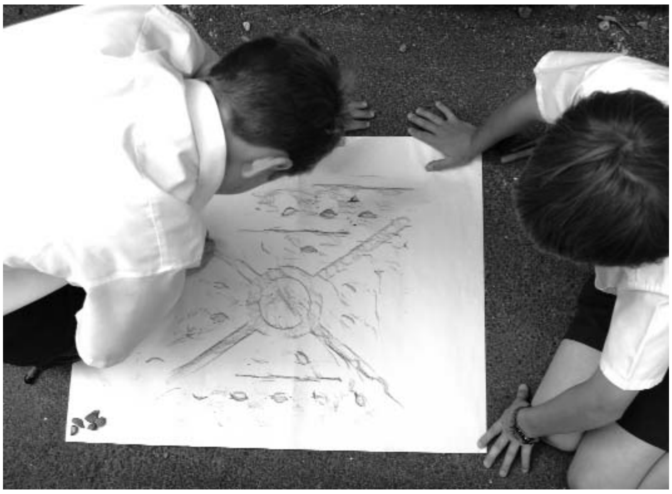
One thing to keep in mind when taking photos of manhole covers is that unless the camera is directly over the cover, there will be some distortion of the shape. Circles will look more oval-shaped, rectangles will become trapeziums or other quadrilaterals, parallel lines may no longer look parallel, and angles will not be their true sizes. This is something to keep in mind if you are using the photos to measure lines or angles.

It might also be an interesting exercise to photograph the same manhole cover from different angles in order to investigate just exactly how the lines and angles do change when observed from different positions.

A manhole cover spotting expedition

Our school site is home to numerous different types of manhole covers. Recently, our Year 3 class and some of our Year 5 children went on Manhole Cover Spotting Expeditions. Their goal was to find as many different 'species' of manhole covers as they could. Each time they located a 'specimen' they kept a record of it by making a rubbing of the cover on a piece of 'Expedition Paper'. In our case, our 'Expedition Paper' was paper from a large pad of flip-chart paper – large enough to cover most manhole covers and thin enough make a good rubbing without being so thin that it rips easily. One benefit of making a rubbing of a manhole cover is that there is no distortion of the design – a rubbing can be taken back to the classroom and measurements taken directly from the rubbing,

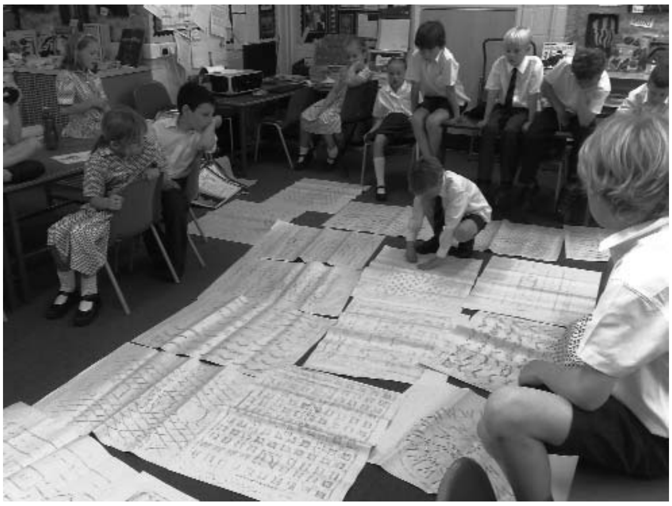
Just one further word of caution: please be safe and only photograph or make rubbings of manhole covers that are in safe locations! School playgrounds and pavements are good, but roads are not.



The children spent the next 15 or 20 minutes spreading out around the area near the playground to find manhole covers and make their rubbings. Groups eagerly came back to show the 'specimens' they had discovered and to ask for more Expedition Paper to use for other manhole covers. At the end of our time outside, the children had collected quite a few different 'specimens' and it was time to take the specimens back inside to discuss them.

Some helpful tips to remember when making the rubbings include removing the paper wrapping from the wax crayon and using the edge rather than the tip to rub, having one or two people help hold the paper in place while someone else does the rubbing, and working carefully to get the fine detail of the design while taking care not to rip the paper. My pupils have also suggested that you choose a calm day as even a light wind makes the large sheets of paper somewhat difficult to manage. They also recommend using darker colours of wax crayons as the rubbings show up much better on the light coloured paper.

Back in their classroom, the children laid their rubbings out on the floor or tables to create a gallery of the Expedition Papers. They were amazed at how many different examples they had found in the relatively small area outside. In the discussion that followed, the children talked about the shapes of the covers they found, which shapes were the most common, and the detail of the covers. Some children said that they thought certain designs were easier to rub because the patterns were more regular than others. There was some debate about whether or not a square was a rectangle so this provided an excellent opportunity to talk about the characteristics of squares and rectangles so that the children finally concluded that squares are a special type of rectangle.



Examining manhole cover attributes

A further extension to this Spotting Expedition is to create a giant Venn diagram on the grass using long ropes.

The labels on each part of the Venn diagram can be written on mini-whiteboards so everyone can see them. Then, the children place their sheets of Expedition Paper in the appropriate sections of the diagram. For example, ‘Four Equal Sides’ and ‘Four Right Angles’ would have all of the rectangles in one section, all of the squares in the intersection of the two circles and the round covers outside both circles. For our children’s set of manhole covers, there were not any that had just four equal sides. In this case, children could be asked what type of shapes could go in that section. Other categories could be created depending on the set of ‘specimens’ in your collection.

This could also be done on a much smaller scale by printing some of the photos, either ones you and your pupils have taken or ones found online. You could also use the photos to fill in a large Carroll Diagram similar to the one below:

	Round	Straight Sides
No lines of symmetry		
At least one line of symmetry		

You can change the category descriptions depending on the photos you have in your collection.

Another use for your photos of manhole covers is to play the game Memory (also called Pelmanism). You will need two photos each of about 12 different manhole covers as well as an identical piece of blank paper or card for each photo. Print each photo and glue to the card. The cards are turned face down, shuffled and laid face down on the table. Two cards are flipped up each turn. The object is to turn over matching cards. If the cards that are turned over do not match, they are both turned back over. This can be played individually or by two or more players but you may find that you need to use more than 12 different photos! A simple set of cards might include photos of manhole covers that are different shapes while a more challenging set might feature all round or all square covers. This is an excellent way for children to learn to look carefully at details. Two manhole covers that look similar may in fact be quite different.

Hopefully by exploring some of these activities, your pupils will become more aware of the mathematics around them every day. The next time you are walking down the street, keep an eye out for manhole covers and consider the possibilities of mathematics in unusual places. I would welcome any ideas you may have!

St Edmund's School, Canterbury

The Harry Hewitt Memorial Prizewinner 2006

Sam Henderson (aged 12)

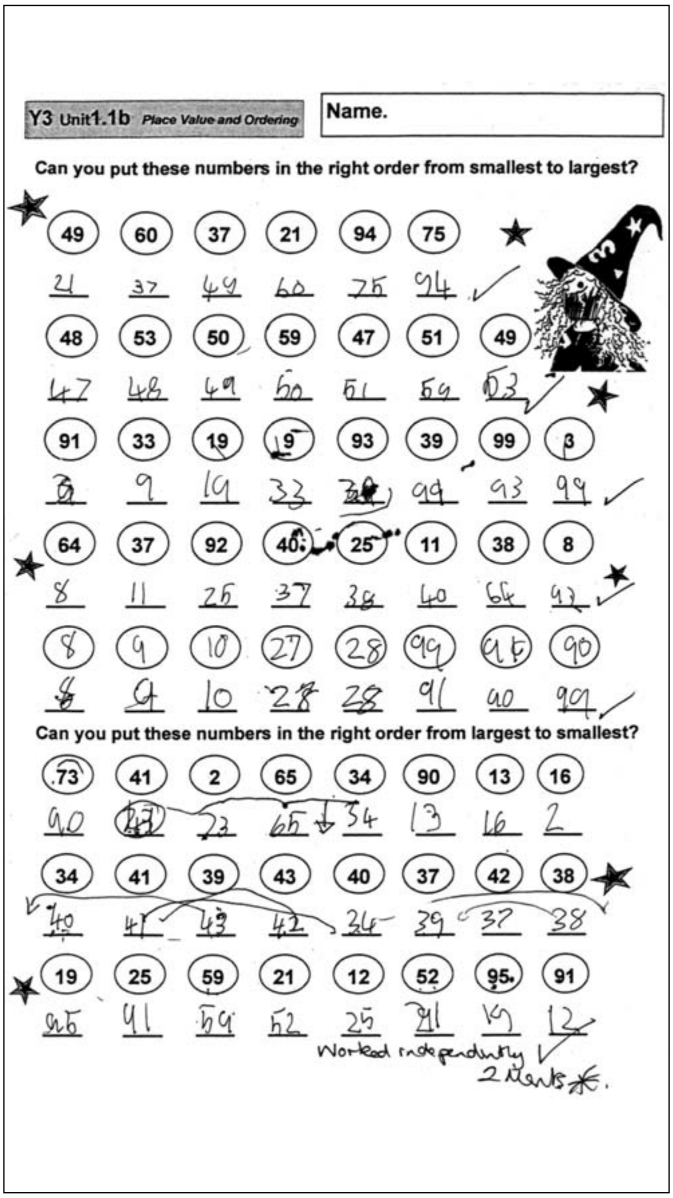
I am a mathematics teacher at St.Joseph's RC Middle School, Hexham. This is a small middle school with 343 pupils of varying ability. Our motto "Striving for Excellence" means we encourage all children no matter what ability to achieve their best. This is especially true for the children who have learning difficulties.

Sam started at St. Joseph's in September 2003 with a variety of learning difficulties. His psychologist report read:-

> "Sam has a diagnosis of ADHD and is on medication. He has a history of delayed spoken language skills but these are now appropriate for his age.
> His literacy skills are also delayed and these are still at about a 7 year old level.
> His numeracy skills are also well behind his peers.
> He has attention and concentration difficulties."

In Year 5 our mathematics groups are organised so that we have two parallel groups, a middle group and a fourth group for pupils who find maths very difficult. I took Sam in the fourth group with 11 other pupils. Sam entered St. Joseph's at National Curriculum level below 2. One of the first lessons I remember is when I asked Sam which number I was pointing at on the number square. The number was 27 and Sam said it was a 2 and a 7. He could not at this stage recognise numbers over 10. He had a teaching assistant with him every lesson to help him and he relied greatly on her to the extent that he wanted her to do the work for him as it was too hard. He was quickly given a special programme of work as he could not tackle the work of the rest of the class. The work of the class at this time was revision of level 2 touching level 3 from ***Target Maths*** Year 4. Sam did have strengths however, his spatial skills were more developed so when we were doing shape or graph work he did well and this gave him a boost.

In Year 6 he had improved, but not enough to catch up with the class so he continued to work on his special programme. I have included a sheet from his Year 6 book. Mrs Storey (his teaching assistant) and myself decided we would try to make Sam a more independent learner so gradually over the year she would leave him to work by himself until he was working almost totally by himself and even started puting his hand up and answering questions. We saw a marked improvement by the end of Year 6 when he moved on to another teacher, Mrs Garraghan and a new teaching assistant, Mrs Stanley.



An example of Sams work from the beginning of Year 6

In year five it was hard doing maths
but now I can do my ten times
table
I can now do the same work as
the class but it used to be difficult
maths is all right.

In Year 7 he has continued to improve and the optional Year 7 national tests showed that he was now working at a National Curriculum level 3A. he is now able to work with the class and he is no longer the poorest in the class. I have included Sam's thoughts on his recent maths work from Year 7 to compare with the work from the beginning of Year 6 and a letter from his parents.

We are delighted with Sam and think that to move nearly 2 levels in as many years is fantastic and for this reason I would like to nominate him for the Harry Hewitt Memorial Prize.

Mrs Maureen Stevens

Woodside
Slaley
Hexham
Northumberland
NE46 1TT

Reference: - Samuel James Henderson
St Josephs Middle School
Hexham

We are writing to give our support to Sam regarding the special math's award.

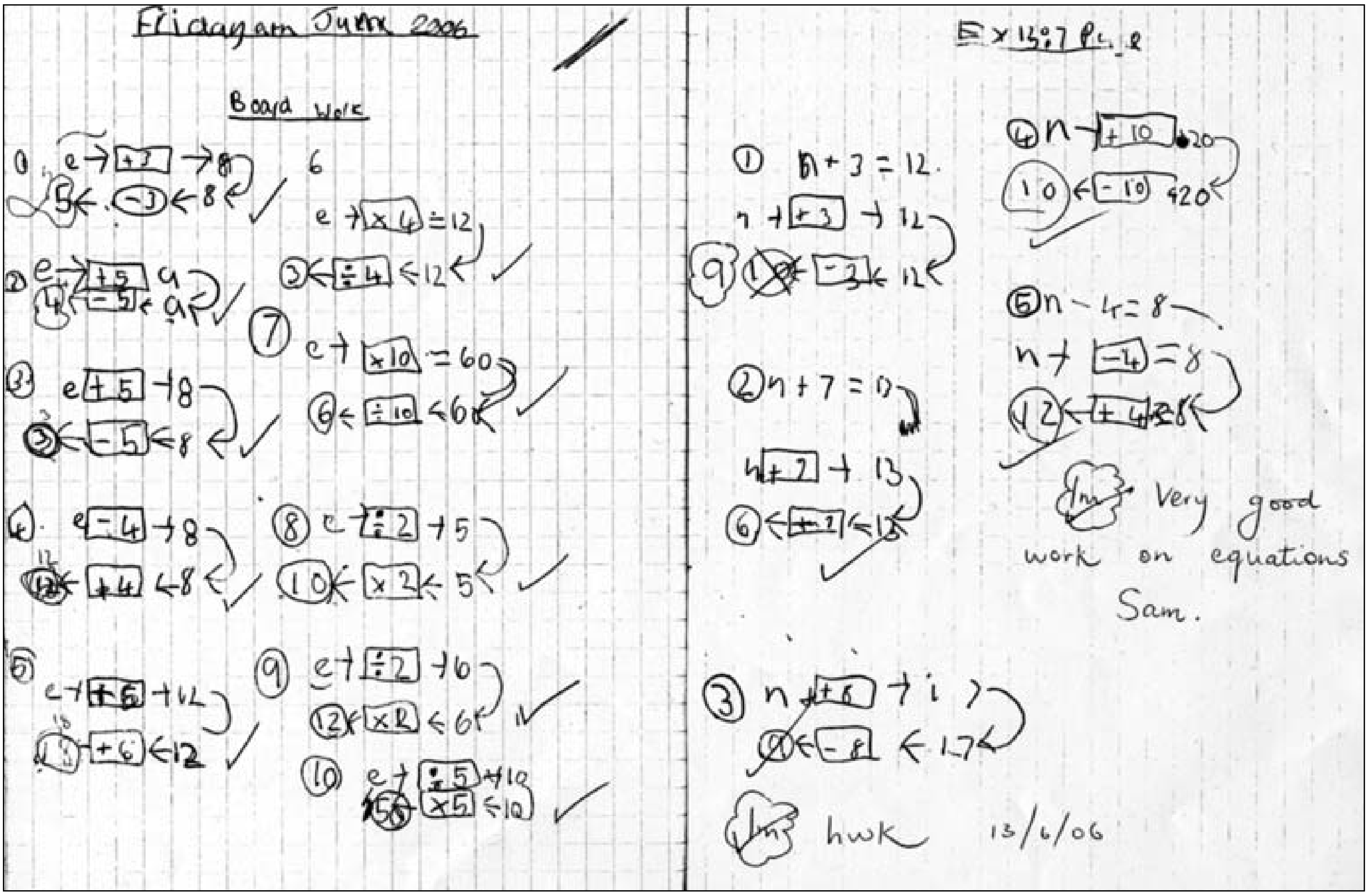
When Sam started St Josephs his understanding of math was very poor. Over the 3 years he has been at St Josephs he has made significant improvement thanks to the hard work of his teachers and classroom assistant. Thanks to their help Sam has become more confident and is willing to try hard to succeed.

Yours faithfully

MAHenderson

R E & M A HENDERSON

Sam Henderson



An example of Sams work from the beginning of Year 7

"Stop interrupting!"

Teachers are inclined to interrupt. Knowing when and when not to intervene, Colin Foster reminds us, is the core of good teaching.

Choosing when to intervene and when to hold back is, for me, one of the most important and difficult elements of working with learners on mathematics. Gauging someone's mathematical and emotional needs at a particular time is something that I cannot be confident about doing, but there is no alternative to trying. No doubt my interventions (or lack of them) are as much affected by my own feelings and thoughts in the moment as they are by my perceptions of the pupils'.

What I do feel sure of is that I get it wrong more often than I get it right. On one occasion, I stand back and watch someone struggle with a mathematical task in a manner akin to watching someone starting to drown, and feeling no compulsion to help! Their confidence and motivation are slipping away, but I am standing on the side-lines doing nothing about it. On the other hand, there are situations when I find myself fussily butting in, pushing a learner to do something the way it is in my mind rather than going with the grain of their own thinking and allowing them the time and space they need to construct their own understanding of the situation. With all the pressures that teachers find themselves under in the current educational climate, I think that for most of us the second of these – the 'back-seat driver' – is much our greater danger: I don't know many lazy teachers.

Offering these things sooner would 'save time' … but what would the learner learn?

I have been trying to find a middle way between these extremes of doing too little and over-helping that allows me to be supportive and encouraging to learners emotionally but significantly less directive mathematically than I might tend to be instinctively. For instance, I have tried responding to opening cries of "I'm stuck" or "I don't get it" or "I'm confused" with the slightly shocking answer of "Good". It is important that this is said with a smile in the context of a good relationship with the learner, and with body language the very opposite of "Do I care?!"; for example, sitting down beside the pupil and appearing available and supportive without actually taking over the task and 'leading' them through it, which is perhaps what they are expecting. I want to communicate that this is *their* task but that I am here to be with them as they tackle it, if they want me.

Initially when I began to try this there was surprise: "He said '*Good*'!", "What do you mean '*Good*'!?" And I sometimes explain that for me getting confused is great – it means you're probably about to learn something. Getting stuck and getting confused are a normal part of making progress in anything. If you never get stuck you're probably not learning much.

Being over-helpful is often motivated by a wish to help learners' confidence. But it can so easily have exactly the opposite effect, since even when you feel you have helped in a very limited fashion, the learner frequently complains afterwards that they could only do it because “You did it for me,” and I find myself saying, “No, I didn’t really do anything”, but with doubts in my mind that what might have seemed trivial input to me was in fact viewed as vital by the learner. Such incidents reinforce in the learner’s mind their uselessness and the teacher’s cleverness and indispensability. Human beings are good at telling when they are not being trusted, and are sensitive to a problem being taken out of their hands and taken over by someone more responsible, even though they – the learner – may still be holding the pen and writing it down.

So I have been trying the approach used by counsellors, in which they try to help someone make sense of their thinking or the problems they are encountering by reflecting back to them the gist of what they are saying. Obviously if this is done in a rigid and automatic way it becomes obvious and irritating, but done carefully it does not attract attention to itself and can allow the person to slow down their thinking and focus on their thoughts more easily than they might be able to do by themselves. It can be very tempting to ‘lead the witness’, slip in an idea of your own, deliberately misunderstand or interrupt them in a ‘helpful’ way, and no-one is saying that these are always illegitimate in the classroom. But teachers seem to develop ‘hinting and nudging’ habits that get pupils through tasks and enable plenaries to be completed more quickly, but which, I believe, inhibit learners’ progress in mathematical understanding in the long run, and I have been keen to try to overcome my tendencies in those directions.

So you could envisage the following sort of conversation between a learner (L) and her teacher (T):

L “I’m confused!”

T “Good! [smiling and sitting down – looking expectantly] Tell me about your confusion.”

L “It doesn’t make sense!”

T “OK.”

L “It says ‘Katie thinks of a number’, but *what* number?!”

T “What number?”

L “It just says ‘a number’, but how are you supposed to know *what* number?!”

T “That’s not very helpful!”

L “I know – this book is so annoying!”

T “Mmm.”

L “... And she adds 6 and timeses by 2 and she gets 26.”

T “... OK ... So what are you thinking?”

L “It could be *anything*!”

T “Anything?”

L “Yeah – how am I supposed to know?!”

T “OK. Like what could it be?”

L “What?”

T “Well what might Katie’s number have been?”

L “You mean like 10?”

T “So you think it might be 10 that was her number?”

L “No, I’m just saying it could have been *any* number!”

T “OK ... So what can you do?”

L “... So say it’s 10. She thinks of 10 and adds 6 ... [gets calculator, works it out] ... 16 ... Oh, yeah, I knew that! ... I’m not stupid! ... Now what? ... Times by 2 ... [does it on the calculator] ... 32 ... So it’s *32*?”

T “It’s 32?”

L “Oh no, I mean ... She gets *26*.”

T “Right – but you got 32.”

L “Right. So it wasn’t 10. I get it – 10 wasn’t her number ... So that means I’ve got to try all the numbers! [exasperated] This question is so stupid!”

T “Very frustrating!”

L “OK, so I’ll try 11.”

T “OK.” [I felt a strong temptation to intervene here, but I’m very glad I didn’t!]

This progresses, with the student trying 11, getting 34, seeing that that’s too much, deciding to try smaller than 10, then noticing that 11 gave 2 more than 10 did, so realising that she wants 6 less than what 10 gave, so therefore going for 3 less than 10 and trying 7 and getting it, with much satisfaction!

This thinking impressed me very much in view of my perceptions of her ability as a mathematician. Had I led the way through this problem, I probably would not have done it this way, and if I had used ‘trial and improvement, I would not have expected a consideration of the differences between the values produced to inform so precisely the trying of 7 at the end. Probably the only thinking required of her would have been the arithmetic: “So what is 26 divided by 2,” etc. I would have done all the strategic thinking and left the ‘sums’ for her. Ironically, this is exactly the bit that she chose not to engage with – turning to the calculator here, even though in other situations I know her to be capable of calculations of this kind. Somehow, her mind appears too full thinking about the other dimensions of the problem to leave any room for her to do the calculations as well, and I can identify with this myself (I recently found myself doing ‘10 + 10’ on a calculator because I had had to think so hard about each of the 10s!)

It seems to me that the time to intervene is now, after the problem is completed (rather than during) – before the pupil moves on to another problem – to look back at what has happened and what can be learned and whether a reverse operation strategy would have any advantages. Offering these things sooner would ‘save time’ and lead to a quicker, more efficient solution of the problem, but what would the learner learn? That teachers know everything, that she is useless on her own, that questions don’t make sense unless someone else talks you through them, that you can’t do anything unless you ‘know the correct method’.

the time to intervene is now, after the problem is completed

Much has been and is being written regarding pupil-pupil discussion in the mathematics classroom. But pupil-teacher conversations are often assumed to be a solved problem. Teachers are helped to develop good questions for plenaries, but are given relatively little guidance on how to handle one-to-one mathematical conversations with pupils. I for one have a lot to learn regarding my informal discussions with pupils about their mathematics, and at the moment my end of term report is telling me that I need to stop interrupting and do much more careful listening!

pupil-teacher conversations are often assumed to be a solved problem

King Henry VIII School, Coventry.

Outdoor teaching helps learning.

Dawn King collects evidence of the value to reluctant pupils of walking through the New Forest with trundle wheels and long tape measures, and arguing about estimates of heights of trees.

Being passionate about outdoor education, I once quipped that teaching mathematics is my 'cunning disguise' to cover up for my 'playing outside'! I organised my first ski-trip in my NQT year, and became Educational Visits Co-ordinator in my fourth year of teaching. I now work in a centre for girls with emotional, behavioural and social difficulties (EBSD). Opportunities are available to us through schemes such as The Duke of Edinburgh's award. But I think learning outside of the classroom is incredibly undervalued, especially in an environment where every action of a teacher has to be justified and evidenced with numerical data.

> **I felt it was almost common sense to assume that taking mathematics lessons outside of the classroom would impact on students' motivation.**

Here I recount my exploration of the mathematical opportunities to be found in the environment and how they can be used to support students' understanding of concepts. By doing this, I hope to give validity to learning mathematics outside the classroom.

Doubly excluded girls

Historically, Pupil Referral Units (PRU) have been dominated by boys. A search of the government website edubase.gov.uk reveals that they are attended by significantly more boys than girls. For example, there are 8 PRUs in Hampshire attended by 350 students of which 256 are boys compared with 94 girls.

Sometimes, however, pupils get permanently excluded from mainstream education but also from Pupil Referral Units. The school in which this study is based caters for some of these pupils. It has been open for three years as a specialist day school catering for girls who have complex emotional, social and behavioural needs.

At the time of the research, all of the 12 girls attending the school displayed extreme behaviour, e.g. in sexual promiscuity with drug and alcohol abuse. Four had Attention Deficit and Hyperactivity disorder (ADHD) and three were on the Autistic Spectrum. The school aims to remove the barriers to learning through nurturing and fostering an ethos of mutual respect.

But how to practically show achievements in such conditions? Data is qualitative, with the danger of subjective analysis and questionable conclusions. One academic view is that becoming totally immersed in situations leads to insights coming to the researcher almost as a matter of inspiration, therefore they need complementing by systematically looking out for the occurrence of particular ideas or events.[1] So I looked out for little 'eureka' moments in an activity to identify the elements which helped learning, looked at how the environment affected their readiness and ability, and removed barriers to learning, and looked at how practical mathematics can strengthen conceptual understanding, and can give a sense of purpose and relevance.

As a group, the girls with whom I worked, here named A, B and C, appeared to enjoy outdoor activities the most. I had formed a bond with them early on in my time at this school, and felt comfortable with taking them off-site. The activities were loosely based on readily available resources, but I had to collect evidence under a few headings, which forced me to look deeper.[2]

Impact on Motivation

I felt it was common sense to assume that taking mathematics lessons outside of the classroom would impact on students' motivation. But motivation was a much bigger concept than I first realised. For example, such work provoked greater curiosity, which lead on to sustainability. During certain tasks such as the estimating of tree heights on the local common, students seemed able to concentrate on the one task for a longer period of time than normal. Perhaps because they had a break between taking each measurement as they walked to the next tree they chose to measure. Some of the distractions which I thought to avoid in planning turned to advantage. For example, whilst working in the forest, dog walkers and such showed some interest in what we were doing. This provided the students with the opportunity to explain the activities to a 'third party'. This gave them the chance to work on their social skills, an important part of their education, particularly given the nature of their needs. It also enabled them to demonstrate their understanding of the activities that they were participating in. I also found that collecting primary data outside improved students' motivation once back in the classroom; this was evidenced by student C's response to completing her scattergraph - she simply refused to leave the room until she had completed it.

The least confident student watched her peer measure the girth of a tree and this gave her the confidence to have a go herself.

Evidence of 'removing barriers to learning'

When asked what they wanted from their school, one student responded with: 'a place where I can feel relaxed and safe'. It could be argued that mainstream schools can be quite hostile environments to a student with emotional, behavioural and social difficulties. The task of the mainstream teacher is to teach large groups of students whilst maintaining discipline, providing challenging yet accessible lessons whilst giving constructive feedback to each student. This is not the recipe for a relaxed teacher. It is almost like an educational conveyor belt. If the teacher is not relaxed, this feeling cannot be passed on to the students. This could be one explanation for why these girls in particular have not succeeded in a mainstream setting. Compared with my previous experience of mainstream schools, the environment of the school in which this study is set is a much more relaxed one. All staff are addressed by their first names and maximum group sizes are 4. Many students describe the school as their 'safe place'. Whilst taking mathematics lessons outside of the classroom brings with it certain physical hazards, it could be argued that it removes some barriers to learning. For example, whilst walking along and apparently chatting idly, students were much more disposed to ask questions. The environment we were working in coupled with the informality of the day enabled everyone to feel relaxed. The tasks were all accessible to the students since they were presented in a non-threatening way. I found it quite remarkable that students did not appear to show any self-consciousness of the fact they were walking through the New Forest with trundle wheels and long tape measures. This enabled all students to become active participants in the tasks. Due to the different roles that needed to be fulfilled - the activities involving estimating tree heights and measuring their girth; the latter particularly required students to work cooperatively with one another. Student C who was the least confident, was the first to volunteer to write down the measurements whilst Student A took the measurements. Once student C had watched student A take a couple of measurements, she was willing to have a go at measuring herself. Watching her peer model the skill gave her the confidence to have a go herself.

She wouldn't leave the classroom before finishing her scattergraph.

Evidence of giving mathematics relevance

Students were able to make links between the fact that pine-trees grew more quickly than oaks by comparing the numerical data. They were then able to look at this on a graph comparing girth and age of tree and see how this translated i.e. that the line for pine trees was steeper which meant that they grew more quickly. We then discussed how this would impact on things such as the furniture industry and explained why so much furniture was made from pine rather than oak. It is often said that mathematics is 'a guide to the world in which we live' and I consider mathematics to be a tool to help us make sense of the world around us. The outcomes of this activity support these ideas.

Evidence of supporting understanding

The nature of the activities was such that they were a real departure from how the students were used to being taught. This made the experience more memorable for the students. This approach would also help students who are kinaesthetic learners by giving them a physical experience on which to attach the concept. Therefore, in the future, should they be required to work out the mean of a set of data, they could mentally place themselves back in the forest, which may improve their retention. This could subsequently translate into improved exam results.

Equals note: As Dawn King's piece shows, all research by teachers, even on a small scale, can offer insights which are in constant need of addressing by all. We welcome all contributions of this kind, whether originating from action research or as part of courses and certificates, especially if there is an effort at presenting it in non-bureaucratic or technical terms.

1. Martyn Denscome (2005) *Doing Your Research Project*, Open University press (p 271).
2. 'Pace counting' and 'Numeracy activities with trees' section of the Association of Field Studies Officers (NAFSO) publication: *Numeracy through the Environment* .

Save the albatross[1]

Albatrosses have circled the world for millions of years. Early explorers of the Southern Ocean came home with stories of monsters, sea sprites and giant white birds… The giant wandering albatross, covering 10,000 km in a single foraging trip is the bird with the longest wingspan in the world... Grey-headed albatrosses from South-Georgia have been satellite-tracked twice around the globe in their year-long sabbatical between breeding attempts. The fastest round-the-world trip took 46 days.

Now 19 out of 21 species face global extinction. Every year at least 100,000 albatrosses are snared and drowned on the hooks of longline fishing boats. The longline fishing boats kill 300,000 seabirds each year, around one-third of them albatrosses.

Albatrosses are creatures of the southern hemisphere which occasionally wander north of the equator. One lived for 34 years in the Faroes with gannets. They are about 2 ft long, with a wing span up to 8 ft.[2]

Birdlife International's Save the Albatross campaign has been adopted by the Volvo Ocean Race[3]. The Race follows the route of the old clipper ships around the world, over a 33,000 nautical miles route used by albatrosses for millions of years. This was, we suppose, the route described by Coleridge in his poem, "The rime of the ancient mariner". The four verses quoted give us some feel for the bird and for the rough weather likely to be encountered on the voyage.

Photo: Michael Gore

The ice was here, the ice was there,
The ice was all around:
It cracked and growled, and roared and howled
Like noises in a swound!

At length did cross an Albatross
Through the fog it came;
As if it had been a Christian soul
We hailed it in God's name.

It ate the food it ne'er had eat,
And round and round it flew.
The ice did split with a thunder-fit;
And the helmsman steered us through!

And a good south wind sprung up behind;
The albatross did follow,
And every day, for food or play,
Came to the mariner's hollo!

1. Extracts from '*Albatrosses: life at the extreme*', Royal Society for the Protection of Birds, *Birds*, Winter 2005
2. Bruce Campbell, *The Oxford Book of Birds*, London: Oxford University Press, 1964
3. (www.volvooceanrace.org)

Teachers' notes

Extracting the mathematics

A study of the albatross offers some useful explorations of applied geometry.

For example:

1. Consider the earth as a sphere and find the circumference of the circle of latitude 30° S.
2. Study the shape of birds – Do they all have the same proportions – look at the ratio of wingspan to length for example. Does this ratio vary with the size of birds? How about planes – their ratio of wingspan to length? This may for some lead to a discovery that in enlargement corresponding lengths, areas and volumes change at different rates. Some difficult concepts here but using simple apparatus and building solids of the same shape can start an exploration of the changing relationships of length, area and volume as the shape grows bigger. (One of my more academic colleagues in the past used to say that the only everyday use she made of her mathematical knowledge was in understanding that if she doubled the amount of cake mixture she was using she did not need a tin of double the dimensions to bake it in.) (See J. B. S. Haldane, 'On being the right size' reproduced in James R Newman, *The World of Mathematics*, vol 2, London: Allen and Unwin, 1960.)

   The extract on the next page from Gibbons & Blofield, *Life Size: a mathematical approach to biology*, Macmillan: London 1971, will prove useful here.

3. Further study might concern speeds:
   What is the average speed of the albatross on its journey?
   What is the direction and strength of the prevailing wind and how does this help the albatross?

Across the curriculum

There is more than mathematics here of course.

Biology:

Consider why the relationship of the mass of a bird – or any other living creature - to its surface area is important (See Haldane and Gibbons & Blofield again).

Geography:

Bring in wider studies of climate, more on winds, a study of fishing patterns, of fish populations.

History:

Start with a study of the development of fishing or the routes of the great explorers.

And then there's **English** of course with a study based on Coleridge's poem.

As an inspirational extra for any of these suggestions the film *Winged Migration*, Writer, Director and Narrator Jacques Perrin, 2001 is recommended. You need 98 minutes set aside to view it but maybe there could be a whole year viewing – or even a whole school. One review does not exaggerate when it comments

"The cinematography is breathtaking. At times we seem to be riding on the back of a bird or as part of a V formation of honking geese. You really get an intimate appreciation of the muscular power and the endurance of these creatures as they make their annual migrations from one end of the earth to the other."

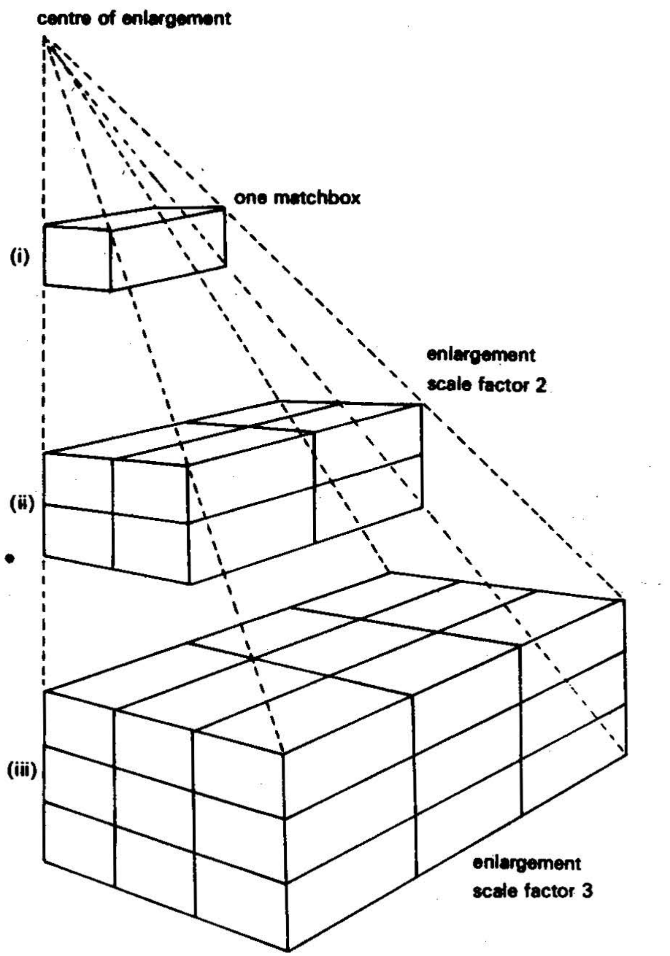
Photo: Michael Gore

ENLARGEMENT

In Figure 7.5 we see two enlargements of a matchbox with different scale factors. How do the heights, surface areas and volumes of (ii) and (iii) compare with those of (i)?

Fig. 7.5

Two enlargements of a matchbox



Let us first consider the surface areas, measuring them as we did those of the arrangements of matchboxes in Figure 7.4.

(i) 2 tops + 2 ends + 2 sides = 1 matchbox surface,
(ii) 8 tops + 8 ends + 8 sides = 4 matchbox surfaces,
(iii) 18 tops + 18 ends + 18 sides = 9 matchbox surfaces.

Considering volumes:

(i) contains 1 matchbox,
(ii) contains 8 matchboxes,
(iii) contains 27 matchboxes.

Tabulating these results

	Height	Area	Volume
(i)	1	1	1
(ii)	2	4	8
(iii)	3	9	27

From these results we see that when something grows, keeping the same shape, its height, area and volume increase at different rates.

GROWTH

Suppose the matchbox solids represent three stages of a growing cell or organism which takes in food and oxygen and gives out waste material through its body surface. In changing from stage (i) to stage (ii) its volume, as we see from the table, gets multiplied by eight. This means that it will probably require eight times as much food and oxygen and produce eight times as much waste at stage (ii). But the surface area is the main region of gaseous exchange, food intake and waste removal and this, at stage (ii) is only four times what it was at stage (i).

Let's look further at 'seeing' and 'doing'.

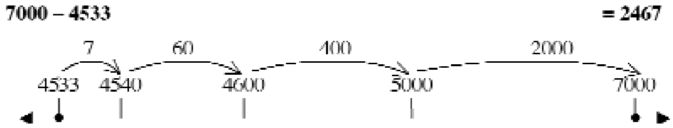
Tandi Clausen-May **expands on a review in *Equals* of her recent work on pupils' learning styles. She offers ideas on how visual images of mathematical tools such as the number line and a novel use of bead strings may relate to personal physical activity.**

Mundher Adhami puts his finger on a key issue in his review of *Teaching Maths to Pupils with Different Learning Styles* when he points out the danger that

> 'the new mathematical models we offer our pupils in the process [of] activity based teaching… may either be ignored or turned into procedures like others, whereas they should be used to allow further refinement and elaboration, by maintaining coherence.' (Adhami, 2006, p 24)

It is certainly true that any mathematical 'model to think with' may be used effectively – or it may be abused by being turned into a meaningless method for getting right answers. A number line, for example, may be reduced to just another routine, to be learnt by rote and followed blindly without any understanding of the meaning of each step. Used like this it will be no more helpful, and it will be considerably less tidy, than a numerical algorithm.

V,A,K – Visual, Auditory, Kinaesthetic – See, Hear, Do – whatever you call it – is just a convenient way to think about the mathematical activity that goes on from day to day in the classroom. Nearly all pupils can benefit from an approach that presents new concepts in different ways, so VAK offers a useful framework with which to classify the *activities* – not the pupils. A number line is just one example out of many.



But Mundher asks,

'Does it really need a number line to think, say, that "4533 add 7 makes it 4540, then 60 makes it 4600 then 400 then 2000, so…"?'

He questions whether a mental image of a number line is actually helpful – and, indeed, for many children, the string of squiggles and instructions may be perfectly meaningful and manageable as they stand. But others may lose the plot at the second digit of the first squiggle. The important numbers to hang on to here are the jumps – the 'add 7', 'then 60', 'then 400', and 'finally 2000'. The 4533, 4540, 4600, 5000 and 7000 are just stopping points on the route. Or if you want to go the other way, of course, you can – 'take 4000', then 'take 500', and so on. This gives different stopping points, but either way, the numbers to keep your eye on are the jumps. The number line gives me a picture in the mind to hang all these numbers onto, so I can keep track of what is going on. My mental image is crude, and not to scale – I have a sense of gathering speed, in fact, so that my first, tentative little hop is just a few units long, but then I am jumping in tens, leaping in hundreds, and finally flying in thousands. But I could not agree more that there is a danger that the mathematical model may be turned into yet another 'procedure' – meaningless, cumbersome, and eminently forgettable!

Interplay between the visual and the kinaesthetic

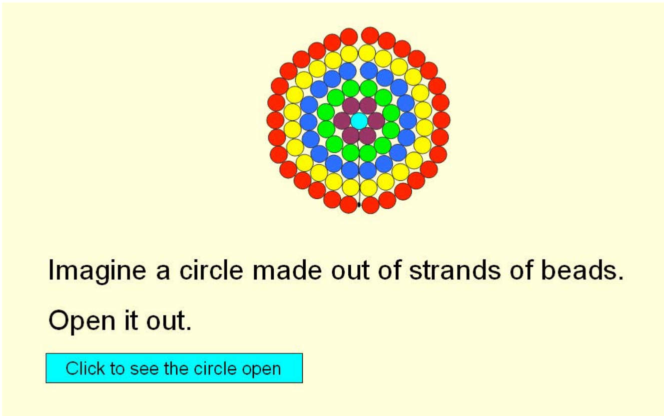
But my image of the number line is just that: my own mental picture, very personal and perhaps individual to me. As Mundher explains,

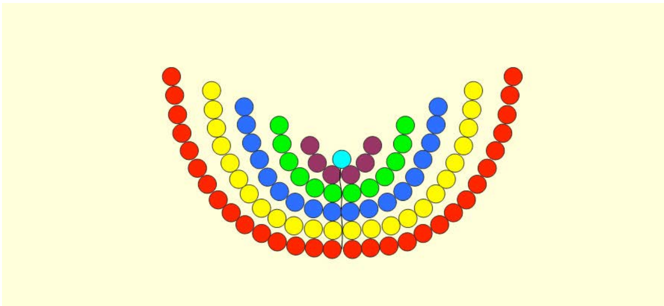
> 'unless what you see makes sense to you in terms of action that you yourself have done, or can easily imagine yourself doing, it is no different from any other phrase or symbol.' (pg 23)

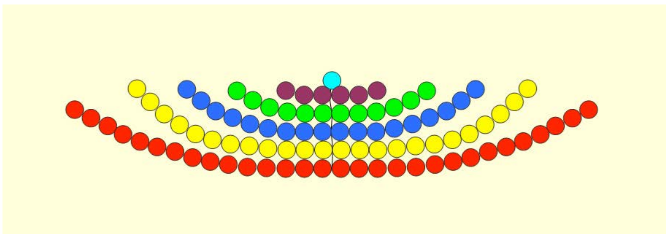
This interface between the visual and the kinaesthetic – the 'seeing' and the 'doing' – is one that we explore every time we ask our pupils to engage in an activity designed to develop their understanding of a mathematical concept. Whether we use the, perhaps fashionable, but long-established and easily grasped 'VAK' terminology, or the language of constructivism to describe what we do, our object is the same. We want to develop the links between the experienced activity and the mental images that offer the models to think with that pupils can use to support their mathematical understanding.

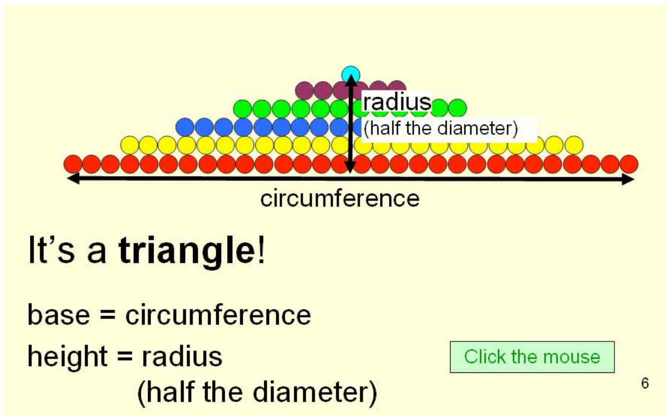
Certainly, direct action by the learner is sometimes essential. Children who have had extensive experience working with the Japanese Soroban, for example, can learn to work mentally, using an imaginary soroban in their heads to carry out complex calculations with astounding speed and accuracy – but their fingers twitch as the physical memory of the kinaesthetic experience with a real soroban supports their visualisation of the movement of the beads on the frame (Markarian et al, 2004, p13).

Similarly, a model for the area of a circle as half the circumference multiplied by the radius that is based on a set of concentric circles of beads does require the learner to have handled a string of beads at some point in their lives. On the other hand, the beads do not have to be present when the model is explained – a dynamic 'movie' can be shown on a computer screen, and can be understood through the visual, rather than a kinaesthetic, experience. (Clausen-May, 2006, pp 42- 44)









What these dynamic visual images offer, though, is not a formula, to be learnt by rote, and then forgotten. It is not a set of symbols: it is a moving picture in the mind, from which an effective approach to the problem of finding the area – first find the circumference, then multiply that by the radius and divide by 2 – may be based. But the pupil does not try to remember a technique: rather, they remember the dynamic model, and then work out the method directly from that. This, I think – and I hope Mundher would agree – is what 'seeing' and 'doing' mathematics is all about!

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NFER

I have a child with autism........

When we heard Jan Brooks talking about her experience of autism in her own child, we recognised the acute observations she had made of his problems and of his needs and the brave and carefully thought out responses she made to those needs. We persuaded her that the wisdom she had gained would be of help to teachers to understand better, and to respond more appropriately to, children with autism and related communication disorders.

Looking back over the last eleven years, being told we had a child with a disability was really hard to accept. We were devastated. At first, we blamed ourselves. I thought it was something I had done. Once we got over the shock, I decided to work my hardest to make him better. I accepted that there is no cure. This is a life-long condition.

As a young child, he appeared normal. He was saying single words like mum and dad. He appeared quite forward in some ways and then between about 12 and 18 months, we noticed a big change. It was like he had hit a brick wall. At first we thought he was deaf, because he didn't look or respond to his name. He simply just stared into space. He would spin around and around, for much of the time, screaming. It was very frightening and we didn't know what to do. We took him to the baby clinic. We explained what was happening and they did some tests and told us that he had 'glue ear' . From this appointment, it was arranged for grommets to be inserted. Nothing changed. The grommets only made him worse, so we took him back to the baby clinic. After more tests, they referred us to the child psychologist. The psychologist came to our home and she stayed for the morning. She watched our son and asked lots of questions.

While she was in our home, our son did 'his own thing'. He ran about constantly, he screamed, spun round and round - ran over to me, pulled my hair and was constantly biting and pinching me for no reason that we could understand. The psychologist told us that our son had a severe speech and behaviour disorder and that she would refer us to Dr Bantock. Dr Bantock diagnosed our son with autism - including severe language and behaviour disorders.

I did not believe them. I knew my son better than any professional.

You cannot imagine how I felt. I was devastated, felt abnormal, felt it was our fault. We were also told that he would never speak or be able to do anything normal. I did not believe them. I knew my son better than any professional. I spent 24 hours each day with him. My son had some speech before this diagnosis, so I decided that he would talk again. We decided that we would try anything, but no drugs.

Never feel alone because your child has special needs.

I started by reading and sharing books with him every day. He would simply stare into space and not take a blind bit of notice of me or the book. He was only doing his own thing - screaming like a wounded animal and acting in what appeared to me, a very strange way. I carried on, night after night I would read to him, showing him pictures. He never made any eye contact - ever - but very slowly, he started to look at the pictures. Every now and then he would look at me for a second. You cannot believe how happy this made me.

We couldn't take him anywhere. Wherever we went he would throw himself on the floor. He would spin himself round and round, eventually he would make himself sick. This happened whenever he didn't want to do anything or go anywhere. We decided that we would continue to try to take him to places but would do it at a slower pace. We would take him to places like the local shops, houses of family, etc., and initially we would take him for a minute at a time. We would do this regularly for a month. Then we would do it for 2 minutes each month until he got more used to it. We did the same thing with buses and trains. At tirst we would only go for one stop and then get off. The following month we would go for two stops etc. After a while he became used to it. Eventually we could take him anywhere, as long as he was told about it, in advance.

All this time, he continued to take out his anger, anxiety and frustration on me. I was the only person with him for 24 hours each day. He would pull my hair out by the roots, bite me and pinch me. He could not tell me what was wrong. In the end, I cut my hair short to try to stop him. Very slowly he began to stop biting, pinching and pulling my hair. This took a long time but I had to be calm and firm. I don't think he understood that it hurt me but gradually he began to understand that it was unacceptable.

After all this, we decided to concentrate on his language - firstly his understanding. I managed to get an appointment to see the Speech and Language Therapist. After a long meeting, it was agreed that he would have three appointments each week and I agreed to work closely with the therapist. We started with small puzzles and pictures of simple familiar things like ball, dog, cat, mum, dad etc. At first, he didn't want to do any of the tasks, but after a while, he started to enjoy it, especially when we both made a big fuss of him when he did well. Very slowly, some words began to come. He started looking at the books and pointing with his finger at the pictures. Occasionally he would say a single word linked to the picture. I was so happy that I cried tears of joy - especially when he looked at me and made eye contact for a second. The eye contact was starting.

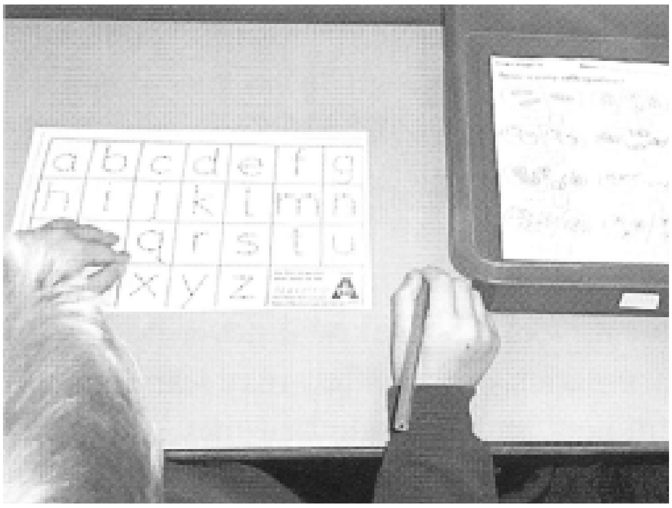
Just because they have the same label doesn't mean that they can all be approached in identical ways.

We decided that we would never give up trying to help him communicate and to make improvements. Over the years we put in an enormous amount of effort and work with him. He is now attending an MLD Secondary school. He is doing well and never fails to amaze me with the information he has learned at home and at school. My motto is 'never give up' or give in when your child has special needs. If you are prepared to put in the effort, be demanding and expect them to learn and behave like others, you will get many rewards - progress is very slow but it does come.

We should not judge them because they do not always understand our world, why things change, our feelings, our pain, our happiness, our ideas, our hopes.

I have a son with autism. I am very proud of him. Never feel alone because your child has special needs - there are thousands of us. I now work in a school for children with autism. I understand the fears, worries and despair of the parents. I have noticed that if you sit back, give the children time and observe them carefully, you will have a better understanding of what works for them. They are all different, unique. Just because they have the same label doesn't mean that they can all be approached in identical ways. Each child is an individual and what works with one child, may not work for another. They all understand and see things in different ways. We do not need to control them, they need to develop inner controls and ways of expressing themselves.

In our school we use many methods - the children work independently at TEACCH (Treatment and Education of Autistic and related Communication handicapped Children) workstations, they work in groups, they learn to wait and take turns, we use Makaton signing and symbols and the Picture Exchange Communication System - all these help the children to communicate with adults and other children. We work together as a team - we have the same routines, boundaries and expectations throughout the school. We use very small amounts of language to help aviod confusion. This helps the children to relax - they begin to understand what is expected of them and they become less anxious. Every class has a teacher, a nursery nurse and a support assistant and sometimes additional 1:1 support if it is needed. We try never to talk across or about the children - they are not deaf and often understand far more than they let on. Their problem is sharing skills, knowledge talents and interests.



Some work at Queensmill School

We work very closely with parents - and the parents work with us to keep the information going. Nobody knows or understands a child like a mother. They try to let us know what is happening at home, and we let them know how the children are in school. What they have learned, read, said, indicated, eaten, etc. In our school parents have regular coffee mornings where they can share ideas, tips, etc. They can also learn how to support their child at home and at school. They are always welcome and have time with staff and the speech and language therapists.

Our children do not like change but the have to learn to cope with it. They also learn better if they have consistency and this is why it is very important for home and school to work together and to trust and respect each other. Parents need to feel that they can trust the staff - their children cannot tell their parents of fears, etc. Their children are very vulnerable. We all try to help prepare the children for what they will need when they leave school - they need to have dignity, to use a toilet, to be able to communicate their wants and needs, to wait, to make choices, to take turns and to have self-help skills. They also need to be able to read, write and have fun like any other children. They need to mix with children who do not have special needs.

We need to be aware of their (and their family's) isolation. Our school shares a building with a mainstream school - children from both schools have inclusion opportunities and this raises understanding and respect of disabilities, and gives good learning opportunities for all the children. It also helps children who are 'normal' to have chances to become more confident, more helpful with others. We all need to learn about and respect the needs of children and adults with autism. They all look normal, (many are very handsome) they all have problems socially and we should not judge them because they do not always understand our world, why things change, our feelings, our pain, our happiness, our ideas, our hopes.

These children are not naughty. They are children who have rights to education, respect and dignity, understanding, fun and love. They are children who have many difficulties understanding the world. We do not know what goes on inside their heads. We do not know why they do some things we think are strange. They do things we do not understand. please think before you judge them or their parents.

Queensmill School