Equals

Realising potential in mathematics for all

for ages 3 to 18+ ISSN 1465-1254

Vol.19 No.2

Ten £10 notes, or £100, are about 1 mm thick. But what about £1 million in £10 notes?

MATHEMATICAL ASSOCIATION
supporting mathematics in education

Realising potential in mathematics for all

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Published by the Mathematical Association, 259 London Road, Leicester LE2 3BE
Tel: 0116 221 0013
Fax: 0116 212 2835
(All publishing and subscription enquiries to be addressed here.)

Designed by Nicole Lane

Editors' Page

We must apologise for the lateness of this issue due to various difficulties but hope it is not too late to give you some valuable advice in these troubled times for education in this country. Today, when politicians are attempting to increase their interference in the classroom and pushing formal testing at more stages of pupils' school lives, the lower attainers in our classrooms become more and more discouraged and disadvantaged. The reason why *Equals* launched the Harry Hewitt Memorial Prize was because there are so many incentives for the high fliers in mathematics in the form of competitions - Olympiads, etc - but nothing of the sort for those whose achievements are not so highly rated. It has always proved difficult to get entries for Harry Hewitt's prize, presumably because teachers cannot believe their pupils with lower attainment levels could possibly be worthy of any distinction, but the excitement and delight of the winners has always been a joy to see.

All young people need to know they are valued. And they need to know that value has nothing to do with possessions: it is not enhanced by owning a t-shirt made by "X" or a pair of trainers made by "Y" (although the advertisers tell them differently). The basics they need to learn in the classroom, coming well before English and mathematics, are how to live in a community peaceably and productively, taking responsibility for their own work and respecting the work-space of others and the resources they are sharing with them. Are they learning these values today? From an incident I heard of yesterday it seems to me that the answer to this question is too often no. A bicycle was left locked up in a courtyard in a street with many shops in the centre of Brighton. Its front wheel, which just protruded through a fence, was slashed. What was the point?

Although wrong, it makes some sense to steal a bicycle because you can then make use of it. But slashing the tyre of a bike belonging to a stranger seems thoroughly mindless vandalism. How are you tackling such problems in your classroom? Certainly it is not easy to deal with them if the teaching approach is basically 'lockstep' – marching the class along together through the exercises in a textbook. Some personalisation of work programmes takes away the sort of competitiveness which can shame the strugglers. And it is only sensitive and understanding teachers who are able to guide appropriately the pupils in a class. Much is being written in these technological days about young people not needing human intervention between them and their phones, i-pads and other technological devices to get themselves educated. How can anyone believe such theories when they look at what is happening around them today - in Egypt or Syria for example? The basic lesson of how to live peaceably and productively with others, taking responsibility for one's own work and using shared resource with care is one that must never be omitted from any time table.

Mathematics in unusual places 6 – Yugoslavian currency

Several years ago, on a trip to Belgrade, **Matthew Reames** became a Yugoslavian multi-billionaire.

A simple exchange with a stranger on the street left me holding an envelope containing 6,505,705,000 dinars. As I'm sure you can imagine, I was quite pleased with my newfound status. But lest you think I was engaged in something less than ethical, let me say that by this time Yugoslavia no longer existed, that currency was no longer in use, and the stranger was a vendor selling souvenirs. But more on this later. First, let's look at large numbers.

What is a big number?

What is a big number? Is 10 a big number? Is 1,000,000? Perhaps it depends on the situation. For example, £10 is a rather large amount to spend on a single pencil, but £10 is perhaps not a lot to spend on buying a new book. Fitting one million people somewhere requires quite a bit of space, but a single drop of water can contain more than a million bacteria.

We often hear large numbers mentioned on television and see reports of large numbers in the newspapers. Footballers' salaries and budget deficits spring immediately to mind. The words *million* and *billion* are often used in the same conversation, leading people to think them nearly equivalent, when in fact, a billion is 1,000 times a million. To compare a million and a billion, consider this: someone with £1 billion can buy 1,000 things that cost £1 million, or if you have a population of 1 billion people, you can group them into 1,000 groups of 1 million.

Just how big is a million? A million is one thousand thousand, or 1,000,000 or $1\text{x}10^6$. A kilometre is 1 million millimetres. A million seconds is just over 11 and a half days.

But how big is a billion? Before 1974, schoolchildren in the UK learned that 1 billion was 1 million millions, or 1 followed by 12 zeroes: 1,000,000,000,000 or $1\text{x}10^{12}$. This is referred to as the *long scale*. In 1974, however, the UK officially switched to the *short scale* and 1 billion is now officially 1 thousand millions, 1,000,000,000 or $1\text{x}10^9$. The UK, the United States and most other English-speaking countries use the short scale, while France, Germany, Italy, and a number of other nations use the long scale. Canada uses both scales – the short scale in English and the long scale in French. A number of countries use neither the long scale or the short scale. An example of an alternate numbering system is the South Asian numbering system in which $1\text{x}10^6$ is called 10 lakh and is written 10,00,000 (note how the commas are positioned here instead of commas every three zeroes). One followed by 9 zeroes ($1\text{x}10^9$) is a hundred crone and is written 1,00,00,00,000. So how much a billion is depends on where you are and to whom you are speaking, but for the remainder of this article, one billion will be understood to be equal to 1,000,000,000.

Place value

How do children learn place value? Sometimes teachers use bundles of sticks or other manipulatives to represent this physically. We often create charts showing ones, tens, hundreds, and so on (and later, extend the chart to the right of the decimal point for tenths, hundredths, and so on). Children generally develop a good sense of ones, tens, hundreds and thousands, and they are able to visualize in their minds how large those numbers are. Base-10 blocks are particularly useful for this: 1 is a small cube, 10 is a rod the length of 10 ones, 100 is a flat square the size of 10 rods, and 1000 is a large cube the size of 10 flats.

Children's understanding of larger values and their ability to visualize those numbers is not as clear as with hundreds and thousands.

Consider a £10 note. It has a thickness of approximately 0.1 millimetre. Ten £10 notes, or £100, are about 1 mm thick. But what about £1 million in £10 notes?

£1 million divided by £10 is 100,000 so you would need 100,000 £10 notes for £1 million.

How thick would 100,000 £10 notes be?

100,000 x 0.1mm = 10,000 mm, that is 10 m

So a stack of £10 notes worth £1 million is 10,000 mm or 10 metres tall! For £1 billion worth of £10 notes, you need 1,000 of those 10 metre stacks. Or, if combined into a single stack, it would stretch 10,000 metres or 10 kilometres high.

Using visualisations like these can help children gain a better understanding of just how large these large numbers are.

Other questions could be:

- How much area is covered by £1 million worth of £10 notes? £50 notes?
- If £1 billion worth of £50 notes was laid end-to-end, how far would they stretch?
- If the school is the starting point for that line of £50 notes, where is the end point?
- In the 2013-2014 UK budget, the Government is projected to spend £97 billion on education. If they paid that amount in notes, how much space would it take?

Or consider population: the population of the UK is just over 62 million people.

- If everyone in the UK gave you £1, what could you buy?
- In the 2013 budget, the UK is projected to spend £31 billion on personal social services. How much is this per person?

How thick would 100,000 £10 notes be?

As the largest UK bank note is the £50 note, you may wish to extend your problems to use Euros (which come in 5, 10, 20, 50, 100, 200, and 500 euro denominations). You might also use photos of Yugoslavian banknotes (see resources). If you had a 500 billion dinar note and wanted to exchange it for smaller denomination notes, how many bundles of 5 million dinar notes would you have if there are 100 notes in a bundle?

A closer look at Yugoslavian currency

At the beginning of 1990, Yugoslavia consisted of six republics: Bosnia and Herzegovina, Croatia, Macedonia, Montenegro, Serbia, and Slovenia. In the early 1990s, the government was taking in far less money that it was spending and it tried to stabilize its economy by printing more money. This fact, combined with shortages of goods, led to tremendous rates of inflation which, in turn, caused the government to print more and more currency. In December, 1993, the *daily* inflation rate was nearly 100%. (Remember that the inflation rates we usually hear are annual rates.) This means that the price of an item purchased on one day was double the price the item sold for the day before. People would put off paying bills for as long as possible because inflation reduced the real value of those bills to nearly nothing. Imagine a daily inflation rate of 100%. If you received a bill for 500 million dinars on 1 December, how many days would pass before that bill was worth only 1 million dinars?

What is a big number? Is 10 a big number? Is 1,000,000? Perhaps it depends on the situation.

In 1988, the largest denomination of currency was the 50,000 dinar note. A year later, the largest note was 2,000,000 dinars. To try to cope the government printed new notes in higher denominations. Then they issued a new set of currency in which 1 new dinar could be exchanged for 10,000 old dinars. They kept printing larger and larger denomination notes. Then they issued a new set of currency. They did this several more times. By the end of January, 1994, 1 novi dinar was worth approximately 1.3×10^{27} 1990 dinars. (That's 13 followed by 26 zeroes). 1 dinar certainly couldn't buy what it used to.

Percentage increase and decrease

Calculating percentage increase and percentage decrease is often part of the mathematics curriculum, and seems to be one area in which pupils struggle. Consider using actual data for pupils to determine the percentage increase in the price to consumers. Pupils could collect their own data throughout the year or they could use historic data. The Food Timeline website (see resources below) has historic food prices for a number of countries, including the UK. There is even a spreadsheet of English prices and wages from 1209 to 1914. The Historic Inflation Calculator (see resources) can be used to determine the inflation rate and price changes for periods in the last 25 years.

Other examples of large numbers in currency

Until recently, visitors to Istanbul might have been shocked by taxi meters showing the fare as several million lira when the largest notes in their pockets were 100 lira. In 2005, the Turkish government revalued the Turkish lira and dropped six zeroes from the denomination. It took several years, however, before taxi drivers replaced their meters.

The highest denomination bank note ever printed is the 100 trillion Zimbabwe dollar note (that is 100 followed by 12 zeroes). Hyperinflation in the period between 2006 and 2009 meant that at one point, a 100 trillion note would not have even bought a bus ticket in Zimbabwe. When the economic system stabilized in 2009, the government redenominated the currency by issuing new currency and dropping 12 zeroes from each denomination.

While my Yugoslavian banknotes are now an interesting souvenir of my trip to Belgrade, they are a perfect reminder of the mathematics found in the most unusual of places.

Matthew Reames is an academic co-ordinator, University of Virginia

Further resources:

The book *How Much is a Million?* by David M. Schwartz and illustrated by Steven Kellogg is a wonderful story to help children visualize millions, billions, and trillions.

How much is a million? Mathematics activity with grains of rice - http://illuminations.nctm.org/LessonDetail.aspx?id=L743

How big is a billion? - Mathematician James Grime has produced a great video describing a billion in the two scales of numbers. It can be found on his YouTube channel, Numberphile. It can also be found at http://tinyurl.com/cag6nko.

Yugoslavian banknotes - http://en.wikipedia.org/wiki/Banknotes_of_the_Yugoslav_dinar

Hyperinflation in Yugoslavia - http://www.rogershermansociety.org/yugoslavia.htm

Famous examples of inflation - http://www.telegraph.co.uk/finance/economics/8388971/Famous-examples-of-inflation.html

What a Pound Could Have Bought You - http://uk.finance.yahoo.com/news/what-a-pound-could-have-bought-you-%E2%80%93-and-what-it-still-can.html

2013 UK Budget - http://cdn.hm-treasury.gov.uk/budget2013_complete.pdf

Historic food prices - http://www.foodtimeline.org/foodfaq5.html

Historic Inflation Calculator - http://www.thisismoney.co.uk/money/bills/article-1633409/Historic-inflation-calculator-value-money-changed-1900.htmlis

Tales from the ARC – part 3

Sometimes in teaching, writes our special correspondent Ken, when you sit back and reflect, a tremendous sense of well being, accompanied by the thought ‘It could not get any better than that’, washes over you.

To me the feeling is the ‘hit’ that provides our profession with the internal compass to demonstrate that we are meeting the needs of our learners. It has not happened so often at the Academy but my best lesson of the year occurred just after the Easter break. It may have been the sleep and renewed enthusiasm but I am not so sure – circumstances prevailed to ensure a brilliant time in the ARC.

Mrs Jones, the TA, had been asking me for about a month to 'do something' with money as she felt her charges needed some life lessons in personal finance and I left before the holiday promising to sort something out. I had an activity, called 'Money-go-round', one that I first tried with a Year 1 class in mind as I felt it would give the pupils in the ARC some concrete experience of money that Mrs Jones could then develop during their 'standard' maths lessons. I was happy as this was the original aim of my time in the ARC – to develop a series of activities that the pupils could access which would then form the basis of a week's worth of teaching by Kathy and her team.

I resourced the lesson and that Thursday I was ready to go but sometimes it is the unforeseen circumstances that conspire to turn a good lesson into an experience that is truly memorable. That day I happened to wear a tie that had been a present from New York. All the Academy students loved it but the ARC pupils, always to be counted upon to be inquisitive and tactile, were all over it from the moment I walked into the room. In the end I had to take it off so they could all admire it. The lesson was due to begin with a time of role-play where the pupils act out buying an item from a shop and paying for it with money from a purse. As John had the tie around his neck I persuaded him to act as the customer who would like to buy it from Georgia and as with all things I have tried in the ARC they responded superbly. This then led to a discussion of where the shopkeeper puts their money at the end of the day and where shoppers get their money from. It was an ideal scenario in which to establish the three key roles necessary for the lesson: shopper, shopkeeper and banker, indeed by now they were keen to get their hands on the money they saw I had!

Money-go-round is a lesson in simplicity, each pupil takes one of the three roles and counts 15 pennies into their purse. At the role of a dice they each pass money from one character to the next: the shopper buys something from the shopkeeper, the shopkeeper takes the money before putting some money in the bank and the banker then gives some money to the shopper so they can keep on purchasing. The beauty here is that after 4 rolls of the dice each player has spent money (subtraction) and received money (addition). At no point are the players allowed to see what is in their purse, they simply take out the number on the dice or take what they are given. At an agreed signal the pupils then tell you how much money is in their purse without counting. At first their answers are all over the place as they have not used any strategy to keep track of how much money is coming and going. Georgia thought she had 30p when in fact she only had 10p!

more and more of them were giving accurate answers when asked how much money was in their purse

What did emerge after their initial attempts to play the game was the idea of conservation i.e. the money at the end had to be the same as the money at the start as it has just been passed around the table. It struck me that this is key developmental step in the understanding of personal finance yet it is one that is missing for many people. The rise of plastic means that money is not concrete anymore – we go to the shops and pay with our cards and leave with more money (because of cash back) that we went in with. It seems to me that a better understanding of conservation (thank you Piaget!) could have gone some way to preventing the global financial crash.

As they started to play the game for the second time, now with 15p instead of 10p, the pupils asked if they could write down what was happening. This proved to be a decisive step for two main reasons:

1. for the first time several of them began to link a mathematical operation with an action i.e. 'If I give him two do I write down -2?' 'Is this an add?' upon receiving 4 p from the shopper.
2. they began to link the giving and receiving of pennies with the fluctuating amount within their own purse.

The more they played the more careful their sums became and it was encouraging to see that each time the game stopped, more and more of them were giving accurate answers when asked how much money was in their purse. After the fourth go they now wanted to talk about what they did with their own pocket money. They asked me what I did with my children and the fact that to receive money they were often given jobs to do at home. Jacob shared that he earned £30 a week which the others thought was a great deal to have, accompanied with a great deal of moaning regarding who they could borrow money from when their original allocation had been spent. With a wise nod of the head Jacob brought the lesson to a very effective close with the words 'When my £30 is gone, its gone, no more money till payday!' Out of the mouth of babes and …….

Ken works in an academy in the north of England

Review - Mary B. J. Clark

Thinking for ourselves
Jill Mansergh and Margaret Jones
Association of Teachers of Mathematics, 2007
ISBN 978-1-898611-46-2

This book provides a variety of contexts in which children are encouraged to think for themselves. The content of the activities is intended for pupils working within levels 1 to 5 of the National Curriculum. It is organised into three sections, each with its own very distinct character:

Section 1 - Questions to promote mathematical thinking skills
relates closely to an earlier Association of Teachers of Mathematics (ATM) publication *Thinkers – a collection of activities to promote mathematical thinking*. This publication was mainly aimed at an older / more able group of pupils than the present publication. This first section encourages pupils to engage in thinking about mathematical statements in a variety of situations, to respond with their own thoughts and ideas and provide reasons for their responses. The authors describe the statements as quite often being ambiguous or giving rise to a number of different answers. Pupils' responses can be used to provide assessment information about the understanding of particular concepts, as well as showing how powerfully children can think about mathematics.

Activities are sorted under headings of ‘Number’, ‘Shape and Space’ and ‘Handling data’ under each of the following focuses:

- Give me an example of … and another … and another …
- Hard and easy
- The same and different
- Odd one out
- Additional conditions. Give me an answer … then ...
- Always, sometimes, never true
- Sorting
- Equivalent statements
- Burying the bone
- Lists
- Ordering
- Find the correct solution
- Agree or disagree
- Tell me more.

Section 2 – Activities to promote mathematical thinking skills

is a collection of longer activities intended to last for at least a lesson. Guidance for teachers emphasises the importance of allowing children to make use of the opportunities to think for themselves. The activities are described as promoting all aspects of Using and Applying Mathematics. The activities are organised by broad mathematical curriculum areas under the headings: Number, Shape and Space, Measures and Data and Probability.

Section 3 – Solving word problems – children asking questions

is a collection of pictorial resources. These can be downloaded from the ATM website for use on interactive whiteboards. The intention is for children to ask their own questions using a given question as a stimulus. Some examples of open-ended questions and more challenging questions are given for the teacher to model with the children. The purpose of this section is to provide opportunities for children to ask their own questions.

While it is clearly stated that the authors’ intention that children are supported through the activities to be able to pose their own questions, they are likely to need a structure to support them in solving questions posed. A problem solving sheet to take them through the process is included.

Resource sheets for duplication for sets of cards and a framework for a problem-solving process are included at the end of the publication. Purchasers also have access to the pictorial resources in Section 3 on the ATM website to enable use with an interactive whiteboard.

As a source of materials to help busy teachers in incorporating more opportunities to encourage children to engage in mathematical thinking, this publication is very useful. It inspires inclusion of this vital aspect of mathematics in lesson planning often driven by the perceived need to ‘cover’ particular mathematical topics. The approach supported by the publication suggests variety in the teaching of mathematics topics, encouraging diverse opportunities for introduction, consolidation and practice of mathematics content.

Through the activities children are helped to make connections within mathematics thus strengthening their understanding of particular content areas whilst developing their mathematical thinking and practising aspects of *Using and Applying Mathematics*. The activities do not require complicated resourcing, just regular mathematics resources that are available in most classrooms.

A couple of examples follow in the frames below to illustrate the richness of mathematical activity that is offered through the use of mathematical statements in the first section of the publication. In each case one statement has been chosen to exemplify the kind of mathematical thinking that the approach described in this publication can promote. Clearly both these examples give children the chance to draw on a range of mathematical knowledge and make use of it in order to rise to the challenge posed by the statement. There are rich opportunities for mathematical communication and thinking in the context, in the first case, of 2D shapes, and in the second, of the operation of division and associated understanding of number.

Agree or Disagree

Children are asked to agree or disagree with a statement and give a convincing argument for their position.

- This shape is a circle because it has no straight sides.

Lists

List or draw a set of objects that satisfies the condition stated. Here children are asked to exemplify a generalisation. It gives them an opportunity for a wide variety of responses at different levels. Encourage children to come up with unique solutions.

- Numbers that leave a remainder of 1 when divided by 5

The second section of the publication presents activities in a more familiar format. They could loosely be described as investigative activities and should usefully add to most teachers' collection of such activities. Included with each are some additional hints to help in getting the most out of the activities.

The third section contains stimulus materials to help improve on a particular challenge that the authors describe. That challenge is to help children frame questions themselves so that their understanding of what is required in answering a question is better developed. The resources offered here provide teachers and children with more structured support for developing questions than the all too common extension activity of the invitation to 'make up some questions of your own for a friend to solve'.

In a nutshell, this is an excellent publication with many stimulating ideas that, while being straightforward to set up in the classroom, should promote rich opportunities to develop mathematical thinking and understanding of a broad range of mathematics content.

Mary B. J. Clark is a freelance advisor

90% of the UK's threatened wildlife lives in the UK Overseas Territories.

RSPB poster, 2013

World's biggest flag
It took 20 people several hours to roll out the flag which measured about 349 metres by 227 metres (1,145 feet by 244.5 feet) about three times the size of a foot ball field

The Guardian 28.05.13

Great teachers 3: Edmond Holmes
In his own words

The obituary of Edmond Holmes, the chief inspector of elementary schools – 'educator, poet and humanist' - in 1936 presented him as 'the greatest of all the "Chiefs" as well as the most famous'.

TheTimes (Oct 16 1936) described him as Educator, Poet and Humanist. In *Equals 19.1* we gave some of his comments on the head of a primary school whom he referred to as Egeria, and whose practice he greatly admired. The second half of his book, *What Is and What Might Be*, is devoted to descriptions of her school.

It seems particularly appropriate to consider Edmond Holmes now when it seems once again teachers' freedom is being constrained for he is first remembered for his successful work in freeing teaching methods from the "strait jacket" in which they had for so long been confined. In what was known as the 'Holmes Circular' elementary school teachers were described as "uncultured as a rule and imperfectly educated".

elementary school teachers were described as "uncultured as a rule and imperfectly educated".

"Such was the old regime. Its defects were so grave and so vital that, now that it has become discredited (in theory if not in practice), we can but wonder how it endured for so long, As an ingenious instrument for arresting the mental growth of the child and deadening all his higher faculties, it has never had, and I hope will never have, a rival. …."

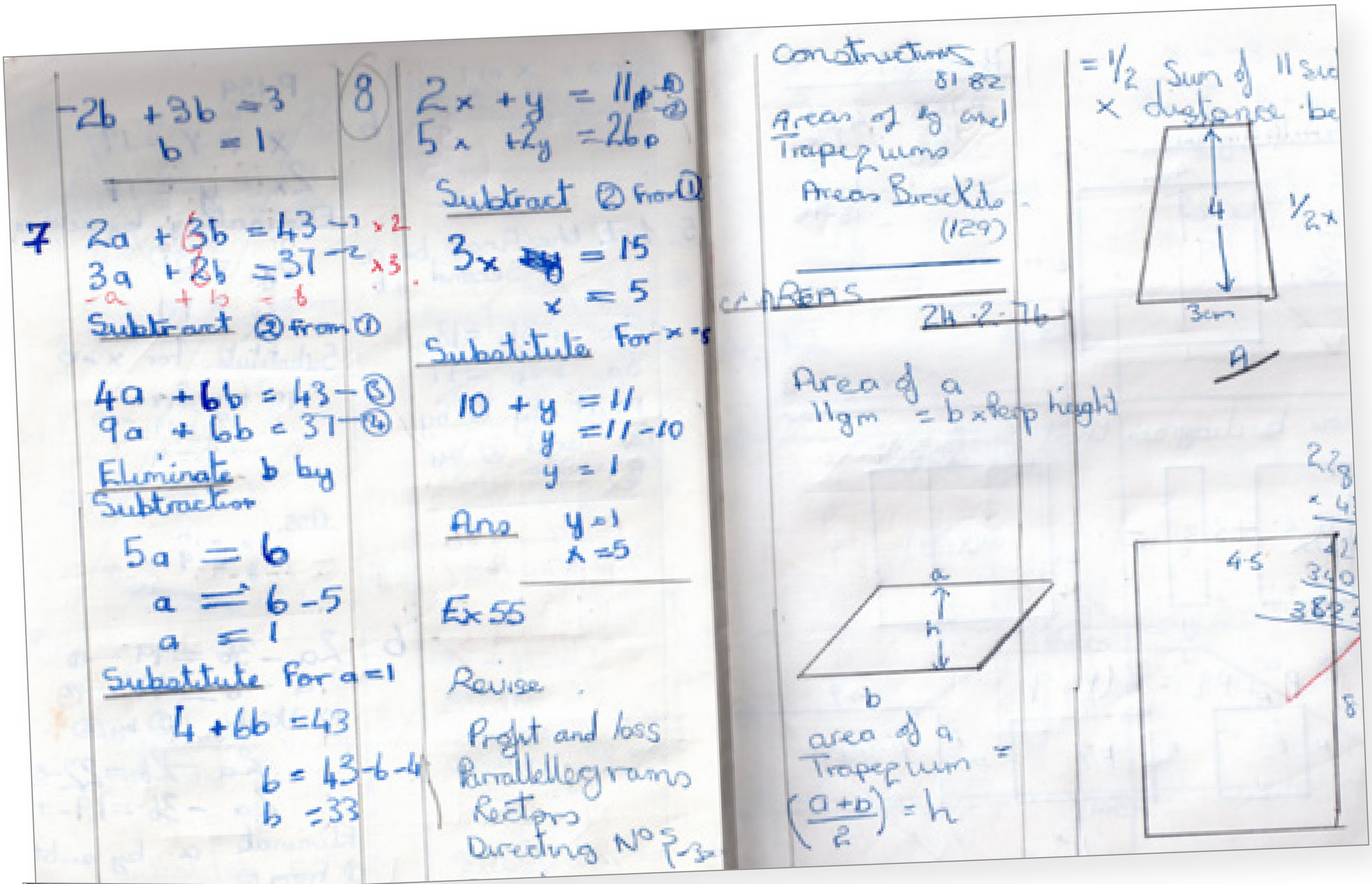
Having for thirty three years deprived the teachers of almost every vestige of freedom, the Department suddenly reversed its policy and gave them in generous measure the boon which it has so long withheld. Whether it was wise to give so much at such short notice may be doubted. What is beyond dispute was that it was unwise to expect so great and so unexpected a gift to be used at once to full advantage. …

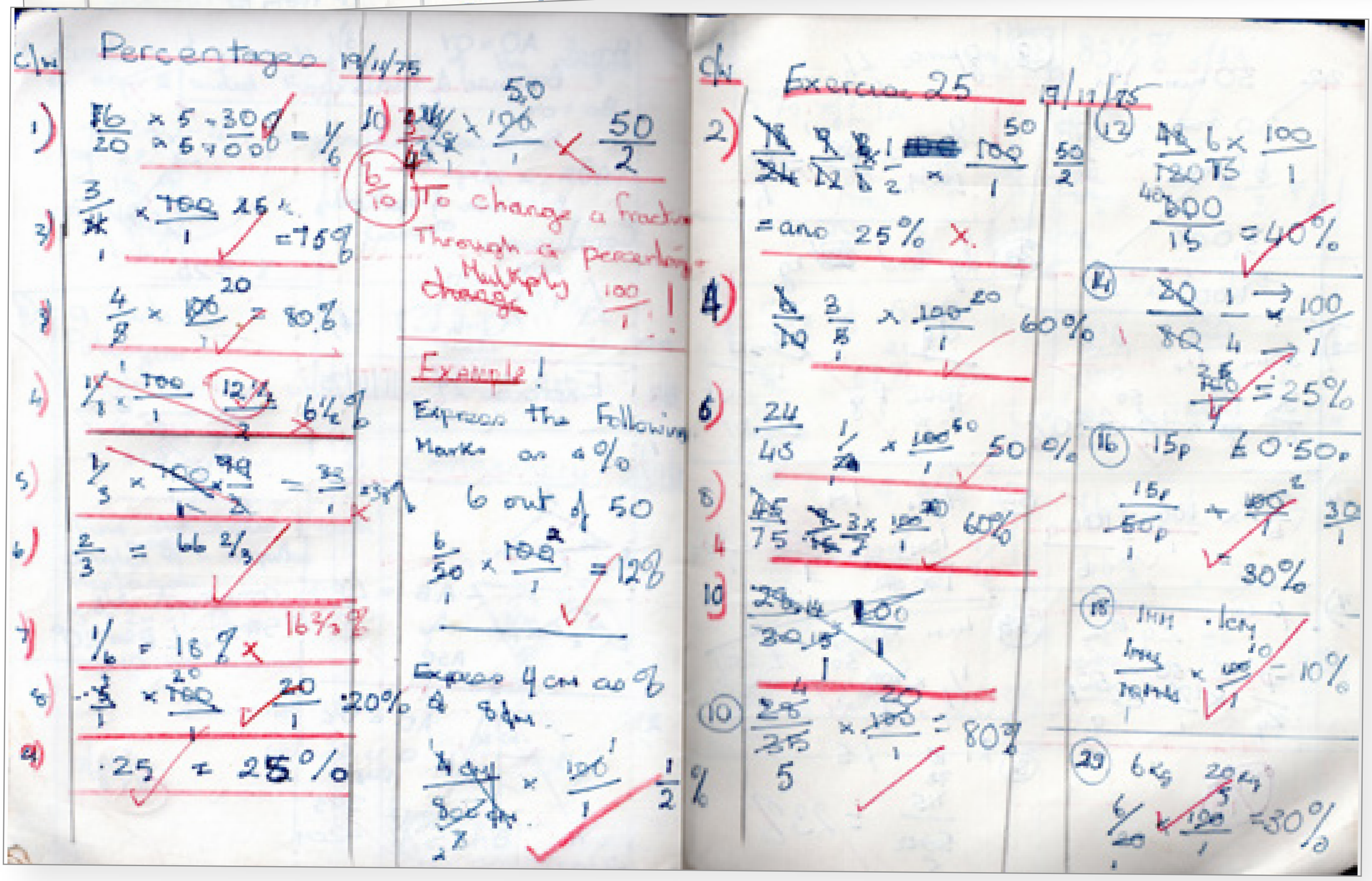
For a third of a century - from 1862 to 1895 – self-expression on the part of the child may be said to have been formally prohibited by all who were responsible for the elementary education of the children of England, and also to have been prohibited *de facto* by all the old unformulated conditions under which the elementary school was conducted. Isn 1895 the formal prohibition of self-expression ceased, but the *de facto* prohibition of it in the ordinary school is scarcely less effective to-day than it was in the darkest days of the old regime.

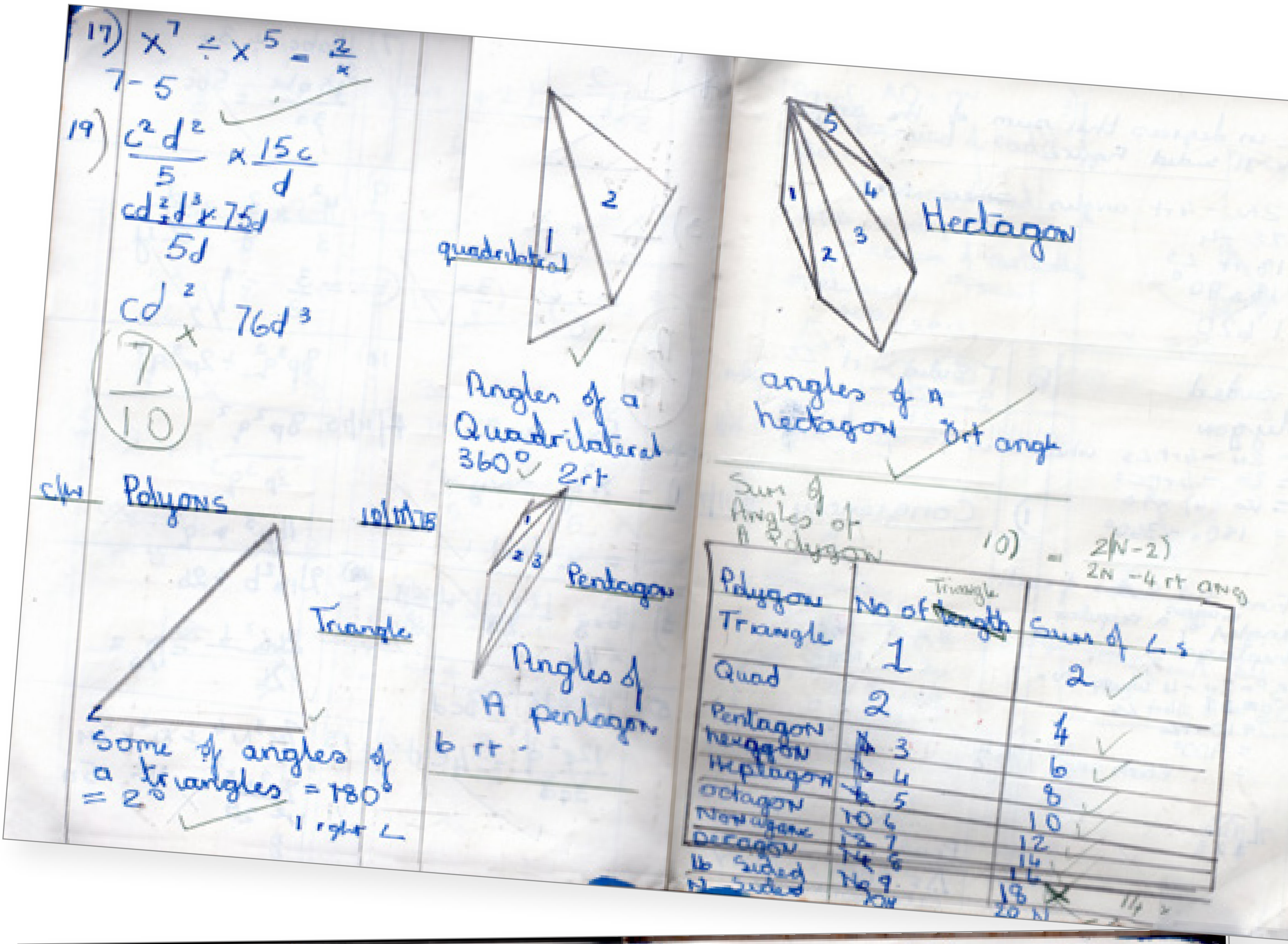
The average UK family throws away approximately £50 a month in food waste

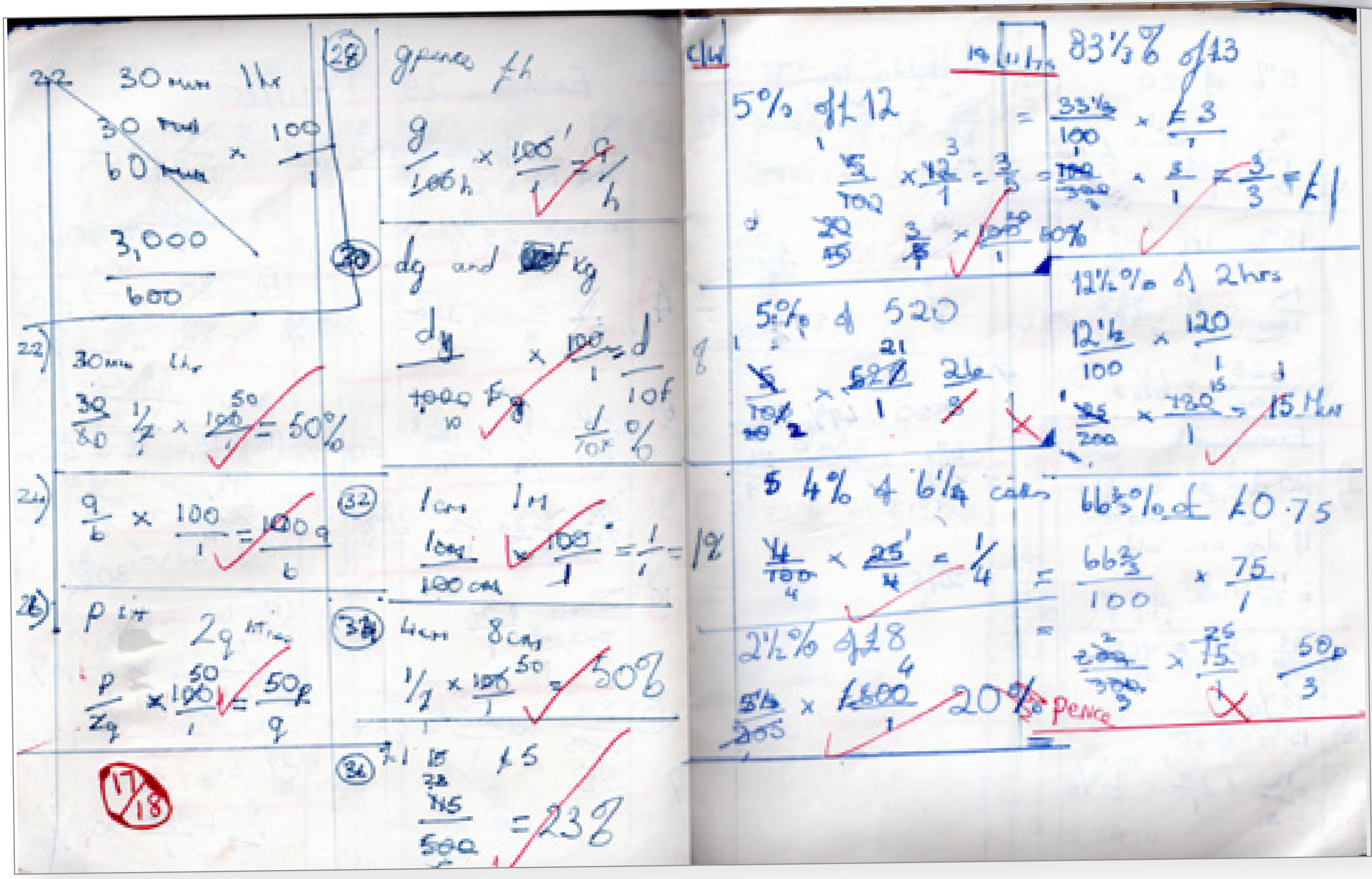
Guardian: Live Green Live Better June 13

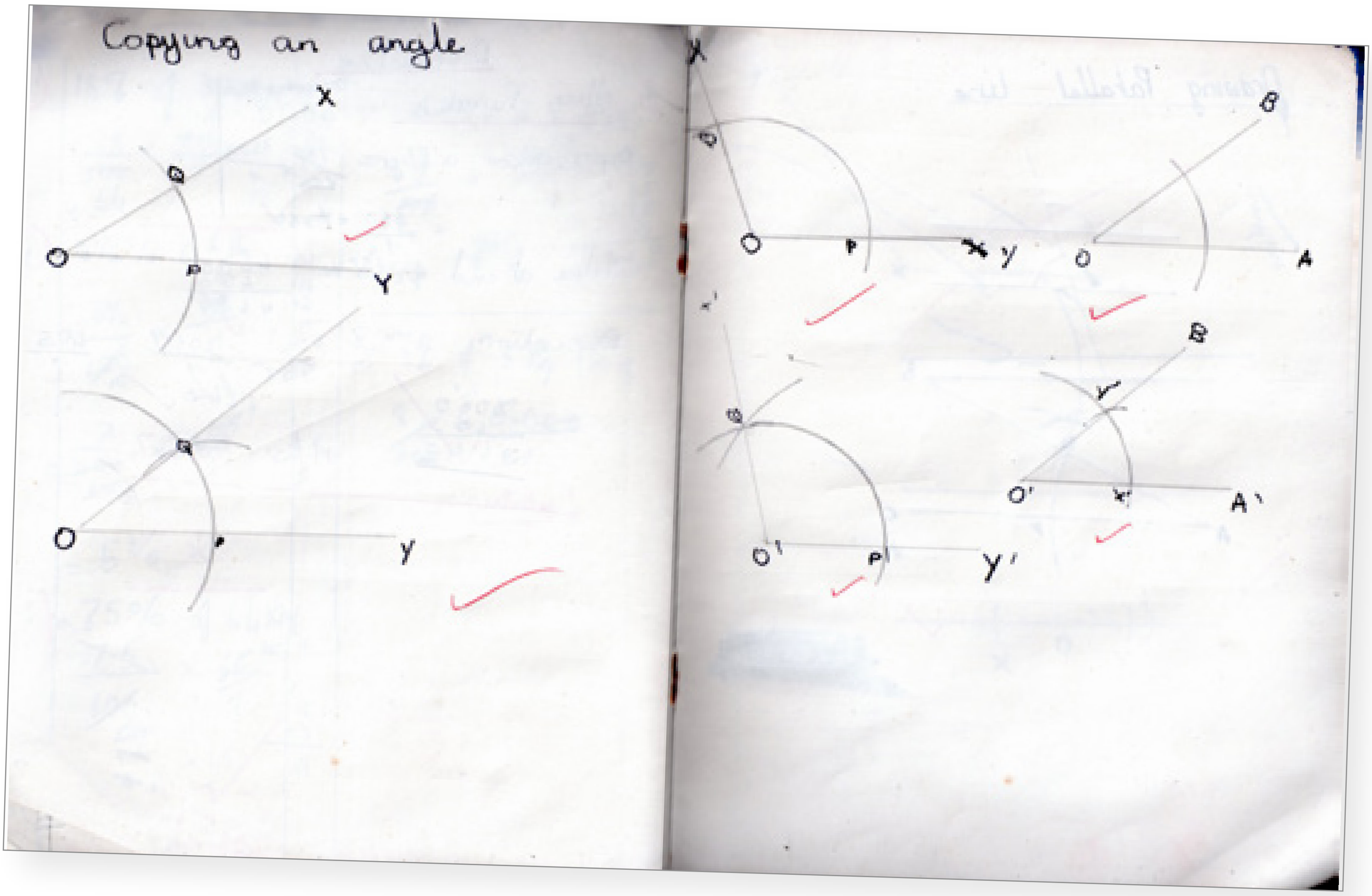
Pages from 1975 - have we come far since then?






Polygons
Triangle
quadrilateral
Pentagon
Hectagon
Angles of a Quadrilateral 360° 2rt
Angles of A pentagon
angles of a hectagon 8rt angle
Sum of Angles of A Polygon
Polygon
Triangle
Quad
Pentagon
hexagon
Heptagon
octagon
Decagon


5% of £12
5½% of 520
12½% of 2hrs
66⅔% of £0.75
2½% of £18
83⅓% of £3

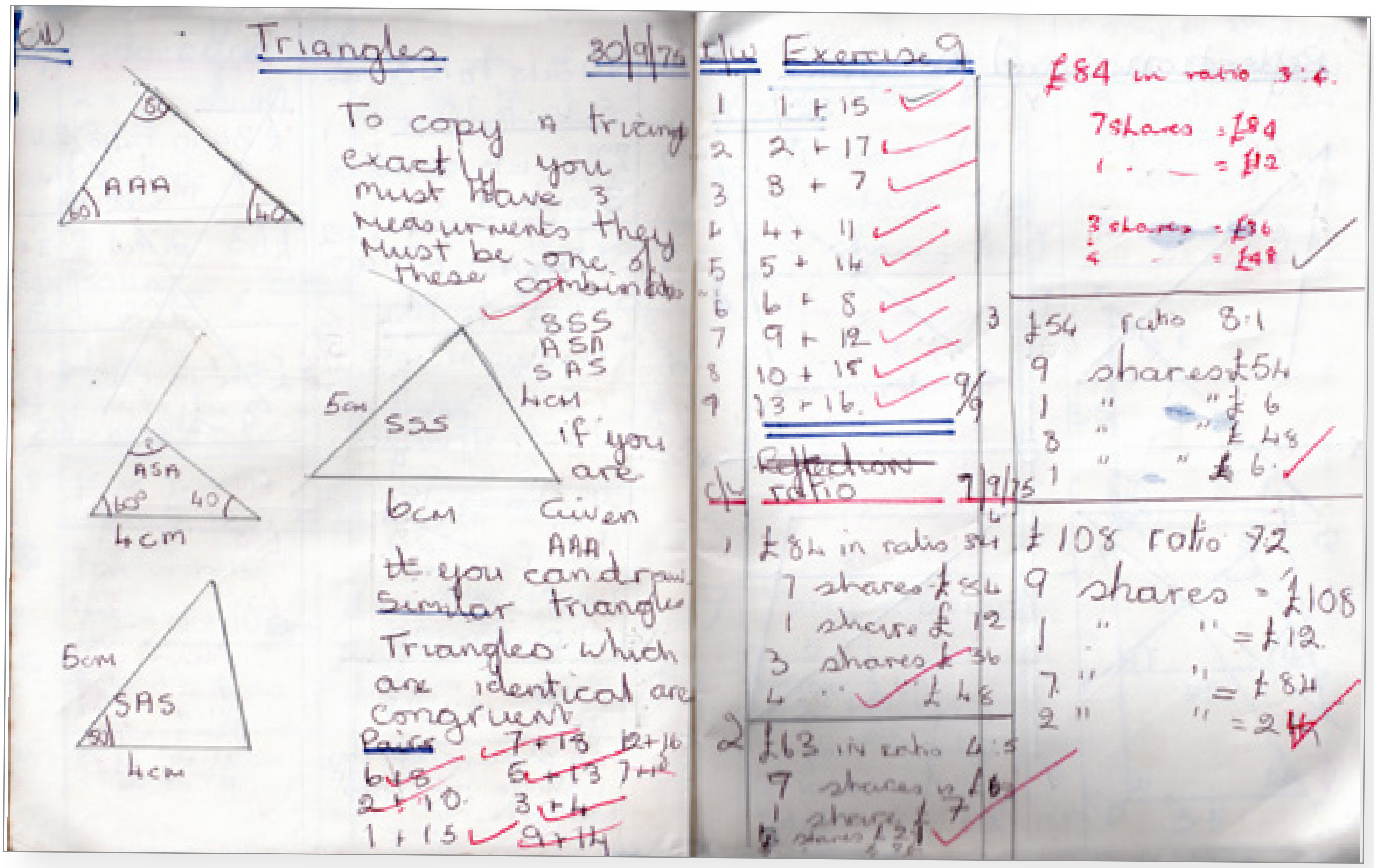

Copying an angle

X Q O P Y

O Q P Y

X Q O P X Y O A B

X' Q O' P' Y' B Y' O' X' A'


CW Triangles 30/9/75

AAA 60 80 40

To copy a triangle exactly you must have 3 measurements they must be one of these combinations

SSS
ASA
SAS

5cm SSS 6cm 4cm

ASA 60 40 4cm

If you are given AAA you can draw similar triangles

Triangles which are identical are congruent

5cm SAS 50 4cm

Pairs
6+8 7+18 12+16
2+10 5+13 7+11
1+15 3+4 9+14

H/w Exercise 9

1	1 + 15
2	2 + 17
3	8 + 7
4	4 + 11
5	5 + 14
6	6 + 8
7	9 + 12
8	10 + 18
9	13 + 16

9/9

~~Reflection~~ ratio 7/9/75

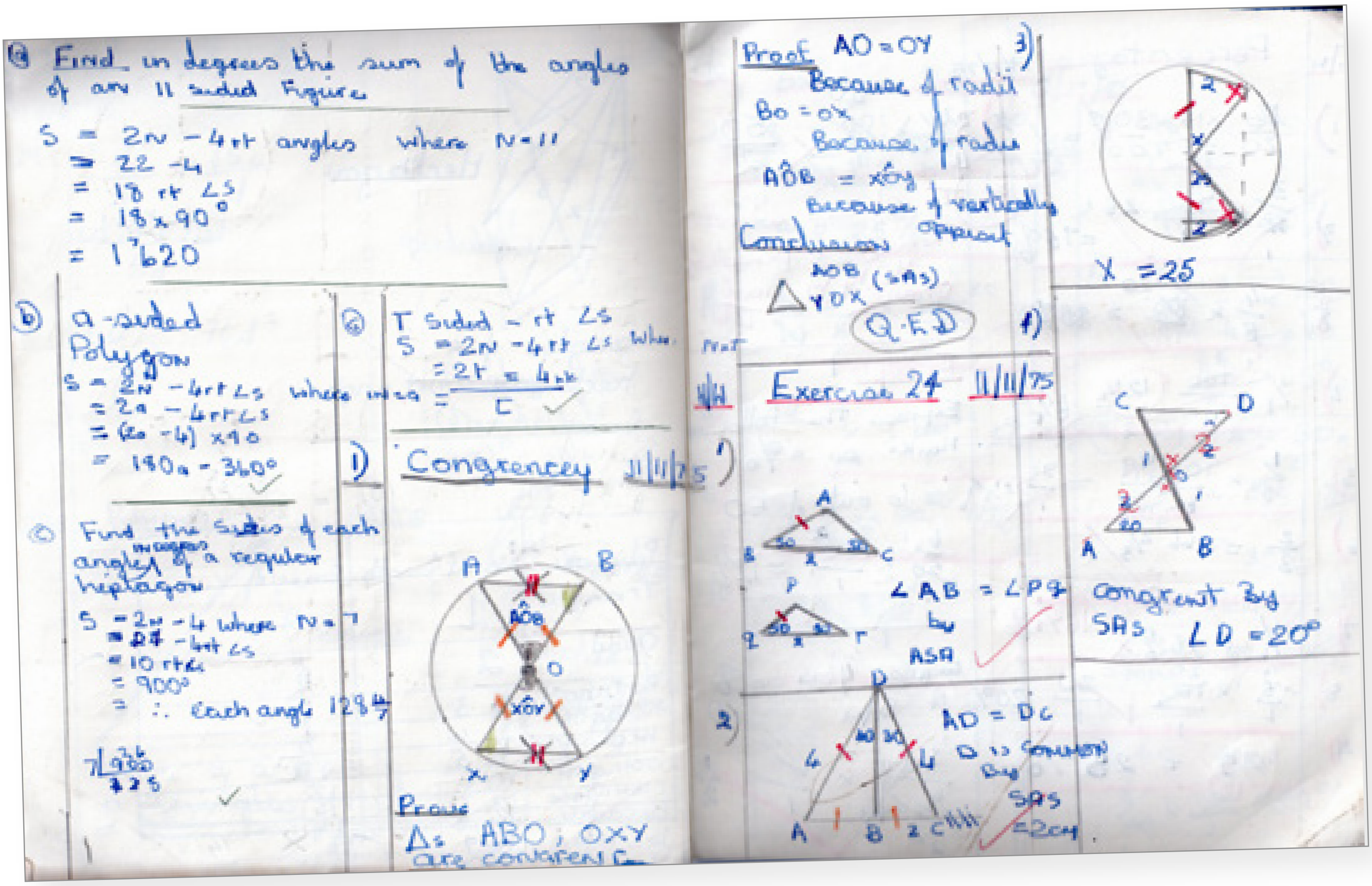
1 £84 in ratio 3:4
7 shares £84
1 share £12
3 shares £36
4 " £48

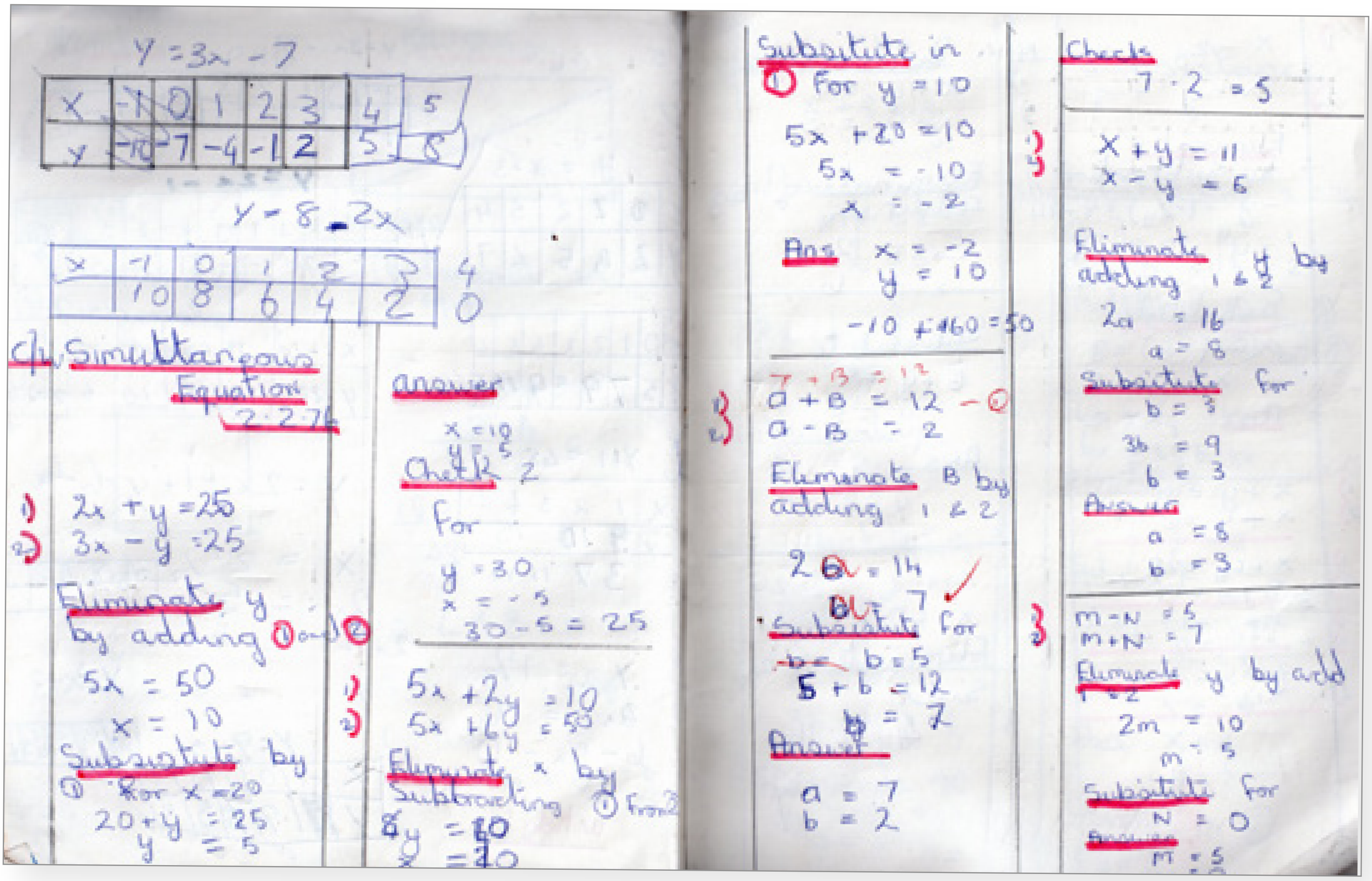
2 £63 in ratio 4:5

£84 in ratio 3:4
7 shares = £84
1 " = £12
3 shares = £36
4 " = £48

3 £54 ratio 8:1
9 shares £54
1 " " £6
8 " " £48
1 " " £6

£108 ratio 7:2
9 shares = £108
1 " " = £12
7 " " = £84
2 " " = 24


Find in degrees the sum of the angles of an 11 sided Figure
S = 2n − 4 rt angles where N = 11
= 18 rt ∠s
= 18 x 90°
= 1,620
Congruency
Proof
Because of radii
Because of vertically opposite
Conclusion
Q.E.D
Exercise 24
X = 25
Congruent By SAS
∠D = 20°
AD = DC
SAS


Y = 3x − 7
Y = 8 − 2x
Simultaneous Equation
2x + y = 25
3x − y = 25
Eliminate y by adding
5x = 50
x = 10
Substitute
answer
Check
Substitute in
Ans x = −2
y = 10
Eliminate B by adding 1 & 2
Answer
a = 7
b = 2
Check
x + y = 11
x − y = 5
Eliminate
Substitute for
Answer
a = 8
b = 3
m − N = 5
m + N = 7
2m = 10
m = 5

Seeding confidence in estimation

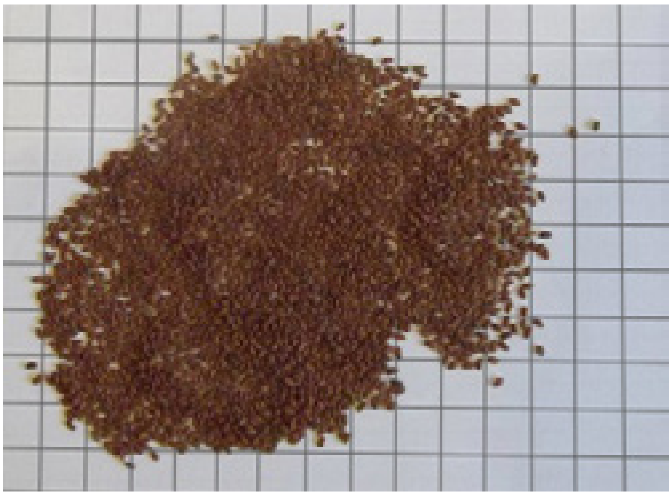
Two schools and two decades ago **Peter Ransom** was having problems teaching Year 9 pupils to estimate.

They wanted exact answers which was not appropriate for certain situations. One of the alpha males in the group was Barry who was very keen on tractors and farming and quite a few in this relatively small class were keen on the outdoor life, so I thought it would be good to capitalise on this.
I brought in several packets of different types of seeds with their contents varying from 50 to 2500 according to the average contents written on the back. The aim was to see whether what was claimed was reasonable. In the pictures that follow I have recreated the work with seeds bought recently as when this was initially done in class digital cameras had not been invented.

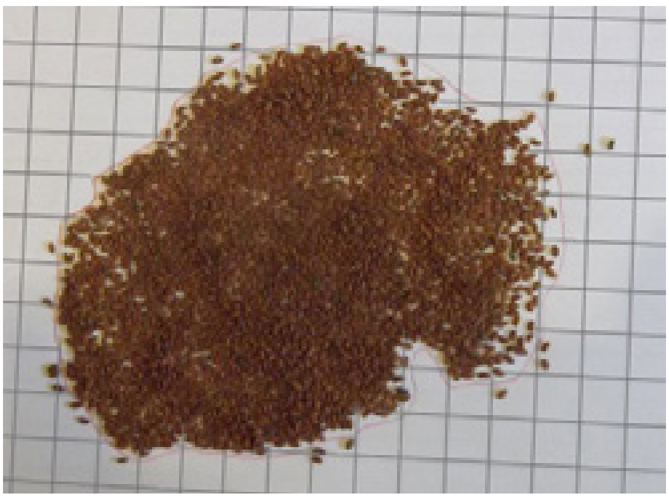
Packets of seeds showing approximate number of seeds in the packet, for a larger version see page 18.

Now there is no problem (so I thought) with peas - few enough to count accurately but not everyone came up with the same number! This was possibly due to losing one or two in various ways – I am sure some were eaten or dropped. One pupil seemed rather red in the face when I asked her how she was getting on. She said 'Pardon' and eventually her partner said she was trying to remove a pea from her ear by blowing with her mouth shut and holding her nose. It was at this point I got a bit worried, but the pea hurtled out and all was well. (Good job I did not have the compasses out that day.)

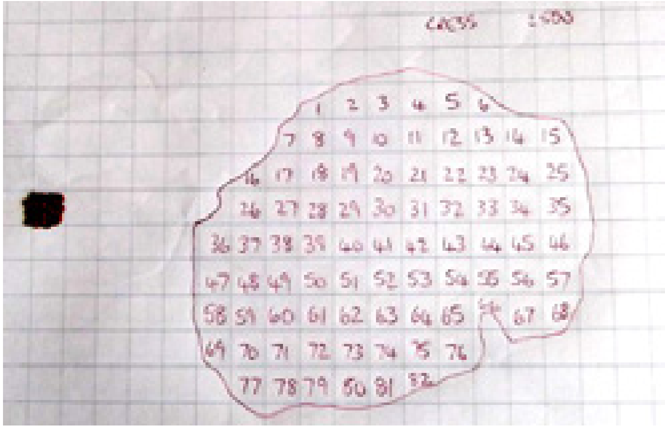
Smaller seeds were spread evenly on centimetre square paper and then the seeds in just one square were counted and multiplied by the number of squares it was estimated they covered. Here is the process in pictures and words:



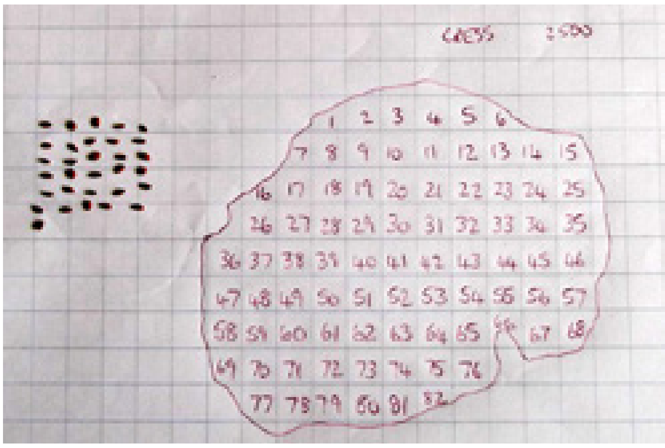
Seeds spread out evenly



Line drawn around seeds

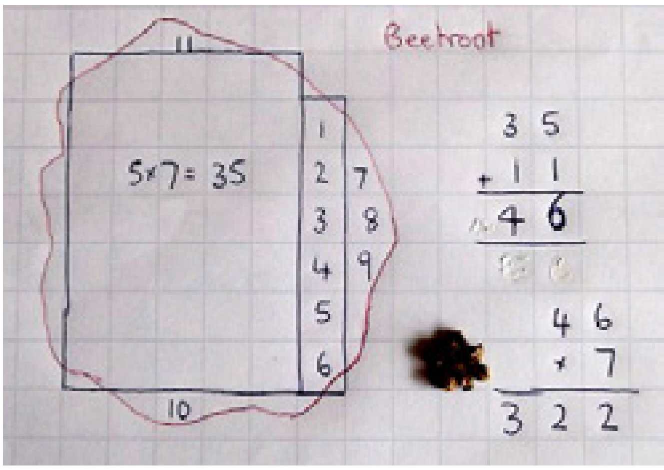


Squares counted and one square filled with seeds



Seeds in one square counted: here there are 31 seeds

82 x 31 ≈ 80 x 30 = 2400, so it looks as though the approximate number of seeds stated is OK.
In fact 82 x 31 = 2542 and this is 2500 to the nearest hundred.



So the beetroot checks out well, 322 rounding to 300 to the nearest hundred.

Rather disappointingly there were only 45 bean seeds in the packet, but this does round to 50.

So we covered some statistics and multiplication and realised that we did not have to count every seed. There was a bonus – they got the seeds to take home and I tried to grow some beans in a jar between blotting paper and the glass side so we could measure the root length and see how it grew. However this turned out not to be a success as the Easter break came and went and when I returned each bean was a 'has been'.

So, spread your seeds around and hope they do not fall on stony ground. No seeds (or students) were harmed in recreating this classroom experience.

Peter Ransom
is President of The Mathematical Association

Internet users by region in millions
(Internet world statistics Jan 2012)

Africa	118.6
Asia	922.2
Europe	476.2
Latin America	215.9
Middle East	68.6
North America	271.1
Oceana/Australia	21.3

Packets of seeds showing approximate number of seeds in the packet

Are they in their element?

‘Gove extols creativity but he has no idea what it is,’ reads **Rachel Gibbons** in a *Guardian* headline (17 May 2013).

The article under this headline was by Ken Robinson, an international advisor on education (amongst a variety of other activities). Robinson maintains that Gove’s new curriculum is ‘a dead hand on the creativity of teachers’. The message of Robinson’s book, *The Element: How Finding Your Passion Changes Everything*, is clear from its title. In it Robinson cites people, such as Paul McCartney, ‘whose talents had been undiscovered by their teachers but who, later in life, became famous through finding their element’. Robinson maintains that we all need to find our element.

Owen Jones in his book, *The Demonization of the Working Class*, details how strong the class system still is in this country and how this prevents many working class children finding their element - or even knowing they have an element to find. Some forty years ago I met Howard who had certainly found *his* passion – he could not wait to join his uncle on his barrow in one of London’s street markets. We both agreed that the last thing he should have been doing for the majority of his schooldays was sitting at a desk in a classroom but that was all he was offered apart from short periods of PE and some time in workshop activities.

He did once find his own way out from behind the desk and into his element by truanting when his fellows were going on a local geography trip. He took himself there too so that he could meet them and sell them cans of foam spray to decorate the school on their return. And today I fear he would fare no better. Howard’s problem and Robinson’s theories both point to the need to offer a personalised education programme to all our pupils. Howard’s number work, for example, should have been closely linked to his future buying and selling on a market stall. There is plenty of material available today at the National STEM Centre for you to plan learning programmes which will enable *all* your pupils to be ‘in their element’ in your mathematics classroom and beyond.

And now, only a few weeks later I find another headline, again in *the Guardian* (17 July, 2013) this time on the front page: ‘Five–year-olds could face national tests’. This time it is Nick Clegg, another politician inevitably ignorant of the details of educational good practice - or what goes on in classrooms - who is suggesting 5 year olds undergo formal testing with outside examiners assessing the results! Everyday - unless they are taking their classes through text books in lockstep – teachers are making individual assessments many times in every lesson throughout their years in contact with their pupils from 5 to 18 and beyond. Without making such assessments how could teachers set tasks appropriate to their pupils’ individual levels of learning lesson by lesson?

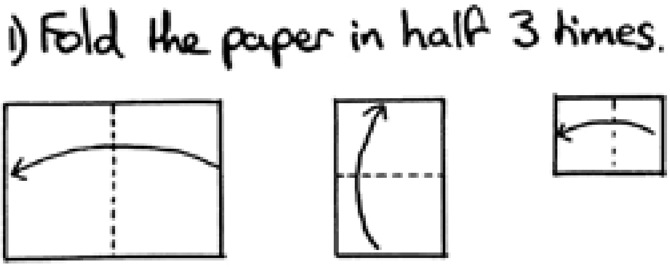
Rachel Gibbons is a retired ILEA inspector

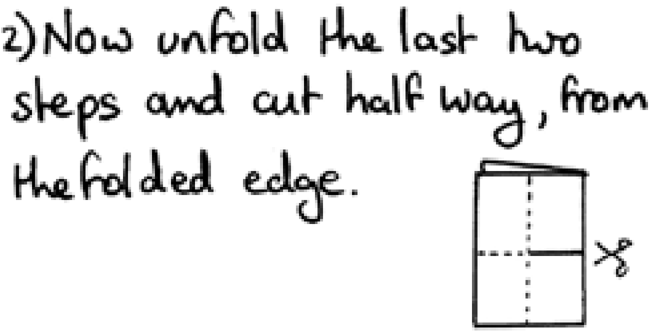
Two-piece tangram, four-page booklet

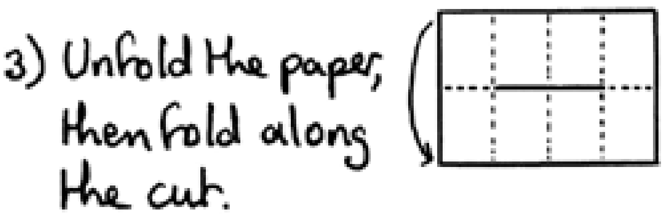
A long time ago in a mathematics faculty far, far away Peter Ransom was shown how to make a small eight sided booklet by Maureen.

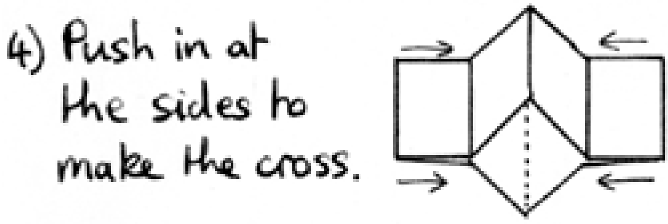
We were both interested in motivating our pupils in mathematics through origami as we wanted to capitalise on kinaesthetic activities that produced something the pupils could take home and would be proud to show their parents. Little did we know how much our pupils would benefit from this activity!

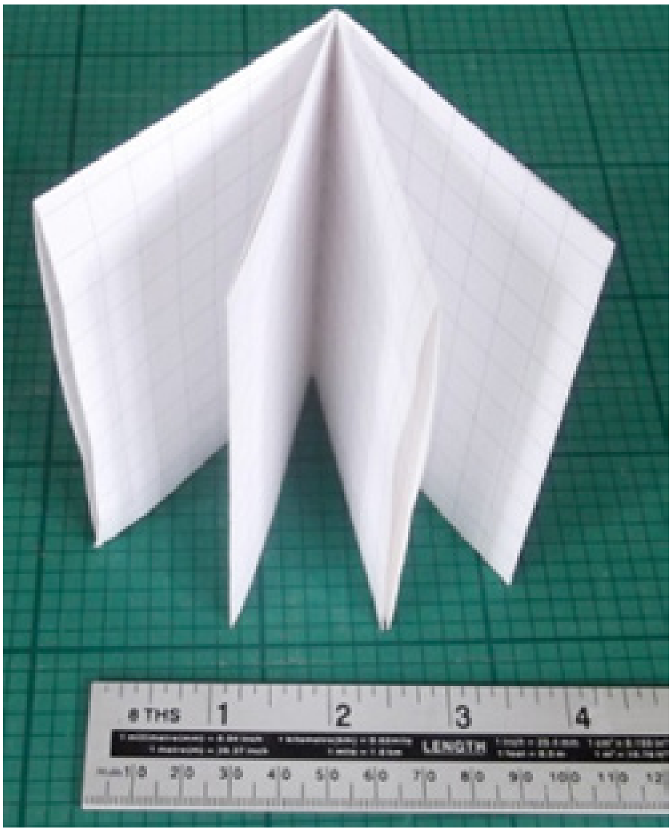
Initially we showed our pupils how to make the booklet. The pictures that follow describe the procedure. You can use any size of paper, but using centimetre squared or centimetre square dotty paper works best for the activity that follows.



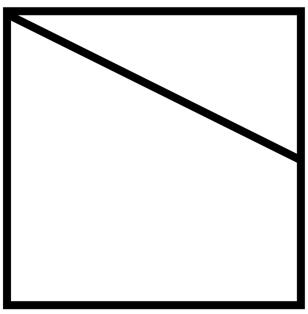








Then the pupils got their two-piece tangram, which is a 4 cm square that has been cut into two pieces by a line joining a vertex to the midpoint of the opposite side. It is better to give them the square that you have cut out from a piece of card than have them draw it as that way you ensure the square is square and by using card they can draw around the pieces if need be.



Using the two pieces of card they put the pieces together along sides that are the same length to make a different shape. They named the shape if they could and tried to convince their partner why it was the shape they named. This helped establish what they knew, the precision of the language they used and helped inform me so they could use the descriptions and explanations given by the pupils to develop more formal language/vocabulary in the main part of the lesson.

I found it useful for demonstration purposes to cut the two pieces from an old cake board as these are thick and generally covered in shiny paper. I could then attach the pieces to the board with Blutack, and pupils could move them round to get different shapes.

Pupils then made some different shapes, drawing and naming them (if they could) in their booklet. This was a useful exercise on seeing how systematic they were (there are 8 different shapes) and what shape names they knew.

The plenary was another opportunity for pupils to come up and explain why the shape is what it was to their peers. This encouraged mathematical communication. Asking open questions is sometimes harder for pupils to answer, so to begin with I tried questions such as "Why is this a square?" rather than "What shape do you have and why is it that shape?"

For homework they finished off and the next lesson they all got a sticky note. They passed their booklet to someone more than two seats away from them. Then they wrote two stars and a wish (i.e. two things they like about the booklet and one thing they think could be improved) on the sticky note and handed the booklet and comments back to the author.

When it came to doing revision, they enjoyed making their own revision notes in similar booklets because this kept the topic notes together in a compact form that they could carry around with them.

This work formed part of a case study *Developing geometrical reasoning in the secondary school: outcomes of trialling teaching activities in classrooms. A Report from the Southampton/Hampshire Group to the Qualifications and Curriculum Authority.* by Margaret Brown, Keith Jones & Ron Taylor © Qualifications and Curriculum Authority, November 2003. ISBN: 0854328092
The full report is available online at:
http://www.crme.soton.ac.uk/research/geomreason.html

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Should Scotland become an independent country?
32% of Britons asked said yes
72% said no
16% didn't know

The Guardian 18.09.13

Sun and moon
Sun rises 0640
Sun sets 1908
Moon rises 1821
Moon sets 0523
Full moon 19 September

The Guardian