Equals

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Realising potential in mathematics for all

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Realising
potential in mathematics
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Editors' Page

In the past, in the old print versions, *Equals* has always had a 'centrespread' giving resources for you to use in your classroom. Now, online, there is no centre to open up in the same way but we still offer mathematical activities for you to try in your classroom. From now on these will be labelled **Try this in your classroom** and the more we all use these and share the results with other *Equals* readers the more we shall all develop our teaching skills and deepen our understanding of mathematics.

Back to Basics?

Soon we shall be coming round to the anniversary of the riots and all those concerned with young people should be asking themselves what went wrong. It is the *causes* of crime amongst our youngsters that must be sought. By the time punishments are being discussed it is too late, the damage has been done. Neither poverty nor bad parenting can be tackled speedily, although of course both these must be faced. But our children belong to a state that believes in settling differences with lethal weapons used in wars Why do we not expect violence?

And what about our education system? Has the way schools are run anything to do with the problem? Is the current league table/testing education scene creating a group who see themselves dubbed failures in their own society: that is, in school? Is the system fascist? How many of the young people involved in violent crime are the failures of this regime of constant testing? Are teachers so busy trying to prepare their pupils for tests that they have no time to concentrate on what really are the basics in a society, that is:

> *taking responsibility for your own work and endeavouring to live peaceably amongst your neighbours, respecting their rights and the resources you all share together?*

These come before English and mathematics and the rest of the school's stated curriculum. In the excerpts you will find in this issue of *Equals* from *What Is and What Might Be,* Edmond Holmes writes of similar objectives for the elementary schools of his era – about a century ago.

We have in this issue recommended a start for background reading for all who teach mathematics and we would also recommend that you make sure a copy of the Cockcroft Report is available for reference for all teaching the subject, especially those who trained for other subjects but have been drafted in to take on groups who have a history of struggling with mathematics. Those groups, after all, contain the pupils who need the most skilled teaching if they are to lose their fears of numbers, develop a sense of pattern and increase their logical skills.

You will also find in the following pages an explanation by Matt Parker as to what he thinks mathematics is but we think we may have to ask him to explain further when he actually considers children to be "doing mathematics" in the classroom. Let us know what you think.

Is Ofsted utterly irresponsible?

What positive responsibility does Ofsted feel for raising the level of education provided for the children of this country? asks Rachel Gibbons.

I had the privilege of spending the last 23 years of my career working for the Inner London Education Authority (Ilea). As a teacher I appreciated the responsibility the local inspectorate took for improving the education provided by the Authority's schools. They ran courses for heads of department, indeed they provided us with the opportunity to develop sets of classroom materials more suitable for our pupils than those we could buy from publishers.

When, as one of Ilea's inspectors, I was myself involved in inspecting schools, I was fully aware of my responsibility for helping to cure any faults I had reported. It is only too easy to go into a school and find flaws, for not even a school run by Sir Michael Wilshaw can be perfect. However, helping to put right the flaws you have reported can be much more difficult. How much responsibility do Ofsted feel for improving a school they have criticised? They must see the best practice in their travels. Why not pass it on for others to learn from?

A former colleague of mine who joIned HMI some 20 years ago tells me that in his years' training it was stressed that members of HMI should always leave a school better than they found it. There is no such mantra now. Doubtless Ofsted will point to their reportsoffer advice but this is not the same as specific suggestions for improvements that might be made in particular schools. No teacher would attempt to use whole class teaching for 100% of their time in the classroom but this is what Ofsted's current methods could compare with. I suggest we all need to campaign for a more positive approach, endeavouring to ensure that each school inspected gets some advice appropriate to its own unique problems and possibilities. What use is any inspection otherwise?

Rachel Gibbons is a retired Ilea inspector

Mathematics Department Bookshelf

However long you have been teaching you can still learn more about better approaches and reading is one of the best ways of doing that so we thought we would open a few books for you as a start to further useful browsing.

The first writer to go to for help of course is **WW Sawyer**. Open any book of his at any page and you will find some mathematical or pedagogical gem. We have many times quoted from his wisdom in *Equals* so shall not do that again today but we urge you to make sure *Mathematicians Delight*, *Prelude to Mathematics*, *Vision in Elementary Mathematics* and *The Search for Pattern* are available to all members of the department for browsing and to those who are so often drafted in to teach the strugglers. Sadly, we think the books are at present all out of print. But if there is a large enough demand it is likely that some will be reprinted – let's see what we can do.

In 2006 **Francis Gilbert** wrote:

> … the introduction of the National Curriculum in schools has meant that a lot of pupils have had their interest in learning destroyed. This idiotic curriculum was an imitation of the grammar school curriculum of the 1950s. Once a child starts to fail in this highly pressurised environment, they drop out completely. There is no alternative offered. No wonder so many schools have to deal with such a lot of bad behaviour

Pupils and teachers have had their territories stripped away from them both literally and metaphorically: playing spaces have been replaced with buildings, and the teacher's ability to choose their own curriculum has been replaced with a centrally imposed curriculum. The pupils' response has been to claim some space of their own: the classrooms and the corridors of the schools, even if that means abusing their teachers in the process.

Yob Nation: the truth about Britain's Yob Culture

It is informative to look much further back. Today, when inspecting, Ofsted do no more than point out the flaws in a school, they offer nothing to help improve the education provided in the particular school they have criticised. It was not always like this. If we go back we find that **Edmond Holmes**, who was the Board of Education's chief inspector for elementary schools, wrote in 1911 *What Is and What Might Be* which is in two parts:

Part 1 **WHAT IS, or THE PATH OF MECHANICAL OBEDIENCE**

and

Part 2 **WHAT MIGHT BE, or THE PATH OF SELF-REASATION**

In the second part he describes in detail some of the best features of one of the schools he has visited and in the first part it is clear that his comments to schools that were failing in some way or other were positively helpful. He is also fully aware of what basic schooling should be all about, which so often gets masked in the testing culture of the education of today:

Activity, versatility, imaginative sympathy, a large and free outlook, self-forgetfulness, charm of manner, joy of heart, – are there many schools in England in which the soil and atmosphere are favourable to the vigorous growth of all these qualities? I doubt it. In the secondary schools of all grades and types, the education given is so one-sided, thanks to the inexorable pressure of the scholarship system, that the harmonious development of the child's nature is not to be looked for. In the elementary schools, from which the chilling shadow cast by thirty years of "payment by results" is passing slowly – very slowly – away, the instinct of the teacher is to distrust the child and do everything, or nearly everything, for him, the results being that the whole *regime* is still unfavourable to the spontaneous outgrowth of the child's higher qualities. There are of course schools, both secondary and elementary, in which one or more of the Utopian qualities flourish with considerable vigour. There are elementary schools, for example, in which children, being allowed by enterprising teachers to walk in new paths without leading strings, have become unexpectedly active and versatile. And there are others mostly in the slum regions of great towns – in which the devotion, the sympathetic kindness, and the gracious bearing of the teachers have won from the childr3en the response of unselfish affection, attractive manners and happy faces.

Margaret Donaldson, when writing about child development in *Children's Minds* in 1978 in a chapter entitled "The School Experience", comments:

> When we make laws which compel our children to go to school we asssume collectively an awesome responsibility. For a period of some ten years, with minor variations from country to country, the children are conscripts and their youth does nothing to alter the seriousness of this fact. Nor is it altered by the intention, however genuine that the school experience should be 'for their good'.

I am not among those who advocate what has come to be known as 'deschooling society'. I believe that we need schools – and never more than now. But the justification of a long enforced period of national service is not something we can treat lightly. The question that must asked, and considered seriously, and reconsidered as knowledge and circumstances change, is whether the school experience really is good for our children – as good as we could make it. And this of course amounts to the same thing as asking whether it is really good for the society that will come into being when the present one is gone.

The question that must asked, and considered seriously, and reconsidered as knowledge and circumstances change, is whether the school experience really is good for our children – as good as we could make it.

We are faced now with something of a puzzle. In the first few years at school all appears to go very well. The children seem eager, lively, happy. There is commonly an atmosphere of spontaneity in which they are encoyraged to explore and discover and create. There is much concern, on the part of the teachers, with high educational ideals. These things tend to be true even in parts of the community which are far from being socially privileged in other ways. However, when we consider what has happened by the time the children reach adolescence, we are forced to recognise that the promise of the early years frequently remains unfulfilled. Large numbers leave school with the bitter taste of defeat in them, not having mastered even moderately those basic skills which society demands, much less having become people who rejoice in the exercise of creative intelligence.

The problem then is to understand how something that begins so well can often end so badly.

Changes to place value thinking during Key Stage 2

As part of the Mathematics Specialist Teacher (MaST) course Pam Fletcher collaborated with a colleague in some research into the understanding of place value in Key Stage 2.

We like to feel that our school is a 'community of enquiry' where each individual is striving to better understand teaching and learning. After a staff discussion, it was decided that for our initial project I should work alongside the Year 5/6 teacher, Wendy. The reasons for this were:

- Wendy had recently moved from KS1 to teach this older group of pupils and felt less confident about the mathematical curriculum in KS2.
- She and I already plan some areas of mathematics together as I teach the Year 5/6 class twice a week.

As mathematical thinking is one of the key areas of the MaST course and as thinking skills are always an area of development within the school, we decided to concentrate on this aspect. Place value was the aspect of mathematics which Wendy highlighted as one of the areas causing difficulties for her pupils. We decided to try to develop mathematical thinking through multiplying a number by 10 and observing the progression of the children's ideas and understanding from year 3 to year 6.

Place value is an extremely important idea and facilitates mathematical thinking in many areas. However many children find it difficult to understand and Sowder (1992)[1] argues that most '*primary school pupils may have a limited understanding of place value'.* Thompson (2009)[2] says that difficulties arise because of the confusion between two different aspects of place value – the 'quantity value' where 45 would be seen as 40 and 5 and the 'column value' where 45 is 4 tens and 5 units. He suggests that in mental and informal calculations children use the 'quantity value'. I have also observed that pupils in our school use this 'quantity value' almost exclusively in mental and informal methods of calculation.

For instance:	'How would you add together 35 and 23?'
Pupil:	'Well 30 add 20 is 50, 5 add 3 is 8 so the answer is 58.'

'Multiply 27 by 5.'

Pupil:	'I know that 20 × 5 = 100 and 7 × 5 = 35 so the answer is 100 + 35 = 135.'

Traditionally teachers have tried to teach multiplication by 10 as 'the digits move one place to the left' rather than 'adding a zero' as this does not work for decimals. Thompson (2009) concludes that 'children in Y2 and Y3 have little or no understanding of what we conventionally call place value' and therefore 'It would appear to be somewhat over-optimistic to expect young children to understand the concept of 'moving the digits one place to the left' which . . . appears to demand a fairly sophisticated understanding of place value.'

After the early whole-school discussions, Wendy and I met after school and discussed the proposed focus for this project. This was decided after joint discussion and looking at the Assessing Pupils' Progress (APP) grids for our classes. We also considered how the project would be carried out and decided that we would jointly plan the lessons and then each teach a lesson to our own class while the other observed. The observation was to be centred on the children's responses rather than the teaching. This joint planning and focus on the responses of the pupils meant that we were both comfortable with the boundaries of the observation and neither of us felt threatened. Finally we planned a post observation discussion in which we would discuss the responses and learning of the pupils. We agreed that we would not discuss the teaching.

The initial discussion covered ideas about 'a learning agreement'. The subsequent lesson teaching and observation gave opportunities for us to experiment and observe and subsequent discussions helped us to set challenging and personal goals for both ourselves and our pupils. We were able to use money from the MaST project to obtain supply cover for one morning so that we could observe each other's classes and to make use of PPA time and TA cover to complete our discussions. This creative use of time also meant that the project was given importance within the whole school plan. Fullan (2001)[3] argues that for a new initiative to succeed there needs to practical evidence that the leadership are behind the project.

1 Sowder (1992) *Estimation and number sense*. In Grouws, D.A.(ed) *Handbook of Research in Mathematics Teaching and Learning* New York; Macmillan. 371-389

2 Thompson, I. (2009) *Putting place value in its place.* http://www.atm.org.uk/journal/archive/mt184files/ATM-MT184-14-15.pdf

3 Fullan, M. (2001). *Leading in a culture of change.* San Francisco: Jossey-Bass.

On reflection our plan had many aspects of a Lesson Study cycle as described by the National Strategies. If we were to continue with this project into further cycles, the Lesson Study idea of only observing one lesson and having a focus group of children might prove useful. We did not interview the pupils after the lesson to gain insights into their learning, but this could prove useful in the future.

During our discussions Wendy and I talked about the need for pupils to develop a deep or relational understanding as defined by Skemp (1976)[4] rather than just superficial or instrumental understanding. If children had been taught either to add a zero or to 'move the digits one place to the left' both of these would be instrumental i.e. a learned rule to get the right answer. We also looked at the articles by Thompson dealing with place value and column versus quantity value. These ideas were interesting to us both and provoked considerable discussion in the staff room. We designed our lessons to see which ideas our pupils would use.

The mathematics lessons were set as problem solving exercises within our 'Companies'. At our school all classes run companies for part of the week using the 'Mantle of the Expert' ideas of Heathcote (1995)[5], Year 3/4 are part of the Virginia Company of 1607 and Year 5/6 are running a bear sanctuary. These enterprises are of course imaginary and use drama techniques to support the children's learning. For the children however the contexts become 'real' and so they would be highly motivated to solve the problems presented. Plenty of time was given for the discussion of children's ideas both in pairs and groups on mixed ability tables and during a whole class plenary.

The pupils were happy to discuss their methods and justify the answers. This was particularly evident in the Year 5/6 lesson where each method for multiplying by ten was evaluated and a consensus was reached as to the most reliable method.

In Year 3/4 many children were able to see the pattern for multiplying by ten and then were able to generalise to any number.

We also tried to give opportunities to the children to see multiple representations of the same mathematical problem by setting the calculations in more than one context – exchange rate for baskets of produce, multiplying lengths and scaling up a recipe for milk.

Within our classrooms most children discovered the rule for multiplying by ten and within Year 5/6 were then able to look at various methods and decide which would be best. They had a much greater relational understanding of the issues by the end of the lessons. There was little evidence of the children using 'column value' (Thompson) in either lesson except during discussions about multiplying decimals.

This collaboration was enjoyable and beneficial for both teachers and pupils. Wendy commented that she had enjoyed planning and teaching the

[4] Skemp, R. (1976) *Relational understanding and instrumental understanding:* Mathematics teaching*, Vol. 77 20-26*

[5] Heathcote, D. and Bolton, G. (1995) Drama for *learning: Dorothy Heathcote's Mantle of the Expert Approach to Education, (Dimensions of Drama*). Heinemann

lessons. She felt that she had a much greater understanding of the progression in this concept from Year 3 to Year 6. It gave us an opportunity to discuss ideas about mathematical thinking which are key to children's learning. I certainly gained a much deeper understanding of the difficulties encountered by pupils when trying to understand place value.

In the future it would be good to continue to work collaboratively and to develop a series of lessons not only looking at multiplication but also at division by ten. We could also probe children's understanding of place value in other situations. Here are some notes about the lessons and what we observed.

Year 3/4 Lesson

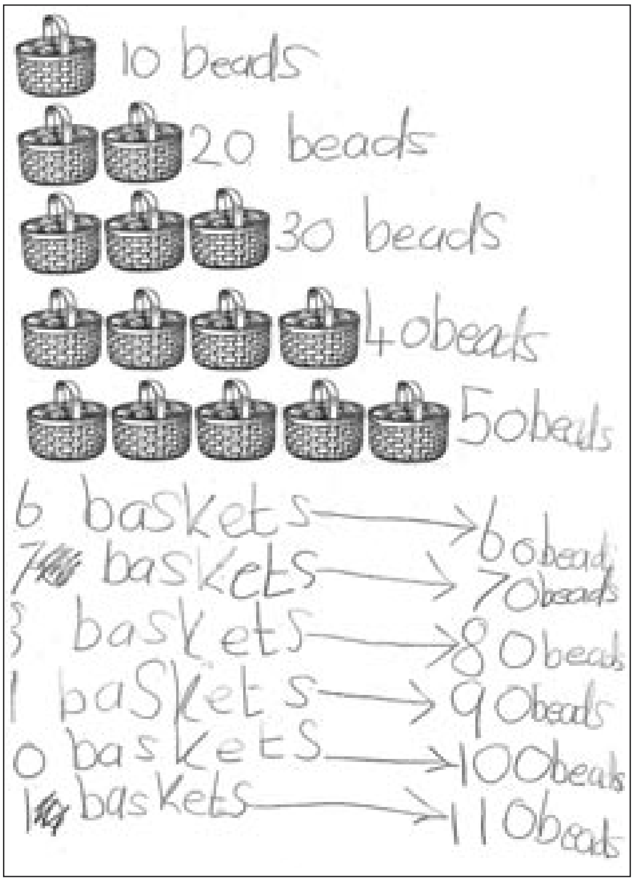
The context for the Year 3/4 lesson was the Virginia settlement at Jamestown in 1607. Initially the settlers were trading with the native Americans. The rate for a basket of fruit was 10 glass beads. The children were asked to work out a chart so that everybody would know how many beads to give.

Pupils had some beads available plus Dienes apparatus. The least able pupils were supported by a teaching assistant.

Many pupils started by counting out the beads but they quickly ran out and so began to calculate. After about 5 minutes there was a whole class discussion about the methods they had used. Here are some of the children's contributions:

- 1 basket is worth 10 beads. There are 10 for each basket so you just keep adding them up. 10, 20, 30, 40
- It's the 10× table
- You just count in tens
- Add a zero to the number of baskets

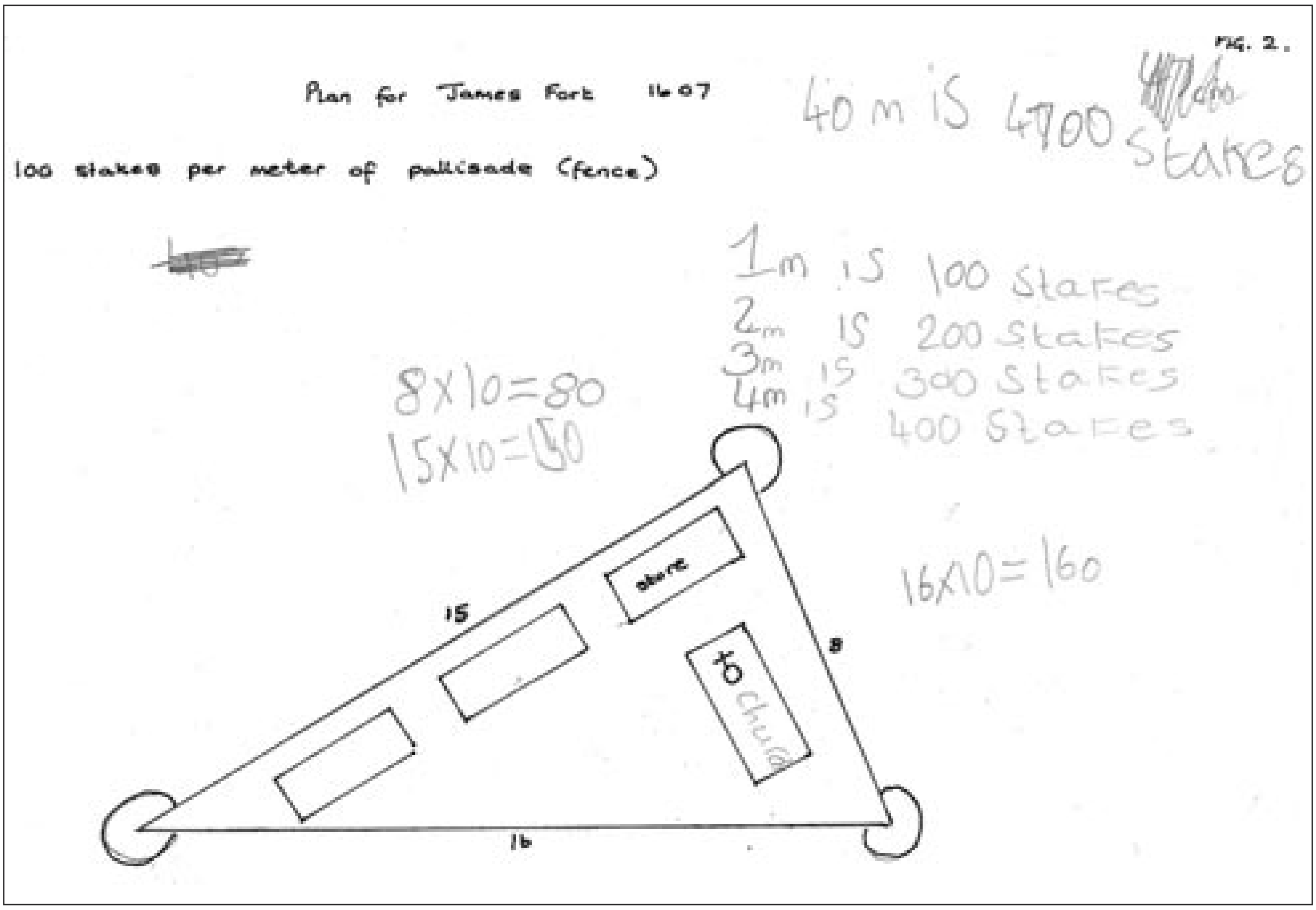
Nobody seemed to want to use the Dienes apparatus.



Once the chart was made, the second problem was introduced:

> 'President Wingfield has been given a plan of the fort we are going to build by the Virginia Company. However the measurements seemed to be 10 times too small. What should the measurements be?'

Here 8 × 10 was easily calculated by everybody but 15 × 10 and 16 × 10 proved more tricky. Some still tried counting up in tens and lost their way, particularly around 100. The general



consensus was that the most reliable method was to add a zero.

Finally we tried working out how many stakes would be needed for the palisade which needed 100 stakes for each metre. Only one or two were able to do this. They either counted in 100's or added two zeros. Nobody mentioned place value or moving the numbers to the left.

Year 5/6 lesson

The Year 5/6 lesson was within their context of a Bear Sanctuary.

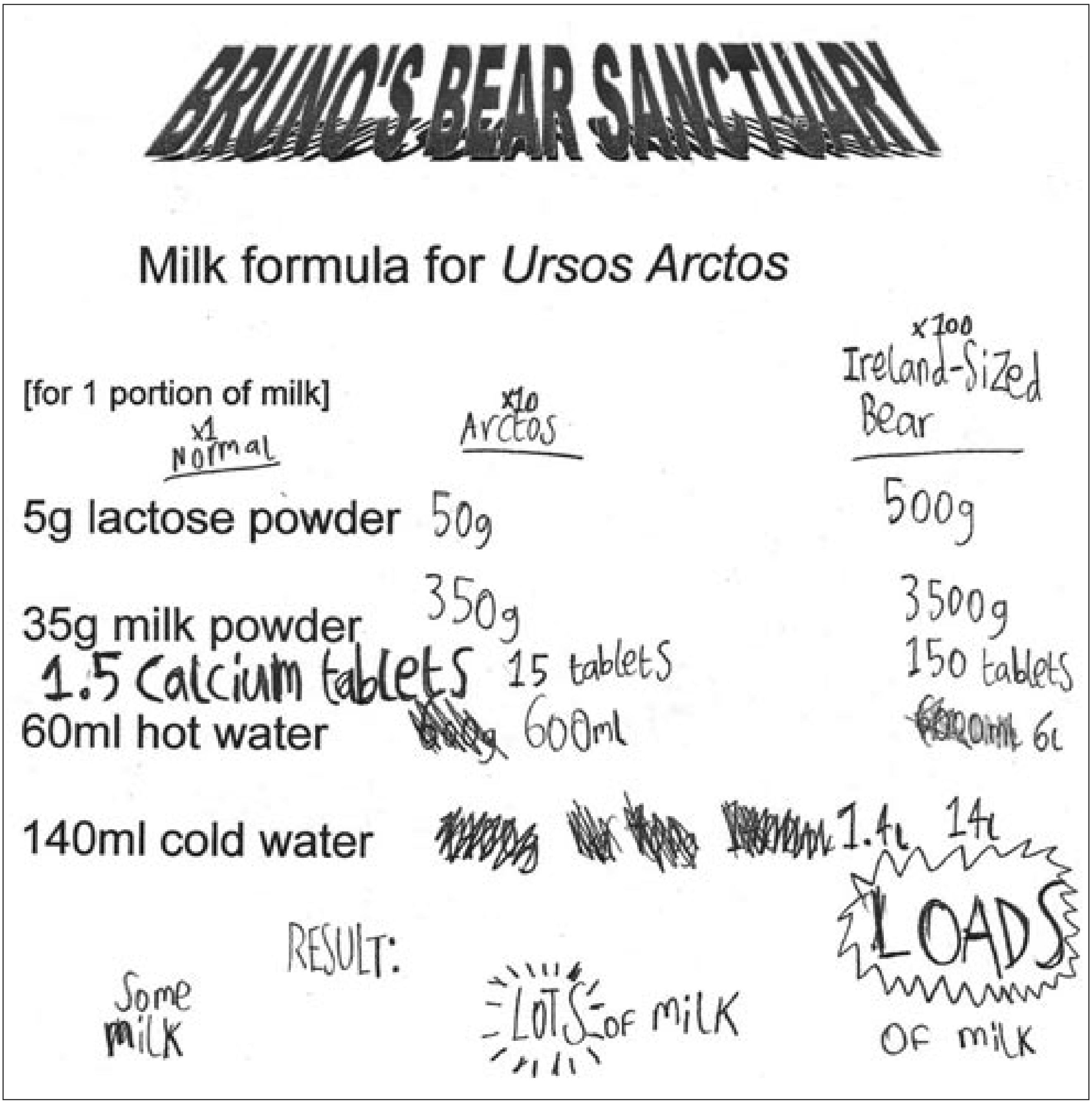
> 'Some of the bears still need milk to drink either because they are very young or because they have problems with their teeth.'

The formula was given for 1 portion of milk:

> 5g lactose powder
> 35g milk powder
> 60 ml hot water
> 140 ml cold water

The children were then asked to work out how much they would need for 10 portions for Ursos Arctos who is a large old bear with bad teeth and who therefore needs 10 times as much milk.

This proved a relatively easy task for most children. The least able were supported by a teaching assistant and had Dienes apparatus available. One of the least able children suddenly saw a pattern and said 'you add a zero'. The pupils who finished quickly were asked to show their results on the measurement chart.

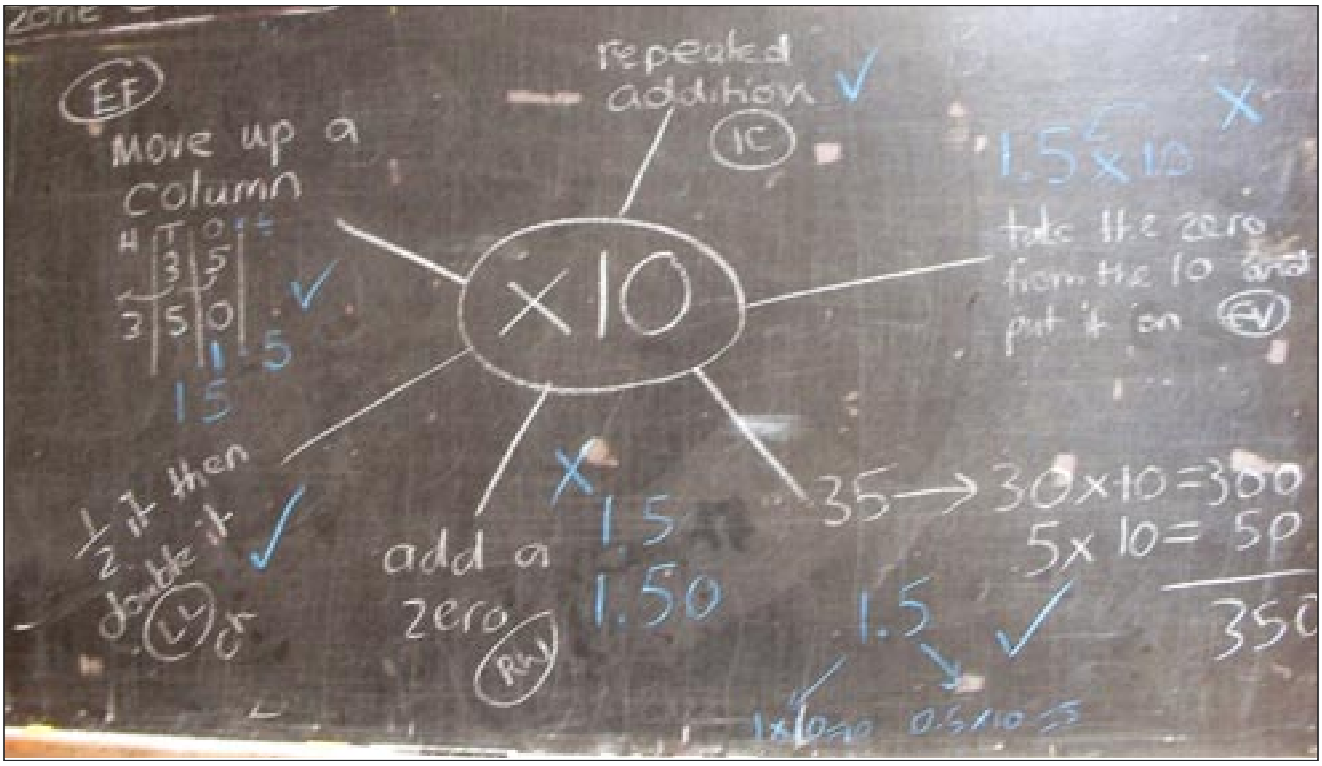


Children were asked how they had completed the calculation. Various methods were voiced:

- I added a zero
- For 35 × 10 I did 30 × 10 and 5 × 10 then added them together
- I moved each number up a column, (moved the numbers to the left.)
- Count up in tens.
- You could do it by adding 35, 10 times

At this point Wendy added an extra ingredient:

> 'Sorry, I forgot to tell you about the calcium. We need 1.5 calcium tablets for 1 portion.' This engendered more thought and some discussion but they soon came up with the answer:



- You need 15 for 10 portions. You have to shuffle it up a column.
- Adding a zero doesn't work for decimals
- 1½ × 10 = 1 × 10 + ½ × 10 = 10 + 5

This time the extension activity was to multiply by 100

- You just need to add 2 zeros.
- Take the zeros off the 100 and put them on the number.
- Move the number up two columns.

Once everybody had had a chance to complete their calculations Wendy brought them back as a group to look at their methods for multiplying by 10. These were displayed on the board. Wendy led the discussion to see whether all of the methods worked when there was a decimal.

Almost all were happy that the 'best' method now was to move the numbers 'up a column' as that was the simplest way to be sure to get the correct answer for all problems.

In our subsequent discussions about these lessons we agreed that we are both very relaxed about the idea that children have their own methods for calculation in mathematics. They have misconceptions, but in a non- judgemental environment where mistakes are okay, they are able to work through their ideas. (This fits with Piaget's ideas about schema and their modification) We wouldn't impose a particular method for any calculation but as far as multiplication by 10, 100 etc. is concerned it seems pointless to try to teach a method which they are not ready for.

Pam Fletcher is an advanced skills teacher at Bealings School Suffolk

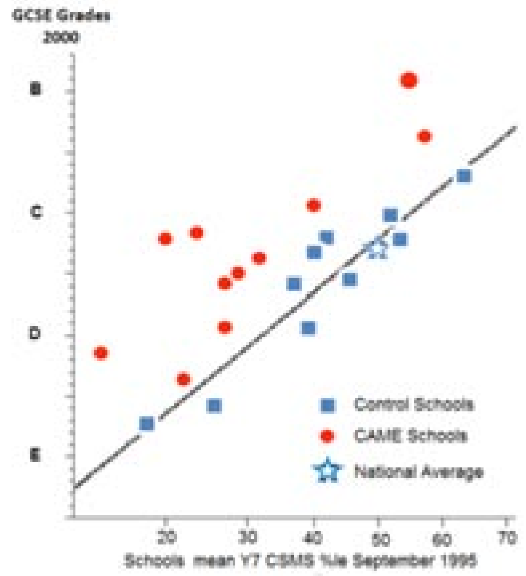
Decimals: The Cognitive Acceleration Approach

In this article, Alan Edmiston explores some of the possible reasons behind the success of the Cognitive Acceleration approach.

Cognitive Acceleration is an umbrella term used to denote a project in applied educational research led by Professors Michael Shayer and Philip Adey from King's College, London for over 20 years. The CA research resulted in many published resources and texts most notably Thinking Maths – CAME – and Thinking Science – CASE. All projects of this type will be referred to as CA.

For many years two CA projects in Mathematics and Science have been successful in raising the attainment of secondary aged pupils. From the 1980s onward data has emerged highlighting the fact that materials under the umbrella term CA can have a significant impact upon the external examination success of pupils aged 16 and, given the issues facing adolescents today, an initiative which helps their attainment in the core subject area is to be welcomed.

The graph below (first presented in the journal *Educational Studies in Mathematics,* 2007) highlights this, showing the impact of CAME in the Year 2000 upon the GCSE performance of pupils who received CAME lessons during Years 7 and 8. For the purposes of this article maths data has been used but many CA interventions have resulted in similar impacts upon pupils across the ability range. Furthermore it is hoped that a new Kings College CA project, in English, directed by Laurie Smith will complement this. The testing in CA research below involves assessing a child's mathematical or scientific thinking level, in Piagetian terms, and allocating the cohort for that school to a percentile similar to those used with young children. For example, in 1992 I discovered that the Year 7 pupils in the school where I was a Science teacher were on the 28th percentile. In real terms this meant that, upon entry to Year 7, 72 percent pupils in the same age range were operating at a higher level than ours. The testing produces a baseline against which any value-added gains can be plotted as the graph below clearly shows.



The data shows that the gain for CAME pupils is 0.8 of a GCSE grade, whereas the control schools have an insignificant gain and do as well as expected given their entry level is above the national average. In addition to this the impact of the thinking lessons at the start of KS 3 is to produce a 0.5 grade gain in both English and Science. This highlights the fact that CA interventions raise the thinking powers of school-aged pupils regardless of subject. To me this highlights the fact that promoting better thinking leads to better learning and ultimately results in better attainment.

This article does not seek to provide an introduction to CA – the references at the end will enable readers to explore the many resources better able to do this – rather it aims to highlight how the approach serves to enhance pupil learning. I hope by doing this to illustrate just how CAME lessons differ from other mathematics lessons for both pupils and teachers. The CAME approach to introducing decimals in Year 4 will be compared to the standard approach used by a co-ordinator with some twenty years experience of teaching mathematics.

Brief summary of the non-CAME introduction to decimals

In order to introduce decimals to children who have had some previous experience of fractions (in particular tenths) I have used the following method:

1) Draw a rectangle on the board and split it into ten sections. Ask a child how we can label each section of the rectangle (i.e. 1/10). Write 1/10 in each section …

1/10	**1/10**	**1/10**	**1/10**	**1/10**	**1/10**	**1/10**	**1/10**	**1/10**	**1/10**

2) Colour some of the rectangles, e.g.

1/10	**1/10**	**1/10**	**1/10**	**1/10**	**1/10**	**1/10**	**1/10**	**1/10**	**1/10**

3) Ask the children what fraction of the rectangle has been coloured
(i.e. 1/10 + 1/10 + 1/10 = 3/10).

4) Explain that 3/10 can be written in another way … i.e. 0.3 (no units and three tenths)

5) Explain the features of the notation:

   - The dot in between the 0 and the 3 is called the DECIMAL POINT and we use it to separate the units from the tenths.
   - We always write in the 0 before the decimal point because it reminds us that the whole number is less than one.
   - We say this number as “nought point three” or “zero point three”.

6) Now give each child a copy of the strips worksheet – they should cut out a strip and colour some of the sections (and then stick it in their books). Next, they should write the following information into their books, changing the numbers according to the amount of strip that they have coloured.

1/10	**1/10**	**1/10**	**1/10**	**1/10**	**1/10**	**1/10**	**1/10**	**1/10**	**1/10**

   7/10 of the strip has been coloured. 7/10 can also be written as 0.7, which we say as “nought point seven” or “zero point seven”.

7) Step 6 can be repeated with the children colouring different numbers on new strips.

Brief summary of the CAME introduction to decimals

The learning takes place over three episodes.

Episode 1

This part of the activity begins with a story, so if you are sitting comfortably I'll begin …

> ***"There was once a giant who was the ruler of a great kingdom of magical creatures like pixies, elves and fairies. That was many, many years ago, before measuring had even been invented. One night the Giant King had a dream about a beautiful palace and when he woke up, he decided he wanted to build himself a palace, exactly like it was in the dream. He wanted to tell his servants, the pixies, exactly how he wanted the palace to be built, but there was no such thing as metres or centimetres, or rulers or tape measures in his kingdom. He has to tell them how big the rooms and corridors are to fit him, so everything should be related to him and his body. What are the easiest body parts to use as measures?"***

The resulting dialogue draws the children into the giant's world and explores using arm length, shoulder length and height and other parts f the body to measure before focusing on the foot. The children will be shown and given the measure of the giant King's foot:

Episode 1 ends with a discussion of the advantages of using the foot as a measure, and come up with ideas of how to use it to measure the classroom or the windows or a table.

Episode 2

This begins with the dilemma of measuring pieces or furniture in the palace that are smaller than a giant's foot. This discussion leads up to the idea of using pixie feet to help with smaller objects. At this point it is a matter of personal choice whether the children are shown that 10 pixie feet = 1 giant's foot or they are left to discover the link for themselves. They now work in groups using cut out feet of both sizes to measure several objects around the room e.g. pencils, rulers, tables. They also need to decide how they will record how long each object is. In addition the children are told that the giant said his foot was the most important and that his measurement must always come first.

The final discussion in this episode is based around how the children recorded their measurements e.g. *1G, 2P* or *1 Giant, 2 Pixies* or 1 2 or even 1,2. This naturally moves towards the idea that we can't just use numbers because if we write 12, it could be 12 giants or 1 giant and 2 pixies. It should be noted that this discussion often takes a great deal of time and that many times pupils will often use a comma to separate the two measurements and also that in Germany a comma is used in decimal notation in the place of a point. The aim of episode 2 is to introduce the idea of using a point to split the giant and pixie numbers in the measurement, and encouraging all the children to record their measurements in this way.

Episode 3

The idea of using the 1.2 notation to differentiate between feet of differing size is now extended to include the fact that the giant King also found out that 100 elf steps fit inside one giant foot step. The children are given some simple decimals as measurements for the palace and they have to decide what they mean e.g. a door 1.27 high could mean 1 giant and 27 elf, or 1 giant, 2 pixies and 7 elves. At this point they can be introduced to the formal labels as tenths and hundredths for the pixies and elves. Discuss the possible confusion of tens and tenths, similarly between hundreds and hundredths. This aspect is consolidated with a discussion of the meaning (in terms of feet) of 0.25, 0.05, 0.006, 11.3, 11.06 or even 1.024?

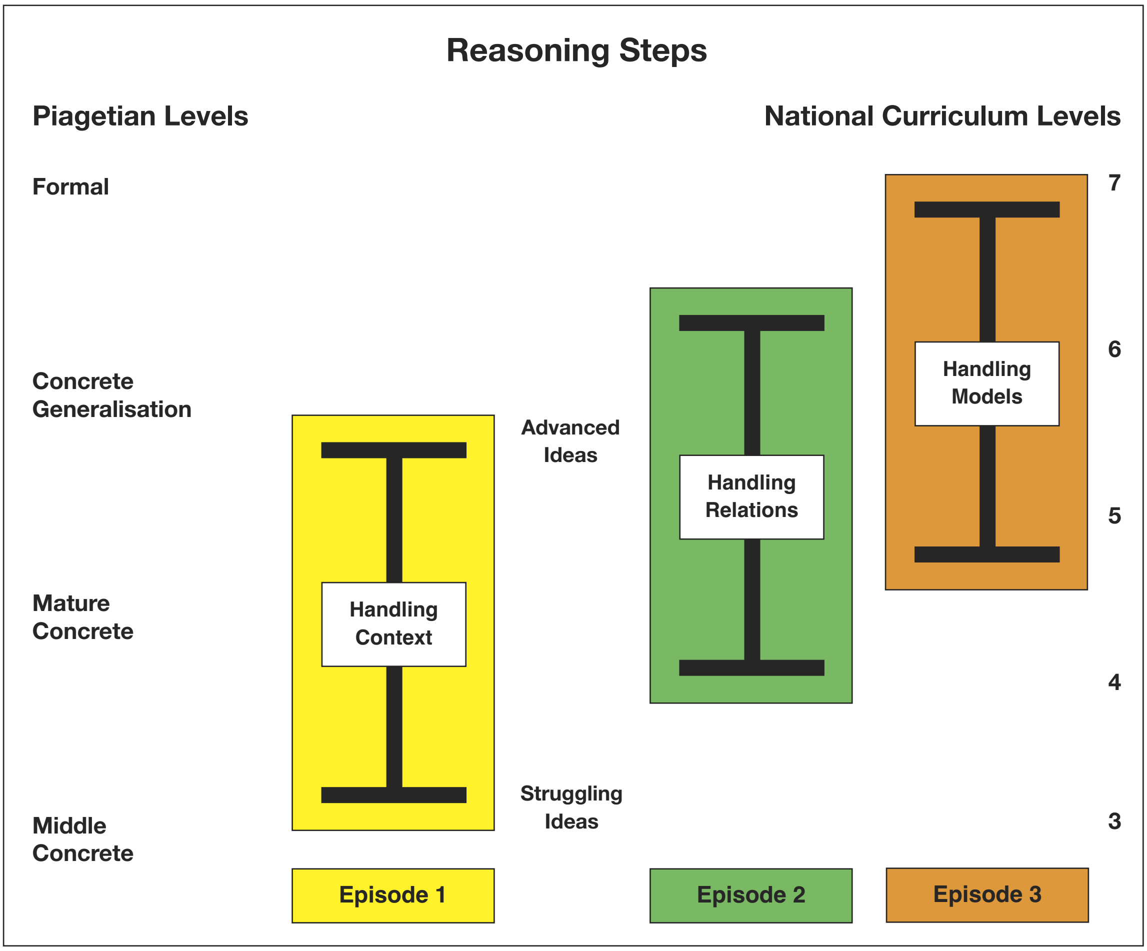
The lesson ends as it began with another story that encourages them to order numbers involving 2 decimal places. "There was a gnome building a wonderful mosaic inside the giant's Palace and the other gnomes had to bring in the pieces for the mosaic in exactly the right order, from smallest to largest. But the pieces were wrapped in brown paper and all they could see was the labels on the outside – can you help them put them into the right order?

1.65	**16.5**	**1.7**	**1.05**	**1.9**	**0.99**	**1.75**

In contrast to a very commonly used method of introducing decimals the CAME approach encourages the pupils themselves to engage in the construction of a concrete model that can be used to visualise the counter-intuitive nature of digit size within both decimal and fraction notation. Indeed in my own year 4 class I often found the pupils referring back to the lesson and using the resulting display to help them work out decimal problems. The learning in the lesson above follows a pattern that is common to the CAME approach and the diagram below illustrates how the three episodes work within the context of the mathematics involved.

It shows a generic model of progression that takes place in any CAME lesson, and was developed for KS 3 – each lesson contains three episodes of learning. Obviously in KS 1 the pupils will not be exposed to level 7 thinking but the aim of the episodes is the same i.e. to structure the pupil exposure to the development of thinking that moves from a context to the handling of mathematical relationships in preparation to handling abstract ideas such as algebra and ratio by the end of Year 6. The National Curriculum levels refer to the Using and Applying Strand. One significant feature of this episodic structure is that it allows lessons of varying lengths of time to be conducted depending upon the engagement of the pupils or the demand of the school day.

In episode 1 the lesson has a low entry point in both thinking and curriculum terms. The mathematics emerges from the context in which the learning is introduced i.e. an engaging story to hook pupil interest. Episode 2 gives the pupils a concrete and practical handle upon the mathematical thinking involved in the lesson, i.e. the link between the size of a digit and its relative size in decimal notation. Episode 3 then encourages them to apply their new thinking within the context of the mathematics involved i.e. decimal notation.



As many teachers who use the CAME lessons readily testify, it is worth stressing the dominant role played by the pupils themselves in the construction of shared mathematical understanding if CAME becomes a regular part of their mathematical diet. The words of Ian Thompson come to mind at this point:

> 'It took mankind a long time to invent such important ideas and this signals the fact that it is going to prove difficult for children to easily understand them'.

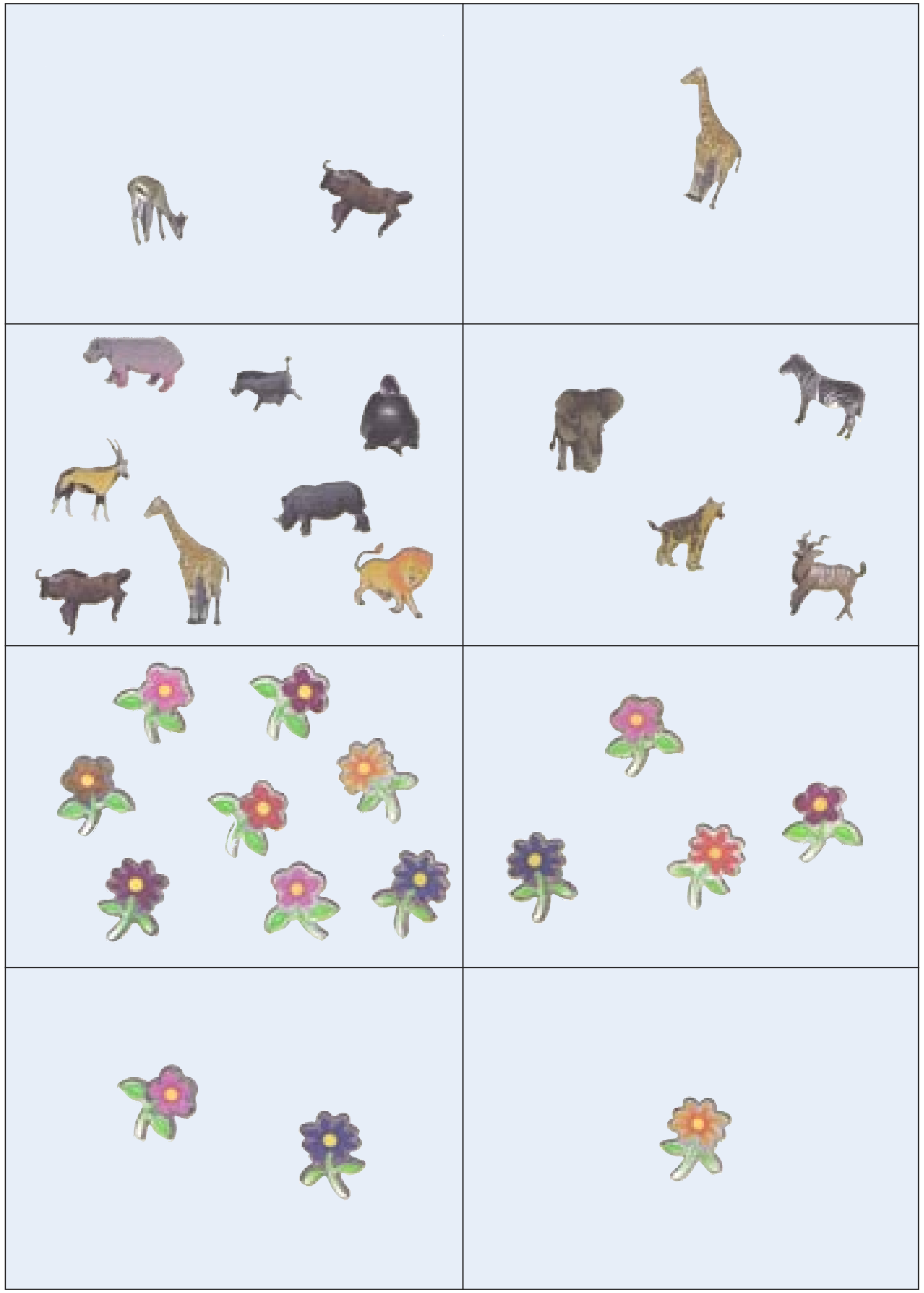
A good case in point is the concept of zero, its common usage did not emerge till the 1500s yet now 5 and 6 year olds are given the idea of zero as an obvious fact. CAME celebrates the notion that the construction of mathematical knowledge is part of what it is to be human and provides pupils with the time and space to reinvent and develop their own mathematical ideas as they explore the world around them. Such thinking foundations can then be built upon during the more common standard mathematics lessons that they experience everyday of their primary lives.

References

For all information regarding the CA approach readers are directed to the website: cognitiveacceleration.co.uk

Ian Thompson (2003) in *Enhancing Primary Maths Teaching*, Berkshire, OUP

Alan Edmiston is a
Cognitive Acceleration (CA) Tutor



Classroom Activity: Funky Fifteens

Rachel Gibbons offers a set of cards useful for starting to sort, count and add, or play a game collecting sets of stars, wild animals, etc.

Enlarge to A4 the sheet below and those on the following three pages, laminate each one and cut up to give 2 sets of 4 cards. Use these 56 cards to collect sets of objects of any number up to and including fifteen or play the game of Funky Fifteens collecting ‘families’ of balloons, wild animals, hearts, stars, etc.

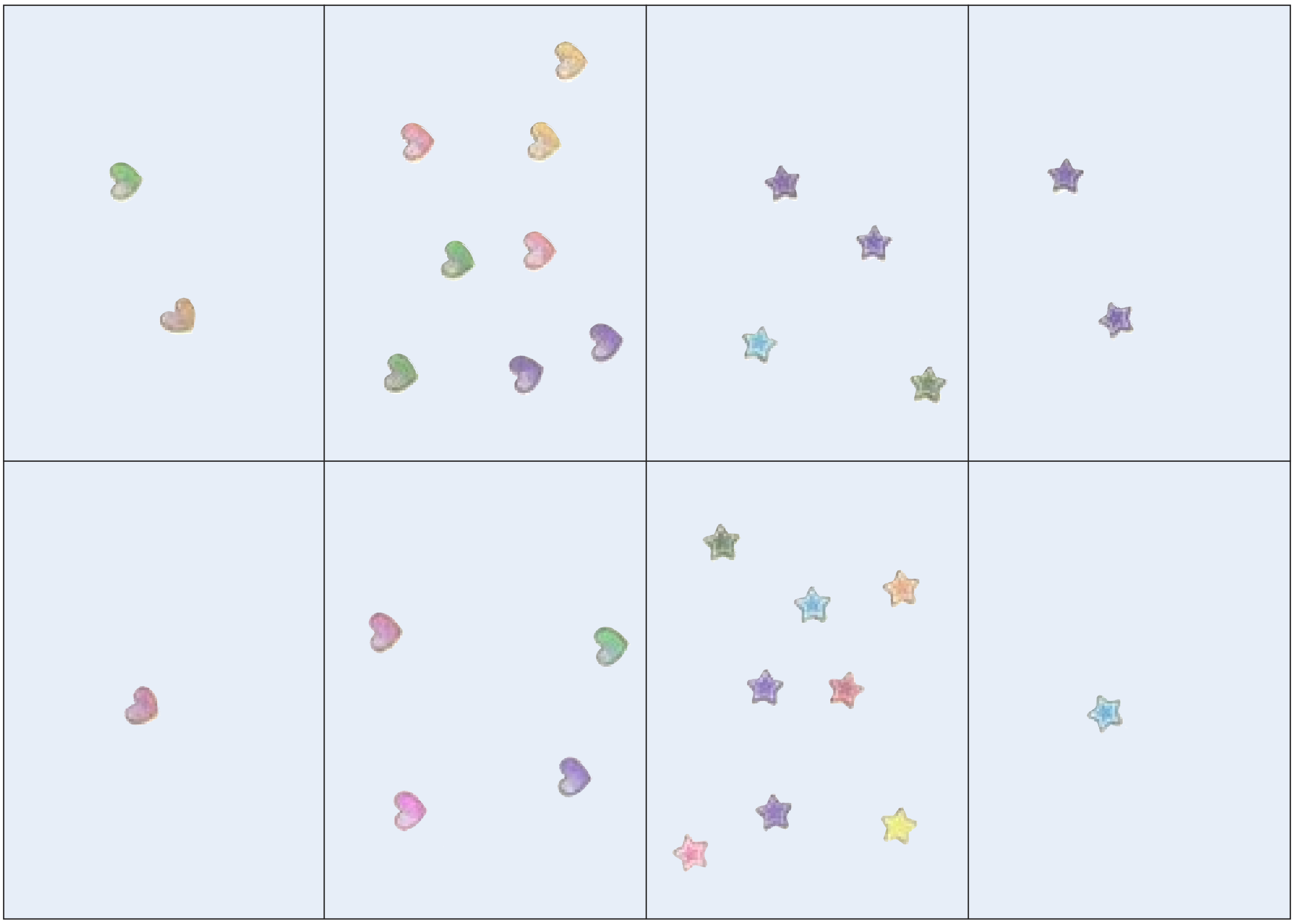
The rules are given below – cut out and laminate these also and keep with the cards to use when playing the game.

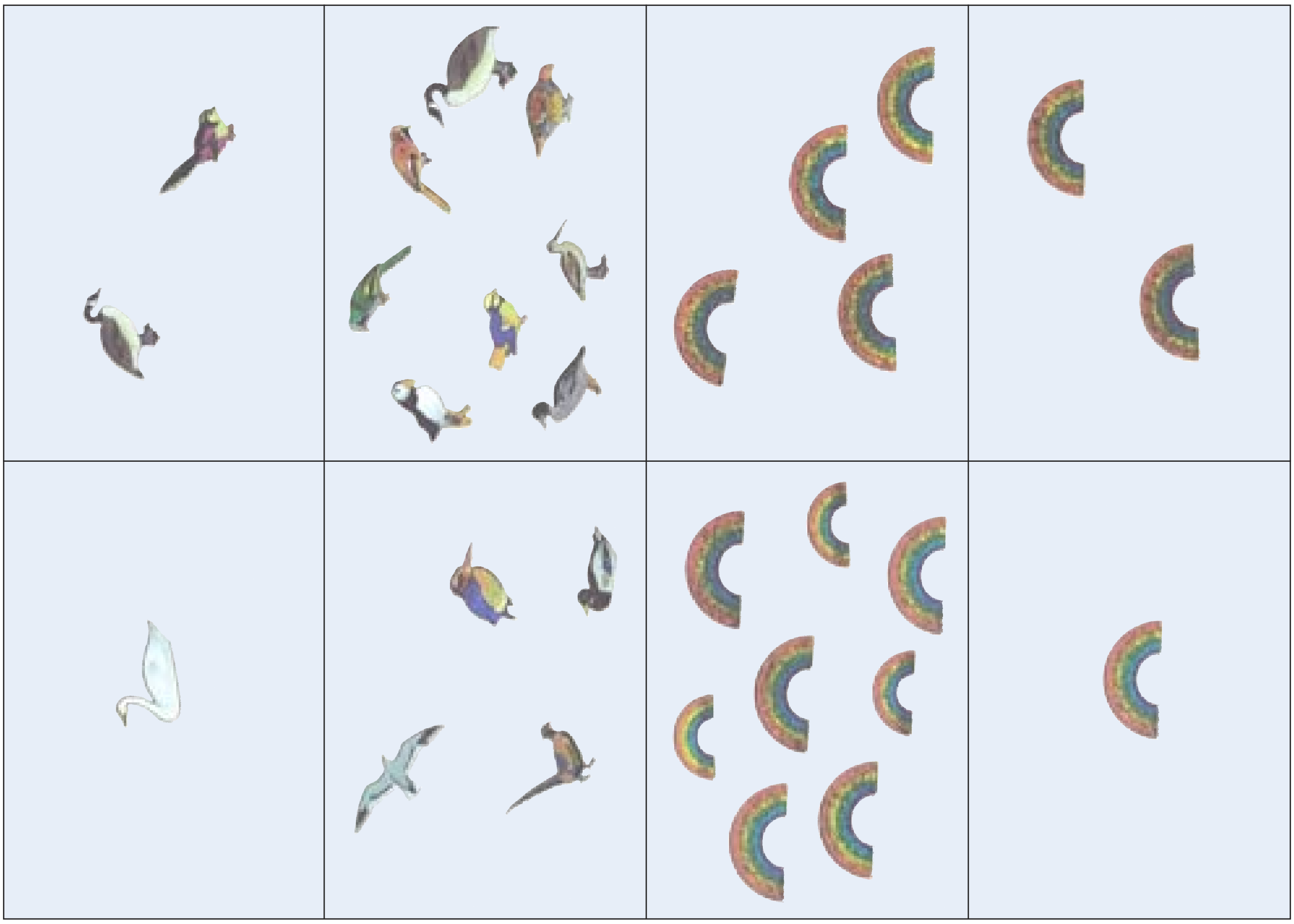
Funky Fifteens

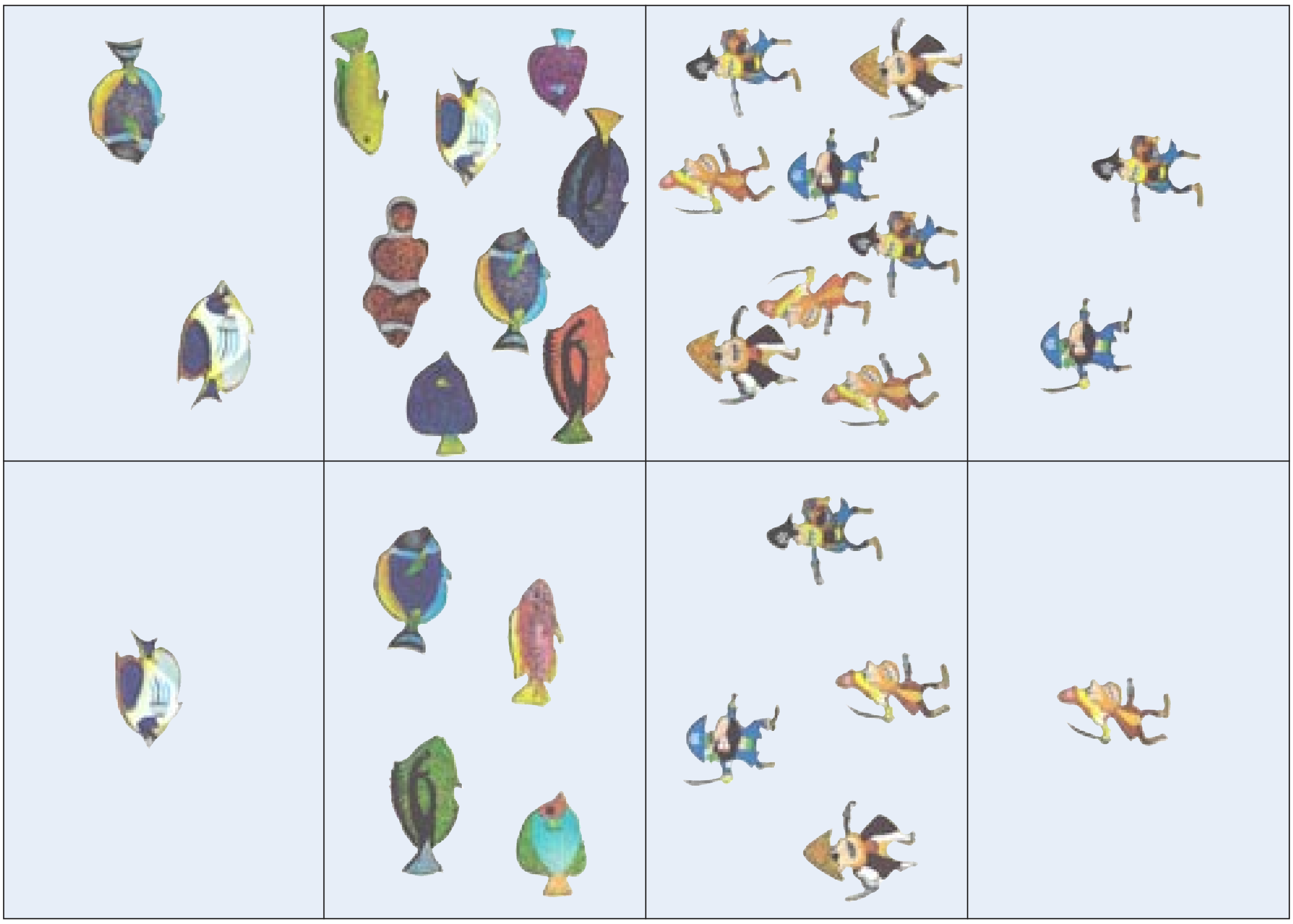
This set of cards can be used for simple sorting, counting and adding activities or for the game of Funky Fifteens which is just like Happy Families only players are collect-ing butterflies, stars, etc., instead of human families with such requests as . . .

“Please may I have 2 hearts.”

“Please may I have 5 balloons.”







Using Dice Here and There

Bernard Bagnall **looks at dice and helps us consider how we can use them to develop children's mathematical understanding from the early foundations of number.**

This is the fifth article in the NRICH series where we share our ideas about how a simple piece of equipment or starting point has the potential to engage learners of all attainment levels and be flexible enough to respond to need. In this article

I guess we are all very familiar with using dice in all kind of environments. Many teachers have their favourite games that incorporate the use of dice and see their pupils enjoying a game while reinforcing some of their basic numeracy skills.

Let's start at the very beginning

Younger children or pupils who need to have their confidence boosted in the world of mathematics are most comfortable with the usual dotted dice. A small group of pupils each roll their own dice (often working on a soft surface to reduce the noise distraction) and have some good worthwhile conversations about what they see.

"What can you see?"
"Have we all got the same?"
"Tell me about yours."

If the teacher also has a dice in front of her she can join in the conversation. The pace of this activity is obviously governed by each contribution, and works well when the teacher doesn't have preconceived ideas of where the conversation will go, although there are lots of prompts that can be appropriate throughout the short session.

"Who has the most dots?"
"Do you think anyone has the same number of dots as me?"
"You have two dots on yours, can we all turn our dice over so we all have just two dots?"
"WOW! Can we do that again?"

So, as the chat goes on, the pupils may feel more confident to add their own thoughts, possibly including ideas that are the basis of number recognition and addition.

A variety of dice

One way of extending the use of dice to help pupils develop their thinking further is to bring in some of the less usual dice – those with a different range of numbers on them. Just showing some 0 to 9 dice in place of the usual 1 to 6 dice is an opportunity for the teacher to ask some open questions.

> "Suppose we swop one of these (1–6) with this (0–9), will that make any difference to the game?"

Then the teacher can go further, depending on the response of the pupil after being given time to think about it. This may lead to some discussion among the pupils as to whether the game will go more quickly, more slowly or much the same. It's always good to add in the option of "the same" when talking with pupils about something being more or being less.

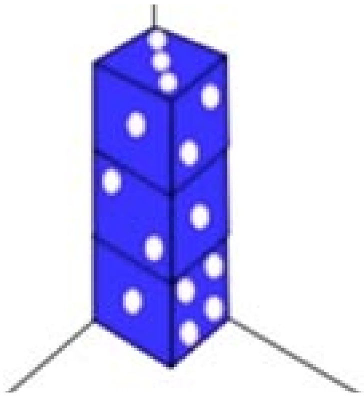
Another dice that can act as a catalyst to this kind of thinking is the dice that goes 1, 2, 3, 1, 2, 3, in place of 1–6. Try it out – you may be surprised! Look out for dice with different numbers of faces often with some quite large numbers. I once had a dice the size of a billiard ball that had 100 faces!

Another way of getting into the larger numbers is by having two 0–9 dice of different colours so that one will determine the tens digit and the other the units.

At NRICH we have a number of activities that rely upon the use of dice. We try very hard to have activities that have a Low Threshold and a High Ceiling (LTHC), so that pupils who are struggling can get into the activity and get somewhere, whereas the confident ones can easily be extended through the activity. You can read more about this type of activity in the article "Using Low Threshold High Ceiling tasks in ordinary classrooms' http://nrich.maths.org/7701

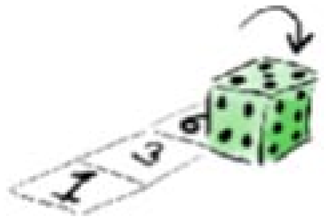
Developing spatial awareness

Returning to the usual dice with dots on there are useful activities that can be used with pupils not only for counting but also to help development of spatial awareness. Imagine some dice placed in a corner. What totals do the dots you can see make?



It's not too difficult to count the dots as long as long as the starting point is remembered, (something which seems to develop slowly with some pupils!). So, in the example above we have 14 dots visible. You could offer the challenge of finding the lowest/highest numbers possible, or asking if the same totals could be achieved with different faces showing – all these can lead to a thorough investigation. Some pupils could just use one dice to start with whilst others could go straight into using three or even more.

Looking at the numbers (whether dots or numerals) on the faces of the dice can be explored by going to the NRICH activity Taking a Dice for a Walk.



This does, however, require some good fine motor skills unless you have one of those lovely very large foam dice measuring about 20 cm! Sometimes we have pupils who do seem to be very slow at the numbers side of mathematics but who seem to be coping much better with the spatial, so moving from Taking a Dice for a Walk to the interactivity Roll the Dice could be a good challenge. Using this it's possible to look at finding all the possible routes.

Exploring probability

Before finishing the most common use of dice in schools apart from games is exploring probability. A simple introduction to this can be found in A Bit of a Dicey Problem and the game Tricky Track.

Many of these activities support children's development of useful recording, and their proficiency in working systematically, both very worthwhile mathematical skills.

> When you throw two regular, six-faced dice and add the two numbers, you have more chance of getting one particular result than any other.
>
> What result would that be?

Enjoy using the dice! Try and get hold of as many different ones as possible – there are dice within dice, spherical dice and blank ones for you to write on.

Some pupils will benefit from the large foam dice, referred to earlier, and there is also a large one that has transparent pockets on each face so that you can insert whatever is appropriate in the pocket for the particular activity.

To find NRICH activities follow the hyperlinks or enter the title in the search bar at nrich.maths.org. Activities that are extended by offering alternative dice are Tug of War and Tug Harder.

The use of dice when Using and Applying with simple mathematical operations can be found in the following NRICH activities:
Dice and Spinner Numbers, Roll these Dice, Scoring with Dice

Bernard Bagnall is a member of the NRICH team at Cambridge University

Doing Maths?

Matt Parker explains what he means by "doing maths".

"It is also important to motivate everyone . . . even those who may never do maths again." This is a quote from myself, taken from the article "Making Mathematics Measure Up", published in the *TES.* Ray Gibbons was worried about my implication that some people will not do maths post-schooling and drew attention to this in a recent Equals editorial, saying "But we are all 'doing maths' for most of our waking hours and Matt Parker should be helping people to see this."

I strongly believe that by over-emphasising the extent to which people are 'doing maths' we are doing a disservice to the subject. By stretching the definition of 'doing maths' too far, we devalue what maths is and why it is so important. Maths is an extremely useful subject – our modern world depends on it – but we often blur the lines of what "Useful Maths" really is.

In my view, there are two "Useful Maths Fallacies" that regularly appear in maths education. The first is that because mathematics is everywhere, people are constantly doing it in their everyday lives. While it is absolutely true that mathematics is unreasonably effective at describing the physical world around us, that doesn't mean we need to know that maths. Just because something you're doing can be elegantly described mathematically, or our innate human behaviour follows some mathematical logic, does not mean we are doing maths. Young children are no-more using maths to sort their toys into boxes than they are using biology to go to the toilet.

We cannot sell maths to pupils on the basis that they are already doing it for all their waking hours. They are doing things for all of their waking hours that can be described mathematically; their innate behaviours may lead them to act according to mathematical principles; but this does not mean they are doing mathematics.

The second is that using technology which uses maths means that you yourself are using maths. This is simply not true, and to try and tell students that they need to learn vectors because they are vital to 3D animation and video games is counter-productive. Students know they can watch a 3D movie without knowing about vectors and we, as maths teachers, look like we're desperately fabricating false uses for mathematics.

There is no questioning that our modern world is built on a foundation of of mathematics. We are constantly using technology that requires mathematics to function. Importantly though, this does not mean we are doing mathematics ourselves. Part of the genius of modern technology is that the user does not need to know what clever calculations are going on behind the scenes. People with zero mathematical knowledge can still benefit from the technology maths makes possible. You no-more need to understand equations to send a text message than you need to understand Newton's gravity equations to not drift off the planet.

Because of the incredible usefulness of mathematics to humankind as a whole, there is an obsession to convince school students that it will also be incredibly useful for them as well. This simply is not true.

As maths teachers know: the vast majority of mathematics taught at secondary school will be of no real use to students in their future day-to-day lives. By the time they've finished Key Stage 3 at age 14, they will have covered most of the functional mathematics they will ever use. Once they're into Pythagoras and beyond, we are no longer teaching them sufficiently-useful life-skills worthy of compulsory education.

This does not mean we should not teach them mathematics. So little of what constitutes a well-rounded education to prepare students for an active and fulfilled role in society has to be functional skills. Other subjects embrace this. English departments don't convince kids they'll need to be able to deconstruct a text to be able to read it, but they do know it will give them a deeper understanding and appreciation. Maths education is not that different.

There are plenty of reasons to teach students abstract mathematics; the development of logical reasoning and thinking skills alone would justify it as a subject. But Mathematics does also have the burden of being vital to modern society and we do need to produce enough future mathematicians, scientists, engineers, medical researchers and so on to keep pushing technology forward. These future maths-users are in our schools system right now, and it is absolutely worth getting all students engaged in mathematics to hedge our bets of finding them.

For students who are struggling to attain at mathematics, it is of course useful to inspire them by demonstrating the role maths plays in their favourite gadgets. It's also useful to encourage them with how much mathematical behaviour they, as humans, already exhibit. But it is important we do not over-step the line and try to convince them they need to learn more maths for either of those reasons.

We cannot sell mathematics as a straw-subject. If we pin the whole point of maths lessons on how useful it will be for students, then we are creating an obvious fiction that will put many pupils off the subject. If we acknowledge how practically useful the subject is to society as a whole, and how abstractly useful it is to individuals, then I believe we will have more engaged students of all abilities.

It is indeed important that we motivate everyone to learn mathematics, even those who will not directly use it again.

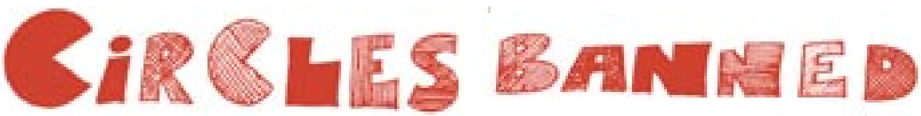
Matt Parker lectures at Queen Mary College, University of London

Assessing Understanding? Try 'Maths-Fiction'

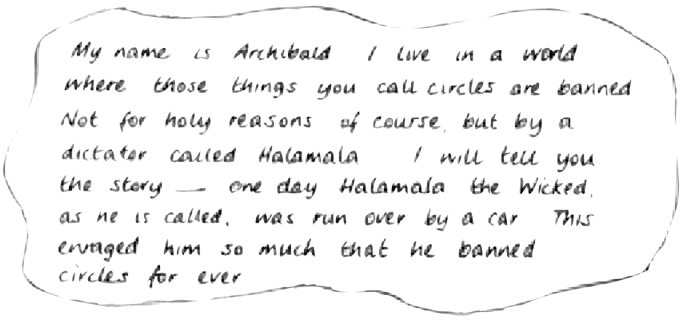
Rachel Gibbons suggests a way of discovering what children think about some geometrical concepts.

To help children progress in any area of the curriculum it is important to discover what they know already. With the experience of the science-fiction of Dr Who behind them, one way of making such discoveries with respect to mathematics is to set them writing 'maths-fiction'. I once learnt much about the geometrical knowledge of a class by inventing **A World without Rectangles** and a kingdom with the edict **Circles Banned.** Such worlds are of course not a new idea – Edwin Abbott created 'Flatland' way back in 1884 (on searching the web I find a film was made of his book as recently as 2007).

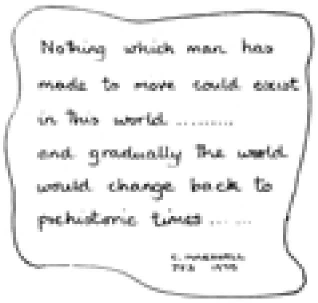
All members of my class had a wonderful time describing strange lands where these shapes were missing – you can see some of their ideas below. Much ingenuity was shown, particularly when writing of a world where circles were banned, in inventing curves of constant width to use for wheels. Some in their worlds without rectangles had cobbled streets because, rectangles being plane shapes, they banned not only rectangles but planes. One girl presented her writing in a notebook which she had cut into a new shape so that it did not have rectangular pages.

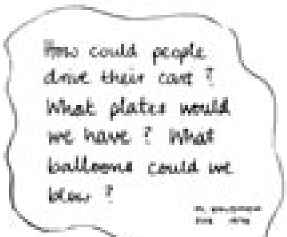


M. Lewis from JFS School wrote:



What would be the effect of Halamala's ban?





How do **you** think the world would change?

1. List things that are circular.
2. Now consider each object on your list:
   a. What other shape could it be?
   b. Would it work as well in this new shape?

Draw diagrams wherever they make your meaning clearer.

This picture appeared in '*The Times*' newspaper in 1970.