Equals

Realising potential in mathematics for all

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Some more fascinating mathematics in unusual places

MATHEMATICAL ASSOCIATION

supporting mathematics in education

Realising potential in mathematics for all

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Designed by Nicole Lane

Editors' Page

We must first apologise for the lateness of this issue of *Equals* which should have been with you some two months ago but has been delayed because of illness and other problems. Now we can get down to business..

In her article on the Eurydice Report (Making Mathematics Measure Up, *TES* 27.01.12) Helen Ward quotes Matt Parker of Queen Mary College:

> "It is also important to motivate everyone …
> even those *who* ***may never do maths again.***"
> (Italics mine.)

But we are all 'doing maths' for most of our waking hours and Matt Parker should be helping people to see this. It is surprising how often mathematical activity is not recognised as such. I remember not long ago reading a comment in one of the newspapers – I forget now which - encouraging readers not to be afraid of the Sudoku puzzles elsewhere on the page because *there was no maths involved in them* - but of course there was! Such puzzles are pure logic and logic is a branch of mathematics. Looking back to beginnings, children's mathematical activity starts very early; they are 'doing mathematics' as they sort their toys, or other objects they find around them, into sets with members having well defined characteristics (and they find this great fun).

Nick Gibb in the same 27 Jan issue of the *TES* suggests that there should be a review of calculator use in schools saying that pupils "can't cope with complicated quadratic equations if they don't know their times tables." The argument about the possible dangers of the use of calculators was, I thought, settled back in the 1980s when Michael Girling defined numeracy as being able to use a four function calculator *sensibly*. Indeed, the calculator, if used strategically, can be a valuable resource for extending children's understanding of number. Furthermore, although it is certainly useful to 'know your tables' it is even more important to understand and remember some more basic number bonds and patterns which will help to fill the gaps in the tables knowledge if and when they appear.

Mike Askew and Rob Eastaway in their *Maths for Mums and Dads* (reviewed in this issue) give many fascinating examples of mathematics cropping up in ordinary everyday life, for example:

> QUICK TIP
> Pastry cutters are usually round, which leaves lots of waste dough or pastry that has to be re-rolled. Why not make tessellating biscuits instead? These days you can find pastry cutters that are triangular, diamond shaped, even hexagonal. Make some hexagonal biscuits without having to waste any of the dough (except around the perimeter). You can do the same with plasticine, but it is more satisfying when you get to eat the finished product.

Noting where mathematics occurs in their everyday lives outside the mathematics classroom, beyond just calculating and measuring, would be a useful activity for teachers to encourage in mathematics lessons - and might well dispel some of the fear that the subject sometimes conjures up. Why not try it with some of your classes?

There seem to be several numeracy campaigns around today some of them run by private companies sheltering behind titles which sound as if they are government backed initiatives – it is as well to look into the origins of whatever scheme you come across.

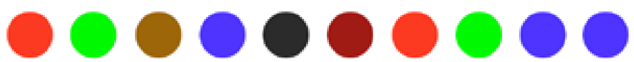
In this issue we are renewing an initiative which has proved worthwhile in the past. We are, together with *Primary Mathematics*, once again offering the Harry Hewitt memorial prize (see page ?). There are plenty of prizes out there for those who achieve easily but too rarely is notice taken of hard work and struggle in face of difficulty. Indeed it is interesting to contemplate that struggle is respected so little that this journal, which started life in the Inner London Education Authority with that title, had to be renamed before anyone else was willing to sponsor it.

Peg and Pin Boards

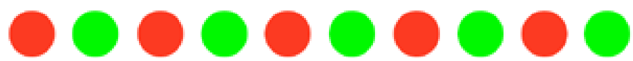
In this fourth article in the NRICH series Bernard Bagnell shares their ideas about how a simple piece of equipment or starting point has the potential to engage learners of all attainment levels and is flexible enough to respond to need.

Beginning with peg boards and developing ideas around pattern

Let's have a look at the activities that you can use with peg boards. They seem to be accessible to quite young children once they have the fine motor skills to handle the little pegs. You can start with them having total freedom to place the pegs wherever they wish. Maybe you would see they created something like this;



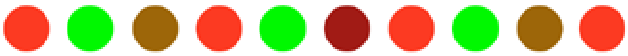
In my experience it's not long before they are placing the pegs in some kind of pattern. When working alongside a child who has started making a pattern we have to sometimes restrain ourselves from pushing the activity to advance quicker than the child requires. So when we see something like this



encouragement and praise are the order of the day.

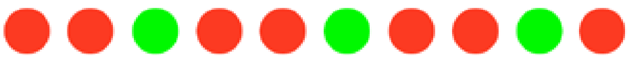
It may be appropriate to join in, (probably on another board next to theirs if the children are happy with that), and create a pattern that is similar to the ones that they have produced. I use the phrase "to dangle a carrot" next, when you are testing the ground to see if the child may move on a little at a time. If the child does not take "the carrot' then we probably have to acknowledge that this is where they are at and they need a good time to consolidate and feel true ownership of what they are doing.

The "carrot" will vary according to what they have been doing. It is likely that their first patterns have been simple and they have been able to say it – "red, green, red, green. . .". If that is the case I would start by saying, "I'm going to make one like yours but this time I'm going to use three colours". Their response determines what happens next and, of course, there is no set rule or dialogue to be had, we are just being the "co-player". (A "co-player" being one who is able to enter the child's play with sensitivity and unobtrusively in order to develop their talk and learning.[1]) So you create something like this;

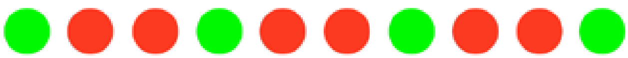


When a child has started much more confidently an alternative "carrot" could be introduced by a conversation starter such as; "Tell me about this," pointing to their pattern. They may respond with something like "It goes red, green, red, green ..." and they tell you confidently about all the other patterns they have done which you see as being similar.

You might follow this up with a comment like, "I like that, may I do the same and have a extra 'red' in each time?" So you generate a pattern like red, red, green, red, red, green, red, red . . . . The child may then take "the carrot" and produce something similar.



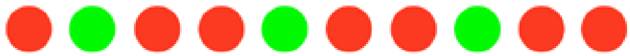
The process goes on and may end up with some patterns like these.



Then there is an opening for all kinds of good talk about what the child has done and sees in what they have produced.

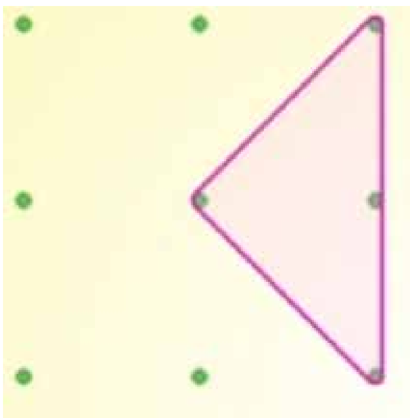
I have gone into a lot of detail with that first example with the strong belief that IT'S NOT WHAT YOU DO IT'S THE WAY THAT YOU DO IT. Let's have a look at a pin/nail board activity for children a bit further on in their development.

Taking it further and using pin boards



Some schools have pin boards with the nails now replaced by plastic. The pins/nails are often much too near to each other to be most useful. Some other schools use a whiteboard interactivity or a computer simulations. At NRICH, we have an interactivity which can be either circle or square based. This can be found at "Virtual Keyboard" found here.

With a small setup with just a 3 by 3 or 5 by 5 pin grid a lot of experimentation can go on with making triangles or squares. Providing the manipulation is not too difficult it's very pleasing for the children to create so many different triangles. After a time to experiment you may choose to come alongside the child observing a recent triangle like:



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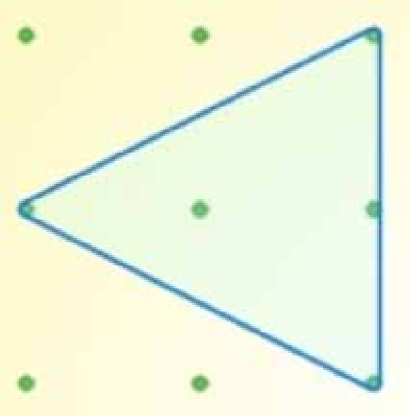

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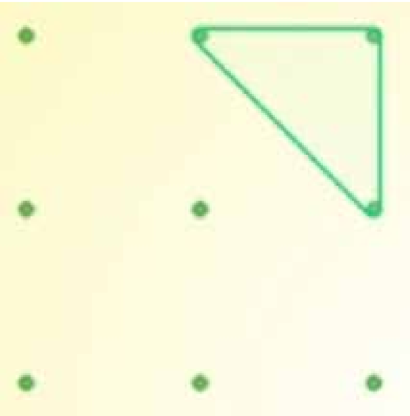
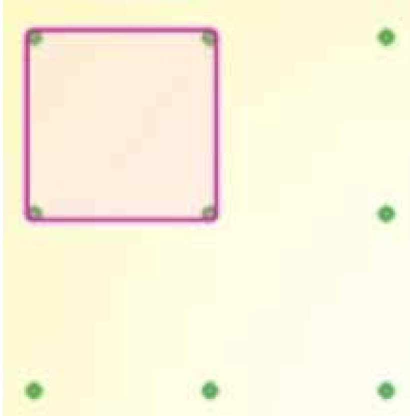


which easily leads us into a conversation with them about the triangle that they have created. It may be useful to have a record of the one they've just made. You may be able to do that with the technology available or else have some dotted paper cut to size that allows you to quickly draw it. It's good to tell them that you are going to make a copy of it, and when you've done it to check it with them that they are happy with it. [I feel that it is important that you are using paper that only shows the number of pins that the pupil has available.]

Questions like "Are there others that might be like this one that you could make?", "Tell me about the triangle," and "Could you make a smaller/bigger one?" all have the potential of engaging with the child in an encouraging way.

You might be going for squares instead, in this case something very similar can occur;



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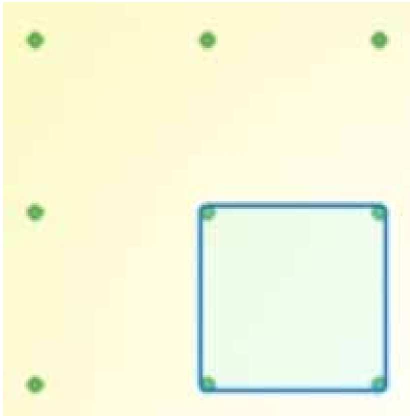

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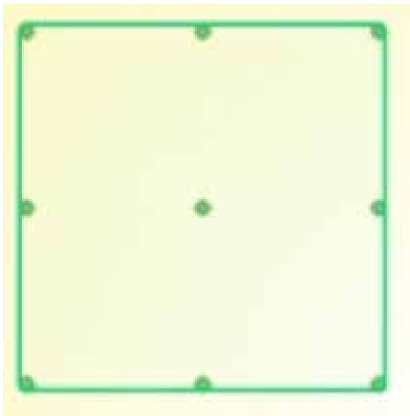
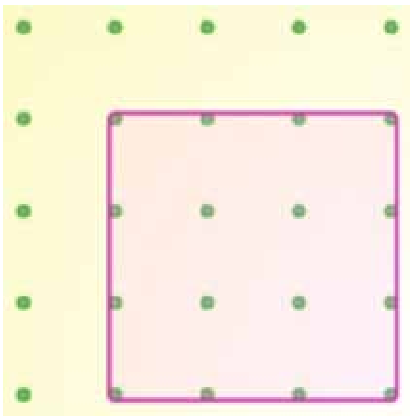


You may decide that a 5 by 5 grid is more useful for squares. So you leave the child to explore and come alongside and talk about their latest creation.



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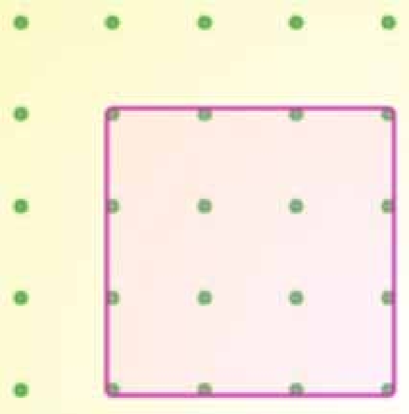
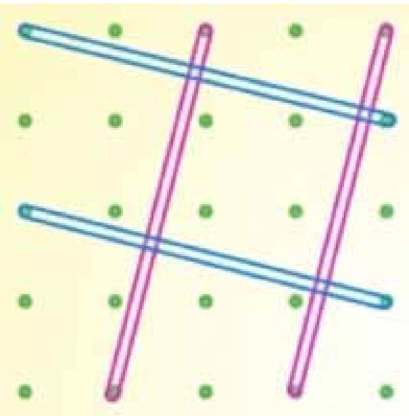


Recording and conversation can go along the same lines as with the triangles.

You may feel that some children could be introduced to the ideas shown below for generating squares.



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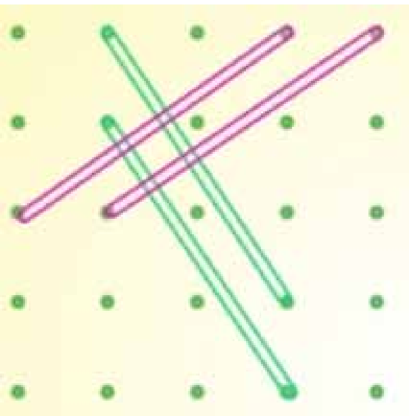
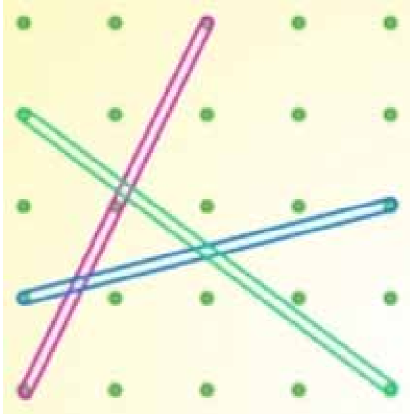


The same idea can of course be used for generating triangles.



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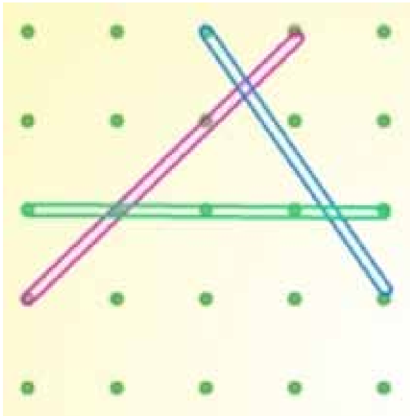


Activities associated with the use of pin boards can be found on the NRICH website.

Board Block

Board Block Challenge

Happy Halving

Inside Triangles

Quadrilaterals

Nine-pin Triangles

Triangle Pin-down

Tri.'s

Geoboards

I would imagine that some, if not all of these would be an excellent resource when helping children develop their concepts of shape and space.

Bernard Bagnall is a member of the NRICH team at Cambridge University

1. R Bayley & L Broadbent in *Like Bees not Butterflies,* Ed by S & P Featherstone A C Black 2010

Reflections on *On Being the Right Size* and the challenges of teaching and learning mathematics in context

Children need a range of environments in which to learn *Using and applying mathematics*, Mary BJ Clark reminds us. It is clearly not enough to learn mathematical knowledge and skills but not be able to apply this learning.

It is a lack of understanding of how to apply mathematics that means that many adults do not recognise the mathematics that they need even for everyday living beyond perhaps being able to use money for a range of transactions. An amusing question to explore at the end of a mathematics lesson is what mathematics the children think they have been learning. I have often found myself faced by a collection of puzzled faces assuring me that we have not been doing real mathematics as number and calculation have not been to the fore.

A challenge for teachers is to plan opportunities for Using and applying mathematics

- within the subject of mathematics itself
- in the context of other subjects in the school curriculum
- in the world beyond school.

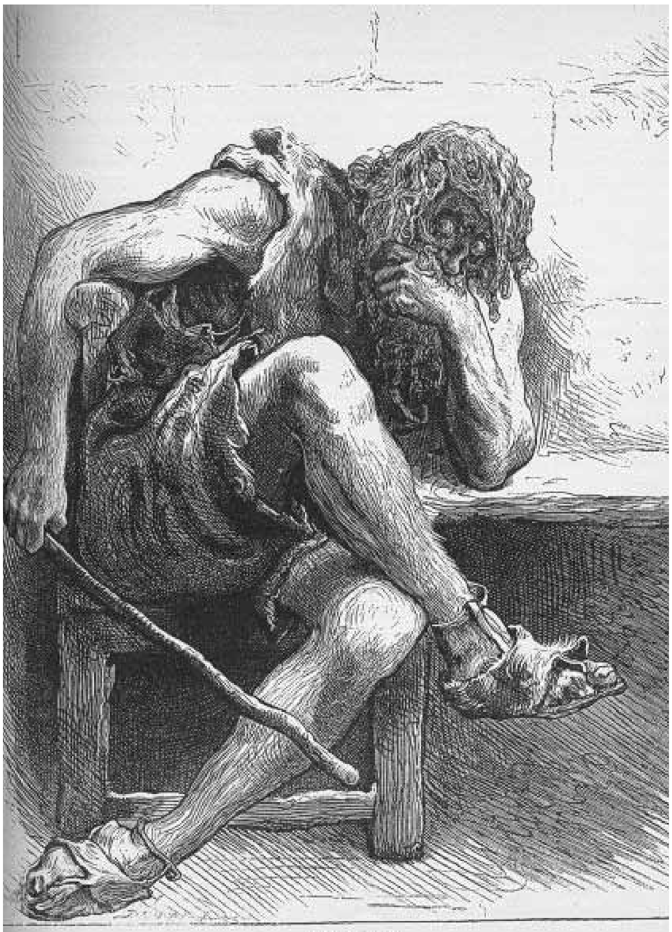
I was recently introduced to an essay, On Being the Right Size, written by a scientist called J. B. S. Haldane. This essay was originally published in 1928 and discussed size in the natural (biological) world and systems. The whole text can be found at http://irl.cs.ucla.edu/papers/right-size.html and browsing on the internet quickly reveals the range of very current thinking that Haldane's work and the theme of this essay continue to inspire. This is intriguing.

However returning to my search for suitable contexts for mathematics learning I have found in the essay a variety of applications of mathematics knowledge and skills that have given me ideas for working with children, particularly on aspects such as area, volume and mass with an application to biology.

The following paragraphs are taken from the first part of the essay. In the frames embedded in this text I have included some suggestions for activities using and applying mathematics that Haldane's comments have suggested to me.

'The most obvious differences between different animals are differences of size, but for some reason the zoologists have paid singularly little attention to them. In a large textbook of zoology before me I find no indication that the eagle is larger than the sparrow, or the hippopotamus bigger than the hare, though some grudging admissions are made in the case of the mouse and the whale. But yet it is easy to show that a hare could not be as large as a hippopotamus, or a whale as small as a herring. For every type of animal there is a most convenient size, and a large change in size inevitably carries with it a change of form.'

‘Let us take the most obvious of possible cases, and consider a giant man sixty feet high—about the height of Giant Pope and Giant Pagan in the illustrated Pilgrim’s Progress of my childhood.



Giant Despair.

These monsters were not only ten times as high as Christian, but ten times as wide and ten times as thick, so that their total weight was a thousand times his, or about eighty to ninety tons. Unfortunately the cross sections of their bones were only a hundred times those of Christian, so that every square inch of giant bone had to support ten times the weight borne by a square inch of human bone. As the human thigh-bone breaks under about ten times the human weight, Pope and Pagan would have broken their thighs every time they took a step. This was doubtless why they were sitting down in the picture I remember. But it lessens one’s respect for Christian and Jack the Giant Killer.’

Light and heavy sticks

Resources:

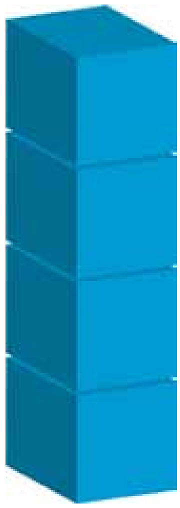
Interlocking 1cm or 2cm cubes

Learning objectives:

To use the language of size

To make comparisons between measures such as length, volume and mass.

Make a stick out of interlocking cubes (the 2cm edged ones are best for this), for example,



and stand the model up on its base cube.

Now increase the height of the stick by a factor, such as 2 or 5 or 10. Try to stand the model on its base cube. Reflect on the stability of the model.

What happens to the stability of the shape if it is not only 2 (or 5 or 10) times as high, but also 2 (or 5 or 10) times as wide and 2 (or 5 or 10) times as thick?

If you multiply the width and thickness of the stick model by a factor of 2, how many of the original models do you need to make the new one?

What happens to the masses of the models compared with the original stick model?

'To turn to zoology, suppose that a gazelle, a graceful little creature with long thin legs, is to become large, it will break its bones unless it does one of two things. It may make its legs short and thick, like the rhinoceros, so that every pound of weight has still about the same area of bone to support it. Or it can compress its body and stretch out its legs obliquely to gain stability, like the giraffe. I mention these two beasts because they happen to belong to the same order as the gazelle, and both are quite successful mechanically, being remarkably fast runners.'

Comparing animals

Resources:
Model animals made to the same scale – mammals such as gazelle, rhinoceros, giraffe

Learning objectives:
To use the language of size
To make comparisons between measures such as length and area

Make use of the models to ask questions and encourage reflection on the attributes of the models in line with the thoughts recorded in Haldane's text.

'Gravity, a mere nuisance to Christian, was a terror to Pope, Pagan, and Despair. To the mouse and any smaller animal it presents practically no dangers. You can drop a mouse down a thousand-yard mine shaft; and, on arriving at the bottom, it gets a slight shock and walks away, provided that the ground is fairly soft. A rat is killed, a man is broken, a horse splashes. For the resistance presented to movement by the air is proportional to the surface of the moving object. Divide an animal's length, breadth, and height each by ten; its weight is reduced to a thousandth, but its surface only to a hundredth. So the resistance to falling in the case of the small animal is relatively ten times greater than the driving force.

An insect, therefore, is not afraid of gravity; it can fall without danger, and can cling to the ceiling with remarkably little trouble. It can go in for elegant and fantastic forms of support like that of the daddy-longlegs. But there is a force which is as formidable to an insect as gravitation to a mammal. This is surface tension. A man coming out of a bath carries with him a film of water of about one-fiftieth of an inch in thickness. This weighs roughly a pound. A wet mouse has to carry about its own weight of water. A wet fly has to lift many times its own weight and, as everyone knows, a fly once wetted by water or any other liquid is in a very serious position indeed. An insect going for a drink is in as great danger as a man leaning out over a precipice in search of food. If it once falls into the grip of the surface tension of the water—that is to say, gets wet—it is likely to remain so until it drowns. A few insects, such as water-beetles, contrive to be unwettable; the majority keep well away from their drink by means of a long proboscis.'

'But it is time that we pass to some of the advantages of size. One of the most obvious is that it enables one to keep warm. All warm-blooded animals at rest lose the same amount of heat from a unit area of skin, for which purpose they need a food-supply proportional to their surface and not to their weight. Five thousand mice weigh as much as a man. Their combined surface and food or oxygen consumption are about seventeen times a man's. In fact a mouse eats about one quarter its own weight of food every day, which is mainly used in keeping it warm. For the same reason small animals cannot live in cold countries. In the arctic regions there are no reptiles or amphibians, and no small mammals. The smallest mammal in Spitzbergen is the fox. The small birds fly away in winter, while the insects die, though their eggs can survive six months or more of frost. The most successful mammals are bears, seals, and walruses.'

Take 24 cubes

Resources:
Interlocking 2cm cubes
cm or 2 cm squared paper

Learning objectives:
To visualise and build cuboids.
To explore volumes and surface areas.
To make links to factors.

From 24 (or another number as you choose) interlocking cubes, make all the different cuboids you can.
Have you found all the possibilities? How do you know?

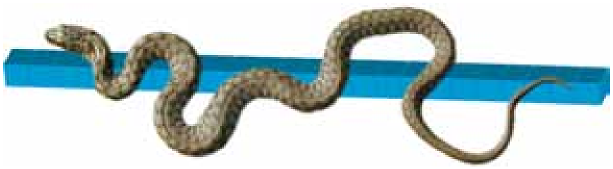
(Opportunity to discuss factors of the starting number, 24, if appropriate. Also to develop the use of a notation to record cuboids' dimensions, for example, 4 x 3 x 2.)

What are the volumes of your cuboids?

What is the surface area of each of the cuboids?

Imagine these cuboids are animals. Make a skin for each animal using centimetre-squared paper.
Which animal has the largest surface area? Which the smallest?

(The cuboid with the largest surface area is the 24 x 1 x1 shape, the shape most like a snake. This creature living in warm places needs a large surface area to volume ratio to help control its temperature.
The cuboid with the smallest surface area to volume ratio is the 4 x 3 x 2 shape which one might describe as a polar bear, that is compact for its volume to help retain warmth in a cold climate.)



Mary BJ Clark is a mathematics adviser

Pages from the Past;

A Victorian Schoolroom

In a year of celebration of Charles Dickens it seems appropriate that we should look at one of his pictures of schooling. This extract reminds us that however our present rulers are degrading the education system it has certainly moved on since Victorian times. David Copperfield describes his first view of Mr Creakle's establishment thus:

I gazed upon the schoolroom into which he took me, as the most forlorn and desolate place I had ever seen. I see it now. A long room with three long rows of desks, and six of forms, and bristling all round with pegs for hats and slates. Scraps of old copy-books and exercises litter the dirty floor. Some silkworms' houses, made of the same materials, and scattered over the desks. Two miserable little white mice, left behind by their owners, are running up and down in a fusty castle made of pasteboard and wire, looking in all the corners with their red eyes for anything to eat. A bird in a cage very little bigger than himself, makes a mournful rattle now and then in hopping on his perch, two inches high, or dropping from it; but neither sings nor chirps. There is a strange unwholesome smell upon the room. Like mildewed corduroys, sweets, apples wanting air, and rotten books. There could not well be more ink splashed about it if it had been roofless from its first construction, and the skies had rained, sowed, haled and blown ink through the varying seasons of the year.

The Harry Hewitt Memorial Prize

The prize is being offered again this year so –

Do you have a pupil who has struggled with mathematics and is now winning through?

If so your pupil could be this year's winner

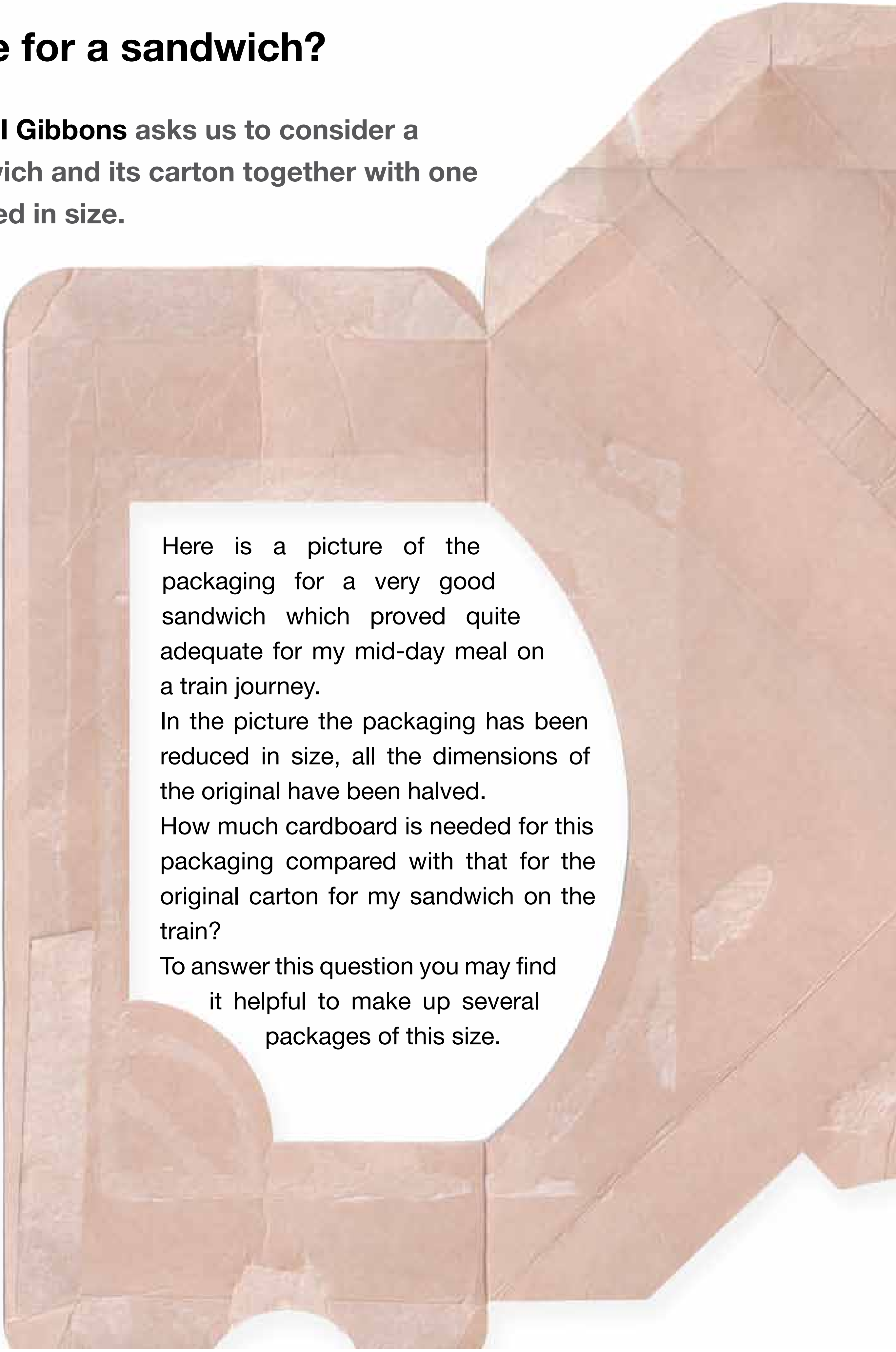
Send to *Equals* a piece of work which you and your pupil consider successful together with:

- your explanation of how it arose
- a description of the barriers that had to be overcome in doing it
- the pupil's age, year in school and the context of the class to which she/he belongs

Entries must be submitted by 31st July 2012

Time for a sandwich?

Rachel Gibbons **asks us to consider a sandwich and its carton together with one reduced in size.**



Here is a picture of the packaging for a very good sandwich which proved quite adequate for my mid-day meal on a train journey.

In the picture the packaging has been reduced in size, all the dimensions of the original have been halved.

How much cardboard is needed for this packaging compared with that for the original carton for my sandwich on the train?

To answer this question you may find it helpful to make up several packages of this size.

If a group of 4 of you work together each making up a carton it may help you to consider how many of these reduced cartons will hold enough food for lunch on the train. Do you think I would have been satisfied with 2 of these reduced sandwiches on my journey? If not how many would I have needed?

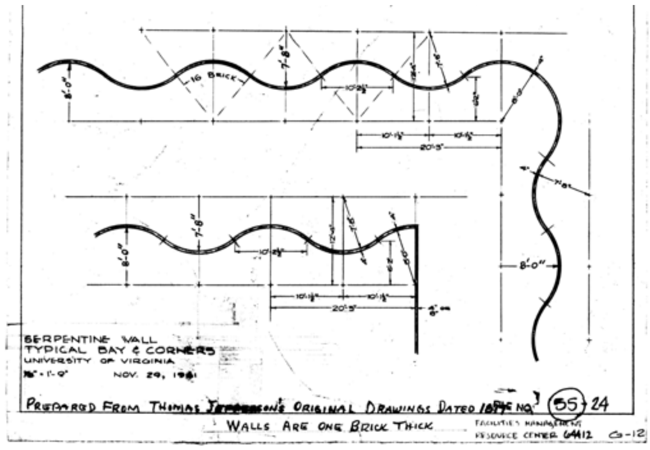
And how about the amount of cardboard needed for the packaging of this sandwich compared with that used for the original?

Mathematics in Unusual Places 5 – Serpentine Walls

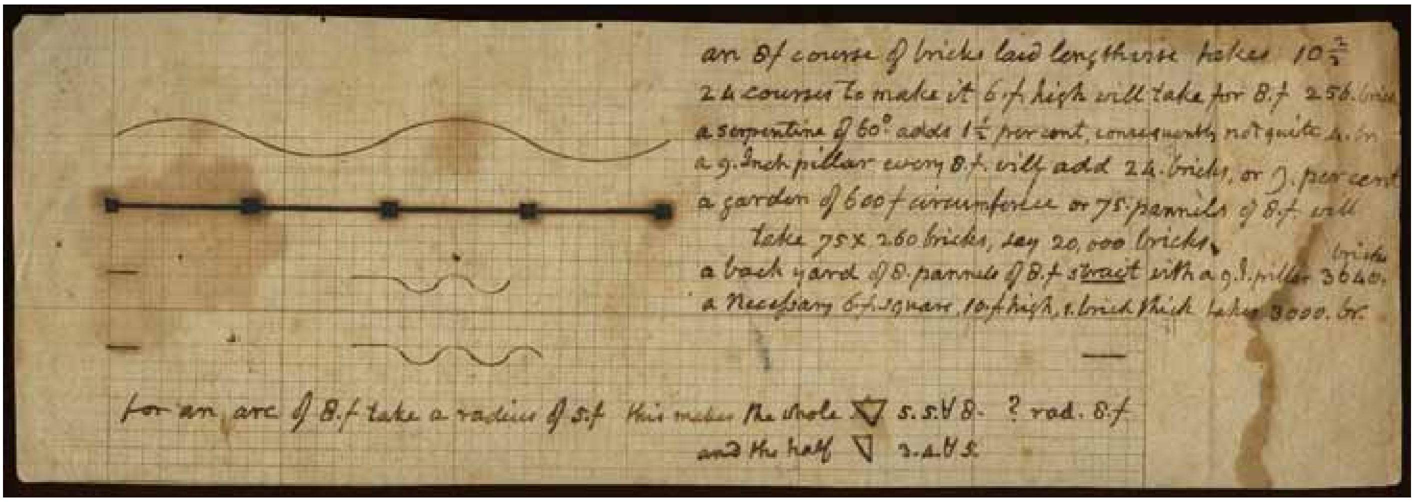
Matthew Reames recently moved from the UK to the United States to study for a degree in mathematics at the University of Virginia where he has found some more fascinating mathematics in unusual places.

I recently moved from the UK to the United States to study for a degree in mathematics education at the University of Virginia. The University was established in 1819, just before University College London, King's College London and Durham University. Founded by Thomas Jefferson, the University along with nearby Monticello are designated a UNESCO World Heritage site. The original Academical Village is still in use today. At the north end of the long, terraced Lawn, lined on each side by 27 small rooms and 5 larger Pavilions, is the Rotunda, a large, domed building inspired by the Pantheon in Rome.

An interesting feature of many of the garden walls of the buildings along the Lawn is their serpentine walls. Though not designed by Jefferson (and probably inspired by similar walls he saw in England), he utilised these walls in his architectural plans, though not simply for their pleasant appearance. The curved shape of the wall's course gives strength to the wall and allows the wall to be just a single brick thick. This allowed considerable savings of bricks compared to a straight wall that would require a thickness of two bricks.



University of Virginia, Facilities Management, Resource Center



Jefferson drawing for the serpentine walls, Accession #1871, 1871-a, Special Collections, University of Virginia Library, Charlottesville, Va

Working one step at a time

One surviving document shows Thomas Jefferson's sketches and calculations for the bricks required for straight walls compared to serpentine walls. This document is undated but certainly dates from before Jefferson's death in 1826. It shows comparisons between a serpentine wall and a straight wall (one brick thick) with a pillar every 8 feet for additional strength. Though some words are difficult to read and reflect some changes in language since they were written, this is what he wrote (I have added certain words in brackets to help clarify the meaning):

> an 8-ft course of bricks laid longwise takes 10 2/3 (bricks).
>
> 24 courses to make it 6-ft high will take for 8-ft 256 bricks
>
> a serpentine of 60° adds 1 ½ percent consequently not quite 4 bricks.
>
> a 9-inch pillar every 8-ft will add 24 bricks or 9 percent
>
> a garden of 600-ft circumference of 75 pannils (sections) of 8-ft will take 75 x 260 bricks, say 20,000 bricks
>
> a back yard of 8 pannils (sections) of 8-ft straight with a 9-inch pillar 3640 bricks
>
> a necessary 6-ft square 10-ft high 1 brick thick takes 3000 bricks

In this document (we could even liken it to an exercise book), Jefferson is setting out his work, one step at a time:

> the number of bricks needed for a straight length 8 feet long
>
> that number multiplied by 24 to make a wall 6 feet high
>
> a calculation that a certain serpentine requires 1 ½ percent more bricks than a straight wall (though at this point, I might be tempted to write, 'Thomas, please explain this step in more detail.')
>
> a calculation that a straight wall requires reinforcing pillars at regular intervals
>
> expanding his original calculations to a garden with a perimeter of 600 feet (his use of the term 'circumference' is interesting here – it may reflect slight changes in language usage

over time)

a calculation of the number of bricks needed for 75 sections of 260 bricks and then rounding up from 19,500 to 20,000

the final two lines may refer to calculations for additional garden features (some of the walls have sections of straight lengths along which smaller outbuildings are built)

It would be interesting for students to discuss what mathematics ideas Jefferson used in his calculations. Some of the terms they might discuss are: multiplication, addition, fractions, percentages, length, height, interval, pattern, perimeter, circumference, estimating, and rounding.

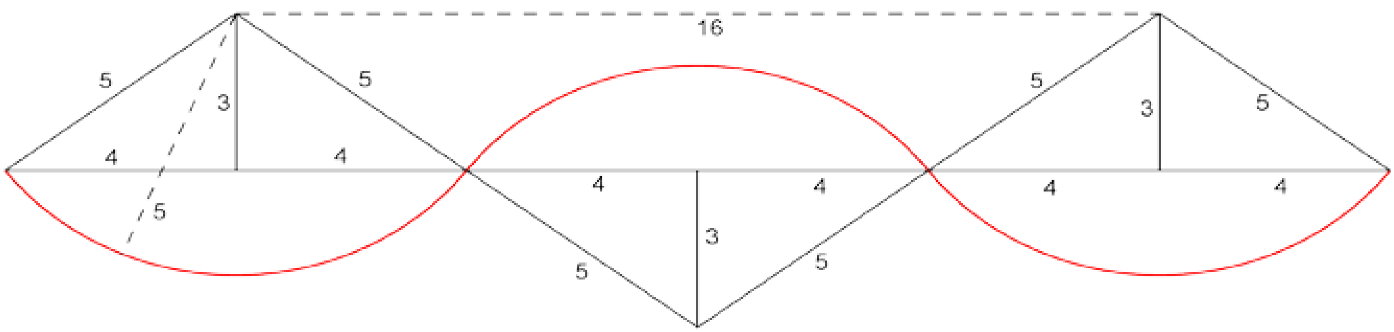
This document does not show the final plan for the walls, but there is a hint of what is to come. Below the drawings are some words and symbols of uncertain meaning but they begin with the phrase, 'for an arc of 8-ft take a radius of 5-ft.' This idea links directly to the walls as they were ultimately constructed.

Thomas Jefferson used feet and inches as his units of length. Why did he use those units? It wasn't just because he was in America (a country that still has not yet officially adopted the metric system).

Drawing the walls

The architectural plans for the serpentine walls show a drawing labelled 'Typical Bay – Based on Jefferson's Sketch Calculations." This drawing (shown here using Geogebra for clarity as the architectural plans are somewhat grainy) comes directly from Jefferson's phrase, 'for an arc of 8-ft take a radius of 5-ft.' A key point to keep in mind is that Jefferson's usage of 'arc' refers to the length of the wall if it were straight, rather than the arc length. So, for a curved wall that stretches the equivalent distance of a straight wall that is 8 feet long, use a radius of 5 feet.

Have children spend time studying this diagram. The red line is the course of the serpentine wall. After looking carefully at the diagram, ask children if they can find where the 8 feet comes from (the length of two of the sections that are 4 feet long) and where the radius of 5 feet is. What else might children notice in this drawing? What maths terms might they be able to discuss? Though the architectural drawings do not indicate it, the triangles are right angled-triangles. Some terms they might mention are: radius, diameter, circumference, arc, angle, triangle, hypotenuse, and length.



Serpentine walls , University of Virginia, Facilities Management, Resource Center

Taking it further

Up to this point, we have been looking at information from a historical document and looking at an architectural drawing. There has been some good mathematical discussion and we have seen how someone used maths to plan and build something. There are several possible extensions, depending on the children in your group. I will mention some here and would love to hear additional thoughts!

Units of measurement

In his calculations, Thomas Jefferson used feet and inches as his units of length. Why did he use those units? It wasn't just because he was in America (a country that still has not yet officially adopted the metric system). Although the metric system was adopted by France in 1799, metrification in the United Kingdom is still evolving. What units would architects today use in their drawings? Could other units be used to create a similar set of walls? What might the drawings or the completed walls look like if architects used meters as their base units?

Pythagoras and serpentine walls

Look back at the architectural drawing above. The sides of the right-angled triangles are 3-4-5. This set of numbers is called a Pythagorean Triple – three positive integers, a, b, and c, such that $a^2 + b^2 = c^2$. What other Pythagorean Triples can you find? What might the wall look like if a different set of Pythagorean Triples was used instead of 3-4-5? Can you find a larger Pythagorean Triple on the diagram above? (Try looking for 6-8-10. How is this set related to 3-4-5?)

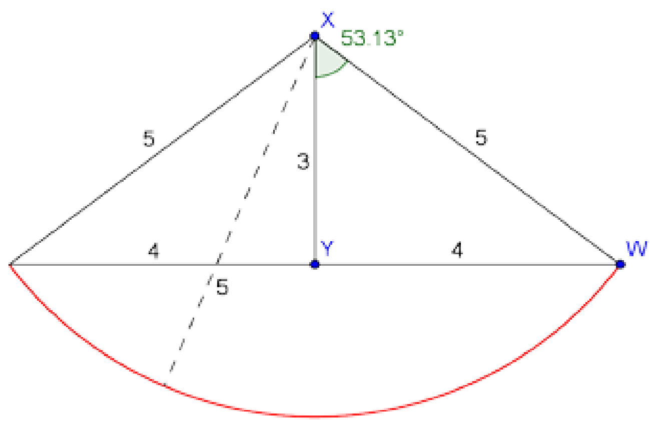
Length of the walls

How long are the serpentine walls? (Or, if they were straightened out, how long would they be?) Some children might choose to solve this using a physical model. The triangles and arcs could be drawn on paper and then a piece of string laid on top of the arc. After the string is laid carefully on top and then start and finish points marked, children can measure the length between the points. A strip of paper could be laid along the curve in a similar manner.

Part of Jefferson's reason for including serpentine walls in his plans was that they used fewer bricks than a straight wall. Ignore for a moment the idea of pillars every 8 feet (as mentioned above) and think of a straight wall using a double thickness of bricks. How many more bricks would be needed for a straight double-thickness wall than for the single-thickness curved wall?

Arc Length and Trigonometry

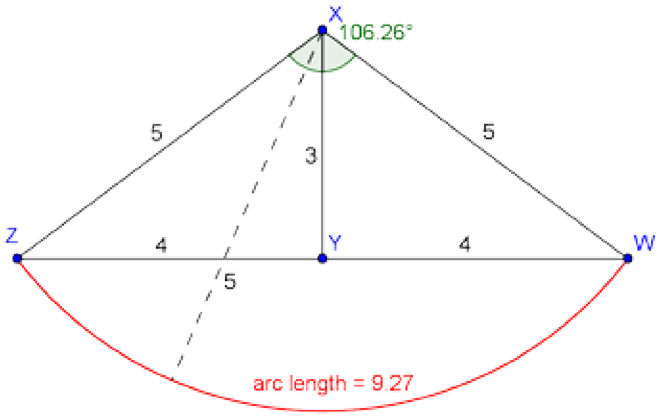
Another way to find the arc length of the serpentine wall is to use trigonometry. If students know the central angle of the arc and the radius, they can calculate the arc length.



Trig – angle of triangle, University of Virginia, Facilities Management, Resource Center

In order to find the central angle, first find angle WXY – this is half of the central angle. The following trigonometric relationship can be used to find angle WXY: $\sin(WXY) = \frac{opposite}{hypotenuse} = \frac{4}{5} = 0.8$

Since sin(WXY) = 0.8, $\sin^{-1}(0.8)$ = WXY. Therefore, angle WXY = 53.13°. The central angle is two times angle WXY (another good point for discussion) or 106.26°.



Trig – central angle, University of Virginia, Facilities Management, Resource Center

To calculate the arc length, use the formula:

$ArcLength = 2\pi R\left(\frac{C}{360}\right)$ where R is the radius of the arc and C is the measure of the central angle in degrees.

$$ArcLength = 2\pi R\left(\frac{C}{360}\right) = 2\pi 5\left(\frac{106.26}{360}\right) = 9.27$$

Constructions and Digital Geometry

The serpentine walls are an excellent example to use both with geometric constructions using pencil, pair of compasses and a ruler, as well as with digital geometry program such as Geogebra (available free at www.geogebra.org). Pencil-and-paper exercises are good for helping children understand the way in which the walls were first designed and to help understand the physical relationships between the components of the drawings. A program such as Geogebra can also help children understand the physical relationships but it is also a way of easily trying out different dimensions of components of the walls. Many of the existing walls use an arc radius of 8 feet instead of 5 feet. A digital geometry program can help students investigate how a change in arc radius impacts the other dimensions.

Volume and surface area

After children have calculated the length of the walls, it is a fairly simple step to find the surface area of the walls (or at least of a section 8 feet long). How might the surface area on one side of the wall differ from the surface area on the other?

Another interesting challenge might be to find the volume of the walls. If the walls are 6 feet tall and 4 inches wide (the width of one brick), what other information is necessary to find the volume? Imagine Jefferson's garden (in the original sketch) that had a perimeter of 600 ft. What volume of bricks would be necessary to build a serpentine wall around the garden? How many bricks are necessary? What information might you need about each brick? (Each brick is about 8 ¼ inches long and about 2 ½ inches high. There is about ½ inch of mortar between each brick.) Once you have figured out how many bricks are necessary for the serpentine wall around the garden, think about a straight wall that used a double-thickness of bricks. Do you have enough bricks to build a straight wall around that same garden? What perimeter could you enclose using that same number of bricks to build a double-thickness wall?

Few would probably be rather surprised to realise that their construction owed as much, if not more, to economy than to appearance.

In investigations like this one (and some of the ones mentioned earlier), there is probably no single correct answer. Of greater importance are the thought processes that children use. Are they using sketches to help their thinking? Are they rounding? Are they estimating? What assumptions are they making? Are they working and discussing with other children? Are they asking questions and justifying their explanations?

A Final Word

Many visitors to the University of Virginia walk along the Lawn and enjoy the wonderful architecture. Few, however, stop to pay much attention to the serpentine walls and would probably be rather surprised to realise that their construction owed as much, if not more, to economy than to appearance. In helping our students understand that mathematics is not an abstract subject, but is found all around us, we can help them discover the mathematics in unusual places.

Matthew Reames is the former Head of Mathematics at St Edmund's Junior School in Canterbury and is now living in the United States studying for a PhD in mathematics education. He welcomes comments and suggestions for Mathematics in Unusual Places, and is happy to provide digital files of the drawings in this article – mreames@gmail.com.

Lifespan of Dolphins

Most dolphins live long lives. The bottlenose dolphin can live over 40 years, and the orca can live to be 70 or 80.

Dolphins washed up on Cornish coast in April 2008

Strandings in the UK have increased from 360 in 1994 to 741 in 2006

In 2008	70 headed for the shore
	26 died
	20 miles - distance from survey vessel using sonar
	700 cetaceans (whales, dolphins and porpoises)

About Books by Rachel Gibbons

Fear of Maths - How to overcome it - Sum Hope³

Steve Chinn, Souvenir Press Ltd 2010

Maths for Mums and Dads: Take the pain out of maths homework

Rob Eastaway & Mike Askew, Square Peg 2010

Do You Panic About Maths?

Laurie Buxton, Heinemann Education 1981

From its contents it is difficult to guess who is being addressed by Steve Chinn in his book and what Sum Hope³ means I do not know. The blurb on the back cover states that the book "is for parents and teachers looking for a way to encourage and help their children." But, starting from its dreary cover, it does not excite me and I doubt if it would any teacher who needs to put a bit more zip into their lessons or any parent who is a little diffident about re-entering the mathematical world believed to have left behind with relief inside the school gates. Nor does Chinn's suggestion of introducing algebra as if it were a foreign language seem to me to be very helpful. Where fear exists it would be better perhaps to go back to Laurie Buxton's 1981 *Do You Panic About Maths?* Laurie's advice was "Don't learn facts, think.

Better still, consult Eastaway & Askew both of whom are well practised in entertaining audiences with mathematics and, in Askew's case, magic. Here you will find plenty of fun. They describe how mathematics lessons have changed (chiefly for the better since most parents were sitting at desks in classrooms themselves). They explain in the first chapter that today's children learn *why* rather than how. There are questions to answer and games to play. The stress is on what can be done and what fun it can be doing it.

A fair society?

... the pay packages of directors of the FTSE100 companies had soared by 49% in a single year to an average figure of £2,697,644. Chief executive officers collect more, an average of £3,855,172.

Guardian 06.05.12

Cameron and his cabinet insist others pull themselves up by their bootstraps even as they themselves swan around in their parents' expensive pairs of loafers. Today almost 40% of MPs went to private school. In 1997 it was just 30%. In terms of social mobility, we are going backwards.

Gary Younge, *Guardian*, 07.05.2012