Equals

Realising potential in mathematics for all

for ages 3 to 18+

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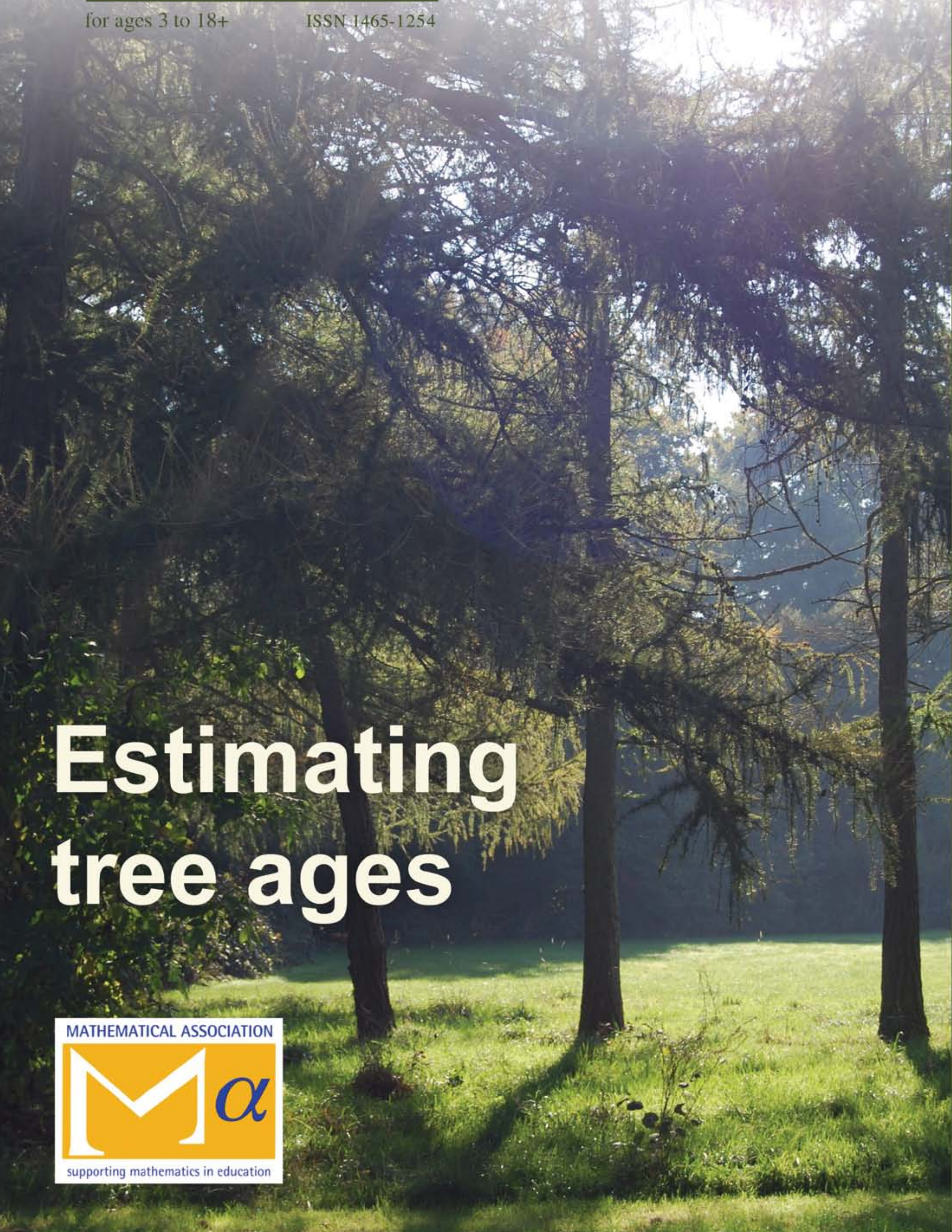

Estimating tree ages

MATHEMATICAL ASSOCIATION

supporting mathematics in education

Equals

Realising potential in mathematics for all

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Designed by Nicole Lane

Editors' Page

After the riots

At the time of writing, a good two weeks after the riots, their analysis is front page news – what caused them? There is an impassioned debate going on about improving the criminal justice system but prison will solve no problems. What is needed surely is a search for measures that will ensure that such a disaster never happens again. An enquiry is being set up to look into causes but it is teachers who must act now because there are few people who can have this influence, particularly now we have lost so much of the out-of-school provision. There seems to be no time for serious thought about how to bring into force measures that will prevent such disasters happening again. Certainly, it is unlikely that any of the offenders whose freedom is curtailed in any way will receive much that is useful in the way of rehabilitation. We put offenders away in a prison or young offenders unit or some other place surely first and foremost to keep society safe.

The question that this journal should be asking at this juncture – and indeed the question for all teachers now - must be: how should our education system change as a result of the riots? Certainly, it is to be hoped that Michael Gove recognises that a return to Victorian education is not the right answer. Melissa Benn's *School Wars*, just published[1], reminds us how the arrival of the comprehensive school brought fresh hope for the 11+ failures and their successors, those pupils who are the concern of this journal. But what has happened since the introduction of the comprehensive? It is clear that considerable discussion is needed considering the practice of good citizenship in the classroom and how behaviour in the small community of the classroom can be made to echo in the larger community of the local streets.

There are plenty of suggestions in this issue of *Equals* about ways to use mathematics across the curriculum. For example some study of Albania seems appropriate if one is going to try any of Matthew Reames's activities concerning tickets and travel and Mary Clark's measurement of trees could take us into a botanical study of the life of trees. In addition Rachel Gibbons asks us to look at our styles of teaching and consider whether we are using what WW Sawyer, that great mathematics educator, called 'lockstep' (marching the whole class together through the same exercise together and giving the speediest workers more practice in procedures which they have already mastered while the rest of the class catches them up). For you can be certain that any class is a mixed achievement class (although, if we set, we risk forgetting this fact) and our presentation of every topic should recognise this.

1. Melissa Benn, *School Wars: the Battle for Britain's Education*, London: Verso, 2011.

Well I never!

A reminder, Fred, for tonight – a small party of twenty, making two hundred light fantastic toes in all, supposing every lady and gentleman to have the proper complement.

Charles Dickens, *The Old Curiosity Shop*, 1841.

11 million people alive today will live to 100 …
The number of carers is expected to rise from 6.4 million in 2011 to 9 million by 2037

The Dilnot Report

What *do you* mean - *estimate*?

Mary BJ Clark reminds us of children's difficulties with estimating a result for a calculation *before* they undertake the calculation.

Many misunderstand the invitation to estimate as a version, somewhat confusingly stated, of a request to work out the calculation with which they have been presented or expected to find within a worded problem. Children's common expectation seems to be that an estimate should, as far as possible, match the answer, and if it does not, then it cannot be a good estimate.

This finding makes us reflect on how many opportunities are in reality built into the teaching and learning of number and calculation. So often the rush is to calculate without following through the process of estimating, calculating and then checking. Estimating is in itself a powerful support for developing that all important feeling for number.

A recent experience of working with children on mathematics in the grounds of their school reminded me again of the often inadequate development of children's estimation skills this time in the context of measuring. The activities I used were: *Estimating tree ages* with a group of KS2 children and *How far is it?* with a group of KS1 and Foundation Stage children. These activities and their accompanying recording sheets follow this article.

For the first of these activities, *Estimating tree ages*, the children were asked to estimate how old they thought particular trees in the school grounds were. Suggestions for the first tree ranged from 12 years to 1000 years. Of course it was not surprising that the estimates varied so widely. What knowledge or experience did they have to call on to estimate in this context? Had they been aware of the history of the school on its site, noticing that the front part of the school dates from 1901 (written on a stone in the frontage of the building), and reasoned that the trees at the front of the building might well be contemporaneous with that part of the building, they would have had something on which to base their estimates. As it was, the children's original estimates just illustrated that they did not have the necessary knowledge and experience to make reasonable estimates at this stage of the activity.

As they worked on the activity, their ability to make reasonable estimates of the tree ages in their school grounds improved. They now had some previous experience on which to base their estimates, and were in a position to explain their accompanying reasoning.

Again, as I was working with a group of KS1 and Foundation Stage children later in the day estimating distances in the activity *How far is it?* I was reminded of the need to establish the meaning and usefulness of estimating.

For this activity the children began by becoming familiar with the length of their personal paces. Then after throwing a beanbag they estimated how many paces it would take to reach their beanbag. The activity was quickly underway with enthusiastic throwing of beanbags. As I watched more closely I noticed that the amazing accuracy of the estimates was attributable to the wiliness of the children in adjusting the size of their paces so that their estimates would match the number of paces they took as they checked. Whoops! Clearly lots more to be done not just on estimating but also on the need for a constant size for a unit of measure, whether non-standard or standard.

Reflecting after these experiences I have been thinking more about the central role of estimation in learning about measures and measurement. Estimating is not about being right or wrong. It is most importantly a way of developing an understanding of what particular amounts and sizes feel like and look like. This idea readily transfers to estimation in the context of calculation.

The message is that time needs to be made for learners to develop their feeling for number and for measures by being supported to see estimation as a fundamental stage in the processes of calculation and measuring. The competitive streak, which misleads children into thinking that estimates should be accurate, must be challenged.

Mary BJ Clark is a mathematics adviser

How far is it?

Mary BJ Clark suggests an estimating and measuring activity.

Make sure the measurement is always in a straight line.

First estimates may be quite inaccurate but practice makes perfect!

Make a note of children's estimates and measured distances on the Record sheet.

1. One child stands and marks their position with one beanbag and throws another beanbag.
    - Each child estimates how many paces it would take to move from the starting beanbag to the thrown beanbag.
    - Then pace the actual distance.
    - Compare the estimate of paces and the actual distance in paces.
    - Repeat the process with another child and a different throw.

2. Now make new distances e.g.
    - Two children standing apart from each other
    - The distance across the playground.

3. Estimate and measure in small steps and larger paces. Talk about why these numbers are different.

Objective: Estimate linear distances in non-standard units (paces and small steps for example)

Well I never!

The first Twilight novel has become the thirteenth book this century to sell 2m copies in the UK.

200,000 people are injured by flip-flops.

A quarter of dogs are overweight.

1 in 36 pound coins are fake.

Guardian 31.7.10

An ostrich egg …

is equivalent to 24 hen's eggs;

takes about 50 minutes to soft boil;

takes more than 2 hours to hard boil;

makes an omelette large enough for 15;

whereas a turkey egg is only 1.5 times the size of a large hen's egg.

Waitrose weekend 11.8.11

Record sheet for estimates and paced distances

Distance estimates (in paces or small steps):						
Measured distance (in paces or small steps):						

Distance estimates (in paces or small steps):						
Measured distance (in paces or small steps):						

Distance estimates (in paces or small steps):						
Measured distance (in paces or small steps):						

Distance estimates (in paces or small steps):						
Measured distance (in paces or small steps):						

Recording sheet for *How far is it?*

Show Me, Don't Tell Me! Part 2

Stewart Fowlie deals three rounds of cards to four children, so they have four lots of three.

The same process works for any number of players and any number of cards for each player.

We write 4 3s or 3 4s as 4.3 or 3.4 (called 4 dot 3 or 3 dot 4) and find the product of 4 and 3 is 12.

The reverse process, considering 12 as 3.4 is called **factorising**: 3 and 4 are the **factors** of 12.

Factorising the numbers from 2 to 12.

2	3	4	5	6	7	8
1.2	1.3	1.2.2	1.5	1.2.3	1.7	1.2.2.2

9	10	11	12
1.3.3	1.2.5	1.11	1.2.2.3

The above lists shows that 2, 3, 5, 7, 11 have only two factors. Such numbers are called **prime numbers** (or just primes). Writing 2^2 for 2.2 and 2^3 for 2.2.2 is optional but should NOT be called 2 squared and 2 cubed until after areas and volume are met some two or three years later but ''two to the two" and "two to the three".

An audience is sitting in 6 rows with 8 in each row. How many are in the audience? The answer is the product of 6 and 8

6.8 = 2.3.8 = 2.24 = 48.

8 men take 6 days to paint the outside of a house.

1 man would take 8.6 days = 2^3 .2.3 = 2^4.3 = 16.3 = 48

In each case the number of men may be interchanged with the number of days. For example, 1 day would be the time taken by 8.6 men.

In a class there are twice as many boys as girls:

the *ratio* of boys to girls is 2 to 1,

written B:G = 2:1.

Another term may be included to give the number of children involved:

B:G:C is 2:1:3.

The ratios are unchanged if each number is multiplied by the same factor:

If there are 16, or 2.8, boys, there will be 1.8, or 8, girls and 3.8, or 24, children.

In another class ratio of boys to girls is 6 to 4

B:G = 6:4 = 3:2.

If there are 12 boys, how many girls are there?

B:G = 3:2 = 3.4:2.4

and 2.4 = 8.

There are 8 girls.

If there are 12 girls, how many boys are there?

B:G= 3:2 = 3.6:2.6 = 18:12

and 3.6 = 18.

There are 18 boys.

If 6 buns cost 90p what would 4 buns cost?

Number of buns (b) to number of pence (p) is 6:90

So 4 buns cost 60p.

How many can you buy for £1.50?

b:p = 2:30 = 10:150

so 10 buns.

A girl cycles 36 miles in 4 hours. How far did she go in 3 hours?

If the number of miles for the journey is m and the number of hours taken is h

h:m = 4:36 = 4:4.9 = 1:9 = 3:27.

So the length of the journey is 27 miles.

How long does it take her to cycle 18 miles?

h:m = 4:36 = 4:4.9 = 1:9 = 2:2.9.

So the journey takes 2 hours.

That's the end of showing! I am sure most teachers today make some effort to explain these results, and ask if the class understands. Many a child will just want to get on with memorising them and not know what 'understand' means anyway.

To make the concepts of sum, difference, product and ratio clear I apply the processes which can be shown using a suit of cards to stand as units as described, and a suit from a different pack to stand for tens. But numbers may also show how many steps you have taken up or down a stair (directed number), the length of something (a measuring number or scalar) where something is in a book (page number). The meaning, if any, of sum and difference of all these numbers should be considered before product or ratio are even mentioned.

Stewart Fowlie is a tutor in Edinburgh

Estimating tree ages

Mary BJ Clark suggests an estimating and measuring activity.

You will need a *Tree ages recording sheet* to note your estimates and calculated ages.

* This is a way of calculating an approximate age of a tree in years:
    1 Measure 1m up from the base of the tree.
    2 At this point measure around the trunk in cm.
    3 Divide this measurement by 2.
* Choose a tree.
* Make an estimate of its age in years. Using the process above, calculate an approximate age of the tree.

Use Make a note of your original estimate and your calculated approximation of the age.

* Choose other trees. First estimate their ages and then try calculating them. Do your estimates improve?

Objectives: Make estimates and understand the meaning of estimation. Measure heights and lengths using appropriate units. Solving problems recognising simple relationships.

Tree ages recording sheet

Tree location	Estimated age	Calculated approximate age

Recording sheet for *Estimating tree ages*

Teaching Reasoning: a key to unlocking potential

Learning to think is as important as learning facts and skills: **Mundher Adhami** considers arguments for this which apply equally to low- and high-achievers.

Currently there is a group of academics reviewing the National Curriculum. Part of the aim is to give teachers more freedom. There is an acceptance that the NC is over-prescriptive. A nice phrase is:

> "In order to bring the curriculum to life, teachers need the space to create lessons which engage their pupils, and children need the time to develop their ability to understand, retain and apply what they have learnt."

It seems worthwhile for all who can to aid this process. The DfE's remit for the curriculum review is on www.education.gov.uk/b0073043/remit-for-review-of-the-national-curriculum-in-england. The responses seem to be on NCReview.RESPONSES@education.gsi.gov.uk .

Here I consider ten key points I think would equally apply to those youngsters who find mathematics difficult and to those who are 'good at it'. All students should persevere and enjoy discovering things they probably feared, or feared to forget.

The 'balanced diet' approach:

1. We believe that a balanced educational diet for all youngsters in school should include at least four ingredients:
    a) **Knowledge** (of accessible facts for the given age group and achievement range)
    b) **Skills** (especially in reading, writing and ways of handling materials)
    c) **Understanding** (of connections between separate items of knowledge and skills)
    d) **Reasoning** (including ability to explore, formulate questions and re-construct concepts)

2. The standard tests and teaching practices cater well for Knowledge and Skills, and teachers' success in these has ensured continually rising exam success over the years in the middle range of attainment, e.g. around grade C in GCSE and level 4 in KS2. On the other hand, weaknesses are evident at the higher levels, since success here often depends on prior full understanding and on reasoning ability. Many high-attaining students do not retain their competencies over time nor do they have a positive attitude towards learning. This also applies to the lower range of attainment for which the mainstream teaching is often ill suited.

3. Good teaching normally recognises that understanding connections and patterns in bodies of knowledge, flexible approaches to a topic, and the ability to reason things out for oneself, greatly help learners. Teachers with a good grasp of the subject on one hand and sensitivity to students' levels of achievement and motivation on the other are the ones we must rely on for the high-attainers we still have. But 'good teaching' is difficult for many teachers. Even attempts at 'teaching for understanding' end up ritualised in pieces of knowledge to be memorised. Witness how National Numeracy Strategy attempts to increase understanding of place value through different methods of adding of 2- or 3-digit numbers could result in asking pupils to know several different methods. Or the attempt to increase understanding of how percentages, fractions and ratio are connected and became a body of knowledge in its own right.

Teachers' subject knowledge

4. The road to improving teachers' subject knowledge seems clear, i.e. having a relevant degree, A-level or further training in the subject. But sensitivity to, and engagement with students might seem beyond training, and the unstated assumption is that it is an inborn trait of character. We believe otherwise, that it is trainable, and propose that the key to both subject knowledge and sensitivity is to emphasise **reasoning**, i.e. the handling of the logic that underlies the main concepts involved in any piece of knowledge. It is this emphasis that ensures a general rise in students' ability within a subject and across subjects. We are effectively talking about raising understanding and competence of both students and teachers.

5. The levels of reasoning in a topic that we propose should accord with the cognitive levels of the students, so that the teacher at any time ensures a match between the logic of the subject matter and the logic of the learner. In addition to this cognitive dimension in teaching there is the dimension of social interactions in the classroom. At any one time and in any one activity there is a range of levels of thinking in any classroom, and a student benefits from interacting with others slightly ahead of them in thinking more than with the elegant full formulations in the curriculum.

6. We suggest that time is allocated in the curriculum for children to reason out concepts in their own ways and natural language, before they assimilate the formal curriculum. Reasoning often subsumes understanding as the starting from scratch, and the careful handling of common misconceptions through teachers' recognition of pupils' views would ensure that solid lasting connections are made in their minds. There should then be little need to repeat the same lessons over the years.

The dilemma in education

7. The dilemma in education is that we are asking teachers to teach in ways few of them have experienced themselves. They have to apply ideas to new situations without having prior experience of them themselves. The practical and economic way through this dilemma is to create a limited but assured experimental environment for teachers with help of sparse but steady input from tutors, and using some materials that have proven themselves in similar situations.

8. The CASE and CAME approaches and materials are examples of available resources that fulfil the need for a reasoning focus in mathematics and science. Developed over the last 30 years with large public investment of funds and academic effort, these have proven themselves in practice in pilot studies and in limited dissemination. The projects are now available for a wider uptake, providing minimal official support, and crucially they involve successful models of professional development for teachers.

9. Our two specific suggestions therefore are:
   a) To add reasoning as an ingredient in the experiences of all learners following the National Curriculum, ensuring classroom activities with such a focus. A definition of reasoning to include reconstructing concepts and exploring misconception is needed in order not to trivialise it.
   b) To ensure that at least 10% of teachers' time is allocated to experimenting with classroom activities that have a reasoning focus, with half that time in collaborative work with other teachers or tutors with experience in such activities.

10. We believe that the two suggestions apply to all age groups and all subjects, but that it is sensible to rely at first on science and mathematics where experienced tutors are still in active service.

Mundher Adhami is a Cognitive Acceleration Associate

Changes to place value thinking during Key Stage 2

As part of the Mathematics Specialist Teacher (MaST) course **Pam Fletcher** collaborated with a colleague in some research into the understanding of place value in Key Stage 2.

We like to feel that our school is a 'community of enquiry' where each individual is striving to better understand teaching and learning.

After a staff discussion, it was decided that for this initial project I should work alongside the Year 5/6 teacher, Wendy. The reasons for this were:-

- She had recently moved from KS1 to teach this older group of pupils and felt less confident about the mathematics curriculum in KS2.
- Wendy and I already plan some areas of mathematics together as I teach the Year 5/6 class twice a week.

Place value was the aspect of mathematics which Wendy highlighted as one of the areas causing difficulties for her pupils.

As mathematical thinking is one of the key areas of the MaST course and as thinking skills are always an area of development within the school, we decided to concentrate on this aspect.

Place value was the aspect of mathematics which Wendy highlighted as one of the areas causing difficulties for her pupils. We decided to try to develop mathematical thinking through multiplying a number by 10 and to observe the progression of the children's ideas and understanding from Year 3 to Year 6.

Place value is an extremely important idea and facilitates mathematical thinking in many areas. However many children find the place value system difficult to really understand and Sowder (1992)[1] argues that most 'primary school pupils may have a limited understanding of place value.'

many children find the place value system difficult to really understand

Thompson (2009)[2] says that difficulties arise because of the confusion between two different aspects of place value – the 'quantity value' where 45 would be seen as 40 and 5 and the 'column value' where 45 is 4 tens and 5 units. He suggests that in mental and informal calculations children use the 'quantity value'. I have also observed that pupils in our school use this 'quantity value' almost exclusively in mental and informal methods of calculation.

For instance: 'How would you add together 35 and 23?'

Pupil: 'Well 30 add 20 is 50, 5 add 3 is 8 so the answer is 58.'

'Multiply 27 by 5.'

Pupil: 'I know that 20x5 =100 and 7x5=35 so the answer is 100+35=135.'

Traditionally teachers have tried to teach multiplication by 10 as 'the digits move one place to the left' rather than 'adding a zero' as this does not work for decimals. Thompson (2009) concludes that 'children in Year 2 and Year 3 have little or no understanding of what we conventionally call place value' and therefore 'It would appear to be somewhat over-optimistic to expect young children to understand the concept of 'moving the digits one place to the left' which appears to demand a fairly sophisticated understanding of place value.'

The observation was to be centred on the children's responses rather than the teaching.

After the early whole school discussions, Wendy and I met after school and discussed the proposed focus for this project. This was decided after joint discussion and looking at the Assessing Pupils' Progress (APP) grids for our classes. We also discussed how the project would be carried out and decided that we would jointly plan the lessons and then each teach a lesson to our own class whilst the other observed. The observation was to be centred on the children's responses rather than the teaching. This joint planning and focus on the responses of the pupils meant that we were both comfortable with the boundaries of the observation and neither of us felt threatened. Finally we planned a post observation discussion in which we would discuss the responses and learning of the pupils. We clarified that we would not discuss the teaching.

For the children however the contexts become 'real' and so they would be highly motivated to solve the problems presented.

The initial discussion covered ideas about 'a learning agreement'. The subsequent lesson teaching and observation gave opportunities for us to experiment and observe and subsequent discussions helped us to set challenging and personal goals for both ourselves and our pupils. We were able to use money from the MaST project to obtain supply cover for one morning so that we could observe each other's classes and to make use of PPA time and TA cover to complete our discussions. This creative use of time also meant that the project was given importance within the whole school plan. Fullan (2001)[3] argues that for a new initiative to succeed there needs to practical evidence that the leadership are behind the project.

On reflection our plan had many aspects of a Lesson Study cycle as described by the National Strategies. If we were to continue with this project into further cycles, the Lesson Study idea of only observing one lesson and having a focus group of children might prove useful. We did not interview the pupils after the lesson to gain insights into their learning, and again this could prove useful in the future.

During our discussions Wendy and I talked about the need for pupils to develop a deep or relational understanding as defined by Skemp (1976)[4] rather than just superficial or instrumental understanding. If children had been taught either to add a zero or to 'move the digits one place to the left' both of these would be instrumental i.e. a learned rule to get the right answer. We also looked at the articles by Thompson dealing with place value and column versus quantity value. These ideas were interesting to us both and provoked considerable discussion in the staff room. We designed our lessons to see which ideas our pupils would use.

The mathematics lessons were set as problem solving exercises within our 'Companies'. At our school all classes run companies for part of the week using the 'Mantle of the Expert' ideas of Heathcote (1995)[5]. Year 3/4 are part of the Virginia Company of 1607 and Year 5/6 are running a bear sanctuary. These enterprises are of course imaginary and use drama techniques to support the children's learning. For the children however the contexts become 'real' and so they would be highly motivated to solve the problems presented. Plenty of time was given for the discussion of children's ideas both in pairs and groups on mixed ability tables and during a whole class plenary.

The pupils were happy to discuss their methods and justify the answers. This was particularly evident in the Year 5/6 lesson where each method for multiplying by ten was evaluated and a consensus was reached as to the most reliable method.

In Year 3/4 many children were able to see the pattern for multiplying by ten and then were able to generalise to any number.

We also tried to give opportunities to the children to see multiple representations of the same mathematical problem by setting the calculations in more than one context – exchange rate for baskets of produce, multiplying lengths and scaling up a recipe for milk.

Within our classrooms most children discovered a rule for multiplying by ten and within Year 5/6 were then able to look at various methods and decide which would be best. They had a much greater relational understanding of the issues by the end of the lessons. There was little evidence of the children using 'column value' (Thompson) in either lesson except during discussions about multiplying decimals.

This collaboration was enjoyable and beneficial for both teachers and pupils. Wendy commented that she had enjoyed planning and teaching the lessons. She felt that she had a much greater understanding of the progression in this concept from Year 3 to Year 6. It gave us an opportunity to discuss ideas about mathematical thinking which are key to children's learning. I certainly gained a much deeper understanding of the difficulties encountered by pupils when trying to understand place value.

In the future it would be good to continue to work collaboratively and to develop a series of lessons not only looking at multiplication but also at division by ten. We could also probe the children's understanding of place value in other situations.

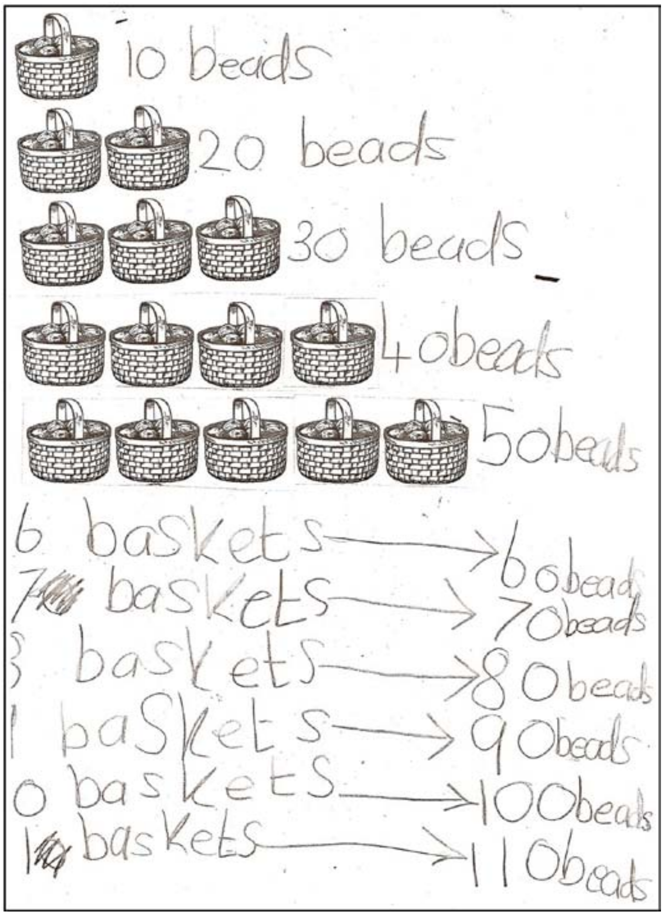
Here are some notes about the lessons and what we observed:-

Year 3/4 Lesson

The context for the Year 3/4 lesson was the Virginia settlement at Jamestown in 1607. Initially the settlers were trading with the native Americans. The rate for a basket of fruit was 10 glass beads. The children were asked to work out a chart so that everybody would know how many beads to give.

Pupils had some beads available plus Dienes apparatus. The least able pupils were supported by a teaching assistant.

Many pupils started by counting out the beads but they quickly ran out and so began to calculate. After about 5 minutes there was a whole class discussion about the methods they had used. Here are some of the children's contributions:



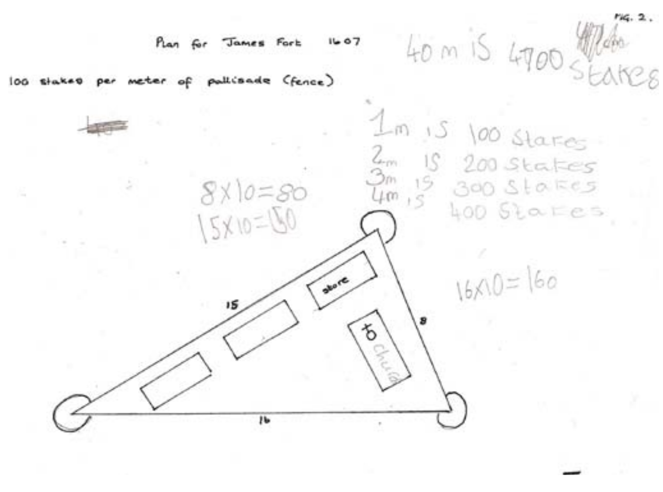
- 1 basket is worth 10 beads. There are 10 for each basket so you just keep adding them up. 10,20,30,40
- It's the 10x table
- You just count in tens
- Add a zero to the number of baskets

Nobody seemed to want to use the Dienes apparatus.

Once the chart was made, the second problem was introduced:

'President Wingfield has been given a plan of the fort we are going to build by the Virginia Company. However the measurements seemed to be 10 times too small. What should the measurements be?'
Here 8 x10 was easily calculated by everybody but 15 x10 and 16 x 10 proved more tricky. Some still tried counting up in tens and lost their way, particularly around 100. The general consensus was that the most reliable method was to add a zero.

Finally we tried working out how many stakes would be needed for the palisade which needed 100 stakes for each metre. Only one or two were able to do this. They either counted in 100's or added two zeros. Nobody mentioned place value or moving the numbers to the left.



Year 5/6 lesson

The Year 5/6 lesson was within their context of a bear sanctuary.

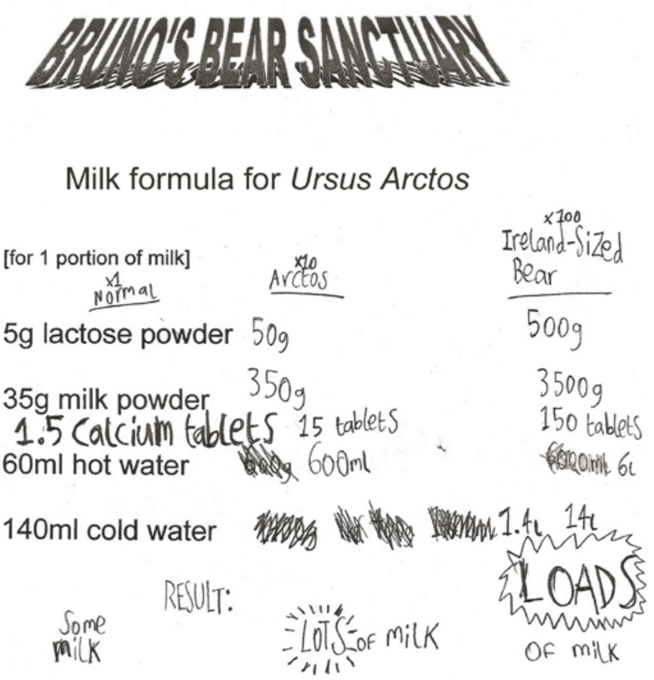
'Some of the bears still need milk to drink either because they are very young or because they have problems with their teeth.'

The formula was given for 1 portion of milk:

5g lactose powder
35g milk powder
60 ml hot water
140 ml cold water

The children were then asked to work out how much they would need for 10 portions for Ursus Arctos who is a large old bear with bad teeth and who therefore needs 10 times as much milk.

This proved a relatively easy task for most children. The least able were supported by a teaching assistant and had Dienes apparatus available. One of the least able children suddenly saw a pattern and said 'you add a zero'. The pupils who finished quickly were asked to show their results on the measurement chart.



Children were asked how they had completed the calculation. Various methods were voiced:

- I added a zero
- For 35 x 10 I did 30 x10 and 5x10 then added them together
- I moved each number up a column, (moved the numbers to the left)
- Count up in tens
- You could do it by adding 35, 10 times.

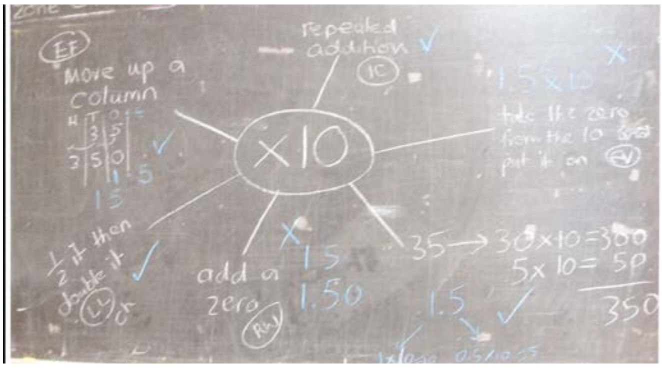
At this point Wendy added an extra ingredient: 'Sorry, I forgot to tell you about the calcium. We need 1.5 calcium tablets for 1 portion.' This engendered more thought and some discussion but they soon came up with the answer:

- You need 15 for 10 portions. You have to shuffle it up a column
- Adding a zero doesn't work for decimals
- 1½ x 10 = 1 x 10 + ½ x 10 = 10 + 5.

This time the extension activity was to multiply by 100

- You just need to add 2 zeros.
- Take the zeros off the 100 and put them on the number.
- Move the number up two columns.

Once everybody had had a chance to complete their calculations Wendy brought them back as a group to look at their methods for multiplying by 10. These were displayed on the board. Wendy led the discussion to see whether all of the methods worked when there was a decimal. Almost all were happy that the 'best' method now was to move the numbers 'up a column' as that was the simplest way to be sure to get the correct answer for all problems.



In our subsequent discussions about these lessons we agreed that we are both very relaxed about the idea that children have their own methods for calculation in mathematics. They have misconceptions, but in a non-judgemental environment where mistakes are okay, they are able to work through their ideas. (This fits with Piaget's ideas about schema and their modification). We wouldn't impose a particular method for any calculation but as far as multiplication by 10, 100 etc. is concerned it seems pointless to try to teach a method for which they are not ready.

Pam Fletcher is an advanced skills teacher at Bealings School Suffolk

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3. Fullan, M. (2001). *Leading in a culture of change.* San Francisco: Jossey-Bass.

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5. Heathcote, D. and Bolton, G. (1995) *Drama for learning: Dorothy Heathcote's Mantle of the Expert Approach to Education, (Dimensions of Drama).* Heinemann

The right size

(see 'Well I never!' page 24)

You will never find a squirrel as big as an elephant or a horse or as small as a mouse and it is all to do with the relationship of volume and surface area. J.B.S. Haldane, a famous biologist, noted how giants ten times as tall as men would not be able to stand because the volume of their bodies would be 10x10x10 that of a man's body while the strength of their legs would depend on their cross-section - only 10x10 that of a man's legs.

(See JBS Haldane, ' *On Being the Right Size*')

Many Ideas with a Few Cubes

Liz Woodham shows how a few interlocking cubes can help focus attention:

pupils' attention on:

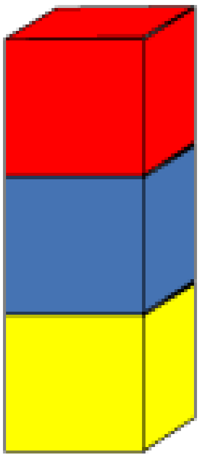
- working systematically
- choosing effective recording methods
- making generalisations

and teachers' on:

- sharing ideas
- precision of arguments.

All that is needed is a large supply of interlocking cubes, for example Multilink.

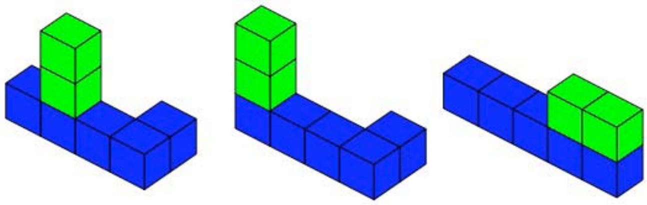
Different towers



3 Blocks Towers (http://nrich.maths.org/137) offers a simple starting point, inviting learners to use three differently-coloured cubes to create as many towers as possible. There are two parts to this problem - firstly making as many different towers as you can think of and then being sure that all possible ones have been made. The first is relatively straight-forward, the second requires some systematic working and logical thinking. Having discussed some ways of making sure all the combinations have been found, each pair can be given three different colours of blocks to replicate the activity and convince you that they have found all posible towers.

All the possibilities?

Two on Five (http://nrich.maths.org/1997) also requires systematic working. This time, seven cubes - five of them of one colour and two of another - are joined according to these rules:



- The five that are of one colour must all touch the table that you are working on;
- The two that are of a different colour must not touch the table.

Again, the challenge is to find all the possibilities.

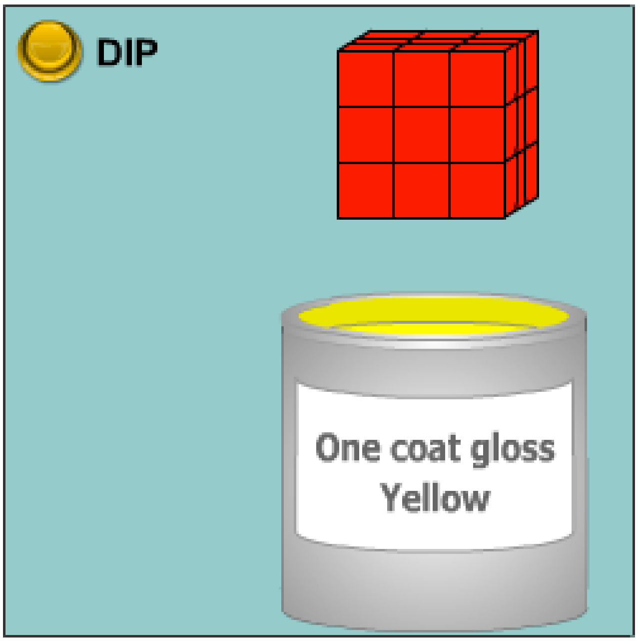
Not only does this give pupils the chance to produce their own system for exploration, it gives a wonderful opportunity for them to explore ways of recording.

Why not?

Brush Loads (http://nrich.maths.org/4911) begins with just five cubes joined so that they don't topple over. Children are challenged to find out how many 'brush loads' of paint (one 'brush load' is enough for one square face) will be needed to cover the cubes, assuming that the base which touches the table is not painted. What is the least number of brush loads a five-cube model may need? The most? Learners are invited to investigate larger numbers of cubes. Can they make conjectures about how to arrange the cubes to get the least and the most number of brush loads? Can they explain their thinking?

Leading on to:

Painted Cube (http://nrich.maths.org/2322), where students are asked to imagine a large cube made up from 27 small red cubes. The large cube is dipped into a pot of yellow paint so the whole outer surface is covered, and then broken up into small cubes.



How many of the small cubes will have yellow paint on their faces? Will they all look the same?

Imagine doing the same thing with the other cubes made up from small red cubes.
What can you say about the number of small cubes with yellow paint on?

Exploring a variety of painted cubes may produce patterns, which learners can describe spatially, numerically and algebraically. They can appreciate the benefits of keeping a clear record of results, and apply their insights from the first case to ask themselves questions about further cases. This problem lends itself to collaborative working, both for students who are inexperienced at working in a group and those who are used to working in this way.

Going a little further:

Castles in the Middle (http://nrich.maths.org/6994) is one of a series of activities which encourages the development of team-building skills such as listening, finding out what others think, giving reasons for ideas and pulling ideas together.

Liz Woodham is a member of the NRICH team at Cambridge University

A computer can't carry your shopping

'Yesterday,' Rachel Gibbons tells us, 'I did a really heavy grocery shop, dragging my shopping trolley on and off the bus and leaving it at the bottom of the steps while I went to unlock the weighty front door and hooked it back to make it stay open.

I had noticed the postman having a quick fag on the steps of the house two doors along and when I got back to the bottom of our steps for the trolley he had arrived there too.

"Do you want me to take that up for you?" he asked.

"Yes please," I said and, when he had got it inside the door, he asked how far up I had to go.

"Three flights", I said and off he set at a speed, with which I, at my slow, over-80's-plod, could hardly keep pace.

I thanked him profusely, of course, as he put the trolley down outside my flat door and I wondered if the break-down of community spirit and, allegedly, ever more misbehaviour of our young people in the streets has anything to do with the increasing 'technologisation' of society. People correspond increasingly by e-mail without the human intervention of the postal system. Banks have cash machines. More and more shops have automatic check-outs, indeed, shopping can be done via the internet without actually visiting the shops at all. News is gleaned via TV rather than newspapers. The younger generation goes about with their mobile phones clutched to their ears as if they are afraid to be out alone among strangers. Single people travelling on trains too often spend their journeys shouting into their mobiles to friends afar, much to the annoyance of their fellow passengers who do not wish to hear about their tedious private affairs.

All in all, there is less and less face-to-face human interaction.

This state of affairs should be treated very seriously by teachers (and Michael Gove incidentally) who all need to recognise that 'back to basics' today means not just mathematics and English, but the skills of living in a community co-operativley. The need for a moral education has been further highlighted over the past few weeks of the Murdoch revelations. Where are our young people to look for examples of a good society? In today's *Guardian* I read that:

> Witnesses in the case have been given strict instructions before giving evidence to tell the truth, although witnesses do not give evidence under a specific oath.
>
> James Murdoch had said that his advisers had urged him to adopt a strategy of telling the truth when he spoke to the committee.

It is in this area of moral education, which is clearly so sorely needed today, that individualised schemes of learning are of great value Having for years been part of an individualised mathematics project I have experienced at first hand the training it can provide (for teachers and pupils) in how to live peaceably and productively as members of a community. In a SMILE (Secondary Mathematics Inclusive Learning Experience) classroom each member of the class has his or her own individual programme but with plenty of built-in joint activities for groups of pupils. No SMILE class can carry on satisfactorily unless all pupils

- accept responsibility for their own work,
- respect other people's work space,
- share amicably all the available resources and equipment and
- take care of them.

In other words, a SMILE classroom is a microcosm of society at large where pupils learn practical citizenship as well as mathematics.

Now that the SMILE materials are archived at the National STEM Centre, University of York, they are freely available on-line to all teachers, parents and whoever else might be interested. All the activities were written by working teachers for use in their own classrooms and, in this writing, participating teachers grew remarkably, deepening their own understanding of mathematics and sharpening their pedagogical skills. Now these materials, revered by the mathematics education community, are waiting for new working groups of today's teachers who, while using what has been handed down to them, will also continue to write their own additions particularly related to their own pupils, so keeping the bank of resources up to date. I suggest you look at what is available to see if it contains something to appeal to those in your classes who are the most difficult to motivate.

Originally the SMILE materials were all produced in varied and attractive formats by the Inner London Education Authority. Today, when education authorities are so often rubbished, it seems particularly important to remember the support for this classroom development provided by ILEA and to ask whose task it is now to back up teachers in such ventures? It is surely the kind of work that the NCETM should be doing. Any group wishing to be involved in the further development of SMILE should contact SAG (the SMILE Action Group) c/o ray.g@nodotcom.no.com to see how SMILE can be taken forward.

Rachel Gibbons is a retired inspector for mathematics, ILEA

Mathematics in unusual places 2: Albanian train timetables

Matthew Reames finds that one of the central points of *Primary Mathematics: Teaching for Understanding*[1] is that making connections within and across topics is an integral part of understanding mathematics.

Additionally, studying the ways ideas are represented is a key part of the process of developing understanding and giving meaning to an idea.

The ability to interpret useful information from new situations is a crucial skill for all to learn, although all too often children find it 'too hard' and give up before they have really made an attempt. By helping children make connections between a new situation and what they already know and understand increases their confidence in themselves and their abilities. By providing such a situation we can also help them to develop for themselves new ways of representing ideas and concepts.

One example is travelling in an unfamiliar area. Strange place names, different timetable layouts, foreign languages – all of these factors can turn a simple train trip into a true adventure! My year 5 class recently spent several lessons investigating the mathematics involved in the Albanian rail system.

But why Albanian train timetables? Surely Britain has enough train timetables to keep people busy for months! This is precisely the reason why looking at Albanian train timetables has so many benefits. The British rail system is huge and complex whereas the Albanian one is small and self-contained. It is possible to get a clear overview of the entire Albanian system and to examine certain aspects in detail without getting too lost in all the information. Also all the children begin the investigation on an equal footing. Though each student in my class was familiar with travelling by train, two were rather expert in local rail timetables. By using a system new to everyone, no one was able to dominate the discussion and everyone felt able to take part. It also allowed children to make connections between what they already knew and what they thought they didn't.

I started the investigation by telling the children that during the recent holiday I had spent several days in Albania and had the opportunity to take the train from the capital, Tirana, to another city. Immediately, we had the opportunity to locate Albania on the map and there was some discussion about its size in relation to the UK and how far I might have travelled. Showing two photos, I then said, "When I went to the train station to purchase my ticket, I was faced with these two signs. What can you tell me about them?"

Figure 1

Figure 2

The children were given the opportunity to discuss the two photos with a partner and then share their ideas with the class. Some ideas were:

They are written in a different language.
Some of the words look like place names.
"Tirane" looks like "Tirana" on our map.
There are more trains between Tirana and Durres than between Tirana and other places.
One part must show when the train leaves and the other when it arrives.
Some of the numbers look like times.
There is a date on one of the pictures.
The first picture has percentages on it.
The list of places could be destinations and the numbers could be prices
The numbers in the 50% column are half of the numbers in the 100% column.
There are 'zona' numbers in one column – maybe this is like zones on the Underground.
The bigger the 'zona' number, the bigger the numbers in the other columns.
Are the numbers prices?
If they are prices, what money are they in?

The children quickly moved from confusion about an entirely new situation to careful examination of it a little at a time in order to make connections with things they knew. What followed from their observations was a very good discussion about the photos. Despite knowing almost nothing about Albanian language or culture, they were able to figure out almost all of the information in the photos. Figure 2 is the timetable of all the trains leaving from and arriving in Tirana each day as of the 21st of February 2011. Figure 1 is a list of ticket prices for all of the destinations from Tirana. The first column shows the zone of the destination in the second column. The next two columns give the prices of single tickets (full-fare and concessions – 100% and 50%) and the final two columns give the prices of return tickets (again, full-fare and concessions).

Some additional points for discussion about the photos might be:

Is the 50% price for tickets always actually 50% of the full cost?
Why might the zone IX and X cost the same for single tickets but different for return?
How much do you save by purchasing a return ticket rather than two singles? Advanced pupils might figure out the percentage saved.
How are the times written? Why is there no am or pm given?
How long does the train take to get to a certain city?
What is the trip of the longest duration? The shortest?
If you go from Tirana to Durres just for the day, which trains should you take to have the longest amount of time in Durres?
If you leave Durres at 9.45, what is the earliest you can arrive in Elbesan? Can you return the same day?

Talking about the ticket prices required some knowledge of Albanian money. The currency used is called the lek. Each lek is divided into 100 qindarka, although the qindarka is no longer used. Any fractions of lek are simply rounded. Knowing that the prices were not in pounds, the children began to speculate about how much 1 lek might be worth. Two immediate observations were, "If each lek is worth about £1, then the ticket prices are very expensive." "At least they are expensive if Albanian people earn about how much British people earn. Maybe they earn more and tickets are less expensive to them."

Already the children were doing mental estimations about prices of tickets as well as about the relative values of the currency. In other words, rather than thinking of them as just symbols on a sign, the children were getting a feel for what these numbers might actually mean.

“What information might help you figure out what you want to know about the ticket prices?” I asked.

“If we knew how much 1 lek was worth in pounds, we could figure out how much the tickets would cost us.” “Or, if we knew how much £1 was worth in lek, that would work as well.”

A quick check online (there are a number of useful websites, but I find http://www.xe.com/ very helpful) showed that, on the day of my trip, £1 was worth 161.9 lek. (As a side note, the XE Corporation’s website has a great deal of interesting information about foreign currency rates, including historic trends available at http://www.xe.com/currencycharts/. Using these graphs, pupils could investigate the value of different currencies over various intervals of time.)

It didn’t take very long for the children to do some metal estimation. “So that means that a lot of those tickets cost less than £1!” “Those tickets are a lot cheaper than here.” “Does that mean that the people there earn a lot less than people here?” “Maybe the tickets don’t cost much because the distances are not very far. How far apart are the places?”

All of these are very good observations and questions that have resulted from the children getting a feel for the actual meanings of the numbers and from a desire to make additional connections to things they already know.

“Here is my train ticket. What can you figure out about my trip?”

After a short discussion with their partners, they came up with several conclusions:

Your ticket cost 110 lek which was less than £1.

It was a return ticket since no single prices on the chart are 110 lek.

You were going either to Durres or to Marmurras since it is a zone III ticket.

Now seemed to be a good time to do some slightly more exact calculations. How much, in pounds, did my ticket cost? Using the exchange rate, children can figure this out in several ways. Some may choose to estimate – using a rate of £1 = 160 lek, the numbers work out much more easily and the accuracy is not reduced very much. One pair of children did some calculations like this:

£1.00	160 lek
£0.50	80 lek
£0.25	40 lek
£0.125	20 lek
£0.10	16 lek
£0.0625	10 lek

Then they could simply add up the necessary values:

£0.50	80 lek
£0.125	20 lek
£0.0625	10 lek

£0.6875	110 lek

They then rounded their answer to £0.69. Notice that some of the pound values in their tables have more than two decimal places. There is no problem with this when doing the calculations – it simply makes the final answer more exact if the rounding is saved until the final step.

Other ways of doing the calculation might involve finding out how much 1 lek is worth (again, using £1 = 160 lek makes the numbers easier to work with) and then multiply that by 110:

£1 = 160 lek => £0.00625 = 1 lek

£0.00625 x 110 = £0.6875 or £0.69

Incidentally, using the actual figure of £1 = 161.9 lek gives an answer of £0.6794317 or £0.68. It is interesting to discuss with children how important it is to use the exact figure rather than an estimate.

At this point, children could do some additional calculations involving ticket prices. Perhaps figuring out the cost in pounds of purchasing two singles rather than one return or finding the cost of the most expensive ticket. Maybe even how many different trips you could take if you had £5 worth of lek.

It is important for children to see information represented in different ways. For this, we used some pages from the European Rail Timetable, published monthly by the travel company Thomas Cook[2]. Despite the name, each month's edition also includes a certain number of ferry timetables as well as some bus information. Each Rail Timetable contains a wealth of information in the form of maps, charts and tables. Here are the relevant extracts we used:

ALBANIA *SEE MAP PAGE 491*

Operator: **HSH**: Hekurudha Shqiptarë.
Services: Trains convey one class of accommodation only. Tickets are not sold in advance, only for the next available departure.
Security: Most visits to Albania are reported to be trouble free, but travellers are advised to avoid the north-east of the country.

Timings: Valid as at May, 2011

HSH One class only — ALBANIAN RAILWAYS — 1390

km													
0	Shkodër	a.	...	...	...	0540	...	...	...	...	...	...	...
47	Milot	d.	...	...	...	0716*	...	...	...	...	...	...	...
	Tiranë	d.	0600	...	...	|	0830	1145	...	...	1420	1700	2010
82	Vorë	a.	0622*	...	...	0830*	0852*	1207*	...	...	1442*	1722*	2032*
82	Vorë	d.	0624*	...	...	0854*	0854*	1209*	...	...	1444*	1724*	2036*
98	**Tiranë**	a.	|	...	...	|	|	|	...	...	|	|	|
102	**Durrës**	a.	0658	...	...	0928	0928	1243	...	...	1518	1758	2110
102	**Durrës**	d.	...	0715	0715	...	...	...	1310	1310	...	...	...
	Durrës Plazh	a.	...	|	|	...	...	...	|	|	...	...	...
138	Rrogozhinë	d.	...	0826*	0826*	...	...	...	1421*	1421*	...	...	...
179	Elbasan	a.	...	1010	|	...	...	...	1603	|	...	...	...
255	Pogradec	a.	...	1333	|	...	...	...	...	|	...	...	...
155	Lushnjë	a.	...	...	0900*	...	...	...	...	1501*	...	...	...
187	Fier	a.	...	...	1015	...	...	...	...	1611	...	...	...
221	Vlorë	a.	...	...	...	...	...	...	...	1750	...	...	...

Vlorë	d.	...	0500	...	...	...	...	...	...	...	...	...
Fier	d.	...	0639	...	...	...	...	...	...	...	...	...
Lushnjë	d.	...	0739*	...	...	...	...	...	...	...	...	...
Pogradec	d.	...	|	...	...	...	...	...	...	...	1350	...
Elbasan	d.	...	|	0640	...	...	...	...	...	...	1716	...
Rrogozhinë	d.	...	0822*	0822*	...	...	...	...	...	...	1831*	...
Durrës Plazh	d.	...	|	|	...	...	...	...	...	...	|	...
Durrës	a.	...	0834	0834	...	...	...	...	...	...	2008	...
Durrës	d.	0615	...	...	0945	1300	1300	1535	1820	...	...	2025
Tiranë	d.	|	...	...	|	|	|	|	|	...	...	|
Vorë	a.	0640*	...	...	1020*	1335*	1335*	1610*	1855*	...	...	2100*
Vorë	d.	0651*	...	...	1021*	1336*	1336*	1611*	1856*	...	...	2101*
Tiranë	a.	0715	...	...	1045	1400	|	1635	1920	...	...	2125
Milot	d.	...	...	...	...	...	1505*	...	...	...	...	...
Shkodër	a.	...	...	...	...	...	1645	...	...	...	...	...

* – Estimated time.

The first image is the timetable entry for the Albanian rail system and the second is a map of the Albanian rail system taken from a larger map of South-Eastern Europe. Before discussing them as a class, the children had an opportunity to discuss with a partner what they could figure out:.

Some of the places on the timetable are the same as on the price chart.
It looks like there are distances in the first column.
The left side has the cities in one direction and the other side has them the opposite way.
Some cities are on there more than once.
Some of the times say 'estimated'.

Again, even though faced with an unfamiliar document, the children started making connections between it and things they already know. As they noted, the timetable is divided into two main parts, the left half being trains in one direction and the right half being the trains in the other direction. On the far left is a column for distances, starting from Skoder, the northern-most city in the rail system. Next is a list of cities (some listed more than once for convenience of listings) with an 'a' for arrival time and a 'd' for departure time.

The time listings are next – note that not every city has a time in every space. This is also a good opportunity to discuss how the times are written since the format here is slightly different from that in the photo of the Tirana timetable. The first train listed starts in Tirana at 0600 and travels to Vore where it arrives at approximately 0622 (the asterisk indicates an estimated time) where it stays for two minutes before starting off again towards Durres where it arrives at 0658. The next train listed starts in Durres at 0715 and does not stop in Durres Plazh going directly to Rrogozhine and Elbesan before ending in Pogradec. This could prove an interesting discussion point regarding the earlier question about leaving Durres at 9.45, arriving in Elbesan and returning the same day.

It is interesting to ask the children to compare and contrast the two timetable representations - this timetable and the timetable from the Tirana train station. Though they both provide some of the same information, the printed timetable gives additional insight. For example, though the train station board lists trains from Tirana to Elbasan, the printed timetable shows that it may not be the same train.

Now, we can return to one of the earlier questions – how far apart are Tirana and Durres? By using the distance information, the children could determine that Tirana is 98km from Shkoder and Durres is 102km. That must mean that the two cities are 4km apart, right? But this did not sit well with what the children had already figured out. Nearly an hour to go 4km?

For this question, we needed to look at the map. Though the map doesn't have the distances labelled (the 1390 refers to the timetable number in the book), we can fill in what we know. From Shkoder to Milot is 47km and from Skoder to Vore is 82km. That must mean that the distance from Milot to Vore is 82km – 47km or 35km.

Now, the distance from Shkoder to Durres is 98km. From the labelled map, we can see that the distance from Shkoder to Durres is the same as the distances from Shkoder to Milot plus Milot to Vore plus Vore to Durres. We could even write it more like a maths problem:

Distance from Shkoder to Durres

=

(Distance from Shkoder to Milot) + (Distance from Milot to Vore) + (Distance from Vore to Durres)

=

47km + 35km + ?

Since we know that the total distance is 98km, this becomes a fairly simple equation. Depending on the ability of children, you might even want to introduce a variable rather than use a question mark.

47km + 35km + ? = 98km

or

? = 16km

Solving this equation does not require an entire lesson in algebra, nor is it truly necessary to write it out exactly like this. What it does, however, is lay the foundation for later connections to more complex algebraic topics. And now we know that the distance between Vore and Durres is 16km. We still don't know the exact distance from Tirana to Durres but we can clearly see that it is more than the 4km that some children thought earlier.

Using a process similar to that used before, we can find the distance from Shkoder to Tirana.

Distance from Shkoder to Tirana

=

(Distance from Shkoder to Milot) + (Distance from Milot to Vore) + (Distance from Vore to Tirana)

=

47km + 35km + ?

This time, the total distance is 102km, so:

47km + 35km + ? = 102km

or

? = 20km

Adding the distances from Tirana to Vore and from Vore to Durres, we can see that the total distance between the two is 36 km. A similar method could be used to find the distance by rail between any of the cities listed on the timetable.

This new fact sparked additional discussion.

It only costs 69p to go 36km!

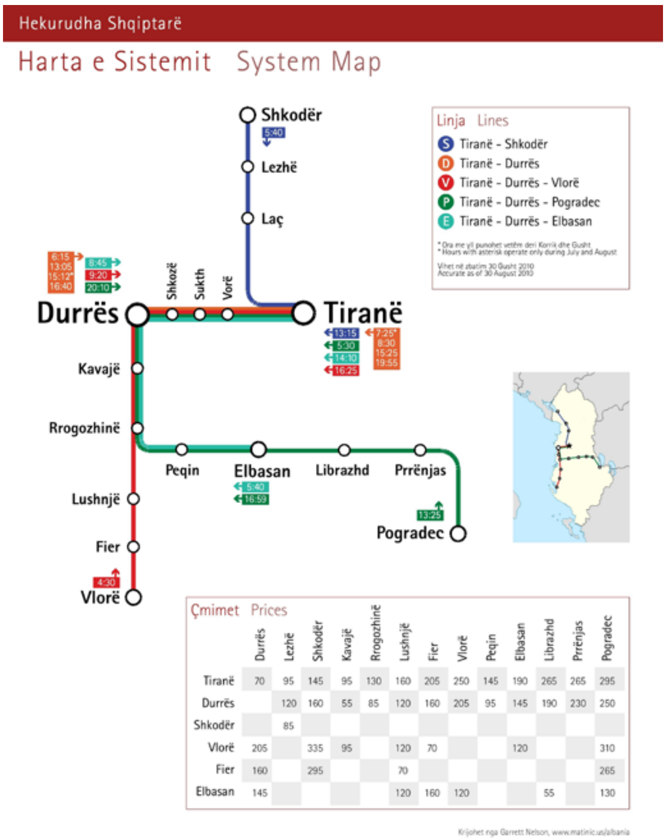
Yes, but it takes almost an hour to get there.

Are the trains really that slow? Maybe there is a big mountain in between places.

At this point, there are some additional possibilities for exploration:

How fast is 36 km/h in miles per hour? What places are 36km away? How long does it take us to get there? How long would it take us to walk from Tirana to Durres? How fast are trains in the UK?

Because the children were having such an enjoyable time with the investigation, I decided to show them one more representation of the data they were using.



The children immediately noticed the similarities between this map (pdf available from http://www.matinic.us/albania) and the map of the London Underground. Different rail lines are in different colours and larger stations are indicated by larger circles. This map also gives departure times from the main stations (though children were quick to notice that these times, as of 30 August 2010, are somewhat different than those from earlier this year). This map also shows a different representation of the ticket prices – as a grid format rather than in columns. In addition, only one price is listed for each destination. A quick check of our earlier data showed these to be the single, non-concessionary price.

One aspect of this chart that the children found interesting was the small map of Albania with the rail network shown. One child noticed that other than Tirana, the capital city, each of the other three end cities was either on the ocean or on a lake. He was quite interested about possible further investigations as to why this might be so and his initial speculation was that it might have to do with tourism.

It proved very interesting to compare the different representations of the data. During the course of this investigation, the children used three different maps, three types of timetables, two price lists and one rail ticket. From these sets of data, they were able to make predictions, test hypotheses and draw conclusions about what was, initially, a completely unknown situation. By using multiple representations of information, they could discuss the most useful representation for a particular situation and they realised that sometimes more than one representation must be used in order to get all of the information necessary.

While most people will not ever take trains through Albania, the skills used in this investigation are important ones for anyone travelling to a new location. Transport timetables, currency exchange, and map reading are all valuable skills, and set in this context they help children to make connections among a number of concepts that are not always taught together. Looking at and making sense of unfamiliar or initially confusing information is a skill that can be applied in almost any aspect of life. Additionally, the use of a variety of representations of information helps children create their own meanings of ideas.

Matthew Reames, former head of mathematics at St Edmund's Junior School, Canterbury, is currently working towards a PhD in mathematics education from the University of Virginia.

1. Patrick Barmby, Lynn Bilsborough, Steve Higgins, *Primary Mathematics: Teaching for Understanding*

2. "Extracts from the Thomas Cook European Rail Timetable, with the permission of Thomas Cook Publishing"

Well I never!

Bird statistics

Birds live at what, for humans, would be fever heat – at a normal temperature when awake of 41 C (106 F), rising as high as 43.5 C (110 F) during exercise. To maintain this temperature and to maintain the energy that it burns at a furious rate, a small bird may take about one-third of its body weight every day; a larger bird, which loses heat less quickly, needs about one-seventh.

The reason in this difference in the rate of heat loss is that heat is lost through the surface, and larger creatures have less surface area in relation to their volume than small ones. Doubling the size of a bird halves the rate at which it loses heat.

Book of British Birds, Readers Digest/AA, 1969