

Realising potential in mathematics for all

Vol.17 No.

for ages 3 to 18+

ISSN 1465-1254

# Graveyard data brings mathematics alive!

MATHEMATICAL ASSOCIATION





Realising potential in mathematics for all

|  | A close fit?   |
|--|--|
|  | For your classroom   |
|  | Graveyard data brings mathematics alive!   |
| Editorial Team:  | Dawn King finds that real-life death data inspires 'switched-off' pupils of varying  |
| Mundher Adhami<br>Mary Clark<br>Jane Gabb  | mathematics abilities.   |
| Rachel Gibbons<br>Caroline Hilton  | More for less: <i>Place your counters</i>  |
| Lynda Maple<br>Nick Peacey<br>Lorraine Petersen  | Liz Woodham suggests various activities involving the use of counters.   |
| Letters and other material for the attention of the Editorial Team to be sent  | Moving from counting to calculating  |
| to: Rachel Gibbons, 3 Britannia Road,<br>London SW6 2HJ  | Margaret Haseler shows how using structured imagery can help develop efficient calculation strategies  |
| Equals e-mail address:<br>ray.g@nodotcom.no.com  |  |
| ©The Mathematical Association  | 'Significant figures' – an invitation to question and discuss mathematics  |
| in Equals is held by the Mathematical Association  | Mary Clark has taken some of the significant figures provided by Rachel Gibbons and suggests some classroom approaches to using these to stimulate                   |
| Advertising enquiries: Janet Powell<br>e-mail address:   | mathematical thinking.   |
| jcpadvertising@yahoo.co.uk<br>Tel: 0034 952664993  | The Harry Hewitt Memorial Award  |
| Published by the Mathematical<br>Association, 259 London Road,<br>Leicester LE2 3BE  | Hand-spans and fingers: a way to tens and units  |
| Fax: 0116 221 0013<br>Fax: 0116 212 2835<br>(All publishing and subscription   | Mundher Adhami explores a way of helping children understand about place value.<br>Having identified that many children struggle with place value, he writes about a |
| Designed by Nicole Lane  | classroom experiment involving a group of teachers.  |
| The views expressed in this journal<br>are not necessarily those of The<br>Mathematical Association. The inclusion<br>of any advertisements in this journal<br>does not imply any endorsement by The | Resources for Applying Mathematical Processes  |
|  | Jane Gabb remembers with affection the GAIM materials of the early 90s, and explores the updated version available free  |

#### Editors' page

Mathematics in unusual places 2 - Curves of constant width or What does a 20p coin have in common with a manhole cover?

they discovered more about circles and their properties.

Taking risks in thinking and discussion works

Matthew Reames describes some recent work he did with Year 5, 6 and 8 children starting an exploration of the attributes of curves of constant width. This context provided opportunities for the children to apply vocabulary, concepts and skills as

Suni McWalter finds devising lessons on data handling for a Year 8 class a rewarding and rich experience. Lo and behold: literacy and functional mathematics as well as creativity are possible alongside meeting the needs of a range of pupils. 3

7

10

12

14

16

19

20

21

23

Mathematical Association.



# **Editors' Page**

It seems appropriate as *Equals* enters a new phase going from print to web with, we hope, a wider readership - to remember how it all started. It was back in the early 70s, in the Inner London Education Authority, that Peter Kaner originally dreamed it up and called it *Struggle*. In the first editorial he wrote:

"... If you could add, subtract, multiply and divide all sorts of numbers to the accuracy of 8 digits (including getting the decimal point in the right place), never get a wrong answer (except when you applied the wrong question), your skill would be worth around £5'.

He went on to estimate that, on this basis, the arithmetical skill of many low achievers on leaving school at that time might be rated at around  $\pounds$ 1.50 and then asked the question:

'If we are still failing to teach arithmetic satisfactorily in our courses for low achievers what should we do?'

He explained that *Struggle* had been created in an attempt to answer this question.

Unfortunately the question still does not seem to have been answered satisfactorily, so *Struggle/Equals* still has a vital part to play. How would you rate the arithmetical skill of your low attainers today?

It is clear from these extracts that, from the beginning, the purpose of *Struggle/Equals* has been in line with that of the Mathematical Association – **to improve mathematics in education**. We of course concentrate on a particular section of the school population: "the other half", the half that does not achieve A\* to C grades - and possibly couldn't, however hard it tried. In the *Education Guardian*, (20.10.10) Warwick Mansell reported that in one school at least "target intervention groups" (TIGs) are being allocated the best teachers and that "higher and lower ability youngsters can receive less support because their results are less likely to affect the school's published scores, it is widely claimed". Readers must recognise that TIGs are for those pupils who might, with some extra help and more skilled teaching, achieve the required C grades in English and mathematics that would make them 'count' in the school's league table statistics.

Back in 1963 on the Newsom Report we read concerning the other half about whom *Equals* is concerned:

'In the teaching situation success depends on more than having a kind and sympathetic interest in young people: most men and women who choose the teaching profession as their career can be counted on to possess that; control is required also, and knowledge, which the pupils respect, and the professional skill to transmit that knowledge to boys and girls whose manner and means of learning may be different from the teacher's own.'

And, now that Michael Gove's proposed changes are attempts to help the financially impoverished who might aspire, with help, to Oxford or Cambridge, and he is, at the same time, encouraging the founding of schools which will encourage those who have parents who already know how to bend the system to their own ends, it looks as if the other half will be almost entirely disenfranchised unless their teachers fight hard for them. We hope, as usual, that the following articles will help teachers to inspire and encourage these pupils to the utmost.

 The Newsom Report: Half our Future – A report of the Central Advisory Council for Education (England) HMSO 1963

# Mathematics in Unusual Places 2 Curves of Constant Width or What does a 20p coin have in common with a manhole cover?

Matthew Reames describes some recent work he did with Year 5, 6 and 8 children starting an exploration of the attributes of curves of constant width. This context provided opportunities for the children to apply vocabulary, concepts and skills as they discovered more about circles and their properties.

The article will be concluded in the next edition of *Equals*. In this the author describes further development of the topic of curves of constant width.

As an enthusiastic traveller, I find one interesting aspect of arriving someplace new is examining the local currency. Though the arrival of the Euro has introduced a certain measure of uniformity into the money of much of Europe, there are still quite a few places that have different coins with interesting and exciting designs. Most coins, at least those currently in use, are circular. There are some notable exceptions. The UK's 20 pence and 50 pence coins spring immediately to mind. The Australian 50 cent coin is a dodecagon. The 2 dollar coins and 20 cent coins in Hong Kong have wavy sides as do 10 cent coins from the Bahamas.

On a recent trip to the Czech Republic, I was examining a handful of coins I had received as change from a purchase. Most of the coins were round but the 2 koruna and 20 koruna coins were not. Closer examination showed the 2 Kč coins all had 11 slightly curved sides and the 20 Kč coins each had 13. These two coins, along with the more familiar 20p and 50p coins from the UK, are all examples of non-circular curves of constant width.

#### Curves of constant width

By definition, a curve of constant width is one whose width (the perpendicular distance between two parallel lines at the widest part of the shape) is the same in any direction. A simpler way of expressing this is to say that a curve of constant width is a shape that can rotate inside a square while maintaining contact with all four edges of the square. Perhaps the most common example of this is a circle.

The children in three of my classes (Years 5, 6 and 8) recently spent some time investigating some curves of constant width. Fear not, the term 'parallel tangents' never entered the discussion though a surprising number of more traditional Year 5, 6 and 8 mathematics concepts did. My pupils were amazed at the variety of mathematics concepts they used during our discussions. One of the best aspects of the investigation was that the children sometimes found that they did not yet know a term or a particular concept that they wanted to use so it was an excellent opportunity to learn the vocabulary or concept in a way directly related to what they were doing.

Here are some possible directions to take the investigations. It is not necessary to do them all or even

to try them exactly as presented. More advanced pupils may find additional things to investigate while pupils who are not yet as advanced may benefit from a discussion that includes vocabulary and concepts they know but applies them in a new context.

#### Start with a circle

'What is a circle?' I asked my Year 5 class this question recently. I got a variety of responses but the general consensus was, 'A shape that is rounded or curved.'

So I drew something like this:



And, predictably, they were not pleased. 'That's not a circle!'

'But it's a rounded shape. I made it curved,' I replied. 'Maybe we need to add something to the definition.'

'Yeah, it needs to be...kind of...**smoothly** rounded,' they said.

So I drew something like this:



'That's not a circle, that's an oval!'

'But it's smoothly rounded,' I said. 'Maybe we need to add something to the definition.'

After some discussion among themselves, they decided that 'with a circle, you can measure the same distance all the way across wherever you measure.' They held a ruler horizontally, then vertically, then at several diagonals. 'It has the same width everywhere.'

So I drew something like this:



And finally they were satisfied.

But I wasn't. 'So this has the same width? What if I measure here?'



'No, you have to measure at the widest point. And when you measure a circle at its widest point, it's the same all the way around. No other shape is like that.'

By this point, the children had been involved in lots of mathematical discussion about circles and other rounded shapes. They had used words such as angle, horizontal and vertical, they had discussed parts of circles such as radius, diameter and circumference, and they had realised the importance of accurately describing a shape. What if I told you that I could draw another shape that has the same width no matter which angle you measure it?'

'No way! You can't do that!'

'Maybe we need to add something to our definition of a circle because I will show you another shape has the same width no matter which angle you measure it. But first, we need to take another look at our circle.'

At this point, I will stop the imaginary conversation between my pupils and me. Instead, I will just describe what we did. Using a PowerPoint presentation, I showed a circle. (The PowerPoint file, including slightly more step-by-step animations, is available by emailing mdr@ stedmunds.org.uk.) By holding a ruler against the screen at a variety of angles, we could indeed measure the width of the circle. But imagine if we kept the measuring device still and rotated the shape inside it.

So I added a square around the circle.



At this point, asking the children to share some things they notice can help bring out some of the important things to notice: the circle fits exactly inside the square and the circle touches the square at exactly four points (and at one point on each side of the square). Depending on the age and level of the children, you may need to prompt or explain different terms – that is one of the purposes of this activity! Referring back to the earlier discussion, ask how we might measure the widest parts of the circle? How are they related to where the circle touches the square? Let's draw them on the circle.



What do the pupils notice now? When you measure across the circle, you can connect the points where the circle touches the square. The two dashed lines are perpendicular to each other.

The horizontal line is parallel to the top and bottom of the square while the vertical line is parallel to the left and right side of the square. If the children do not already know or use these terms, this is a wonderful time to discuss them. What is parallel? Perpendicular? What is a right angle? How many degrees? Show me with your hands. Point to another example of parallel or perpendicular. A major focus of this investigation is to use a variety of mathematical terms and concepts in a new way and to learn new terms and concepts along the way. When the children need the terms to explain their observations, this is the time to explain them!

Now, imagine that I rotate the circle clockwise. (What is a rotation? Which way is clockwise? How can we measure rotation?) Remember that we are keeping the measuring device (the square) still while rotating the circle inside it.



What has happened to the lines we drew? Why? Are they still perpendicular to each other? Why? Are they

still parallel to the sides? Why not? Can we still draw lines across the width of the circle that connects the points where it touches the square?



Now what do you notice about the new lines? Are they perpendicular to each other? Parallel to the sides as we saw before? How are they the same as or different from the first pair of lines? Where is the intersection point of the pairs of lines?

What if we rotate the circle again? Will we still be able to draw lines across the width of the circle that connects the points where it touches the square?

What do you notice? What is different from before? What is the same?



At this point, it was useful to remind the children the question we were investigating: is there another shape has the same width no matter which angle you measure it? The purpose of the square around the circle was so that we could measure the width of the circle at its widest point.

No matter how we turned the circle, its width was still the same and it still touched the square once on each side. Another way to look at it is that the circle will rotate freely inside the square while remaining in contact with the edges of the square.

## Making a circle in a square

Children can prove that the circle will rotate inside the square by using a compass to draw a circle. Then, by cutting a set of L-shaped right angles from sheets of card, they can make a square in which to rotate the circle. The benefit of using two L-shaped right angles is that circles of various widths can be measured. The diagram below shows how we made our sets of L-shaped right angles. The sheet of card is cut as shown on the dashed lines. The grey bit in the centre is then put to the side.



The L-shapes can then be carefully placed round a circle as shown below. The children can then rotate the circle freely inside the square while observing that it will still remain in contact with the square frame.



As I mentioned at the start of the article, investigating curves of constant width provided an excellent

opportunity for my pupils to take part in mathematical discussion and investigation. As far as I know, the heading 'Curves of constant width' has yet to appear on any primary mathematics syllabus. What my pupils discovered, however, was that they had to use and understand a huge number of concepts during their investigating. When they had questions or encountered something new, they were far more receptive to learning

a new term, concept or skill when it was in this context than if they had learnt it in isolation.

## St Edmund's Junior School, Canterbury

In the next edition of *Equals*, Matthew Reames' article will be continued to show how he developed the idea of curves of constant width further.

# Taking risks in thinking and discussion works

Suni McWalter \* finds devising lessons on data handling for a Year 8 class a rewarding and rich experience. Lo and behold: literacy and functional mathematics as well as creativity are possible alongside meeting the needs of a range of pupils.

I am going to explain how I created a set of lessons on data handling for a challenging Year 8 set. Although the pupils were in set 1 out of 4 they were working at level 4-6 which is below expectations for that Year group.

When designing a sequence of lessons for a topic it is important to take into account the diverse needs of the pupils within the group. I considered the following key points:

- All learners are different and are capable of some achievement;
- Knowing individual pupils well is essential to good differentiation;
- We cannot assume pupils will always be operating at the same level;
- We have to be aware that different pupils learn in different ways;
- Possible misconceptions need to be considered;
- Prior knowledge must be considered.

The plan built on prior learning of reading simple bar charts and ordering data. The knowledge base of the module was to design a collection sheet, collect data, construct and interpret graphs and tables. The pupils would analyse the data, communicate what they had collected and reflect on this. The lessons were ordered to suit the time of day with lesson 1 being more of a thinking lesson and lesson 5 being more activity based.

I felt that data could be a creative and practical topic and I planned in paired work and use of ICT, which was used as an incentive for those who might find it more difficult to engage.

I was trying to incorporate different types of activity:

- Visual representations using Mymaths, Smart board or PowerPoint.
- Thinking time by allowing pupils time to write answers on individual white boards.
- Discussion time they could confirm answers before taking a risk and answering questions.
- Group work
- Activities using the interactive white board.



Other considerations were:

- Differentiation the pupils were able to work to their own level. If I thought someone was not working to the standard of which they were capable I made them think harder about their questions, or asked them to think about a different way of showing their results.
- Real life applications we talked about making the questionnaires realistic and relevant to the population that they were aimed at. Many pupils went on to talk about healthy eating, fitness and social networking.
- Literacy the questions had to be discussed and written and then communicated
- ICT I took a risk and had a whole lesson in the ICT suite, as well as having pupils use the interactive white board to further their learning.
- Functional mathematics writing their own questionnaire, understanding how to word the questions, carrying out their research, working out the sample size that would be fair, collating and presenting the data and interpreting the results

Here is a brief outline of the lessons and my reflections on them.

#### Lesson 1 – What makes a good question? :

First we discussed terminology (e.g. primary source, questionnaire). This was followed by exploration of what makes a good questionnaire using a spider chart and some examples of unsuitable questions. This included a poor questionnaire which needed to be corrected.

I had intended that they should start their own questionnaire but the above took more time than I had anticipated.

Reflecting on work I realised that the pupils found this first lesson difficult. They were not used to having a discussion in mathematics and we needed this time to understand the terminology that we would be using. I realised that it is important to plan for the discussion to ensure that all will engage with it. One strategy that I used later was to have mixed gender groups. The questionnaire that had been given out to be corrected was level 6. This meant that the quality of answers varied due to the range of ability of the pupils and this gave me an insight into their levels of understanding. In the lesson I took a risk by including paired work because I prefer to feel that I am in control and I know that it is easy for pupils to talk off task. This was an important classroom management technique that I felt I needed to develop and the response made this a worthwhile risk.

#### Lesson 2 – Developing their own theme

To keep the pupils motivated and involved in designing the questionnaire I made it a creative lesson by giving them the opportunity to develop their own hypothesis. Areas they chose to explore included holidays, music, football, social networking sites and TV. They were allowed to pick their partners and where I had doubts as to the combination I made them aware that if they did not stay on task I would change the grouping. To get the best out of them I needed them to be interested. They were all engaged and the promise of the ICT lesson kept them all on task. This highlighted to me the value of positive reinforcement and encouragement.

#### Lesson 3–Looking ahead before the nitty-gritty

The starter showed various ways of presenting data so that pupils could start to think about how they might present their data, including some practice exercises after discussion. I had the pupils answering questions on the interactive white board so that I could assess if they knew how to read and draw certain graphs. This was a very hands-on lesson.

In reflecting on this lesson I realised that although I was out of my comfort zone again, pupils responded well; the lesson was a stepping stone between the previous ones and the future lessons as we discussed how they might choose to display their data using ICT. The bookwork that was completed allowed me to assess their confidence. I addressed common mistakes such as bars touching on bar charts, reminded them to label both axes and showed how a dual bar chart helps to compare data. Some excellent work was achieved. I used Mymaths because I have gained more confidence in using ICT and if I can use it, I will, as it tends to get the attention of the class. As I was able to keep reminding pupils about using the IT suite I found that their commitment to working in the lesson was very good.

#### Lesson 4 – Collecting data within the classroom

This lesson needed to be particularly well planned as pupils would be moving around the classroom. They had to sort out tally charts and sample groups; they had to ask and then record these answers. I took a risk with this lesson as I had all pupils moving, including those who tend to get over excited. Next time I might have half the groups sitting down and the other half up and about as this would lead to a less chaotic lesson. I did stay calm and was able to pull pupils together at the end. I feel that this lesson really worked because of the relationship I have with them and their awareness of my expectations.

# Lesson 5 - Presenting data using ICT tools Reflection

The students were taken out of a familiar environment and into one which was exciting and distracting. It involved a lot of planning and only worked because of that planning as I was also out of my comfort zone. I was worried as I was being observed and really would have preferred to have been observed at any other time than a Friday period 5.

I didn't change what I had planned to do as my pupils were looking forward to the lesson so I took the risk. During the lunchtime before the lesson I went into the ICT suite to see which computers were working so that I could sort out a seating plan. I didn't want to be hit with any surprises about machines not working that had been allocated to a group. I kept the pupils who I thought might get distracted in the main area and the ones who could be trusted in the other part of the room. I put up a slide of expectations so that the pupils knew what I expected of them. We started off in the mathematics room and clear instructions were given. I took squared paper, rulers and coloured pencils in case pupils messed around on the computer so that they could draw their presentations manually if necessary. I used a couple of pupils who needed more direction to help collect all the data for the slide show at the end.

#### Evaluation

Throughout all these lessons I found that because it involved discussion and group or paired work the lessons weren't like fixed structured lessons and so timing was very important, as was pace.

It was an even bigger risk for me to take the pupils out of their usual environment to the ICT suite but it worked. So I learned that taking risks is very important. Taking the Year 8s to the ICT suite was my worst nightmare but I felt it had to be done because I wanted to make the topic interesting and relevant to all the pupils.

I was not confident with ICT when I first went to the school and this batch of lessons gave me a great boost of confidence. I not only used Mymaths as a starter or plenary but I used the interactive white board and had the students come up to the board to move the scales, bar heights etc. I found that I got excited that this was working and the pupils enjoyed the flexibility in the classroom.

I also learnt that over-planning was vital for a large, bright and challenging class. Extra work sheets, exciting lessons and above all patience. All of this allows one to take risks. It is the lessons that are the most fun that take the most planning.

Now that I have more experience and a better understanding of teaching larger classes I realise that there is more to differentiation than just adapting work sheets for lower ability.

\* **Suni McWalter** was on the Graduate Training Programme in Windsor and Maidenhead last year. She is now working as a mathematics teacher in the Windsor Girls' School where she trained.

Vol. 17 No.1

# A Close Fit?

It seems whatever we choose to do from day to day we need to measure to see if something will fit. Here are two more ways of measuring. Check the thumb-fits of all our class mates - and as many adults as you can persuade - and write about what you find. If any of you has a small baby you can check with the hole do that too and add notes on your findings.



As for the glove box measurements, can you find gloves to fit all your class? Check and write about what you find.

Spring 2011

10



#### You Could Support Vital Work Like This

#### 1,200 Children In Two Weeks

MSF sent me to Leer, southern Sudan, as part of an emergency team. In January, it had become clear that there was a 200% increase on the previous year in the number of children coming to our centre there with severe acute malnutrition.

First we helped treat more children in Leer, a town in Unity state. Our next task was to get an accurate picture of the need across the state. In just over a week, we assessed 1,200 children - aged between six months and five years - using MUAC bands to measure their upper arms and quickly establish how many needed treatment for malnutrition. We found that there was a high number of children with severe malnutrition in Bentiu (a town 125km north of Leer). We then set up a further feeding centre there.

## What Does It Take To Set Up A Feeding Centre?

We were setting up a feeding centre in Bentiu from scratch. Our first job was to recruit 60 local staff, including nurses, feeding assistants, logisticians and a community health worker. Intensive training followed for all. MSF expects the highest standards. A MUAC band is a quick way of identifying children with severe acute malnutation. When a child's arm is this thin it will be in the <u>red</u> zone <u>without</u> treatment they could have just days to live.





Vol. 17 No.1

Equals

# **Graveyard data brings mathematics alive!**

Dawn King finds that real-life death data inspires 'switched-off' pupils of varying mathematics abilities.

#### Pupils with many acronyms

As an outdoor enthusiast and mathematics teacher in an emotional, behavioural and social difficulties (EBSD) school at the time, I was delighted to read Alan Edmiston's article called 'Cemetery data as a setting for real mathematics' (*Equals* 16.2). The school I first used this idea in was a unique one as it was an independent day school providing an alternative education for girls with complex needs. Specifically, the girls who attend the school have not only been subject to permanent exclusion from mainstream education but also from specialist Pupil Referral Units (PRUs).

The school has capacity for sixteen girls and all of the girls attending the school display extreme forms of behaviour,

in particular, promiscuity arising from early sexual experiences combined with drug and alcohol abuse. Several of the girls suffer from Attention Deficit and Hyperactivity disorder (ADHD) and some are on the autistic spectrum. The school aims to remove barriers to learning through nurturing and fostering an ethos of mutual respect. The ability of the girls varies widely and many of them have large gaps in their knowledge due to extended absence from school caused by exclusions or non-attendance.

#### Not a dull place to start

I believe that data handling can potentially be a very rich part of the curriculum, unfortunately, it all too often becomes dull and repetitive. Therefore, I was inspired by Alan Edmiston's article because it drew on data that was readily available and it interested pupils. It was particularly appropriate because previously, I had taken

One of the great things about this activity was the quantity of data.

8 of the girls to visit a graveyard as part of a school camping trip. The lesson was especially useful because the maximum class size in this school is 4; this made the collection of interesting primary data rather challenging.

In the summer term of 2010, I decided to use Edmiston's idea with my pupils. The lessons had been preceded by some classroom work on statistics: calculating averages, tallying Smarties, doing a traffic survey and producing bar charts etc. I began the activity in the classroom. I used photos from a camping trip the previous summer where we visited a graveyard at twilight as a starting

point and asked pupils what sort of mathematics they could find to do in a graveyard. This gave rise to fairly simple ideas such as: 'calculating how old

people were when they died'. This enabled me to assess at what level the girls were comfortable working. I had printed the data provided onto images of headstones and gave each of the students a set of cards and asked them how they might choose to sort them. Their first reaction was to sort them by male and female. I then asked them if they could sort them further which led to them sorting them into piles of dates.

One of the great things about this activity was the quantity of data. Whilst it did take a lot of time to prepare the resource, it was useful for the girls to have a large quantity of data to deal with as it highlighted the need to be systematic with the collection as well as the need for perseverance. It gave them a realistic experience of what it could be like to deal with data. Once the data was sorted, we discussed how we might display it and they suggested bar charts or pie-charts. I introduced



the concept of a dual bar chart to enable a comparison between males and females.

The following day was bright and sunny so I took four students out to a graveyard nearby. The graveyard provided lots of opportunities to think mathematically. They asked many questions such as:

- · How many graves were there in the whole graveyard?
- · What was the average age of death?
- · What was the youngest age and what was the oldest?
- · What could be possible causes of death?

These questions gave rise to rich conversation about things such as sampling. The girls demonstrated genuine interest and intrigue which was rare for them.

Once they had collected what they felt was enough data, we sat on a picnic table to work on displaying the data. They were able to look at the data and spot the most common months that people died in and because it was based firmly in real life, they were even able to draw meaningful conclusions about why they thought there were more deaths at certain times. The lesson gave the opportunity to make some cross-curricular links especially to history. The graveyard we visited had graves of victims of the Titanic tragedy as well as graves of soldiers who served in the First and Second World Wars.

I noticed that these girls, all of whom had been identified as having behaviour and social difficulties, were not in the least bit self conscious sitting in the middle of the common having a mathematics lesson and their behaviour was impeccable throughout.

#### Classroom activity in mainstream school

At the end of the summer term, I left the EBSD school to return to mainstream teaching. I am a firm believer that teaching mathematics outside the classroom supports learning. However, taking large groups of pupils (often around 30) away from the classroom and managing the activities single-handed is practically impossible and it would be a very rare occurrence to be able to have extra staff on hand to help. Therefore, I began to consider what it is about outside mathematics that helps so that I could try to apply some of these strategies and ideas in the classroom.

Edmiston conveniently provided a version of this lesson that could be taught in a classroom also. Therefore, I repeated the classroom lesson with a top set Year 9 class. The lesson took place in the week before October which provided the ideal opportunity to create a Halloween display. This time, when I asked students what sort of mathematics they could find to do in a graveyard, they came up with some really imaginative ideas such as: estimating how many graves in the whole graveyard by counting how many in a certain area and working out the total area of the graveyard, looking at patterns in the lay out, looking for symmetry, parallel lines and examples of different angles. Once again, a real insight for me into the level of mathematics for my pupils.



These pupils tackled the card sorting activity in much the same way that the previous ones had, by dividing them into males and females and then into months of death. As I looked around my classroom to see every pupil totally engrossed in the task, leaning over the work and tackling it with gusto, I felt thoroughly proud! When they came to display the data, I gave them complete free rein. My role was not to tell them what to do but support them in displaying the data however they chose. I was excited about the variety of different ways they chose to display their data. Some groups created a stem and leaf diagram comparing males and females in each month. Another group decided to tally the data, display it in a bar chart and then generate various questions about the data. I even got a cumulative frequency curve showing how many people had died by each month. They cut out large headstones from card to display their work and we made a spooky graveyard display in my window facing out, complete with fake cobwebs.

These pupils were clearly engaged by this task and there was further evidence of this as three groups came back at lunch time to finish their work. The slightly macabre nature of the lesson really appealed to them and they were really enthused. Letting pupils guide the lesson and choose how they wish to do things can be a scary experience for most teachers, not least due to curriculum constraints and the fact that so many teachers are uncomfortable if they are not totally in control. However, I found this lesson a useful assessment task. In fact, in hindsight, I think it would also have provided a really good opportunity for some peer or self assessment. I will continue to look for ways to bring the outside inside as a way of bringing mathematics to life and engaging pupils.. Once again, I would like to thank Alan Edmiston for his article and if anyone has any other ideas like this that they would like tried out, I would love to hear from them: <u>dmk@henry-cort.hants.sch.uk</u>

Henry Cort Community College, Fareham, Hampshire

# More for Less Place your counters ...

Liz Woodham suggests various activities involving the use of counters.

In this edition, I have selected a range of activities from the NRICH website which make use of counters. As in the previous articles in the series, I hope to illustrate how this simple piece of equipment or starting point has the potential to engage learners of all attainment levels and be flexible enough to respond to need. This time, most of the suggested activities are games, chosen to help focus pupils' attention on:

> Identifying and classifying patterns Forming convincing arguments based on findings

And teachers' attention on:

Assessing understanding Making mathematical connections Precision of arguments

## Where shall I start?



You may want to begin with the investigation Bracelets (<u>http://nrich.maths.org/79</u>) which engages pupils in number patterns, symmetry, factors, multiples, algebra and shape work, and gives them opportunities to be creative. Using 18 counters to start with, can they design



bracelets, making sure there is a 'hole' in the centre for someone's wrist?

How about using more counters?

Might it be possible to design two identical bracelets with a given number of counters? What is the smallest number of counters that allows you to do this?

Whatever design you create, how many counters would you need for the 'next design up'?

What are good numbers of counters to use? Why?

Would differently-shaped counters lead to different explorations?

## What next?

If you wish to continue with a spatial focus, try playing Square It (<u>http://nrich.maths.org/2526</u>) against the computer. Players take it in turns to click on a dot on the grid – the first player's dots will be red and the computer's will be blue. The winner is the first to have four of their own colour dots that can be joined by straight lines to form a square. Having played against the computer, pairs can have a go against each other using a grid and counters.

Not only is this game a fantastic way to reinforce the properties of a square (pupils are often surprised when the computer wins by drawing a tilted square), there is also the strategy to think about. Asking about the best place to start the game can lead into a systematic enquiry focusing on the number of possible squares from each dot. Does the strategy differ on a larger/smaller grid?

## Why not?

Introduce another strategy game, Nim-7 (<u>http://nrich.</u> <u>maths.org/1204</u>). Players take it in turns to take either one or two counters from a pile of seven. The player who takes the last counter wins. Challenge learners to find a winning strategy. Does it matter who has the first turn? What happens when you start the game with more counters?

How would you play to win if the loser is the person to take the last counter?

You could always investigate other versions of Nim which use counters (<u>http://nrich.maths.org/1209</u>).

## Leading on to:

For a dip into calculation, probability and strategy, introduce learners to the games First Connect Three (<u>http://nrich.maths.org/5865</u>) and Playing Connect Three (<u>http://nrich.maths.org/5864</u>). Whole-class discussion can focus on emerging strategies, observations, explanations and justifications.

Are there some numbers that we should be aiming for? Why?

Which numbers on the grid are the easiest to get? Why?

Which numbers are most difficult to get? Why?



## Going a little further:

Where Can We Visit? (<u>http://nrich.maths.org/746</u>) offers opportunities to explore fundamental ideas about number theory in a simple context.

Counters in the Middle (<u>http://nrich.maths.org/6978</u>) is one of a series of activities which encourage the development of team-building skills such as sharing reasoning, allowing everyone to contribute and valuing those contributions, and coming to a consensus.

NRICH, Cambridge University

# Moving from counting to calculating

Margaret Haseler shows how using structured imagery can help develop efficient calculation strategies.

The National Strategies research project report, *Children* who get 'stuck' at level 2C in mathematics (2010) lists a number of common persistent difficulties which pupils have in mathematics and which are seen as barriers to their progress. One of these is a lack of efficient calculation strategies. Most class teachers know pupils who still need to count on or back on their fingers to add or subtract even a small number and in many cases this remains the only strategy available to them when calculating. Using fingers for addition and subtraction is almost universal among those children who struggle with mathematics.

To develop efficient calculation strategies a pupil must memorise a bank of number facts. Without this, the burden on the working memory when carrying out anything other than a simple calculation is too great. For many children with special needs, this can be a problem! And while understanding mathematics should not become in itself a test of memory, often our approach to teaching mathematics depends on a good auditory-verbal and semantic memory rather than on developing understanding.

How can we help those children for whom memorising anything is going to be difficult if not impossible? The answer might lie in how we present number. Pupils with learning difficulties often have stronger visual-spatial and procedural (motor) memories, so by presenting pupils with clear structured images of number such as those afforded by Numicon shapes.. The structured images help to support memory for number facts.



Looking at each Numicon shape, we can see that each pattern is arranged in arrays of 2. By depicting numbers in this way, Numicon shapes provide a 'structured' image of each number. Because it is easy to see the incremental increase with each number in the sequence from 1 to 10, we can see why each number occupies its position on the number line. For example, we can see why 7 is between 6 and 8 – because it is 1 more than 6 and 1 less than 8. The properties of the number are also apparent. Because the pattern is arranged in twos, we can see that 7 is an odd number. More importantly, structured images such as these allow us to see how numbers are constructed and this is what helps children when they are calculating. In the case of 7, we can clearly see that the 7 pattern is made up of the 6 and 1 patterns, 5 and 2 patterns and the 3 and 4 patterns. The ability to flexibly split 7 into different subsets is the basis for calculating and inherent in this skill is the relationship between addition and subtraction. So by knowing the Numicon patterns for the numbers 1 to 10, a child has access to all the addition facts for the numbers up to 10 - and there are about 100 of them - as well as an understanding of the relationship between addition and subtraction. Of course children need to know the Numicon patterns very well but learning them is in no way as hard as having to memorise a set of number facts. Teachers are constantly surprised at the speed with which the majority of children in the Foundation Stage learn the Numicon patterns and can then use them to do simple addition and subtraction in a practical setting. Children then go on to internalising the patterns and are able to combine them with a knowledge of how our number system works to develop calculation strategies e.g. if 4 and 3 equals 7, 14 and 3 will make 17.

Here are several activities I have used successfully with both KS1 and KS2 pupils. For all of these it is important to encourage pupils to use the Numicon patterns to work out the answer rather than resorting to counting on/back on their fingers. Later, the Numicon number line may be a sufficient 'in between' step before encouraging pupils to visualise the patterns.

 Have a selection of shapes on the table. Choose one of the shapes and ask pupils to find 2 shapes to match your shape. Once pupils are confident, this can be extended by asking pupils to say as many addition facts as they can. *How many different ways can you make 8?* Pupils could also be encouraged to use a variety of addition words 6 *plus 2 equals 8...* 4 add 4 equals 8.

7 seven eight nine

A pupil chooses a Numicon shape and then fills the shape with pegs. The pupil then chooses another 2 shapes to match this and places them on top of the pegs. Another layer of pegs is added with another 2 shapes to match on top and so on. So if the first shape was 8, the next layer might have a 3 and a 5, the third layer might have a 6 and a 2, the fourth layer might have two 4 shapes. If appropriate, children can then say the number sentences they have constructed. E.g. *I can make 8 using a 3 and a 5, or a 6 and a 2 or two 4s*. Or they might see how many pairs they can remember once their block of flats is complete by recording the number sentences. By having the block of flats visible, the child can look to see if a particular pair has been forgotten.

An extension to this activity could involve using known addition facts to generate subtraction facts. So using the 8 block of flats, how many subtraction facts related to 8 can the pupil remember?

Once pupils are confident in building their blocks of flats, they can be encouraged to visualise the shapes. For example, *How many different ways can you think of to make 6? Close your eyes and imagine building a block of flats for 6.* Again pupils can check their answers by combining the 2 shapes suggested and seeing if they match the 6 shape.

## 3. Play What's in the bag?

Pupil 1 puts two shapes in the feely bag and says what the total is. Pupil 2 has to guess what shapes could be in the bag. So if a 4 shape and a 2 shape are in the bag, Pupil 1 says I've made 6 with 2 shapes. Pupil 2 chooses all the possibilities e.g. either 5/1, 4/2 or 3/3. Pupil 1 takes one of the shapes out of the bag and Pupil 2 identifies which shape is left in the bag. At first, Pupil 2 may need to find all the pairs using a set of shapes on the table. S/he could then graduate to looking at the 6 shape and visualising all the pairs. As they become confident, gradually encourage pupils to visualise the Numicon shapes. Encourage mathematical reasoning and use of appropriate language by asking pupils to explain how they know what the remaining shape is. 4 must be in the bag because if you've already got 2, you will need 4 more to make 6. Extend this activity by providing a clue about one of the shapes in the bag instead of identifying it e.g. I've made 6 with 2 shapes. None of my shapes is





*odd*. The game can be extended further still by playing the game with 3 shapes in the bag instead of 2.

The following activity is taken from the *Closing the Gap* teaching manual and while appearing simple, it encourages children to use known addition facts to begin to reason about number.

4. Turn it over - a game for 2 or more players

Have 2 sets of Numicon shapes 1-5 in a basket. Arrange numeral cards 2 - 10 in a 3 x 3 grid face up. Pupil 1 chooses 2 shapes, combines them and says the total. The corresponding numeral card is then turned over (so if the 2 shapes total 8, the 8 card is turned over). Pupil 2 takes his/her turn. As more cards are turned over, there are fewer possibilities left so pupils have to think carefully which shapes to choose from the basket in order to turn over one of the remaining cards.

A competitive element can be introduced where the winner is the first to turn over a row of 3 cards. Alternatively, each pupil can place a counter on the cards they have turned over. The winner is the person with the most counters.

Once children are confident with constructing and deconstructing Numicon patterns, they can use these to develop efficient calculation strategies. Each time a new idea is introduced, the Numicon shapes provide a strong consistent visual model which in turn supports children's developing mental imagery. Below is one example of how Numicon can be used along with the tens number line when bridging through a multiple of 10 to solve 28+5:



Providing a visual model of the bridging strategy is an essential 'in-between' step for some children to illustrate how an empty number line works. Activities such as these help pupils with special needs in a number of important and significant ways:

- By encouraging children to use the patterns found in Numicon shapes, children move beyond counting in ones as a calculation strategy.
- Most Numicon activities are designed to enable pupils to work either in pairs or trios. Because of this, they are ideal to use as independent activities, thus helping children who struggle with mathematics to become independent, confident learners. Working with others also encourages opportunities to use mathematical language in a meaningful context.
- Because all Numicon activities provide the opportunity for self-correction, confidence is increased and anxiety levels decreased – a vital change around for the majority of pupils who struggle with mathematics,

It is important to remember that Numicon is suitable for pupils of all abilities. Activities can be easily differentiated to meet the needs of a wide range of abilities, including those who are working above age related expectations.

Numicon

The world's rainforests are being destroyed at the rate of two rugby pitches a second. If this continues, half the remaining forests will be gone by 2025 and the rest by 2060. Stoppingc the deforestation now would reduce carbon emissions by 17%.

TES Magazine 7 May 2010

# **'Significant figures' – an invitation to question and discuss mathematics**

Mary Clark has taken some of the significant figures provided by Rachel Gibbons and suggests some classroom approaches to using these to stimulate mathematical thinking.

Returning to a digest of research on mathematics education  $5 - 16^*$  published in 1995, (still obtainable as I found after a quick internet search), made me think again about questioning to stimulate good quality mathematical thinking.

One of the research studies quoted refers to the power of using statements to provoke mathematical discussion: 'when teachers make statements rather than ask questions, pupils can display more complex thought, deeper personal involvement, wider participation, greater interconnectedness, and richer enquiry. So for example, rather than asking "Are all squares parallelograms?", it might be more profitable to ask pupils to discuss the statement "All squares are parallelograms."'

With this in mind I returned to thinking about the regular *Equals* feature of 'significant figures'. We have a collection of these for this issue:

80% of marmalade is bought by people over the age of 45.

We watch a day's worth of television every week.

83% of teenage girls do not eat enough fruit or vegetables.

More babies are born on Christmas Eve than on any other day for the year.

The challenge here is for pupils to think about and discuss



And then, because I was tempted myself to think about questions I would want to ask and find answers for, I started to explore this for my own satisfaction. Here are some of my questions as I thought my way into the implications of each statement in turn:

# 80% of marmalade is bought by people over the age of 45.

What would I need to find out to discover whether this statement is true?

This is a big question so first think about how to find out whether it is true for some people I know.

What questions would I need to have answers for?

Who do I know who eats marmalade?

Who buys it?

## We watch a day's worth of television every week.

Do I think this true for the people I know?



Would it be different for different types of people?

Why?

Is it true for me or for my family members?

How could I work out whether it is really is true for anybody I know?

# 83% of teenage girls do not eat enough fruit or vegetables.

How much is 'enough fruit and vegetables'?

Do I think it is true for teenage girls I know?

What information would I need in order to be able to check whether the statement is true for teenage girls I know?

More babies are born on Christmas Eve than on any other day for the year.

Do I think this is true?

How could I check it?

What information would I need?

Where might I get that information?

Over to you ... what do your pupils identify as questions to ask in order to find out about just one of these statements? Of course if time allows further of the statements can be explored but it is definitely worth investing time to investigate just one statement in depth rather than to cover more and lose that opportunity for deeper thinking.

\*Mike Askew and Dylan Wiliam, *Recent Research in Mathematics Education 5 – 16*, Ofsted Reviews of Research, HMSO 1995

# **The Harry Hewitt Memorial Award**

Do you have a pupil who has struggled with mathematics and is now winning through?

# Celebrate the success in Equals

We are once again offering a prize of a £25 book token for the best entry we receive. The winning entry will be published in *Equals*.

To enter choose, from a pupil who has been struggling, a piece of work that you and the pupil consider successful and send the original to *Equals* together with:

- \* an explanation as to how it arose;
- \* a description of the barriers the pupil has overcome in doing it;
- \* the pupil's name and age, the name and address of the school and the context of the pupil's set / class / stream.

## Entries should be received by 31 March 2011

(Send all work and information to Equals, 31B Brunswick Square, Hove BN3 1ED)



# Hand-spans and fingers: a way to tens and units

Mundher Adhami explores a way of helping children understand about place value. Having identified that many children struggle with place value, he writes about a classroom experiment involving a group of teachers.

Children's difficulties with place-value start with two-digit numbers early in their school lives. Many children with, and without, special educational needs in secondary schools have such difficulties, and some mainstream pupils go through the hoops of doing calculations with multi-digit numbers without knowing whether their answers are sensible or not.

I recently worked with a Year 4 child who had no idea what '11' meant and how to add 1 to it. Two-digit numbers are confusing, especially if they are between 10 and 19. That is probably because the reading of the number is from right to left, e.g. 19 is nine-teen as compared with the larger numbers which are read from left to right as in 29 - twenty nine. Also 'eleven' and 'twelve' do not fit the pattern and have no 'teen' in their names or any indication that they relate to the 1 and 2 in the units place. The words do not fit the marks on the page, and what does a number mean anyway? Colleagues confirm that such problems are common in primary classes, and teachers who have been teaching Year 8 pupils in 1-to-1 sessions found some in the same predicament.

Even at the end of a good lesson on tens and units which a group of us observed recently, only 2 out of 10 Year 2 children agreed that 58 is smaller than 60. It is likely that this is because one number has a 6 and a zero, and the other has a 5 and an 8, and many children tend to combine numbers without regard to their place value.

But the activity we trialled<sup>1</sup> did attempt to give meanings to the digits that all children could make sense of at the start of their learning. After the activity many of them were starting to understand a bit about what each digit means.

A group of 10 children was given a 'real-life challenge' to find out whether pieces of furniture in the classroom would fit into a gap. 'This furniture in this room is messy, could you help us make it better? But you shouldn't say anything to Jane for now!' They were all engaged. The task required them to compare lengths and they were asked not to use rulers but instead were shown how to use hand spans and finger widths as measurements.



A teacher demonstrated a hand span and putting palms together, marking the ends on a strip of paper, tearing it to that length. She then showed them how to repeatedly mark a middle finger on that strip of paper. Children were given strips of paper and pencils to help them to do their own. Now the children had different strips of paper,



each equal in length to about 10 of their middle fingers, marked in lines. Each pair had to decide which strip to use. (This needed more thinking.)

Working in pairs, children used the strip or one real hand span and fingers to measure the width of the gap and a chosen piece of furniture to check if it would fit. They were asked to write the measurements for the gap and one of the 5 pieces of furniture: a short cupboard, a tall cupboard, a drawer unit, a round table and a sand box.

In the final discussion each pair gave their measurement for the width of the gap and their piece of furniture and said whether it was too big or it fitted. Since each pair was accompanied by a teacher, even the children who weren't numerate did manage to measure and to use each hand as 10 to write their numbers. The class was involved in giving meaning to each measurement, and to make the comparisons. The teacher at the end asked why one pair gave 53 as a measurement for the gap while another gave 78. One child straightaway said that Gill's hand is small than Saba's. The teachers asked the two girls to compare the hands and showed the class that was true. Although it became obvious that this part of the lesson was more suited to older children we confirmed the potential of the lesson.

It is clear that 'tens and units' and how they are written as numerals are not simple skills that can be guickly learned, but rather an incrementally developing recognition of conventions. They are concepts to be gradually built up from experience and concrete objects. Most schools have blocks of small wooden cubes, 'longs' of 10s together with the 'flats' for 100 and the large cubes for 1000.<sup>2</sup>



Some teachers use these blocks, called base-10 blocks or Dienes' apparatus, but it may be that others find them cumbersome to organise. We now have literally a hands-on experience that combines personal measurements and ratios, and a link between counting and length measurement. It did seem in this first trial that the approximate nature of the 10 fingers was not an obstacle, but we need more trials.

#### Cognitive Acceleration Associates

- The trial was in an Islington School in London 16th Nov 2010. I 1 worked with a group of 5 teachers of nursery, reception, Y1 and Y2 classes. We started with an idea that a hand span of a person is about 10 times the width of their middle finger. Each of the teachers confirmed this for themselves on strips of paper, and quickly recognised its potential. One teacher suggested that there may be a confusion because we use a palm for a 5 in counting fingers, so introduced the two palms tightly placed together to show both the hand span and the 10 fingers. We looked for a context that is meaningful to children and settled on fitting pieces of furniture into a gap.
- Illustration from an article by David McLaren in MA's magazine 2. Mathematics in School November 2010

## Time is running out 280 \$85 The amount of CO<sup>2</sup> in the atmosphere in every parts per million before the start of the Industrial Revolution 430

The amount of CO<sup>2</sup> in parts per million In the earth's atmosphere now

#### 150,000

The number of people killed by climate change since the 1970s, according to the World Health Organisation

The Guardian 31 Oct 2006

The economic damage caused by tonne of carbon that is emitted

£25bn

The size of the environment sector in Britain

#### £150bn

The potential cost of making developed countries' buildings and infrastructure resilient to climate change



# **Resources for Applying Mathematical Processes**

Jane Gabb remembers with affection the GAIM materials of the early 90s, and explores the updated version available free at: <u>http://www.nuffield-foundation.org/applying-mathematical-processes</u>

Anyone reading this who was teaching mathematics in the late eighties and early nineties, when we were battling with the first version of the National Curriculum, might remember some brilliant materials which came out of a research project at King's College, London. They were called GAIM (Graded Assessment in Mathematics) and the resource consisted of 40 investigations and 40 practical tasks plus some other assessment materials. Their particular value from my point of view was in including examples of pupils' work and suggesting a level for that work. This was the first exemplification of standards that I remember and it was very useful in coming to a judgement about a pupil's level, and in developing an understanding of progression in Using and Applying mathematics. Incidentally these materials are now available on line. They have been scanned in and can be found in a variety of formats at:

http://www.nationalstemcentre.org.uk/elibrary/ resource/87/activities-investigations\_

and:

http://www.cognitiveacceleration.co.uk/resources/gaim. html

Originally published by Thomas Nelson, and quite costly as I remember, they are now freely available to all.

More recently, the Nuffield Foundation together with the Clothworkers' Foundation, funded a project to update the materials and the results are now on their website, again freely available. There are 20 activities, roughly split between investigations and practical problems.

Each activity includes:

- teacher guidance including key vocabulary, mathematical processes likely to be covered, the resources needed, probing questions and possible extensions
- pupil sheets
- progression information an editable resource illustrated by examples of pupil work
  Some include:
- spreadsheets to support the activities & interactive pupil resources

All of the activities are suitable for all secondary pupils, and they lend themselves to pair and group work.

Some of the contexts have been updated (e.g. 'Sending cards' is now 'Sending texts') but the mathematics is the same. Others will be instantly familiar to those who worked with GAIM.

One that caught my eye (perhaps because it was one I hadn't used) was:

**Emergency Shelter** – design a shelter, given 3m x 4m of material, to protect 3 people from rain and wind. The pupil sheet shows photographs of people in disaster areas with the shelters they have put together, making this task immediately relevant and topical.

The progression table exemplifies what the mathematical processes are likely to look like in this task and illustrates

this with examples of pupils' work. The progression is clear, though no levels are indicated for particular pupils' work.

A full list of the activities available is given below.

It would be good to hear from teachers who have used these materials in their own classrooms. There is nothing as powerful as first hand experience. If you would like help in putting an article together, please get in touch with Equals and one of the editors will support you in making your experience accessible to all our readers.

Woking

Here is a full list of the activities available:

## 9 Practical explorations

#### Beach guest house

Time Up to 2 hours

Spreadsheet

Simulation of a booking system for a small guesthouse. Pupils have to manage the bookings and, as far as possible, arrange to give people the accommodation they request.

#### **Cemetery mathematics**

**Time** 1+ days; 1 hour upwards for preparation Pupils can experience collecting primary data from a local graveyard or cemetery and then set and test their own hypotheses.

#### Design a table

#### Time 2 to 4 hours

Pupils are asked to design a table for a group of 5 people for daily use. The table must be extendable to accommodate 8 to 10 people for some occasions.

#### **Emergency shelter**

Time 1+ hours

The task is to design an emergency shelter, using a 4m x 3m rectangular piece of tent material, to protect three people from wind and rain.

#### **Every second counts**

#### Time 2+ hours

Pupils explore how far away they could travel in one hour.

#### **Fashion entrepreneur**

#### Time 1 to 2 hours

Scheduling jobs in a fashion workshop. There are six people in the workshop and a series of jobs to be completed in one day.

#### Money bags

#### Time 1+ hours

Designing a wallet or purse, and explaining the rationale underlying the design.

#### **Reaction times**

#### Time 1+ hours

Pupils have to develop an experiment to measure reaction times and use it to test people's reaction times. They can use any equipment available in the school/ classroom, and will need to consider the reliability of their experiment and how any data collected will be analysed and presented.

#### **School holidays**

#### Time 1+ hours

Pupils consider what factors might affect the choice of dates for school holidays and use these to determine the holiday dates for an alternative school year.

#### **11 Investigations**

#### **Average limits**

Time 1+ hours

#### Spreadsheet

Pupils explore limiting values of an iterative process, using arithmetic, algebra or spreadsheets. Pupils can move from identifying patterns to forming, verifying and proving conjectures.

#### **Co-primes**

#### Time 1+ hours

Starting with a definition of what it means for integers to be co-prime, pupils investigate how many positive integers are less than and co-prime to any given positive integer.

#### Corner to corner

#### Time 1 to 2 hours

#### Flash & PDF interactive

Pupils investigate how different numbers of squares can be joined corner to corner, and the effect their arrangement has on the area of the rectangle that encloses the squares.

#### **Fire hydrants**

Time 1 to 2 hours

Flash & PDF interactive

Pupils experiment with the placing and number of fire hydrants required in a city with square blocks that form a rectangular grid.

#### Golden mazes

#### Time Up to 1 hour

#### Flash interactive

Rooms in a rectangular maze of rooms have bags with a varying number of gold coins. Pupils explore the effect of the route on the number of gold coins that can be collected.

#### Hide the spies

Time Up to 1 hour

### PDF interactive

Pupils determine where spies should sit in a park that has a square grid of benches, interspersed by bushes, so that they cannot see each other. They investigate how many different arrangements are possible.

#### Paper sizes

#### Time 1 to 2 hours

Pupils study paper sizes in the A and B international series, exploring relationships within each series and between the series.

#### Sending texts

#### Time 30 to 45 minutes

This investigation involves determining the number of text messages sent if four people send texts to each other, and then extending this for different numbers of people.

#### Stacks

#### Time 2 hours

Flash interactive

Pupils explore, analyse and describe the patterns generated by moving counters between two stacks according to a fixed rule, always doubling the size of the smaller stack.

## Symmetry

**Time** Up to 2 hours PDF interactive Pupils make different symmetrical shapes, using one or more of three given shapes.

#### Three dice

**Time** 1 to 2 hours Flash interactive

To maximize their chances of winning a bingo-style game, pupils must decide which numbers are most likely to occur when three dice are thrown and the scores are added.

