Equals

Realising potential in mathematics for all

for ages 3 to 18+

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Realising potential in mathematics for all

Letters and other material for the attention of the Editorial Team to be sent to:
Rachel Gibbons, 3 Britannia Road, London SW6 2HJ

Equals e-mail address: ray.g@ukonline.co.uk

Advertising enquiries: Janet Powell
e-mail address: jcpadvertising@yahoo.co.uk Tel: 0034 952664993

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Editors' page

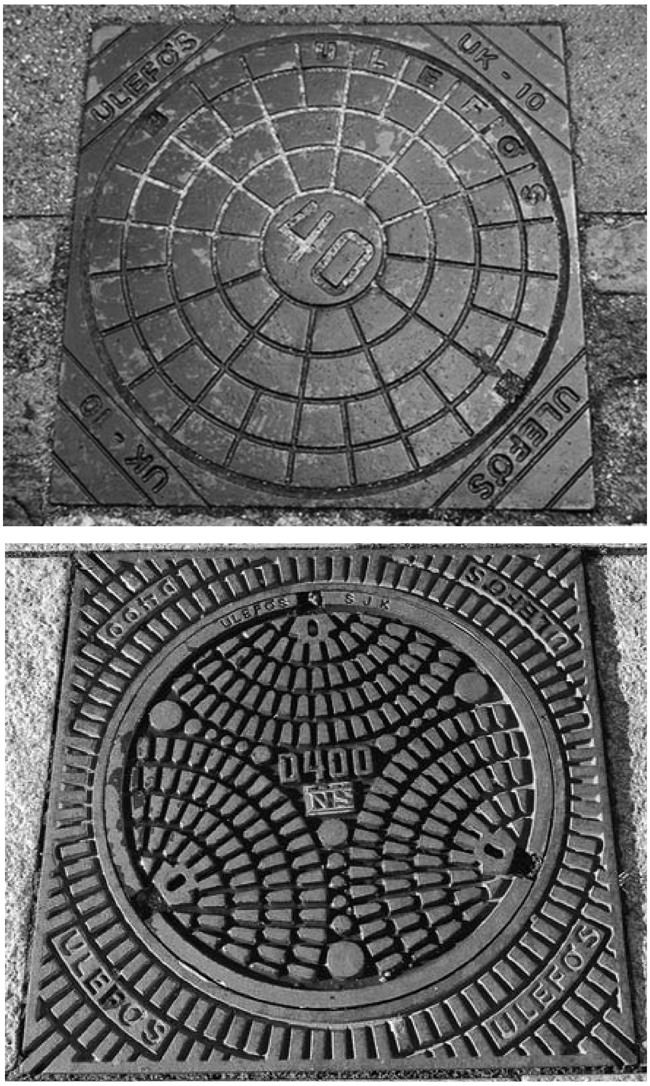
This may be the last paper issue of *Equals* you will get. But never fear – we are going on line. You can find us on the Mathematical Association's web site at m-a.org.uk with open access to all and we hope this will mean that *Equals* will reach a wider audience. We write for teachers of those who find mathematics difficult and these teachers are not always the ones who feel drawn to mathematical magazines. Please tell any teacher of this description who may be lacking in confidence either mathematically or pedagogically that we are here to help. We have stories of what happens in real classrooms, written by teachers who have cracked the code in some area of classroom practice. If you have learnt from something that happened in your classroom it is likely that other teachers can learn also. Tell us about it. And if your outline is sketchy we will, with your permission, polish it up.

It is difficult to predict in what direction the present government will try to steer schools. Unfortunately politicians, like everyone else, think that because they have themselves for years spent their youth sitting at desks in classrooms that they know what should happen in the classrooms of today – indeed it seemed at one time that Michael Gove might be trying to return us to the days of Victorian education with the reciting of tables and other rote learning – but there does seem to be a recognition that it is the quality of the teacher that is of first importance. Dylan Wiliam in a book reviewed in these pages quotes research which shows there is more in-school variation than between-school difference, in other words, it is the teacher that counts.

The other important element is of course the attitude of the child which can be so seriously affected by labelling and testing. The lower half of the attainment range has been losing out for years through lack of self-esteem, a member of a lower set, perhaps the bottom set. In the next issue of *Equals* we shall be reviewing *Learning Without Limits* which seems worth quoting from at this point:

When young people's learning is dominated by judgements of ability, their sense of identity may be profoundly affected, not just while they are at school, but beyond into adulthood. Readers of this book will no doubt be able to bring to mind people they know whose lives have been affected by being written off as incapable of serious academic achievement at crucial points in their education.

We are used to seeing the slogan: SMOKING DAMAGES YOUR HEALTH. Maybe we should start to display another: **SETTING DAMAGES YOUR EDUCATION.**



Manhole covers, see *Mathematics in Unusual Places* page 3

Mathematics in Unusual Places 1 – Manhole Covers

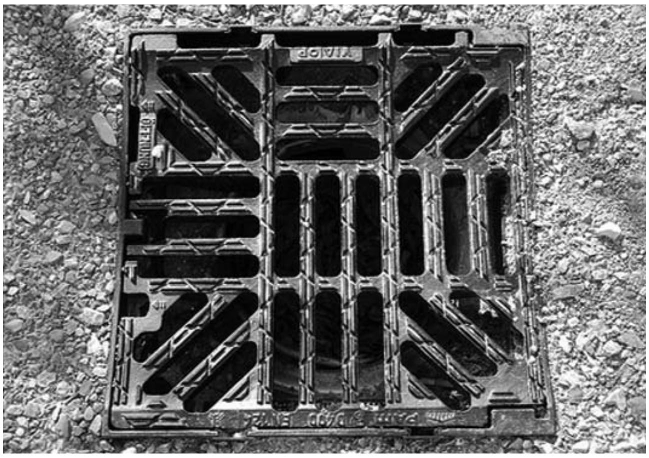
Manhole covers are more than just heavy pieces of metal that stop people from falling into holes in the ground Matthew Reames explains. They come in a variety of shapes: circular, square, rectangular, hexagonal and more - and can be an exciting way of exploring mathematics.

The designs on the surfaces of the covers range from simple circles or squares to groups of parallel lines and some rather complex shapes. Some covers have patterns involving translations or reflections while others have line symmetry or rotational symmetry. A quick look at the collection of photographs of manhole covers at http://tinyurl.com/manholecovers will show some of the huge variety of shapes, sizes and patterns throughout Europe. (As an aside to purists, the collection includes photos of both manhole covers and drain gratings.)

Using different manhole covers is a real-life way of incorporating maths skills into your lessons. What is the area of each cover? The perimeter? Can you find a pair of manhole covers that have the same perimeter but cover different areas? Do rounded corners make any difference to the calculations? Do some covers have corners that are more rounded than others? The area calculations are fairly straightforward with rectangular covers but become slightly more difficult for round covers. Other shapes such as hexagons, ovals, and triangles each have their own challenges.

Children who are studying circles and Pi may benefit from the opportunity to use what they have learnt to measure and calculate the area and perimeter of round manhole covers. They can have a very good discussion about the best way to measure the circumference of a round cover, something that is not nearly as simple as measuring the perimeter of a rectangular cover. Investigating round covers becomes even more interesting if you have round covers of different sizes. Compare the diameter, circumference and area of each circle. Can you find rectangular manhole covers with the same area or perimeter as the round covers?

Looking more closely at the covers, have the children examine the surface patterns. Is there line symmetry? Rotational symmetry? Can they find examples of translations, reflections or enlargements? Some surface designs are intricate tessellations while others are complex swirls. Are any of the designs completely random? Do any covers have a completely blank surface? Why do you think the designers created the patterns that they used?



A note on photographs

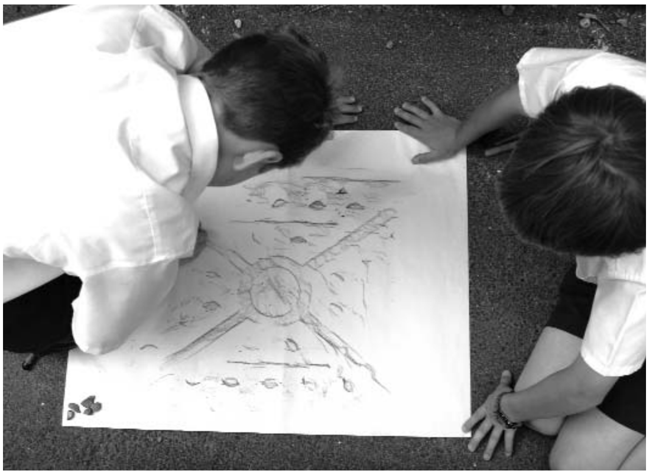
One thing to keep in mind when taking photos of manhole covers is that unless the camera is directly over the cover, there will be some distortion of the shape. Circles will look more oval-shaped, rectangles will become trapeziums or other quadrilaterals, parallel lines may no longer look parallel, and angles will not be their true sizes. This is something to keep in mind if you are using the photos to measure lines or angles.

It might also be an interesting exercise to photograph the same manhole cover from different angles in order to investigate just exactly how the lines and angles do change when observed from different positions.

A manhole cover spotting expedition

Our school site is home to numerous different types of manhole covers. Recently, our Year 3 class and some of our Year 5 children went on Manhole Cover Spotting Expeditions. Their goal was to find as many different 'species' of manhole covers as they could. Each time they located a 'specimen' they kept a record of it by making a rubbing of the cover on a piece of 'Expedition Paper'. In our case, our 'Expedition Paper' was paper from a large pad of flip-chart paper – large enough to cover most manhole covers and thin enough make a good rubbing without being so thin that it rips easily. One benefit of making a rubbing of a manhole cover is that there is no distortion of the design – a rubbing can be taken back to the classroom and measurements taken directly from the rubbing, something that is not as easy to accomplish with a photograph.

Just one further word of caution: please be safe and only photograph or make rubbings of manhole covers that are in safe locations! School playgrounds and pavements are good, but roads are not.



The children spent the next 15 or 20 minutes spreading out around the area near the playground to find manhole covers and make their rubbings. Groups eagerly came back to show the 'specimens' they had discovered and to ask for more Expedition Paper to use for other manhole covers. At the end of our time outside, the children had collected quite a few different 'specimens' and it was time to take the specimens back inside to discuss them.

Some helpful tips to remember when making the rubbings include removing the paper wrapping from the wax crayon and using the edge rather than the tip to rub, having one or two people help hold the paper in place while someone else does the rubbing, and working carefully to get the fine detail of the design while taking care not to rip the paper. My pupils have also suggested that you choose a calm day as even a light wind makes the large sheets of paper somewhat difficult to manage. They also recommend using darker colours of wax crayons as the rubbings show up much better on the light-coloured paper.

Back in their classroom, the children laid their rubbings out on the floor or tables to create a gallery of the Expedition Papers. They were amazed at how many different examples they had found in the relatively small area outside. In the discussion that followed, the children talked about the shapes of the covers they found, which shapes were the most common, and the detail of the covers. Some children said that they thought certain designs were easier to rub because the patterns were more regular than others. There was some debate about whether or not a square was a rectangle so this provided an excellent opportunity to talk about the characteristics of squares and rectangles so that the children finally concluded that squares are a special type of rectangle.

Examining manhole cover attributes

A further extension to this Spotting Expedition is to create a giant Venn diagram on the grass using long ropes.

The labels on each part of the Venn diagram can be written on mini-whiteboards so everyone can see them. Then, the children place their sheets of Expedition Paper in the appropriate sections of the diagram. For example, 'Four Equal Sides' and 'Four Right Angles' would have all of the rectangles in one section, all of the squares in the intersection of the two circles and the round covers outside both circles. For our children's set of manhole covers, there were not any that had just four equal sides. In this case, children could be asked what type of shapes could go in that section. Other categories could be created depending on the set of 'specimens' in your collection.

This could also be done on a much smaller scale by printing some of the photos, either ones you and your pupils have taken or ones found online. You could also use the photos to fill in a large Carroll Diagram similar to the one below:

	Round	Straight Sides
No lines of symmetry		
At least one line of symmetry		

You can change the category descriptions depending on the photos you have in your collection.

Another use for your photos of manhole covers is to play the game Memory (also called Pelmanism). You will need two photos each of about 12 different manhole covers as well as an identical piece of blank paper or card for each photo. Print each photo and glue to the card. The cards are turned face down, shuffled and laid face down on the table. Two cards are flipped up each turn. The object is to turn over matching cards. If the cards that are turned over do not match, they are both turned back over. This can be played individually or by two or more players but you may find that you need to use more than 12 different photos! A simple set of cards might include photos of manhole covers that are different shapes while a more challenging set might feature all round or all square covers. This is an excellent way for children to learn to look carefully at details. Two manhole covers that look similar may in fact be quite different.

Hopefully by exploring some of these activities, your pupils will become more aware of the mathematics around them every day. The next time you are walking down the street, keep an eye out for manhole covers and consider the possibilities of mathematics in unusual places. I would welcome any ideas you may have!

St Edmund's School, Canterbury

Pour Relations

Another puzzle from *Professor Stewart's Hoard of Mathematical Treasures*

This is a traditional puzzle that goes back to the Renaissance Italian mathematician Tartaglia in the 1500s, but its solution has systematic features that were not noticed until 1939. There are many similar puzzles.

You have three jugs which respectively hold 3 litres, 5 litres and 8 litres of water. The 8-litre jug is full the other two are empty. Your task is to divide the water into two parts, each of 4 litres, by pouring water from one jug to another. You are not allowed to estimate quantities with your eye, so you can only stop pouring when one of the jugs involved becomes full or empty.

Note that if you want more information about this type of puzzle you will have to refer to the book itself:
Stewart, Ian *Professor Stewart's Hoard of Mathematical Treasures*, London: Profile Books, 2009

My Money week

Jane Gabb explores the Pfeg website and finds some useful resources for teaching pupils about personal finance.

Did you know that 28th June – 4th July was 'My Money week'?

Recent pupil surveys show that young people feel ill-equipped to cope financially and of the five Every Child Matters outcomes, economic well-being is perhaps the least successful. Of course economic well-being is not just learning about money, but being given good information about future career choices and the pathways towards them. We also know that looked-after children are particularly vulnerable in this area.

The Personal Finance Education Group (Pfeg: http://www.pfeg.org/index.html) has put together some useful resources for primary and secondary schools. They can be found at http://www.mymoneyonline.org/ There are free activity packs for your pupils which can be ordered or downloaded, one for primary and one for secondary. They contain lesson plans and ideas which are intended to be fun as well as informative. From the resources you could plan a lesson or a whole school event. There are interactive videos and online polls which can be accessed from this site.

For pupils in Key Stages 3 and 4 there is an online interactive game called 'Fortunity' which challenges players to make financial decisions and choices in a fun environment. There is also an assessment activity 'Fincap', which teachers can use to assess their pupils' knowledge about financial matters. This is linked to lesson ideas, so that pupils' needs can be addressed. The materials are presented in a comic strip format and cover real life scenarios designed to be relevant to teenagers. As a teacher you can try these out to see if they are suitable for your pupils (see the Teacher Guidance for instructions and access codes). There is quite a lot of reading involved, but at the end of each quiz the pupil is given feedback on how they did in terms of a percentage score and a comment on their attitude to financial matters.

> **There is quite a lot of reading involved, but at the end of each quiz the student is given feedback on how they did in terms of a percentage score and a comment on their attitude to financial matters.**

For Key Stage 3 the assessment quizzes centre around a character called Noo who is 14 and wants to buy a new mobile phone. There are 5 quizzes which cover:

- Jobs
- Banking
- Credit
- Exchange rates
- Debt
- Savings
- Tax
- Consumer choice.

For Key Stage 4 the five quizzes involve a family and cover:

- Career choices
- Taking out a loan
- Buying a car
- Savings
- Interest
- Insurance
- Investing money.

To help secondary teachers plan financial capability activities as part of PSHE, citizenship and mathematics lessons there are also teacher handbooks.

Pfeg staff are available to help you to plan your work on financial capability. Pfeg is an independent charity which is funded by a variety of organisations.

Woking

Getting to grips with 'equals' – a balancing act

Margaret Haseler **thinks the notion of equivalence and the word itself should be introduced early to all children. She suggests stages including 'more than and 'less than' and using different numbers represented by shapes and a balance.**

Writing the answer to a sum after the sign for equals is only a partial use of the sign. Equivalence is probably one of of the most important mathematical ideas that children are likely to meet in primary school. Children see and use the = symbol almost every day, making it one of the most frequently used symbols in recorded mathematics – it crops up everywhere!

Considering children meet it every day, the = symbol is probably the least understood of all the mathematical symbols which children will use throughout their primary years. For many children, = means 'write the answer' and although that will work some of the time, e.g. with simple calculations such as 8 + 5 = □, it doesn't work with a more complex calculation such as 6 + 3 = 2 + □. A common mistake when solving this calculation is to put 11 in the box (perhaps reasoning that because there are a couple of + signs, three numbers and an = symbol, you have to add up all the numbers and put the answer in the box.)

That the equals sign only means 'write the answer' is a misconception that starts at the very beginning.

The word 'equals' (represented by the symbol =) means 'is equivalent to' or 'has the same value as'. It does not mean 'makes' or 'is the same as'. Often these words/phrases are used by adults because we want to make things easier for children. But for a child, the meaning of the word 'same' implies that it looks the same, which in the context of 5 + 2 = 7 is clearly not the case. If the term 'is the same as' is used, it needs to be qualified by ...'but may look different' – all of which is a bit of a mouthful! Much better to just use the word 'equals' or 'is equivalent to' from the start. (And if a child can say the words Tyrannosaurus Rex, s/he can say the word 'equivalent'!!)

An understanding of = as 'equivalence' is important not only to solve missing box calculations such as the one above (which of course lead into algebraic notation) but also in getting to grips with a number of other important mathematical ideas. For example, we use the = symbol to show:

- equivalent fractions ($\frac{2}{4} = \frac{1}{2}$)
- the relationship between percentages/decimals/fractions (75% = 0.75)
- to describe measures (1 litre= 1000 millilitres)
- to describe coins (2 five pence coins = 1 ten pence coin).

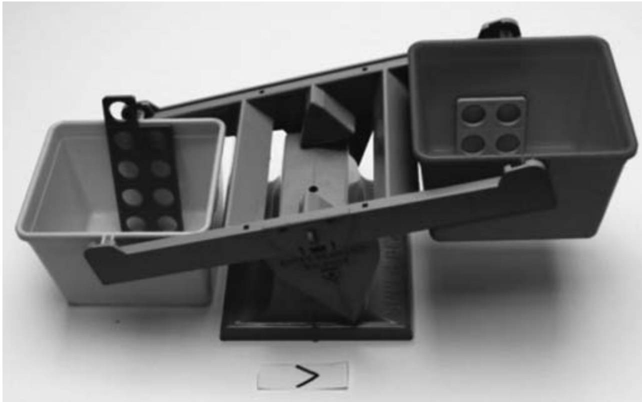
All of the above - fractions, measures and money – pose difficulties for many children, particularly those with special needs, and a major contributory factor to these difficulties is a misunderstanding of equals. Seeing = as equivalence from the beginning is really important!

Yet despite the fact that children see and use the = symbol every day and that understanding many mathematical topics is dependent on a secure understanding of it as equivalence, the symbol = does not get as much prominence in terms of teaching time as the operational symbols of + and -. There may be a number of reasons for this but a significant factor is the tendency to introduce = alongside +. So when children are being introduced to the '+' symbol they should be introduced to '=' at the same time usually when they need to record an addition number sentence.

All too often, when young children observe their teacher recording an addition, straight after writing the = symbol, the teacher will record the answer to the calculation. Thus, it is not surprising that children believe that = means 'write the answer'. The misconception about its meaning has started very early on – in fact from the very beginning for most children.
Those who work with special needs children know how much harder it is to deal with a misconception later on than it is to teach an idea correctly from the beginning so it is important to think carefully how we introduce '=' to young children. The Primary Strategy introduces the '=' symbol in Year 1 but does not offer any accompanying teaching ideas on how to do this. In fact we need to teach 'equals' explicitly, not only so that misconceptions don't arise in calculations but also to ensure a solid understanding of equivalence to help with future topics such as fractions, measures, money and of course algebra.

Stages in introducing the idea of equivalence.
The following uses ideas from the Numicon teaching programme which combines action, imagery and conversation in any mathematical activity. Using a balance with Numicon shapes to teach equivalence helps children to 'see' how the idea of equivalence works. It also provides children with hands on opportunities to test hypotheses, thus developing reasoning skills whilst at the same time practising number facts. It should be noted that a key element of the Numicon teaching approach is that children should be confident in using the language associated with mathematical symbols before recording the actual symbol in a written number sentence. This process is set out in more detail in both the Numicon 'Firm Foundations kit and Kit 1' but for the purposes of this article, we will assume that this initial stage has been covered and the children are confident with showing, describing and recording additions as follows: 6 + 2 equals 8 and 8 equals 6 + 2.

Stage 1: Recording number sentences using < and > symbols.
Using a balance, place a different shape on either side, 4 on one side and 9 on the other side. (The word shape here is used for a number of linked units). Record as 4 < 9 or 9 > 4. Make sure children have plenty of opportunity to use both signs and to say the corresponding number sentences e.g. *'9 is greater than 4'* and *'4 is less than 9'*



Encourage reasoning by posing questions such as *If I change the 9 shape for a 3 shape, what happens to our number sentence? Can I still use the > sign?*

Stage 2: Understanding equals as equal value

2.1: When the two signs < and > are secure, place identical shapes in each side of the balance scales and ask the child to describe what has happened. *Yes, the 7 on this side balances with the 7 on the other side. Can we use < or > now? In mathematics we use the word 'equals' for 'balances'.*

2.2: Remove one of the shapes and ask the child to find two shapes, or collections, to 'balance' or 'equal' the 7 shape, encouraging the child to say the number sentence. Record this using numeral cards, a card with + and a card with 'balances' or 'equals',

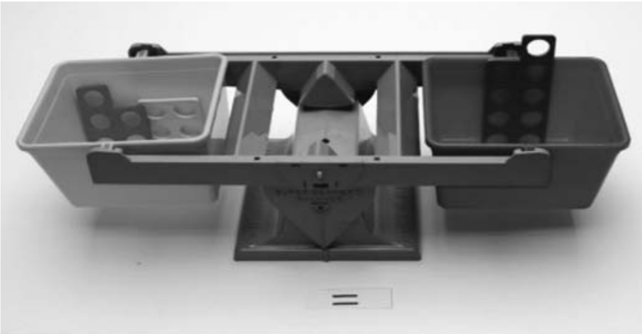
3	+	4	equals	7

Vary the position of the single shape on the balance, sometimes on the left and sometimes on the right so children have opportunities to record the number sentences as *3 + 4 equals 7* and *7 equals 3 + 4*.

From their previous experience, most children who are familiar with Numicon will correctly choose two shapes to match the larger shape. However, those who are less confident can try out different shapes until they are successful.

Being able to try out different pairs allows the child to self-correct and this in turn helps to develop confidence and perseverance, which contributes to a positive self-image of him/herself as an independent learner.

Stage 3: Recording using the = symbol
Replace the 'equals' word with the sign = and repeat the activities above, again recording the number sentences each time.

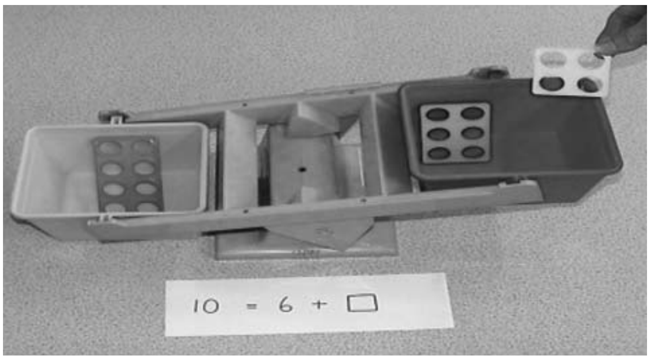


Include activities such as *Can you use the balance and the Numicon shapes to show this number sentence: 8 = 7 + 1?*

The idea of equivalence can now be extended to include 'missing number' calculations:

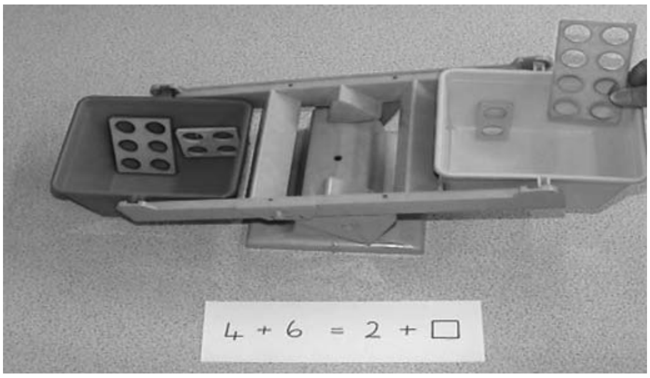
1. Place a different shape in each of the balance pans, 10 on one side and 6 on the other side. Ask the child *What do I need to do to make each side equal?*

   Record the above question as 10 = 6 + □ or 6 + □ = 10, depending on whether the larger shape is on the left or the right.



   Include activities such as *Can you use the balance scales and the Numicon shapes to show this number sentence 10 = □ + 1?*

2. Progress to two shapes in one side and one in the other, 6 and 4 in one side and 2 in the other. Record as 6 + 4 = 2 + □



   Ask: *If I change the 4 shape for a 3 shape, what do I have to do to make each side equal?* Replace one of the shapes on the other side with the shape which is 1 less. Questions like this help children to develop their reasoning skills through the use of relational understanding. By combining relational understanding with an understanding of equivalence, we can then lead children to be able to say without having to calculate that the following statements are correct:

   6 + 8 = 5 + 9
   Ask children to use Numicon shapes and the balance to solve:

   □ + 3 = 4 + □
   Is there only one answer? How many different possibilities can you find?

Children can carry out many of the above activities in pairs or threes, one child putting shapes in the balance, the second child recording the number sentence while a third child might solve the equation. Paired or group activities such as these provide opportunities for children to discuss the mathematics they are doing in a meaningful context, thus developing their use of the appropriate language. And, as with all Numicon activities, this provides valuable opportunities to assess their mathematical thinking.

Numicon

Group work is great, providing…

Mundher Adhami has written much to promote group work and learners' talk. Here he places some caveats based on earlier psychological writing with implications for structuring classroom activities.

The current drive to encourage classroom talk is not a baseless fashion. Discussion and argument in mathematics and science lessons are both motivating and enlightening, something often missing in the traditional 'telling-and-practice' mode. This is evident not only for the large number of youngsters disenchanted with schooling who see no relevance of lessons to their lives, but even for those who do play the game, and who realise the need to get the qualifications so valued by society. Inner motivation and genuine enlightenment are often missing even in individual investigation mode. So anything that gives pupils a greater role in lessons, whether in teacher-pupil talk, or between pupils in groups, is to be highly valued.

It is easy to show that talk does not automatically lead to learning useful things or developing the ability to think for oneself.

So what is the caveat? Where is the holding back? Isn't it a trivial point to argue that class-room talk, discussion and arguments are not a panacea on their own? Surely we all know that! We know that there must be a relevant context for the talk. So what else?

I argue here that classroom talk is best seen as a diabolical issue, i.e. apparently benign but with the devil in the detail. We should strive for more group interaction and whole class discussion, but also for the thoughtful framing of such interactions. They are necessary but not sufficient. I would go further: that unless some conditions apply, talk and group interactions can be a hit-and-miss affair, as likely to be harmful as useful. Just like all teaching!

The haphazard-ness of talk

It is easy to show that talk does not automatically lead to learning useful things or developing the ability to think for oneself. Look at the excitement of teenager girls or boys in groups talking about what they fancy, or on the mobile phone about what they are doing; there is always exchange of information, but not always any use or growth. Much of it would be a waste of time, irrelevant wrangling and posturing, or even misleading and harmful. The blubbering talk of bigots and other idiots, and the gossipy talk throughout history have done much harm. Development of language did allow progress, but also allowed regress. Just like any technology or facility, talk is not value-free. Values come from a higher plane.

That would seem to rule out autonomous learning without guidance and direction. It would lean towards 'telling' and training, and even towards suppressing of peer talk! Not so, unless you wish to produce clones for mindless mass production, or foot-soldiers for war, rather than free citizens. Teachers or anyone in authority cannot influence learners while developing their autonomy without interaction at their level, least of all to know the routes by which to do the influencing. For most people that interaction would involve talk, including talk amongst peers, into which to intersperse input.

Hence the puzzle: you cannot trust peer talk, but you cannot do without it. Solving such 'puzzles' is the essence of the professional job of the teacher, as opposed to their technical job of child-minding, and the training in rule-following of the curriculum.

One answer to 'what kind of classroom talk to promote?' comes from the didactic orientation: you only encourage task-focused, or curriculum-framed talk. But that is an oxymoron, since talk is in the vernacular while curriculum is formal. Unless authentic, such talk is neither motivational nor enlightening, but rather a wooden question-and-answer interchange, and in groups a half-hearted effort at finding correct answers.

Another answer comes from the investigative orientation: we should allow all exploratory talk, in informal or formal ways, prior to sharing ideas. It is clear such approach is nearer to the aims of children's autonomous and collaborative development implicit in current reforms. This is what is in need of elaborating, since it is crucially dependent on the task design and the form of questioning, and how all that ties up with the curriculum! Learners may have a heated exchange but may not make any progress.

The issues of task-design and forms of questioning, and combining authentic talk with genuine learning, are too big to elaborate in this article. What I suggest however is something more basic: how to judge when some interaction, on whatever focus, is useful or not? You could see this is what a non-specialist inspector or teacher needs in evaluating a scene in a lesson, and what all teachers need in deciding when and how to intervene.

Conditions and implications

One set of conditions may provide us with a framework for making a judgement on whether an episode of group work or interactions amongst pupils is fruitful, and to what degree. It comes from an old friend of pupils and teachers, Jean Piaget (1896-1980), whose wisdom is often sidestepped in teacher training, like most other social psychology and epistemology, which deals with how we acquire knowledge. Part of the reason must be that much of Piaget's work is untranslated, or badly translated from the original French.[1]

Piaget delineates three requirements that need to be present in exchange at any level between peers in order to contribute to learning, which he calls in psychological jargon 'cognitive reorganisation'. Recast in practical terms for any exchange at any level these are:

1. They engage with each other, comment, respond, listen and exchange ideas. This is opposed to a series of utterances or statements.
2. They use subjective terms and meanings. This is opposed to not agreeing meaning of terms.
3. They conserve their ideas until they are convinced of the need to change them. This is opposed to appeasement, emotional reactions, or other politics.

We can even make it simpler and say that for talk to be useful people must be a) talking about the same thing, b) using words which they all understand in the same way, and c) saying what they genuinely believe. It all seems very straightforward until you think about the many pitfalls in each phrase.

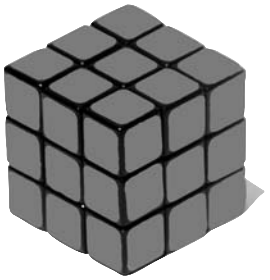
This simple 3-part frame goes beyond evaluating an episode of interactions in a group of pupils around a piece of mathematics, i.e. whether the peer-talk is helping, worthless or even harmful. It helps in both design of activities and in guiding the teaching. The activity must start by being at an accessible level for most learners in the group; they must have time to agree terms and meanings they use in talking about it and the challenges placed before them, and they should feel at ease in changing their minds, or sticking to their ideas.

But we know, don't we, that there is more to peer-group work than some pupils talking around a table.

Cognitive Acceleration Associates

1. This issue is discussed at some length in Paul Cobb and Heinrich Bauersfeld (Eds.1995) *The Emergence of Mathematical Meaning*. Hillsdale New Jersey: Lawrence Erlbaum. This book has 8 chapters on integrating psychological and sociological perspectives. Chapter 3 is by Cobb entitled 'Mathematical learning and small group interactions: four case studies' pp25-130. On page 108 Cobb discusses Piagetian perspectives, based on a 1967 article by Piaget on Logical Operations and the Social World, in French, in Etudes Sociologiques of Geneva, Librairie Droz. It looks as if Cobb relied for that on Roggof, B (1990) *Apprenticeship in Thinking: Cognitive Development in Social Context*. Oxford England: Oxford University Press.

Manhole cover, see *Mathematics in Unusual Places* page 3



Painted Cube

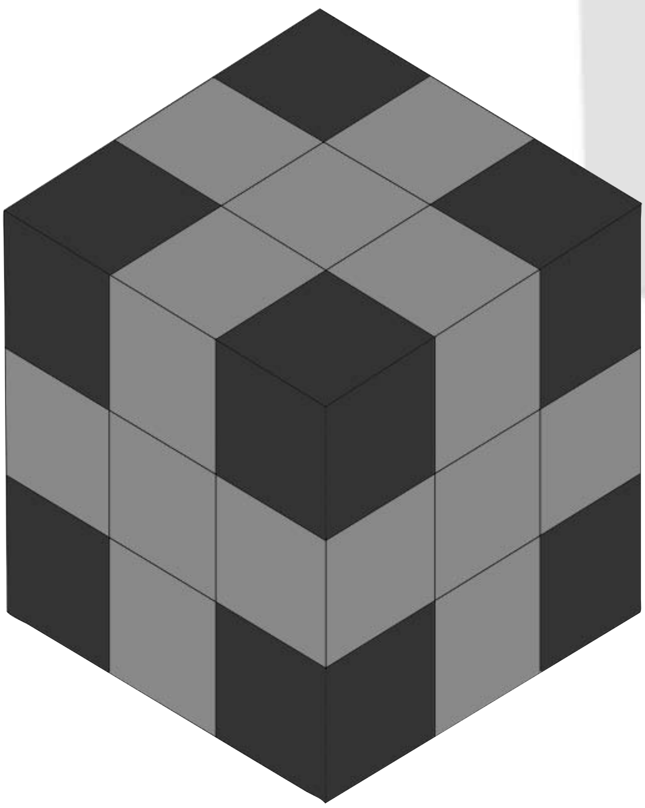
A cube is made of smaller cubes and then painted blue on all faces.

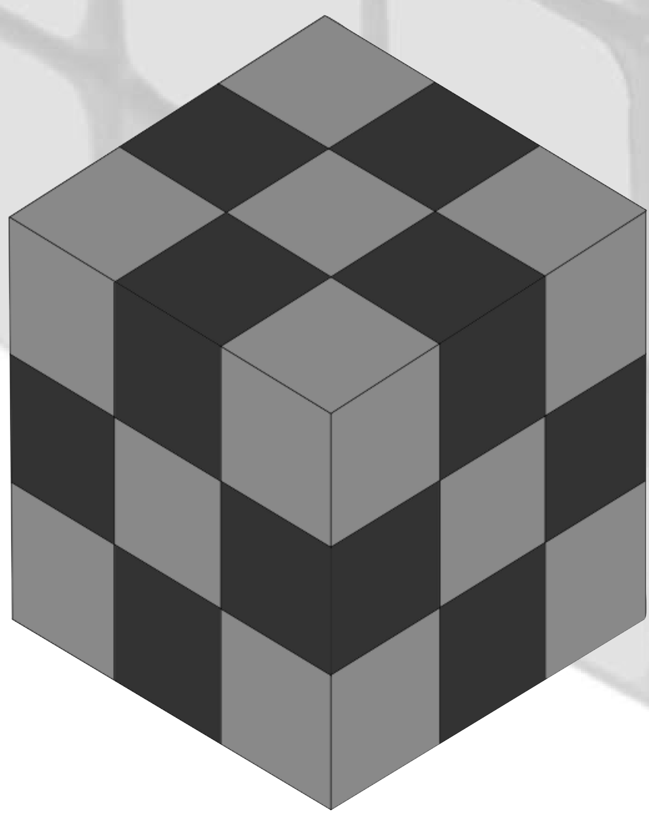
When you look at one face you see 9 squares.

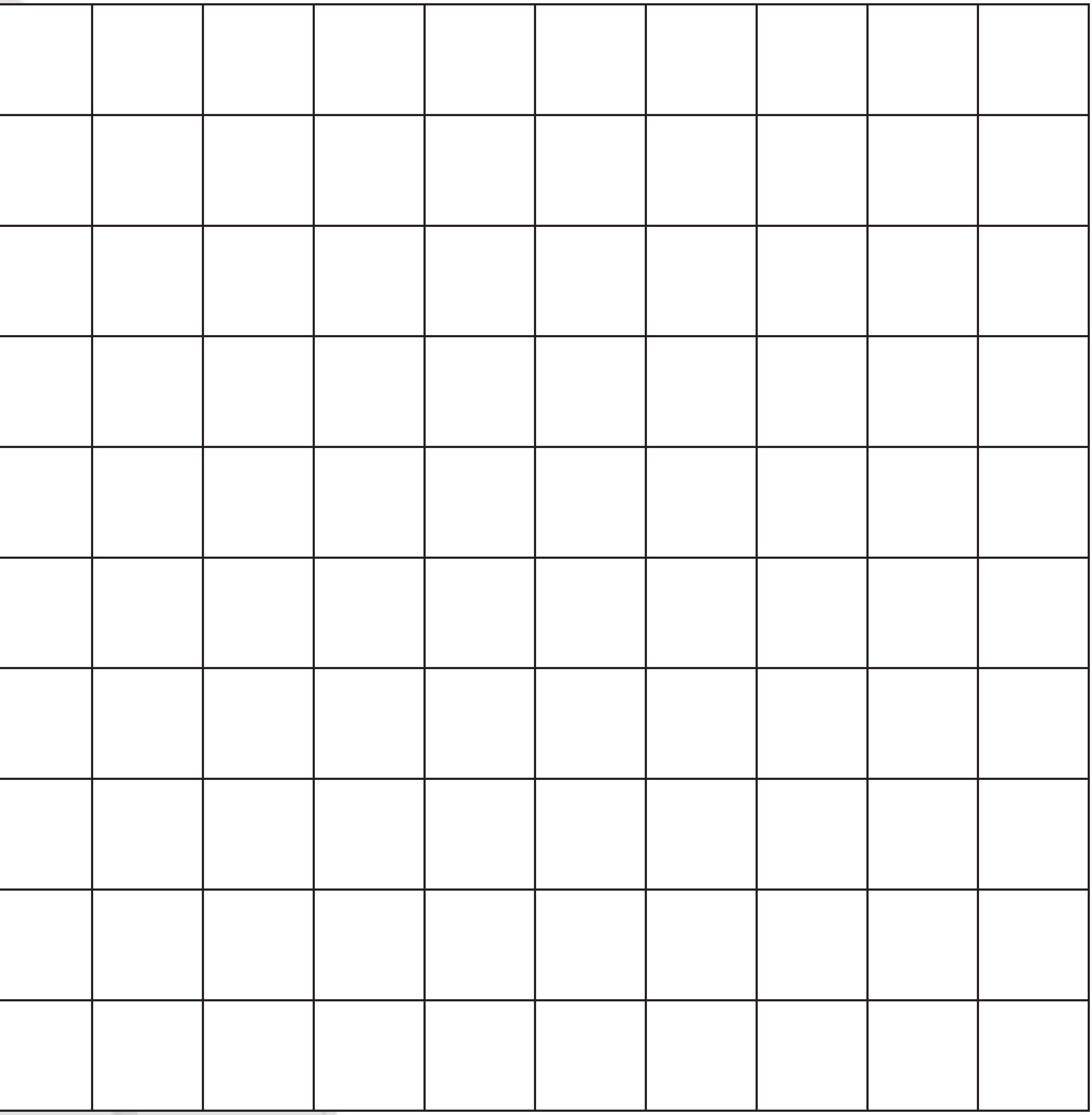
How many smaller cubes are needed to make the larger cube?

How many of these cubes have no blue faces?
How many have 1 blue face?
2 blue faces?
3 blue faces?
4 blue faces?
5 blue faces?
6 blue faces?

These images may help:







Grid for max box problem using multilink cubes

from Activities in 3D – Cuboids, see page 16

Making improvements through lesson study

Sile Bourne, an Advanced Skills Teacher, describes the experience of a mathematics department that she has been supporting through a lesson study approach.

The advent of Functional Skills and the new GCSE curriculum requires us all, as teachers, to reflect on the relevance of our subjects and the effectiveness of our teaching. For pupils, 'the new qualifications mark a shift in emphasis, from simply learning how to do something, to actually choosing and using skills to solve problems' (*Delivering functional skills: Lessons learnt from the pilot* - Qualifications and Curriculum Authority 2009).

A local mathematics department has begun to use an innovative approach to professional development to experiment with some new ideas and teaching strategies in a bid to improve the effectiveness of their teaching. 'Lesson study' is an initiative developed and fostered in Japan, but is now used more widely. The lesson study approach assumes that teachers' teaching and pupils' learning can always be improved regardless of the age or experience of either. It was felt within this department that, in recent years, pupils had become over reliant on the 'teachers providing all of the information' in a mathematics classroom. Furthermore, it was thought that many pupils were unwilling to engage with multi-stage tasks, to challenge themselves in order to break problems down or to search their 'toolkit' for the appropriate mathematical technique needed when presented with more complex tasks.

The strength of the lesson study process lies in the collaborative approach used, whereby teachers assume collective responsibility for any changes suggested and improvements made. Conversely, working alone, teachers can be reluctant to try new strategies and they may feel an isolated sense of responsibility for any subsequent effects when they do make changes. Within the lesson study model, groups of teachers identify an area of weakness in pupils' learning in their lessons. They then experiment with developments in teaching that are likely to have an impact on this aspect of pupil learning.

Currently in the early stages of using a lesson study process, this department has recognised the need to incorporate more 'real-life' activities into their planning, and to increase pupil engagement in the classroom by creating opportunities for pupils to discuss ideas, explain their understanding, present convincing mathematical arguments, make sound mathematical decisions and ultimately solve complex problems. For the department, this represents a journey from which change will be a gradual process. For some of the teachers, this approach is encouraging them to think more laterally, to pass some of the 'control' to the pupils, and to consider delivering their lessons in different ways. The first cycle for this department focused on introducing a unit on circles to Year 9.

During a typical lesson study cycle, a group of teachers:

(i) identify the medium/long term objective of the process;

(ii) collaboratively plan a single lesson, by pooling the ideas and resources from all members of the group. It is important that every participant has equal status within this developmental phase and that all ideas and approaches are explored before arriving at an agreed lesson plan;

(iii) teach the agreed lesson plan and observe any noticeable changes within the classroom with regard to pupil engagement, pupil behaviour and progress made by pupils;

(iv) collaboratively review and critique the lesson taught. Since all members of the group jointly 'own' the lesson, the dialogue here is not about the individual teacher – it is about the learning for which they all have collective responsibility;

(v) implement the suggested action points from the review when this topic is next taught.

Following the department's first lesson study cycle, we are confident that we can make some improvements to our practice with subsequent phases. The value of lesson study lies in successive cycles of collaborative planning and reviewing. This should serve to encourage staff to be more experimental with their ideas and candid in their evaluation of what works and what doesn't work in the classroom. Further opportunities for a greater number of peer observations are needed as these can provide staff with valuable feedback on the lesson. The review dialogue can be further enhanced if there are opportunities to video the lessons taught. The play back in the review phase would provide for a more in-depth and valuable analysis of what has worked and why.

Currently in the early stages of using a lesson study process, this department has recognised the need to incorporate more 'real-life' activities into their planning, and to increase pupil engagement

The lesson study approach presumes that the observer's focus is on a small number of pupils only within a classroom (i.e. case pupils). This is designed to direct attention towards the learning of the pupils, and to deflect attention away from the teacher, which, in turn, should enable more constructive dialogue in the review phase.

In the medium or longer term, we would like to create smaller lesson study teacher-pairings with teachers of similar ability groups working closely together rather than the current larger lesson study group. This should allow the lesson plans to be more specific in the detail.

Below are some pertinent quotes regarding lesson study taken from *Getting Started with Networked Research Lesson Study*, published by the National College for School Leadership:

"The process has been found to help teachers – experienced and less experienced – to "see things differently" (project member), to be able to critically view their own practices without being blinded by familiarity or 'blinkered by…assumptions about [their] immediate settings' ". (Desforges, 2004)

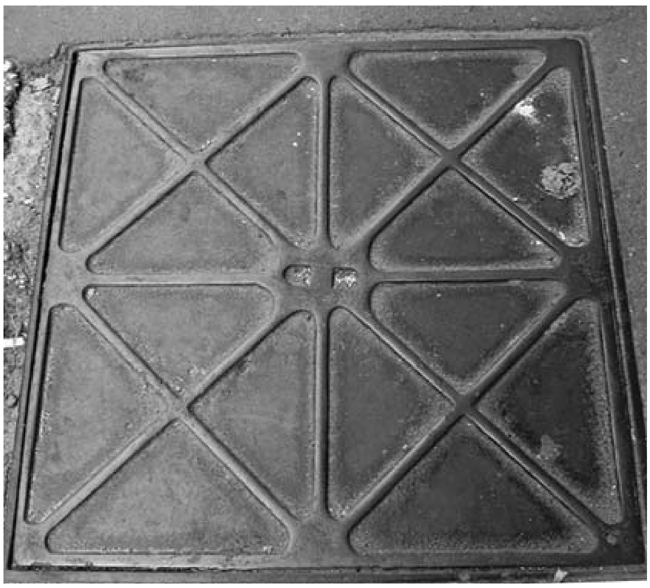
"Teachers in their first three years of teaching have found the process has given them an opportunity to engage in 'deep' professional learning, not offered by existing models such as the standard diet of the induction year".

"What's very powerful is that people felt that because they'd planned together, it made it okay if it went wrong…"

"The Research Lesson Study process encourages risk-taking in a culture of professional learning from what does not work, as well as what does"

Significantly, lesson study is not concerned necessarily with the creation of a "perfect lesson" – it is the teacher collaboration that is the key element.

Charters School
Royal Borough of Windsor and Maidenhead



Manhole cover, see *Mathematics in Unusual Places* page 3

Activities in 3D – Cuboids

Jane Gabb suggests some activities which can be used to help pupils work systematically, as well as giving them important experience of working practically with 3D materials in problem-solving contexts.

This is a collection of ideas for using multilink cubes to explore the properties of cuboids. There are also opportunities to explore working systematically and visualising. The activities work well with pupils who are working at National Curriculum levels 3-5. Experience tells me that pupils have difficulty working in 3D and I believe that they do not get enough practical experience. The activities below can all be tackled practically, and then extensions can be given which require more abstract thinking. However, it is only with the hands on experimentation that this further level is possible.

Starting point

- If you have 24 multilink cubes, how many different cuboids can you make?
- How could you record them?

This provides an activity for looking at working systematically. If you want to pursue this, model how to determine the 3 dimensions of the cuboid but don't suggest any particular order to write the numbers in. Then set the class off to work practically with 24 cubes to find as many different cuboids as they can and record the 3 dimensions on paper. Wander round the class and note the different ones, then write them on the board, again in no particular order. Make sure that you have duplicates, but with the numbers in a different order. (There are 6 different cuboids: 1x1x24, 1x2x12, 1x3x8, 1x4x6, 2x2x6, 2x3x4 **but don't tell them this yet!**)

When you have a list on the board, preferably with all the possibilities and a number of duplicates, start a class discussion with the questions:

- Are these all different?
- How could we find out?

Someone may suggest making them all to decide practically. Accept that this is a possible way forward, but ask if anyone has any other ideas.

If none are forthcoming, ask:

- Why is it difficult to tell if any are the same?

Take one example and ask about the order of the numbers in it:

- What order could we write these in that might help us?

If someone suggests starting with the smallest or the largest and putting them in order, ask them how that would help.

Amend the list so that each one is in order (either smallest to largest or the reverse).
Now go back to the question:

- Are these all different?

Pairs should now be easy to spot and the duplicates eliminated. Emphasise that we have made our work easier by being systematic. Check that they understood what this means:

- What have we changed that has made it easier to see if cuboids are the same?

The next questions are:

- Have we found all the possibilities?
- How can you tell?
- What could we do to make it easier to see?

Hopefully someone will suggest writing the list in a systematic way – similar to my list above. If not, you may need to prompt them:

- Which one should come first in the list?

When you have a list in a systematic order, ask:

- What do you notice about the numbers?

And if necessary:

- What relation do they have to 24?

A pupil in one of the classes where I did this activity said 'They don't add up to 24.' This observation helped another pupil to see that the 3 numbers multiplied together make 24.

Test this out with all the sets of 3 numbers, and demonstrate what it means in terms of a cuboid – it's good to show this with the 2 x 3 x 4 cuboid and it can help to have the different layers in different colours – the four 2 x 3 layers in blue, yellow, blue and yellow for instance. Then you can show the 2 x 3 face and demonstrate that there are 4 of them.

Going back to the question 'Have we found all of them?' ask them to see if they can find any new different ones, but only give them a few minutes. Some may suggest that there aren't any more and they should be asked to explain their thinking. Point out how the systematic list has helped in this activity.

Option:
If working systematically is not part of your agenda for this lesson, demonstrate right at the beginning that you want the numbers which define the cuboid in order. Then when you collect up the results, put the 6 different ones in a systematic order ready to go on to the questions about whether all the possibilities have been found. You might want to stress at each of these points that you are showing them how to work systematically, by asking:

- Why is it a good idea to write the numbers in order?
- Why have I written the possibilities in this order?

Second activity

- How many of these 1 x 2 x 3 cuboids could fit in a given box? (The lid of an A4 paper box is good for this activity – the 1 x 2 x 3 cuboids fit snugly into a 5 x 5 array and you can get 4 layers in.)

Of course you are not going to give them enough multilink to make all the cuboids, so they have to find some way of working it out.
The discussion is about how they tackled the problem.

Third activity
This is an old chestnut – the max box problem. At this level it is tackled practically. As they have been working with multilink it makes sense to give them a 10x10 grid of 2 cm squares so that they can use the cubes to explore the volume of the boxes made practically if they need to. (See centre spread for the grid.) The NRICH site has an activity 'More Christmas Boxes' which provides a good explanation of the task and useful pictures.
When they have had a go there can be a class discussion on how many cubes will go in each of the different boxes made: (1 x 8 x 8, 2 x 6 x 6, 3 x 4 x 4, 4 x 2 x 2). It is useful to explore each one and to look at how the numbers are derived:

- When you take a single square out of each corner, why isn't the base 9 x 9?
- Why is the length of the base always even?

An extension to this task might be to suggest a bigger square as the starting point (15x15 or 20x20) and see if they can work out the volumes without doing the task practically.

Fourth activity
This is also an adaptation of an nrich activity: 'All wrapped up'. The adaptation is that the cuboid has to be made with multilink cubes. Questions which may come up during this activity:

- Can the paper be cut? (the class should decide)
- Can the cuboid be at an angle to the edges of the paper? (Yes)
- How much overlap should there be? (As little as possible)

Visualisation
I have used this as the plenary to a lesson with some of these cuboid activities and it gives an opportunity for pupils to visualise a 3D shape – in this case a cube. This is another familiar activity – the painted cube. If you google 'painted cube' you can find useful pictures which can help if pupils have difficulty visualising – you can project a picture for a short time and then return to the questions.

Script:
I want you to imagine that you have some plain wooden cubes and you make them into a larger cube. If you look at the face of your larger cube you can see 9 squares which are the faces of the smaller cubes you have used.

- How many cubes have you used to build it? (When I did this recently we had a number of different answers to this first question: 27, 54, 81, and we spent some time exploring these through the pupils' explanations of their reasoning. Time ran out and we never got as far as actually painting the cube!)

When they have agreed that there are 27 cubes needed, go onto the next stage.

Script:
You now paint your large cube blue all over the outside.

- How many of the cubes have 1 face painted? [6] How do you know? [one on each face – 6 faces]
- 2 faces painted? [12, one on each edge, 12 edges]
- 3 faces painted? [8, one at each vertex, 8 vertices]
- 4 faces painted? [0]

It's useful to keep a tally of all the answers and then to add them up. It should come to 26!
Then say:

- But I thought there were 27 cubes altogether!
- What has happened?

They should be able to realise that the one in the middle doesn't get painted at all, but this may take a little while.

Winston Churchill School
Woking

More for Less

Liz Woodham and Jennifer Piggott from NRICH suggest lots of activities using squares as their starting point. All of these activities, complete with teachers' notes can be found on the NRICH website. (http://nrich.maths.org)

Squaring it
Our simple starting point in this edition is squares of paper. We have chosen a small number of problems from the NRICH website, linked in some way to area, that have the potential to engage and challenge learners but be flexible enough to respond to need. In particular we hope the activities will help to focus pupils' attention on:

- exploring relationships
- posing their own questions
- representing their findings
- developing convincing arguments.

And teachers' attention on:

- giving space for exploration
- encouraging pupil talk (see the article 'What's All the Talking About?' http://nrich.maths.org/6662)
- listening and encouraging peer assessment of mathematical arguments.

Where shall I start?
You will need squares of paper – some coloured some white. So, the obvious place to start is by making squares of paper and preparing arguments to convince others that what you have is a square and that it is accurate. For example, "The shape has four equal sides" is not enough.

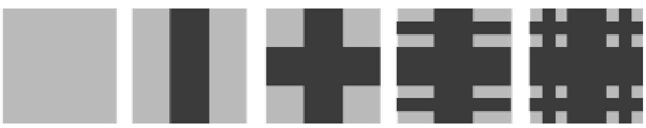
Hand out some paper and ask the group to make squares that they could convince the rest of the group are squares.

There will be lots of methods for making squares and lots of ways of demonstrating convincing arguments. One aim is to be able to efficiently produce squares for future activities. One learner may measure the short side of an A4 sheet and transfer that measurement to the two long sides and fold across. Why is this a square? Would it be more accurate to copy the distance than measure it? Why?

What next?
Let's look at halving our squares. The problem ***Halving*** (http://nrich.maths.org/1788) is a good place to start if you are looking for ideas. In the Teachers' Notes to this problem you will also find a PowerPoint file which has an animation of halving which would make a good introduction to this activity.
Encourage your learners to explore halving squares, suggesting that they make up halving patterns of their own. This can lead on to learners producing an exciting wall display of halving made from cutting two differently coloured squares. Encourage them to explain how they know what they have produced is two halves. How about quarters or other fractions? See also ***Diminishing Returns*** (http://nrich.maths.org/6700).

Why not?
Work on ***Light Blue Dark Blue*** (http://nrich.maths.org/public/viewer.php?obj_id=2105)

Ask learners to look at the pattern.
Discuss what they see and how it is made.
Hide the image.
Can they recreate it without looking at the image again?
(Look, cover, draw, reveal …)



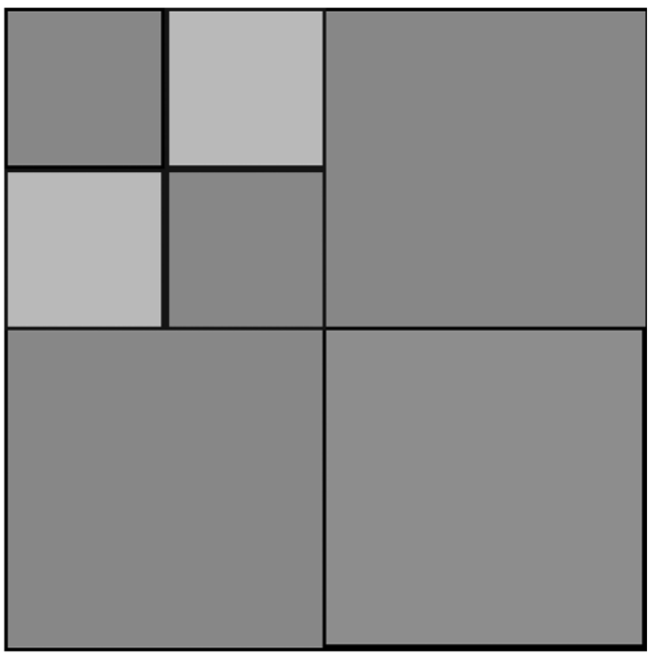
Ask:
- What fractions can you see?
- Is there a pattern to the fractions of area in each image?
- Can you convince us that your fraction is correct?

Learners could also make up other patterns of their own for partners to extend and the work can be used to make an attractive and meaningful display.

And, on the way, a good discussion:
… might come out of looking at the image ***Baravelle*** (http://nrich.maths.org/public/viewer.php?obj_id=6522).
Show the image for a minute, then hide it before asking learners to describe what they saw to others in the class. Ask them what questions they would like to answer, for example:
- What fraction of the image is blue?
- What is the connection between the largest and the next size triangle?
- How do you know they are squares?
- What fraction of the area of the larger square is the next size square and how could you convince me?

Learners can then work on answering a question of their choice.

Leading on to:
…work on dissecting squares and considering related fractions. For example, a good starting point is:
Squares, Squares and More Squares
(http://nrich.maths.org/public/viewer.php?obj_id=2104)



This square has been dissected into seven smaller squares.
Can you use two different colour squares to recreate this dissection?

Can a square be dissected into any number of squares?
What fractions of the area of the original square can you make?
Create a display of all the possible dissections and areas in a way that shows how you have worked systematically.

Of course there is always:
The NRICH website has a wealth of tangrams, which can be used to emphasise the ideas of conservation of area. From how to make them (***Making Maths: Making a Tangram*** http://nrich. maths.org/5355) to tangram. The 2006 advent calendar features twenty-four of the NRICH site's tangram problems (http://nrich.maths.org/5522).

Going a little further:
Symmetry challenge (http://nrich.maths.org/1886). You can either use squared paper or small coloured squares to explore this problem.

On the Edge
http://nrich.maths.org/public/viewer.php?obj_id=2401

NRICH
Cambridge University

Twenty-two pupils under the age of five are excluded from school each day.
TES Magazine 7 May 2010

Assessing Pupils' Progress case studies

Jane Gabb draws readers' attention to some accounts of how a variety of schools have introduced APP in mathematics.

On the Secondary Framework website there are some very useful and interesting case studies from schools which have been tackling the question of how to introduce Assessing Pupils' Progress.
This will take you to the Chailey School Case Study: http://nationalstrategies.standards.dcsf.gov.uk/node/284417. The others can be found by following the links from there.

Although each school has taken a different approach, a common theme is that using APP has led to a change in the way teachers are approaching mathematics in their classrooms. This involves both the process of what happens in classrooms and the ways that mathematics is presented. Working with APP is leading to a wider variety of mathematical diet, with more open-ended tasks and more opportunities for pair and group work. Connections are being made between the different aspects of mathematics and with other areas of the curriculum. Teachers are developing their use of probing questions and are relying less on written tests and more on assessment for learning. This is coming about because they see that what they have been doing previously does not provide them with enough evidence of what pupils are able to do, what their difficulties are, and how best to take them on from where they are.

On the next page is a summary of each school's situation and a key aspect of the way they are working. None of the case studies is very long and they are easy to read. It is always valuable to hear what real teachers and departments are doing when faced with initiatives such as this, and I think these are a very useful addition to the section on APP.

Summary of each school	**Aspect covered in case study**
Chailey School is a small rural comprehensive secondary school catering for around 850 students from 11 to 16 years of age. The school is situated in Sussex on the outskirts of South Chailey, near Lewes and draws its pupils from a wide catchment area. The proportion of students from higher-income households is above average. It is a specialist language college and has recently been awarded a second specialism in humanities. In 2006 Ofsted reported that the school was 'good with outstanding features'. Ofsted also noted that the students 'enjoy their education a great deal' and that this is reflected in 'very good attitudes to learning, excellent behaviour and good attendance'. There are five full-time teachers of mathematics in the department.	Changing teaching to improve pupil engagement
The English Martyrs School is a large Catholic comprehensive school catering for nearly 1600 pupils including a sixth form of over 300 pupils. It has specialist arts college status. The pupils come mainly from a number of Catholic primary schools spread over a wide area where the extent of social and economic deprivation is greater than the national average. The standards on entry to the school are above average while there is an average proportion of pupils with identified special educational needs. The mathematics team comprises 12 full-time mathematics teachers plus two trainee teachers.	Amending the Scheme of Work to include a wider variety of teaching and learning activities
The Holy Family Technology College is an 11–19 Catholic school catering for 1110 pupils, including a sixth form of around 210 pupils. Students come from a wide catchment area within the London Borough of Waltham Forest and beyond. Four-fifths of pupils are from minority ethnic groups, of which Caribbean and African are the largest, each comprising about one sixth of the pupils. One quarter of students speak a first language other than English. While students come from a range of socio-economic backgrounds, the proportion eligible for free school meals is close to the national average. The percentage of pupils with identified learning difficulties or disabilities, or with statements of special educational needs, is lower than the national average. The school has been a specialist technology college since 2000. There are 11 teachers in the mathematics department.	6 week project – 6 teachers, 20 pupils – additional teaching activities, dialogue, probing questioning
Kirk Balk School is a larger than average 11–16 school, with nearly 1200 pupils. The proportion eligible for free school meals is similar to the national average but the degree of social disadvantage is higher than average. There are very few pupils from minority ethnic groups. A higher proportion of students than average have learning difficulties and/or disabilities though the proportion with a statement of special educational needs is similar to the national average. The school was awarded Technology college status in 2002.	Themed learning journeys
Lipson Community College is of above average size with over 1350 pupils and it has specialist status for the performing arts. It serves a local community where fewer adults than average have participated in higher education. The proportion of students eligible for free school meals has fallen and is now broadly in line with the national average. The proportion of pupils with identified special needs has steadily increased and is well above average. Most students are from a white British family background; the percentage speaking English as an additional language is similar to the national average. The mathematics department includes ten full-time teachers, two part-time teachers, a one-to-one tutor and three teaching assistants. The department is without a sole subject leader. The College has an Assistant Principal in the capacity of Executive Leader of Mathematics, as part of a distributed leadership model at middle leadership level.	Moving away from the textbook, more group work and activity-based work

Malmesbury School is an 11–19 comprehensive school of around 1250 pupils serving a rural community in Malmesbury in Wiltshire and its surrounding villages. The majority of pupils are from a white British background and a smaller than average proportion of pupils are eligible for free school meals. The proportion of pupils with learning difficulties is also below average. The school has specialist school status in science, mathematics and computing as well as a performing arts specialism, and in 2008 was awarded 'High Performing Specialist School' status. The mathematics department comprises seven full-time mathematics teachers plus three other teachers who also teach other subjects. It is a strong department with all the teachers being graduates in mathematics or related disciplines. Approximately 70 per cent of Year 11 pupils gain a GCSE at grade C or above in mathematics.	Strong Assessment for Learning basis – approaches to end of unit assessment
Varndean School is a larger than average 11–16 school in Brighton and Hove catering for around 1300 pupils. It gained specialist status for technology in 1998, for music in 2006 and for applied learning in 2007. It has a smaller than average but increasing proportion of students from minority ethnic groups and a larger than average proportion of students with learning difficulties and/or disabilities, mainly associated with behavioural, social and emotional issues. The mathematics team of seven specialist mathematics teachers includes an advanced skills teacher (AST) and a teacher who is a Lead Professional in Brighton and Hove.	Classroom dialogue and more open activities

We at *Equals* would welcome contributions from our readers on how they are implementing APP processes and what they are learning from this.

Woking

Let's Get Back to Basics: The 3 Cs of Education

We need to re-assess what is really the core of education, suggests Rachel Gibbons.

Recently I have been having an argument with the editors of the *TES* about current content, suggesting that the editor has lost his way. In the past the paper contained exciting maths extras, science extras and all sorts of other discussions about curriculum matters. Now all one can read about is the categorisation of schools and who should run them or accounts of teachers who have abused their pupils. All very newsworthy items, no doubt, but of no help to teachers who are trying to move education forward. Serious thought about education must focus on the three Cs: **the child, the curriculum and the classroom**.

The child must be the teacher's first consideration: the children - their own self-esteem, their belief in their ability to learn and their continued interest in learning. When we consider the curiosity of the infant, the endless questions about the world they find themselves in, we must question what it is that kills that passionate desire to learn. Failure and the setting of tasks beyond their ability must be elements in this loss of curiosity. Testing must also have a negative effect, especially on those who do not expect to do well in the tests, those with whom *Equals* is especially concerned, in the lower part of the attainment range.

John Holman, the director of the National Science Learning Centre is quoted in the Guardian as saying that preparing pupils for exams might lead to good grades but this could be "at the expense of long-term learning and comprehension." Challenges must, of course, be chosen to extend each child appropriately and it is often tempting to narrow the range of challenge of the mathematics presented if classes are set. Moreover, labelling is certainly very dangerous. As noted in the editorial, children who have been put in a lower set may feel the ill effects of that labelling not only throughout their school days but for the rest of their lives. In addition, research has shown that pupils do better if teachers do not place too much stress on grades.

The curriculum is the second element of the core. Maybe the focus of the curriculum needs a rethink. I suggest we should go back to what Pope told us: "the proper study of mankind is man". And who can properly deal with the problems of obesity unless they have some knowledge of how their bodies work? The majority of our pupils will someday find themselves caring for their own children and perhaps also providing care for a disabled or elderly relative or friend. All these tasks will be better faced with a sound knowledge of the human body. Surely this is the starting point for the curriculum. English and mathematics must come next in order of importance because they are the languages needed to study biology.

Then, provided that the child has learnt how to learn, how to acquire further skills when needed, how to find further information, how to test whether the information found is reliable, how to work on logical problems, the work-skills that have been so much talked of recently can be learnt when they are needed. All these activities need a certain level of confidence and it is also important that the teacher helps every pupil to build this. Children who are put in lower sets are not likely to do well here or later in life.

Now that the STEM Centre is up and running at the University of York, there is a wealth of good materials available to all teachers - much, like the SMILE material, with built in assessment systems - waiting to be used by teachers and pupils. The SMILE material in particular has a good "map" of the mathematical tasks available which has been built up by teachers for their own pupils, some written by those teachers and some being taken from the texts they had available in their classroom cupboards. They need to be kept up to date by groups of teachers pruning out old dead wood and grafting on the best of their own ideas.

The classroom, the place where the teacher and learners meet, is the third element of education's core and it may include the space outside the school building, its playing fields and other areas where pupils are taken for learning-trips. The walls should be covered with colourful displays for learning through looking and these should have a plentiful scattering of posters done by pupils illustrating their own ideas. Classes may still need to have their attention drawn to the display; it is surprising how blind we can be to our surroundings. The best classrooms have always been rich in resources and today, with all the technology available, they can contain the whole world and beyond.

The classroom is of vital importance because it is the place where teacher and pupil meet, the place where pupils can share ideas, learn to live as positive, purposeful members of a community, taking responsibility for their own work and respecting the work and work-space of others, learning to put their best efforts into joint projects as members of a team, sharing and caring for equipment and never being afraid to ask questions.

We teachers are inclined to like the sound of our own voices but the classroom is not a place where the teacher should be continually holding forth. The lecture is the most wasteful method of facilitating learning. Even with the best of lecturers, members of the audience can get so carried away by polishing in their own minds some gem from the lecturer's lips that they lose the next half dozen gems completely and when they start listening again they can find themselves utterly lost. So make your classroom the place where you listen to your pupils, respect their ideas and encourage them to solve problems for themselves.

Two thirds of the average May rainfall fell over the bank holiday weekend.

TES 7 May 2010

Review

by Jane Gabb

Assessment for learning: why, what and how?
An inaugural professorial lecture by Dylan Wiliam
Institute of Education
www.ioe.ac.uk/publications
ISBN 978-0-85473-788-8
£5

Speaking recently Dylan Wiliam appeared heartened by his conversations with ministers in the new government. They seemed to have a positive attitude to educational research, and to taking this into account in their administration of education. If this is so, then what Wiliam outlines in this booklet will be very pertinent to that endeavour.

The lecture on which the booklet is based outlines an argument which few people who understand and care about education will disagree with:

- Raising achievement is important
- Investing in teachers is the solution
- Formative assessment should be the focus
- Teacher learning communities should be the mechanism.

I will assume that no-one reading this will need convincing about the first point. Wiliam argues that, because of the short-term nature of political timescales, there tends to be a focus on things that are easy to change, rather than on those which have been shown to make the most difference to young people. Research shows that in-school variation is greater than between-school variation, leading to the assumption that we should be concentrating less on the structures underpinning schools, and more on improving the quality of teachers. Hence the second point that investment in teachers will bring about the changes needed to raise achievement.

Wiliam then examines the cost effectiveness of these options:

- Reducing class size
- Increasing teacher content knowledge
- Using formative assessment
  and shows that the last of these costs the least and has the largest effect on learning.

The next section refers to the research on Assessment for Learning and outlines what Assessment for Learning involves in the classroom, identifying five essential aspects of formative assessment which involve the teacher, the learner and his/her peers.

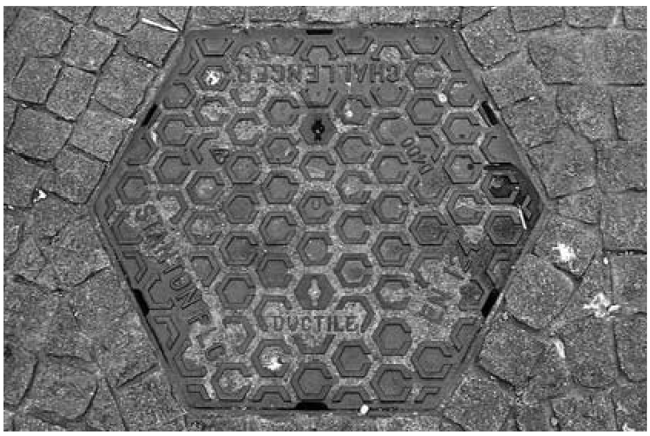
At the beginning of the section entitled 'Improving teacher practice' he expresses puzzlement about why educational research has had so little impact on classroom practice. He argues that the problem comes from the insight that knowing something is not the same as being able to do it and that what teacher educators need to address is the lack of understanding of what is needed in the classroom. In order to affect change, teachers need to reflect on their own practice in systematic ways, to build on their knowledge base, and most importantly, to learn from mistakes.

Wiliam then outlines a professional development process which takes into account what we want teachers to change and how we help teachers to change. He identifies five aspects of the process which seem to be particularly important: choice, flexibility, small steps, accountability and support and goes on to expand on what these mean in practice.

Finally he reports on what he and his colleagues have learnt about teacher learning communities. These include the duration of a project, who the teachers are, the size of the group, the frequency and format of meetings.

So, who should read this book? Anyone who is involved in teaching will gain something, even if it is just an understanding of why it is so difficult to improve one's practice as a teacher. I would particularly recommend the booklet to those who have responsibility for CPD in a school or teacher education in a wider sense, because they are the ones who need to be implementing the ideas in this book.

Woking



Manhole cover, see *Mathematics in Unusual Places* page 3