Equals

Realising potential in mathematics for all

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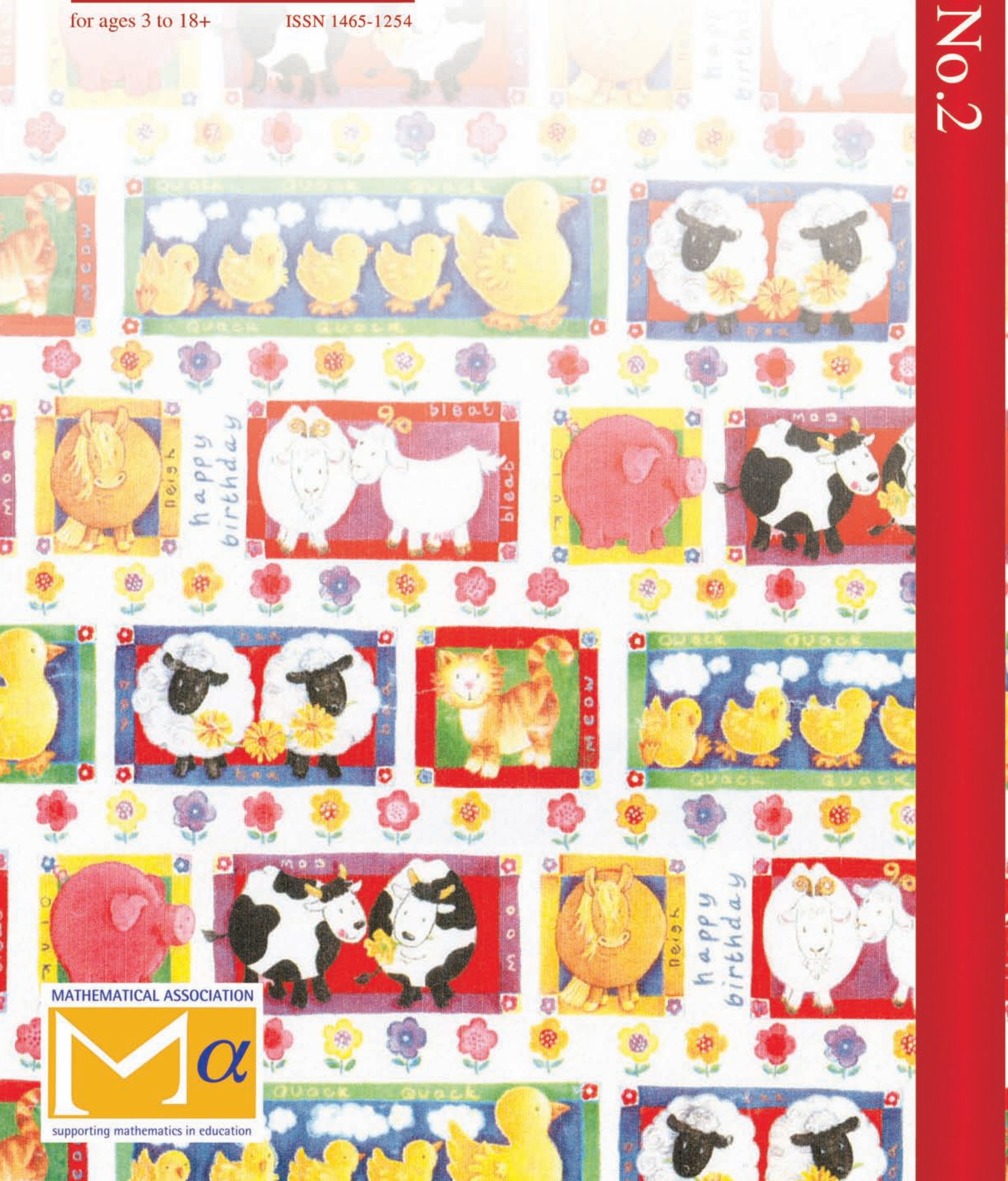

MATHEMATICAL ASSOCIATION

supporting mathematics in education

Equals

Realising potential in mathematics for all

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Editors' page

At the time of writing the G20 has just taken place and education must be an important consideration if the world is to move forward, improving the global economy and developing its resources and knowledge base. We include some information on the members of the conference with a couple of questions about the participating countries that might be considered with pupils. As we have suggested before, to become properly participating citizens, our pupils all need a sufficient understanding of number to make sense of the information presented to them in the media about the world around them. We are very grateful to CircaMaths[1] for permission to use some of their exciting material for our centrespread.

There are plenty of other suggestions for class projects in this issue of *Equals* on topics ranging from a prehistoric stone circle at Avebury to a piece of contemporary wrapping paper. Some of you reading this issue will be teaching in special schools, others may have a wide range of achievement in your leaning groups. Most of the suggestions in the following pages would be suitable for either type of learning group. If you are teaching in mainstream and agree with a comment in a recent TES that setting is cruel, you should find material suitable for a learning group with a very wide range of achievement. However, you will then have to decide whether you vary the tasks you give to different members of your class. Whether you organise them to work as a whole class, in groups or individually, their learning will be personalised. Whatever your approach, whatever your organisation, all members will learn differently. At the end of a lesson each member of your class will have learned something different from the rest – there will be common threads but the whole will vary from to pupil to pupil. And this is why you need to construct some form of map of mathematics on which the mathematical journeys of your pupils can be recorded in detail.

1. www.circamaths.co.uk

More for Less

Using very simple equipment, a set of digit cards from 1 to 9, Liz Woodham & Jennifer Piggott explore some innovative ways of using them to explore numbers. They suggest ways of introducing and extending the activities including using some of the resources of the nrich website.

One to Nine

A simple piece of equipment or starting point has the potential to engage learners of all attainment levels and be flexible enough to respond to need. In this article we have chosen number cards to help focus on pupils' attention on:

- exploring the mathematical potential of a situation
- posing their own questions
- developing systematic recording

And teachers' attention to:

- nurturing ideas
- assessing understanding
- making connections

You will need sets of nine cards 1-9, possibly some dice, and the games Make 15 and Shut the Box from the NRICH website. More information on magic squares can be found on the NRICH website here: http://nrich.maths.org/public/viewer.php?obj_id=87 and here http://nrich.maths.org/public/viewer.php?obj_id=2476

Where shall I start?
Well you might want to find out what your learners know and how creative they are.

Why not hand out the nine cards to learners working in pairs and ask them to make a poster to share with the rest of the group about what they know or can say about them?

They might notice the number of odds, evens and primes or that they sum to 45, or that their product is a multiple of 10.

- *How could they know the product is a multiple of 10 without having to work it out?*
- *Are there any quick ways of adding 1-9 that might shed light on quick ways of finding the sum of 1-19 or 1-99 and so on?*

What next?
How about using up to three cards at a time and the number operations to make the biggest/smallest number they can, or their birthday (31 = 6 x 5 + 1)? How many different ways can they make 10? Can they make up some problems of their own or make up a game to play that uses the nine cards? Remember this means that they cannot repeat a digit.

Why not?
Work on Finding Fifteen (http://nrich.maths.org/public/viewer.php?obj_id=2645)

Ask the class to sort the nine cards into three piles and find the totals in each pile.

- *How could they know they are right?*
- *What is the total of the totals for the three piles and what do the numbers 1-9 add up to – what is the connection?*
- *Did anyone end up with three piles all with the same total?*
- *Is that possible?*
- *Is it possible to do this in more than one way?*
- *How can we find out?*
- *Can we work systematically to capture all the possibilities? What would be a good way to represent them?*
- *Why can't a pile contain 8 and 9?*
- *Why not 1 and 2?*
- *What will work with 1?*

Here is a list of possibilities:

159 249 348 456

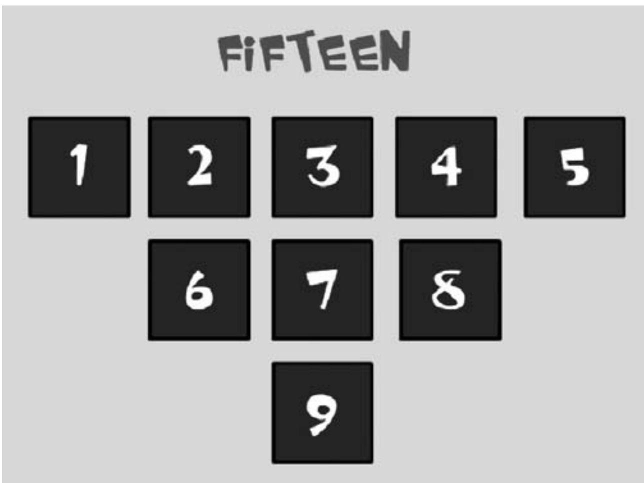
168 258 357

267

- *How do we know we have them all?*
- *How many triples have 1, 2, 3, 4, 5…?*
- *Which ones can we put together to make the three piles of three cards each totalling 15?*
- *Is there anything you notice?*

There are only two ways to make 15 when one number is 1, 3, 7 or 9
There are three ways to make 15 when one number is 2, 4, 6 or 8
There are four ways to make 15 when one number is 5

Leading on to:
Playing the game Make 15 (http://nrich.maths.org/public/viewer.php?obj_id=1223)



- *Can you make any connections between what we have been doing and this game? How does it help in developing a strategy?*

- *How about making a magic square with the numbers 1-9?*
- *What is the connection?*
- *What about noughts and crosses (http://nrich.maths.org/public/viewer.php?obj_id=1224) and the game "Online (http://nrich.maths.org/public/viewer.php?obj_id=1220)?*
- *What do all these things have in common?*

Going a little further:
Sealed Solution
(http://nrich.maths.org/public/viewer.php?obj_id=1177)
Shut the Box
(http://nrich.maths.org/public/viewer.php?obj_id=6074)
Dozens
(http://nrich.maths.org/public/viewer.php?obj_id=559)

NRICH team
Cambridge University

Guided by an Old Hand. Is that what education is about?

How can an 'outdated', badly-translated dead Swiss scientist have a major impact upon a science teacher from Sunderland? And make him focus on pupils across ages overcoming difficulties in mathematics? Alan Edmiston reflects on this with his own children in mind.

Sometimes you need time away from your classroom, like in the half-term holiday, to reflect on your practice. It was in the caravan site shop in the Lake District on 25th October 2008 that I read the headlines from the Daily Mail which set me reflecting on my path in teaching. On the front page was a report highlighting the findings of some research[1] on the thinking of today's Year 9 pupils compared with those from 30 years ago. The sad finding is that when faced with a complex task, one that requires little prior knowledge, they fare significantly less well than they used to. This is in contrast with the official ever-rising achievement levels regularly reported on schools and their pupils. Here we have a measure that is not transient and politically driven but one that explores how pupils interact with and make sense of the world around them, without relying on taught procedures.

Knowing that a National Curriculum level 6 will gain a child a Grade C in their GCSE exam, I first reflected on my own children. I know that my eldest daughter will finish Year 6 with a strong level 5 in mathematics. Would that mean that all her secondary school would have to do to meet their target is to help her make just one level progress in her subsequent 5 years of education? Inversely does that mean that she progressed through 5 levels in her first 6 years of schooling?

Of course I no longer believe in the linear model of knowledge acquisition inherent in the design of the National Curriculum, and do not take seriously the official pronouncements on standards. I gladly acknowledge that the ideas of Jean Piaget have given me a more complex model of learning and thinking ability, built upon developmental foundations. More importantly I have something that can inform my observations of real children in the classrooms I visit each week. Thank goodness for Piaget through whom my eyes have been opened to what is really taking place around me.

Do you still remember Piaget, the genius Swiss biologist, turned developmental psychologist and theorist of knowledge construction in the mind, adopted by UNESCO in the 60s? Possibly only vaguely!

On starting teaching 20 years ago I had heard of Jean Piaget's ideas but felt they were more relevant to crusty old lecturers than to real teaching.

His work had little attraction in the classroom and his theories were better suited to the intellectual realm at university. Even during PGCE training we were more concerned with the National Curriculum than with how children learn. Once in school this was replaced with the need to get to grips, managing time, reports, discipline and much more.

The same feelings are here today, as I found in a straw poll of teachers I was working with recently.

- *I cannot remember much, he talked about foundations of learning I think.*

- *He was someone I had to know about at University and now I know nothing about him!*

- *Piaget is someone who is frequently quoted and mentioned in education. His ideas seem to suggest that children are complex beings (though similar developmental paths appear) who vary, yet some aspects of modern education seem to insist that all are measured on the same scale.*

- *Children built upon previous learning and needed those first building blocks. Learning through doing. Children cannot learn out of their ability level.*

- *A French man interested in cognitive development, had lots of theories and thoughts on children's development*

- *Tried to find answers as to how children learn*

- *Have not thought about him since University - something to do with developmental stages and a spiral curriculum*

In contrast to those views I was fortunate to discover Piaget first-hand as a participant in a training programme (I attended simply to escape a rather traumatic Year 9 class) early on during my teaching career. In 1991 I participated in a pilot (Cognitive Acceleration through Science Education) professional development programme with a dozen other teachers in Sunderland. I remember one particular session where one of the CASE authors, Philip Adey, was talking about the concept of floating and sinking from a Piagetian point of view. Back in school the more practical aspect of this project was a battery of tests that were carried out as part of the research side of the programme. At that stage what had the greatest impact on me was how strongly the results of the (Piagetian reasoning) tests correlated with the personal judgements we were making about the pupils in our care. Without even realising it my journey with Paiget had begun.

Five years later Piaget re-appeared as I became involved (as a rather nervous biology teacher) in running the mathematics equivalent of the CASE project, CAME. What fascinated me now was the teachers I was supporting. I found that even though my knowledge of KS 3 mathematics was minimal I was able to engage with them fruitfully. I now realise that this was due to the Piagetian analysis inherent in the design of each CAME lesson. In practical terms it meant that I had a hierarchical frame of reference that clearly illustrated the mathematical steps that the pupils would take as they progressed through the lesson. Being familiar with each lesson it meant that I could draw upon this framework when talking about the pupils' thinking and how this could manifest itself in their responses to the tasks facing them.

By 2002 some 11 years from my first meeting with Philip, by now Professor, Adey I found myself in KS1 *(again I was feeling anxious and out of my professional depth)* supporting infant teachers on a project called Let's Think through Maths. Through the intellectual lens of Nunes and Bryant I began to see Piaget in a new light. What helped me at this point was the way they had begun to explore many of his mathematical observations of young children concluding that 'these are basic premises of Piaget's theory and we believe there is much evidence to support them.'

Thanks to Piaget I found I now had a language to help me discuss children's thinking, it was this that was able to guide my first tentative steps into the teaching of Year 1 and 2.

This trio of historical episodes brought me to the conclusion that I have to acknowledge the effect Piaget has had upon me. The impact has been one of perception, a sort of illuminating presence that is able to guide the way I look at children's learning in the classroom. The main tangible benefits are the ability to use the descriptions he made of what pupils can do to provide an insight into the level of thinking a child is operating at. I find this of use in two areas of my work and the following anecdotes will highlight the impact of this on me on a day-to-day basis:

1. About 3 years ago we were worried about our youngest child's academic ability in comparison with her eldest sister. At the time we were due to visit France for the first time and as we prepared they both naturally played with the foreign currency. As I listened I to her play I noticed she had sorted her spending money into 2 groups, which she happily told me were the Euro and not-Euro piles. What struck me was that mentally she was clearly able to organise and make sense of the world around her something that I had begun to realise it is essential for pupils to do if they are to benefit from the instructional curriculum they would face in school.

2. This working knowledge is guiding my current work trying to link the Assessing Pupil Progress project and the Piagetian levels that underpin the CAME approach. The aim is to provide a taxonomy to help teachers make effective observational judgements of their pupils in mathematics learning. I am really enjoying this as the majority of Piaget's work is built upon a solid foundation of observation. It is these observations that provide effective help to teachers trying to equate pupil statements with curricular levels.

1. The report was on an academic paper by Michael Shayer (the originator of the CASE and CAME approaches)

CAME

Understanding the score – Ofsted's report September 2008

Jane Gabb finds plenty of food for thought in this recent Ofsted report on what they found about mathematics in schools. Throughout this article direct quotes from the report are in italics.

Introduction

This report is recommended; it is easy to read and contains some very good examples of good and not so good classroom practice, always explaining why it was good, or how it could have been improved. It is based on mathematics inspections between April 2005 and December 2007 in 84 primary and 108 secondary schools. In places it paints a rather bleak picture of mathematics teaching and learning, despite the apparent rise in attainment as measured by national tests.

Part A of the report focuses on the inspection findings and Part B discusses the issues in mathematics and barriers to improving learning. It is from Part B that this article draws its discussion points; in particular those sections which explore the lack of opportunities that learners are being given to use and apply their mathematics.

Understanding versus getting the answers right

The fundamental issue identified concerns pupils' mathematical understanding and how it can be better developed. The report says that too often, pupils are expected to remember methods, rules and facts

- without grasping the underpinning concepts;

- without making connections with earlier learning and other topics;
- and without making sense of the mathematics so that they can use it independently.

Pupils were interviewed as part of the report. They confirmed the narrow nature of much of the teaching but they also showed how much difference a teacher's enthusiasm can make.

The report found that pupils wanted to do well in mathematics, knew it was important, but were rarely excited by it. They were generally not confident when faced with unusual or new problems and struggled to express their reasoning. Their recall of knowledge and techniques was stronger than their understanding.

The report says *'Too many secondary pupils expect to find learning mathematics difficult and seem to accept that this is so. They know the difference between being proficient at carrying out techniques and understanding the underlying mathematical ideas. They recognise that they often learn methods by following teachers' illustrative examples and working through many exercises, obtaining correct answers without really understanding why.'*

An example of this is illustrated by this account of an algebra lesson:

<table>
<tr><td>Weaker factors: right answers but insecure learning</td><td>A Year 8 lesson in which pupils learnt a method for solving simple equations of the form 2x + 5 = 13 and 5x – 7 = 8 but with superficial understanding. Although the technique was initially demonstrated correctly, pupils' thinking was not developed in a way that would support further learning.</td></tr>
<tr><td colspan="2">The teacher demonstrated correctly the technique of adding to or subtracting from each side of the equation to create a simpler equation, such as 2x = 8 and 5x = 15, and then dividing by the coefficient of x.

Pupils were set an exercise with around 20 similar questions. The teacher gave help as needed until most had answered several questions. The answers were read out and pupils gave themselves a mark out of 20, with many scoring full marks. Noticing that every question had the same format, and that several pupils had omitted their working, the inspector tried out some variations with a few pupils. These pupils tackled 3 + 18x = 42 with confidence. When asked to explain how they arrived at their (incorrect) answer of x = 8, they said they had subtracted 18 and divided by 3. Their choices were based on the position of the numbers 3 and 18 in the equation, and not their meaning.

By setting all questions in the same format, pupils took a short cut to the answers, and did not think about the method they had originally been taught. Critically, the teacher gained a false impression of pupils' learning, believing they could now solve simple equations, whereas this was in fact restricted to a particular subset of such equations. Pupils could not extend their approach to any other equations.</td></tr>
<tr><td>How might it be improved?</td><td>To improve learning in this lesson the teacher, when first demonstrating the method, could have checked that pupils understood each step by selecting examples in which the positions of the numbers within the equations varied. Following this by independent work that included a range of equations would allow any misconceptions to be exposed. Insisting on good presentation of solutions would help reinforce the need for logical thinking.</td></tr>
</table>

The report continues:

'The vast majority of pupils of all ages are capable of more.

Even those with little experience of solving problems showed that they were able to do so, with coaxing through a mix of encouragement and prompting, when inspectors presented them with challenging problems. They were willing to engage in discussion, although many struggled to use appropriate mathematical language to explore problems and express their ideas.'

Some keys to good practice are given here:

- *knowing what questions to ask to probe understanding and to identify and tackle pupils' misconceptions*

- *knowing how to use visual representations and practical resources to enhance understanding*

- *selecting a rich variety of examples, exercises, practical activities, problems and extended investigations that challenge and extend pupils' understanding*

- *understanding the role of 'big ideas' in mathematics, such as the number line, place value, multiplicative reasoning, and inverse processes*

'Effective teachers anticipate pupils' likely misconceptions and are skilled in choosing resources and particular examples to expose misconceptions and check that their understanding is secure.

When asked, most pupils recognised the difference between just getting answers right and understanding the work. Nevertheless, many of those observed in lessons were content to have the right answers in their books when they did not know how to arrive at them. They frequently replicated steps in a method without thinking and sometimes altered answers, or waited until the teacher read them out before writing them down. This view that mathematics is about having correct written answers rather than about being able to do the work independently, or understand the method, is holding back pupils' progress.'

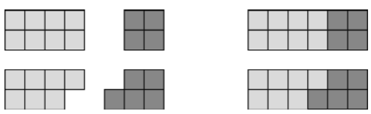
Using and applying mathematics

The report found that pupils are often left ill equipped to use and apply mathematics. They rarely investigate open-ended problems which might offer them opportunities to choose which approach to adopt, or to reason and generalise. There is not enough emphasis on mathematical talk and as a result, pupils struggle to express and develop their thinking.

In secondary schools the inspectors found that teachers seldom plan explicitly for 'using and applying mathematics' and it is very rare for schools to assess this aspect of pupils' learning separately, even though this is a statutory part of the Key Stage 3 teacher assessment.

Some teachers set investigations that do not allow pupils to actually investigate as this example shows:

Weaker factors: pseudo investigation	The way tasks are framed can close down opportunities for pupils to investigate mathematics. In this example, Year 5/6 pupils were nominally 'investigating' what happens when different combinations of odd and even numbers are subtracted. They had previously found rules for adding.
The teacher had presented the task as one of identifying 'the correct rule' by asking: 'Does odd minus odd give an odd or even answer?' Confident that a rule existed, pupils simply tried one example and inferred general rules from single examples. The teacher's approach meant that pupils never engaged with the possibility that there might be no consistent rule. In the previous lesson they had been guided to record three rules for addition (O+O=E, O+E=O, E+E=E) but reasons why the rules worked and links between the rules were not made clear.	

How might it be improved?	The teacher's questions could have been phrased in an open way: 'What happens when you add or subtract two odd numbers?' followed later by: 'Does this always happen?' Learning would have been better if the teacher had given the pupils greater independence by not assuming that a rule had to exist and by providing practical equipment such as interlocking cubes so that they could represent odd and even numbers visually. Pupils could then illustrate their explanations and justify rules. They could also have been encouraged to look for unifying ideas, for instance when adding two even or two odd numbers, the sum is always even:  The teacher might have benefited from guidance on teaching approaches for such tasks and about what aspects of using and applying mathematics pupils could develop through the activity.

It struck me as I read the report that many of the recommendations about good practice are what we promote in Equals. You do not have to look very far in any issue to find teaching ideas which suggest practical approaches to mathematics, which promote pupils' thinking, and which advocate pupils working collaboratively in groups, exploring open-ended problems.

It may be an unusual experience to find that one is in tune with Ofsted, but where mathematics teaching is concerned, I find myself in full agreement with their findings and their recommendations.

Throughout the report there are examples of prime practice, so I thought it would be good to finish with a selection of these.

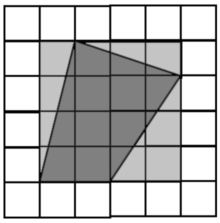
Here is an example of how mathematics can be used in a real situation:

<table>
<tr><td>Prime practice: real enrichment</td><td>Year 6 pupils investigate projects and bid for money from governors in the style of a popular television programme.</td></tr>
<tr><td colspan="2">Groups of Year 6 pupils thought up ideas, consulted the rest of the school, and then planned their projects, including a healthy eating tuck shop and outdoor play. They carried out research through questionnaires, collating their findings, using ICT very well. They researched costings, knowing they were expected to prove best value by comparing prices. The pupils devised criteria to ascertain which projects went forward to the judging panel, which comprised five governors, the chair of the Friends of the School and the headteacher. For this, they created presentations that gave a rationale, statistical analysis and justification for their project, including graphs and charts for visual impact, to convince the panel to part with their money.

All the groups were granted at least some of their funding and soon several schemes were in train. The pupils overcame practical problems as they arose, for example acquiring old supermarket trolleys to customise into a tuck shop, helped in this design and technology project by a local secondary school. The project met its aims including the application of skills in calculation, problem-solving, communication, collaboration and ICT in a real-life context. Pupils enjoyed the contribution they made to the projects.</td></tr>
</table>

Another example shows how a teacher addressed pupils' understanding:

Prime practice: building understanding	Conceptual approaches to the teaching of area meant that Year 5 pupils could do much more than find the area of a rectangle using a formula.



A primary teacher emphasised that the area of a shape was measured by the number of 1cm squares it could hold. By drawing rectangles to the correct size on squared paper, she had helped pupils to give meaning to the numerical answers. They had initially counted squares. She checked carefully that pupils had recognised the rows and columns of squares in their rectangles and could use them to calculate the area of a rectangle more quickly. She introduced triangles and many other shapes through geo-boards. Pupils devised their own strategies for composite shapes, including halving to get triangles, and discussed them with other pupils.

A further example showed how ratio and proportion can be introduced in a way which engages pupils.

Prime practice: discussion	An interesting approach to ratio and proportion with Year 6 pupils with lots of discussion.

The teacher engaged pupils throughout the lesson by incorporating many activities and encouraging discussion and argument in pairs until an answer was agreed. A reverse approach to solving problems was effective in getting pupils to think about clarity of expression. The teacher put one cup of fruit juice and two cups of water in a jug and one cup of fruit juice and three cups of water into another jug. The contents of both jugs were poured into a bowl, which, by then, contained 2,800ml of the mixture.

The teacher posed the question: how many millilitres of fruit juice are in the bowl? Pupils worked in pairs with jottings on mini whiteboards. Many struggled at first, argued with each other, but eventually worked out that 2/7 of the mixture would be juice. Pupils were then asked to write a question, in words not just numbers, to match the problem they had just solved. As the lesson went on, middle-attaining pupils in the group completed more, similar questions and higher-attaining pupils were given some requiring much deeper thinking.

This report can be found at: http://www.ofsted.gov.uk/Ofsted-home/Publications-and-research/Browse-all-by/Education/Curriculum/Mathematics/Primary/Mathematics-understanding-the-score

Royal Borough of Windsor and Maidenhead

What *are* the Basics?– or *All* Learning is Personalised

What Rachel Gibbons taught throughout her many years in the classroom was labelled mathematics on the timetable but there was something more basic that her pupils were learning.

Mathematics, the subject I taught -with passion- is considered even today to be one of ‘the basics’ but I would now define the basics differently. I grew to recognise over the years that there was something much more basic that my pupils must be learning throughout my lessons (without which I could not have taught the mathematics) and that was how to live peaceably together, concerned for each other’s well-being, respecting each other’s work-spaces and sharing and caring for the resources provided for the use of all.

This area of what might now be called practical citizenship is basic to every lesson on the timetable whatever its label. After all, without it I knew that precious little mathematics would be learned by anyone in the classroom.

Although mathematical mistakes never angered me, I do remember being very angry many years ago with one group of pupils. I was teaching at the time in a Jewish school and a member of the mathematics department came to me complaining that some of his class had called him a ‘Paki’. That a group of Jewish pupils should behave in this way seemed to me, as I told them, particularly reprehensible. At this time I was a member of the group of teachers developing SMILE, which we now translate as the Secondary Mathematics Inclusive Mathematics Experience. In a SMILE classroom every pupil had his or her own programme of learning. These programmes, of course, intersected and overlapped; there was group work and the occasional whole class activity. A SMILE classroom was more similar to life outside school than the room in which the teacher spends a great deal of time “holding forth ” to the whole class and then expects everyone to proceed with steps locked together in good military fashion.

I am freshly aware of the excitement an enthusiastic and informed lecturer can stir up

Having listened last night to a lecture on Darwin and his legacy by Professor Steve Jones, I am freshly aware of the excitement an enthusiastic and informed lecturer can stir up. “Holding forth” - if it is inspired - can certainly encourage learning. Steve Jones is a geneticist and surely one could not get more basic than the gene? Yet I was permitted to go through the whole of my formal education without ever studying biology. I am sure there are many more to whom this happened and perhaps some schools still allow some of their pupils to be equally lacking in the real basics which must be surely be a knowledge of our own bodies; in other words, human biology. Perhaps if we had recognised what really are the basic blocks of knowledge which our young people should be acquiring we should not now have in our midst fathers of 13 years old and the highest teenage pregnancy rates in Europe.

If we do accept the functioning of our own bodies - i.e. human biology - as the vital basic block on which all our formal education should rest, then English and mathematics, the two languages in which human biology is described, must come next in importance. And if we think of mathematics as fulfilling this descriptive role we may approach its teaching differently, especially for those whose study of it may be minimal.

Among the articles in Equals we try to ensure that there are many on applied mathematics – examples of numbers and patterns that describe and explain to us more of ourselves and of our world. The arguments about personalised learning change their meaning in this context too. We could say that all learning is personalised because it is all about ourselves and our own individual points of view of the world in which we find ourselves.

We could say that all learning is personalised because it is all about ourselves and our own individual points of view of the world in which we find ourselves

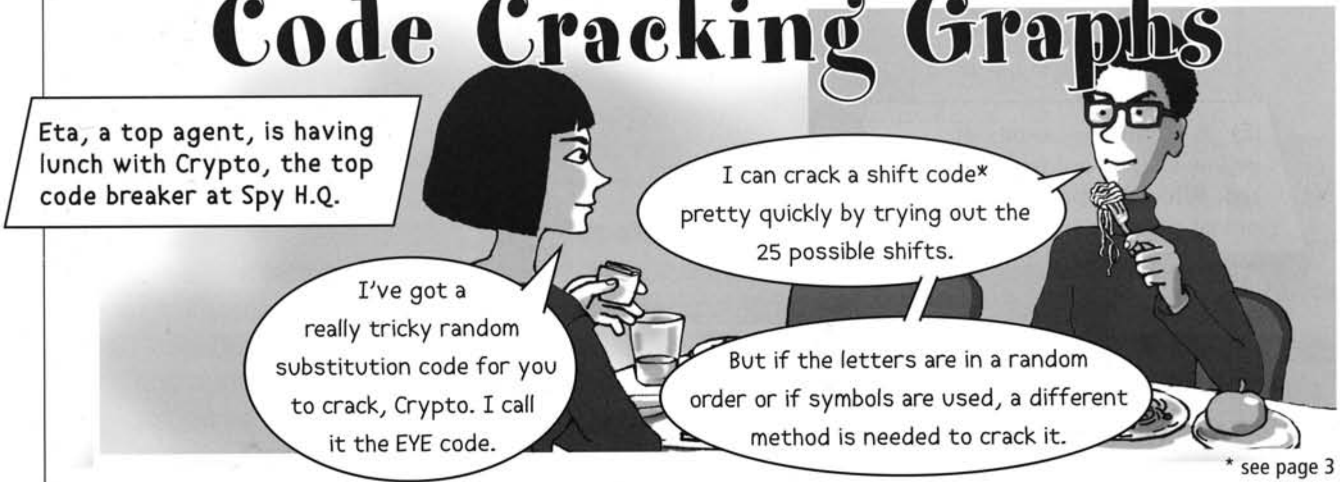
Having just been reading for review Ian Thompson's 2nd edition of Teaching and Learning Early Number, which has rich examples of real classrooms filled with living children - and therefore much invaluable information for practising teachers - I consider that *Equals* is more valuable now than it has ever been. The descriptions of observed classrooms and real children’s approaches in Ian's book are of great value. However, the book contains over 200 pages and I found myself asking how many of those teachers who would benefit from it will find time (with all the paperwork they have to contend with these days) to read it? Perhaps *Equals* and the other Mathematical Association and ATM journals can provide as rich a content but in short enough reading tasks to fit into teachers’ over-crowded timetables.

London

From *Circa Maths Magazine*

There are 400 000 000 000 000 000 000 000 000 possible arrangements of the alphabet in a random substitution code. To break these you need a cryptographer who knows his...

Code Cracking Graphs

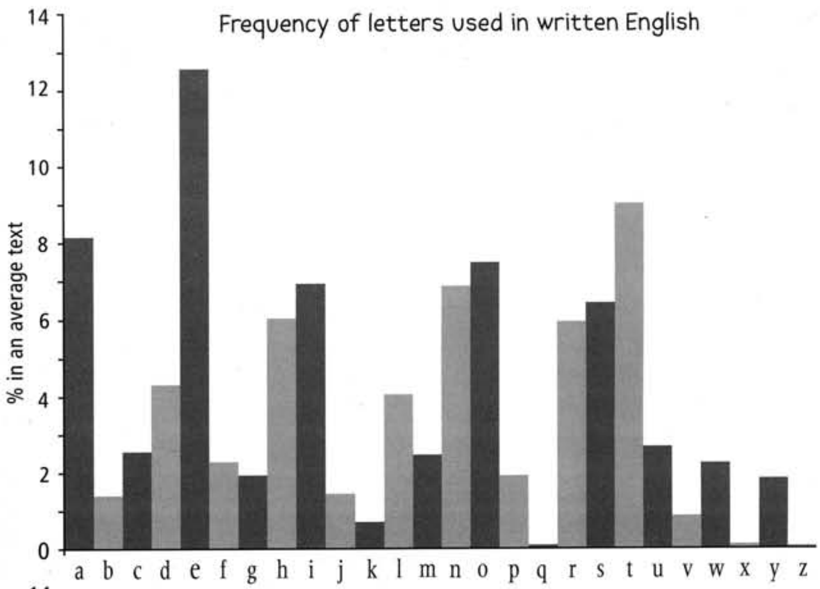


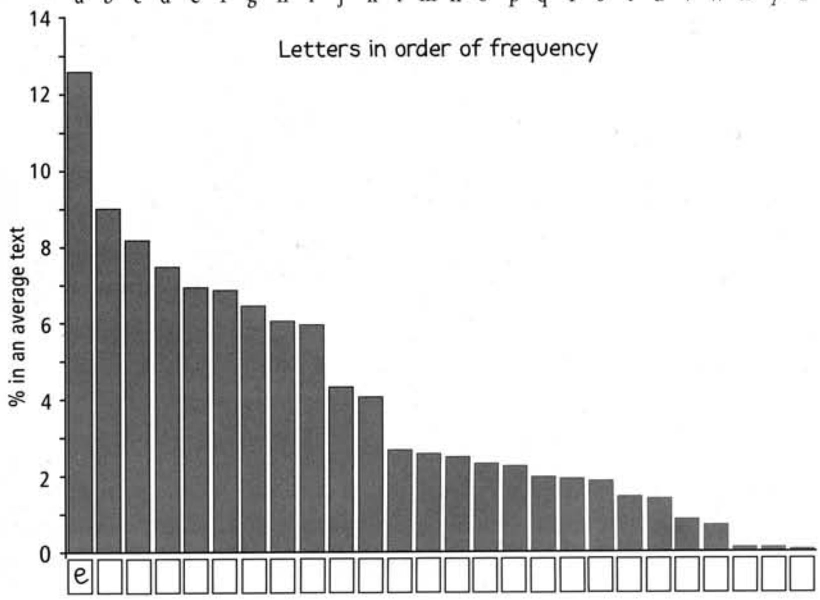
* see page 3

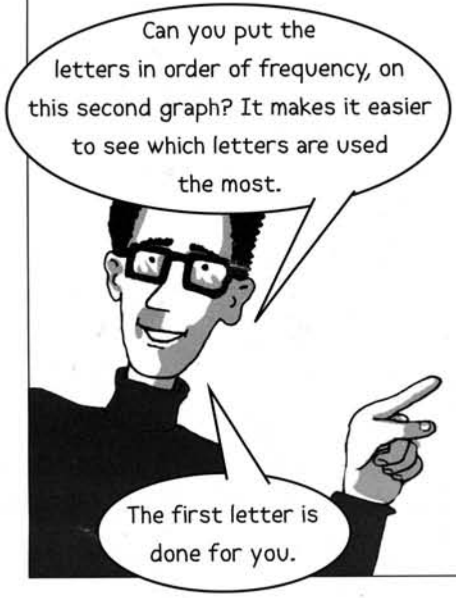
Which letter do we use most?

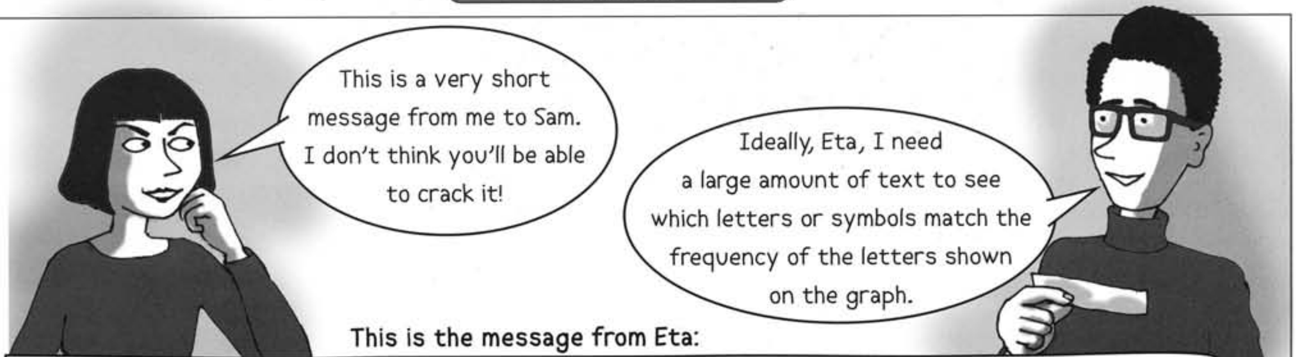
In a random substitution code any letter can stand for any other letter. The only way to crack these codes is to search for clues. Cryptographers know that certain letters are used more often than others in written English. For example, 'e' comes up twice as often as the letter 'h'. The letter 'e' is often at the end as well as in the middle of words. This is true for other languages too, such as French, German and Spanish.

A frequency graph like the one on the right shows how many times each letter of the alphabet appears in a long piece of English text.

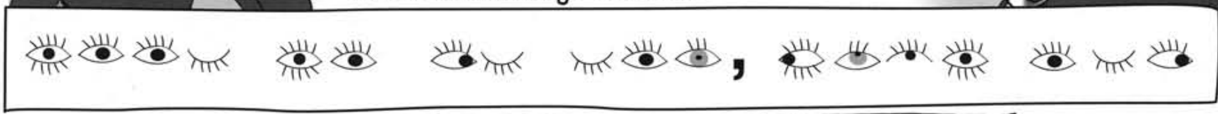


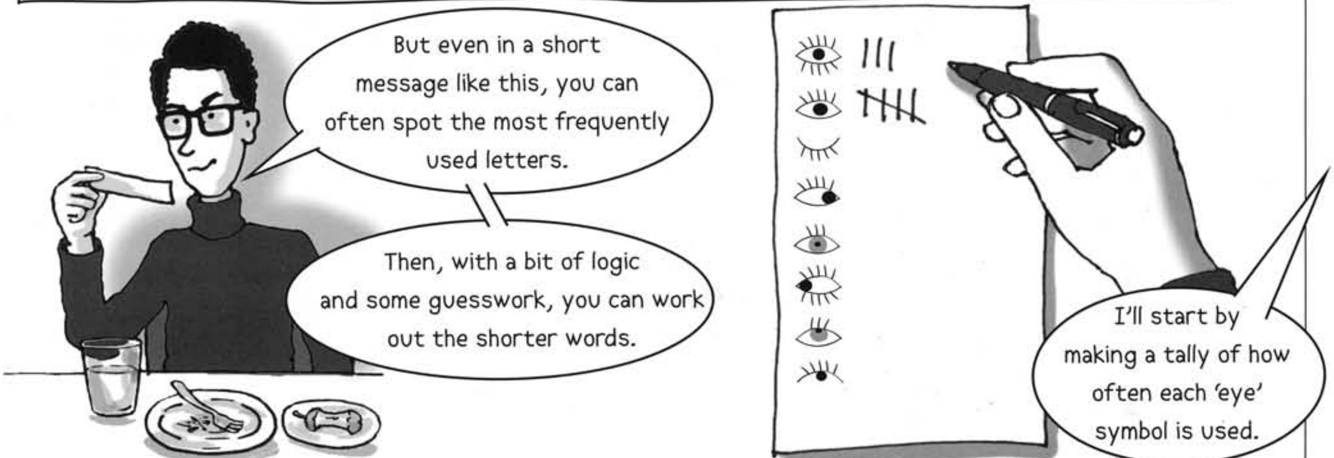






This is the message from Eta:



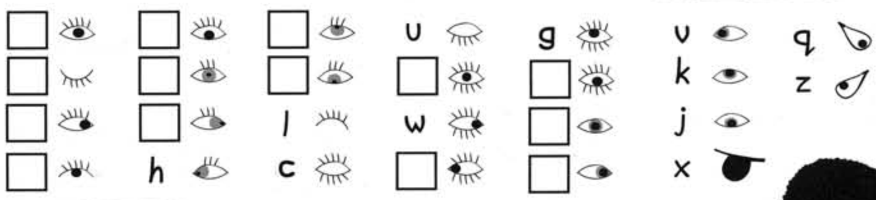


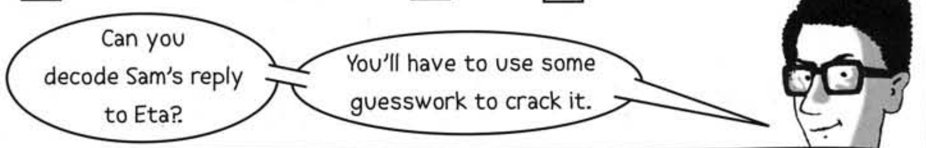
Make a tally and then write in 'e' for the symbol most frequently used. Can you make a guess at what the message says?

What's the second most frequently used letter? Put it in. Can you make out the message now? Be careful, the third most used symbol isn't the third most used letter on the graph.

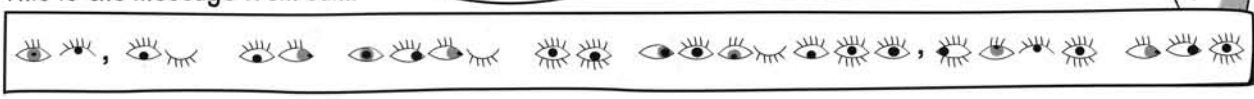
Message from Sam

All 26 symbols of Eta's code are shown here. Use what you have learnt from the first message to write in the letters they stand for. They are NOT in alphabetical order. The least used letters have been given as there isn't enough text to be sure of what they are.





This is the message from Sam:



Write your own message in EYE code. Get a friend to try to decode it.

The London Summit

(with data from *the Guardian*, 27 March 2009)

The illustration gives information about the size (population) and wealth (GDP: Gross domestic product) of the members of the London Summit last March.

1. Discuss the meaning of GDP.
2. Study the data.
3. List the countries in order of size.
4. List the countries in order of wealth.

Argentina

President **Cristina Fernández de Kirchner**

Big issues Protectionism; tax haven suppression; financial regulation and IMF reform
Key quote “The crisis is similar to the … fall of the Berlin Wall, with the difference the wall fell on this side this time”

GDP per person	Population	Public debt, % of GDP
$9,843	40m	51%

Australia

Prime minister **Kevin Rudd**

Big issues More stimulus packages and closure of non-viable banks
Key quote “What's been a breath of fresh air has been the return of US global leadership on the financial crisis."on this side this time”

GDP per person	Population	Public debt, % of GDP
$50,053	22m	15.4%

Brazil

President **Luiz Inácio Lula da Silva**

Big issue Concerned about protectionism; reform of developed nations’ financial systems
Key quote “For the first time ever, emerging countries will go to a G20 meeting in a morally superior position to that of the wealthy countries”

GDP per person	Population	Public debt, % of GDP
$8,169	194m	40.7%

Canada

Prime minister **Stephen Harper**

Big issue Banking regulation, protectionism; more funds for the IMF
Key quote “If we pursue stimulus packages, the goal of which is only to benefit ourselves or to benefit ourselves, worse, at the expense of others, we will deepen the world recession, not solve it"

GDP per person	Population	Public debt, % of GDP
$46,799	34m	62.3%

China

President **Hu Jintao**

Big issues Voting rights on the IMF; financial regulation
Key quote “Despite its severe impact on China's economy, the current financial crisis also creates opportunity for the country”

GDP per person	Population	Public debt, % of GDP
$3,577	1.33bn	15.7%

France

President **Nicolas Sarkozy**

Big issues More regulation; tax havens
Key quote “France was the first to say she would not let a single bank fail. I said not a single depositor would lose a centime. France kept her word”

GDP per person	Population	Public debt, % of GDP
$48,293	62m	67%

Germany

Chancellor **Angela Merkel**

Big issues IMF reform; extended regulation of markets
Key quote “We do not think much of the idea of a new package of measures"

GDP per person	Population	Public debt, % of GDP
$45,999	82m	62.6%

India

Prime minister **Manmohan Singh**

Big issues Increased IMF voting rights; protectionism; market reform
Key quote “The crisis was not made in our country but elsewhere... Due to the interdependency (of the world economies), we are in the same boat"

GDP per person	Population	Public debt, % of GDP
$1,122	1.2bn	78%

Indonesia

President **Susilo Bambang Yudhoyono**

Big issues More support for developing nations; IMF reform
Key quote “(We want to attend the meeting) so that we can voice Indonesia's interests, so that our economy could be managed and our people could be protected, though there are obstacles"

GDP per person	Population	Public debt, % of GDP
$2,393	231m	30.1%

Italy

Prime minister **Silvio Berlusconi**

Big issues Coordinated market regulation
Key quote "Coordinate our action with each other instead of hampering each other in our efforts"

GDP per person $40,298 | Population 60m | Public debt, % of GDP 103.7%

Japan

Prime minister **Taro Aso**

Big issues Free markets; better regulation of finance sector
Key quote "I have renewed confidence that the world's largest and the second-largest economies can work together by joining hands"

GDP per person $37,644 | Population 128m | Public debt, % of GDP 170.4%

Korea

President **Lee Myung-bak**

Big issues Protectionism; low-carbon growth
Key quote "The recovery of the global economy will depend on actions the US government takes"

GDP per person $20,955 | Population 49m | Public debt, % of GDP 32.7%

Mexico

President **Felipe de Jesús Calderón Hinojosa**

Big issues Has been accused of protectionism for introducing tariffs on US goods; IMF reform
Key quote "We obviously cannot act in an isolated to deal with a crisis of this scope, which requires a firm, timely, global response"

GDP per person $11,103 | Population 107m | Public debt, % of GDP 20.3%

Russia

President **Dmitry Medvedev**

Big issues Protectionism; IMF reform and voting rights; Nato expansion
Key quote "In general, if you ask what we are proposing, it is simply a more equitable international financial system"

GDP per person $14,688 | Population 141m | Public debt, % of GDP 6.8%

Saudi Arabia

King Abdullah bin Abdul Aziz Al Saud

Big issues IMF reform and extra funds; energy security
Key quote "I think that if these international gigantic economies cooperate, they will be able to get through this crisis"

GDP per person $22,291 | Population 26m | Public debt, % of GDP 13.5%

South Africa

President **Kgalema Motlanthe**

Big issues Credit for emerging economies; IMF reform
Key quote "No country can respond to the global financial crisis in isolation. Government will work with trade unions and business to curb its most dire impact on the economy"

GDP per person $6,648 | Population 49m | Public debt, % of GDP 29.9%

Turkey

Prime minister **Recap Tayyip Erdogan**

Big issues Coordinated EU stimulus; protectionism. May host next summit
Key quote "I'm not a diplomat. I'm a politician"

GDP per person $11,194 | Population 70m | Public debt, % of GDP 37.1%

UK

Prime minister **Gordon Brown**

Big issues New global stimulus; IMF reform
Key quote "We will have to take action in London to make sure that the banking system is reformed, to ensure ourselves that our financial institutions can come to the aid of the poorest" are obstacles"

GDP per person $44,720 | Population 61m | Public debt, % of GDP 47.2%

US

President **Barack Obama**

Big issues Coordinated stimulus plans across the G20; offshore tax centres
Key quote "It's very important to make sure that other countries are moving in the same direction, because the global economy is all tied together"

GDP per person $47,335 | Population 308m | Public debt, % of GDP 60.8%

Other participants

The **European Union** is the 20th member of the G20 and it will be represented by the Czech Republic, which holds the presidency. Two other countries will be there: **Thailand**, as chairman of Asean (the Association of South East Asian Nations), and **Ethiopia**, as chairman of Nepad (the New Partnership for Africa's Development). **Ban Ki-moon**, the UN secretary general, and **José Manuel Barroso**, president of the European commisison, are also attending. Because the guest list has been expanded, it's officially known as "the London summit".

Packing Cases

As part of the planning of an imaginary journey to Peru, Emily Fletcher explores with her class how to arrange regular blocks of different sizes to get as many as possible in a given container. The class then tries to improve the packing, reducing the wasted space and working more systematically, possibly assigning a numerical value to each block.

Materials required
Large wooden blocks and trolley

Episode 1: Covering flat space

Whole class introduction:

Lesson first trialled with groups of average and able year 3,4 pupils - but of use for the whole ability range.

I suggest that the blocks are the cases that we are taking to Peru, as part of our imaginary expedition. 'We will have to put all of our cases under the seats in the truck – lift up the cushions and put the cases in.' I then showed them the block trolley and explained it was the same size as the space under the seats. We then talked about when Mum or Dad has to pack the boot of the car to go on holiday.

'Can you pack the blocks into the trolley so we can get as many in as possible. Remember not to fill it too full or we won't be able to shut it.'

Partner work within groups of 6

Arrange as many blocks in the trolley as you can.
Can they all fit inside?
Try a different arrangement with these blocks. Can you fit in anymore?
How is this possible?
Can you cover the surface with no gaps?
The children filled the trolley randomly although because of the shape of the trolley they managed to do this without leaving any gaps. (The blocks fit exactly with 4 single blocks across the width and 6 down the length. There is space for 2 layers)

Whole class discussion:

- How many cases will fit?
- 11
  OK if you try again can you fit in any more?
- Yes you can fit in 15.

Ben – It depends which size blocks you are using!

- Is order of placing blocks important? Is height important?

Ben – It depends which size blocks you are using!

- What do you notice about them? What differences?

If we use big blocks we can only fit in 6

Isis – What counts as a case?

– Perhaps the square block is a case.
– Should we give the blocks names?
– Tiny block
– Square block
– Super block

Extension:

– How useful are numbers in describing the pieces? Can we make a list with numbers for the pieces?
– Can we give the blocks numbers?
– That's easy!
– Super block = 4
– Square block = 2
– Tiny block = 1

Discussion

Write totals from each type of block for the different groups.
Are these the same? Discuss any differences.

Holly – If the tiny block was a case we could fit in 24.
– How do you know that? We've only got 12 tiny ones.

Edward – Well the super block is worth 4 tiny ones and you can get 6 super blocks in.

Rory – How many 4's in 24?

Isis – Or 12 square blocks because they are 2.

Saskia – Yes because there would be 2 layers of 6.

Taylor – If we had half blocks we could get 48 in and if they were quarter blocks we could get 96 in!

I tried this lesson again with a group of 6 less able year 3 pupils.
The 'hook' was the same and the children were very enthusiastic.
They started by just using the largest blocks and discovered that the trolley would hold 6 blocks.
'Can you get any more in?'
William – I think we could fit in 8 or 9.
They tried again another way and still fitted 6.
Several other attempts produced the same results
Connor – I don't think it will work because there is no space left over so there can only be 6.
How many square blocks can you fit in?
The guesses ranged from 7 to 18.
William – If you split up the superblock you get a square block so if there are 6 super blocks there will be 12 squares.
Evan – I've changed my mind to 12 because you can fit 6 on the bottom and 6 on the top.
They then tried practically and found they were correct.
What if we did the same with the tiny blocks?
Again estimates varied but
Jay – 24 because a tiny block is half a square block and 12 + 12 = 24.
At which point the rest agreed with his idea.
Jessica – We could get more in because they are smaller. We could fit twice as many tiny ones in.

At this point I was very happy with the outcome and thought they had worked really hard and that we had had a very successful lesson.

BUT

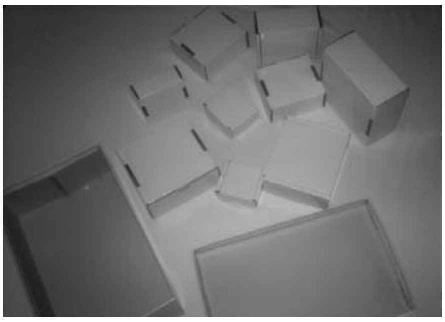
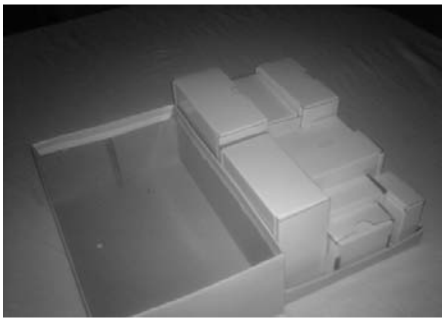
William – I still think we could fit in more super blocks if we had another go! I think we could get at least 8 in.
I really enjoyed this lesson it was great fun. I like thinking and doing things.

Predicting how many will fit.

If you had a box which was 5 blocks long and 3 blocks wide, what combinations of blocks could fit in 2 layers.
Try other sized boxes. What about a 3 x 3 box?
Not attempted

Reflection:

- How did we work on the problem? (A general talking point is on the need for looking at the whole problem first, then try a solution, and be prepared to try again from the start. The end result could be a general way to solve similar problems).
- How useful are the numbers?
- What connection is there to other problems?
- What is a general good approach to packaging? (A related point is 'How to give advice to someone else on how to fit packages into a box? Imagine you are working for a mail-order company can you write instructions of the steps for filling the boxes efficiently?

Achieving valuable outcomes for children, families and communities

Some years ago Denis Mongon was a member of this journal's editorial team. Now, with Charles Leadbetter, he has produced a report - *Leadership for Pubic Value* - for the National College for School Leadership. We urge you to consider how the extracts that follow may give new insights for the more effective teaching of mathematics.

Educational settings developing an interactive approach generating both wider social value and businesses, artists, and sports

Four hypotheses :
Schools are more likely to be effective if

- they draw the community into their work, for example by engaging parents more in school life or engaging local employers and public agencies
- they reach out to work with their immediate social networks and particularly with families
- [they] work with the wider community, beyond the families directly involved with the schools to help generate social capital and cohesion
- [they] create value for their local communities, for example by providing their facilities as a base for community activities.

The report shows how a set of schools and centres are exploring new ways or working intended to allow them to do a better job for the young people and families they serve directly and also for the communities they work with and within. … In an instrumental sense the job of the education organisation is made easier the more effectively it can mobilise resources and support from its community. It is rarely enough simply to invite these resources – parents, local employers, social businesses, artists and sports clubs – into the buildings to work.

Clearly it makes sense to reach out to parents and the direct social networks that surround the setting because these have a direct bearing on the climate in which teaching and learning is conducted and so for educational attainment.

……………………………………………

There is also a clear agenda for schools and centres to develop how they work, which will enhance their relevance to the communities they work in. This is one reason why approaches to personalised learning are a vital link to a more community-based approach. More personalised learning based on a wider curriculum, alternative forms of assessment, more collaborative, exploratory and real world learning, is more likely to make it possible for the community to engage with the school and to make the school more relevant to the community.

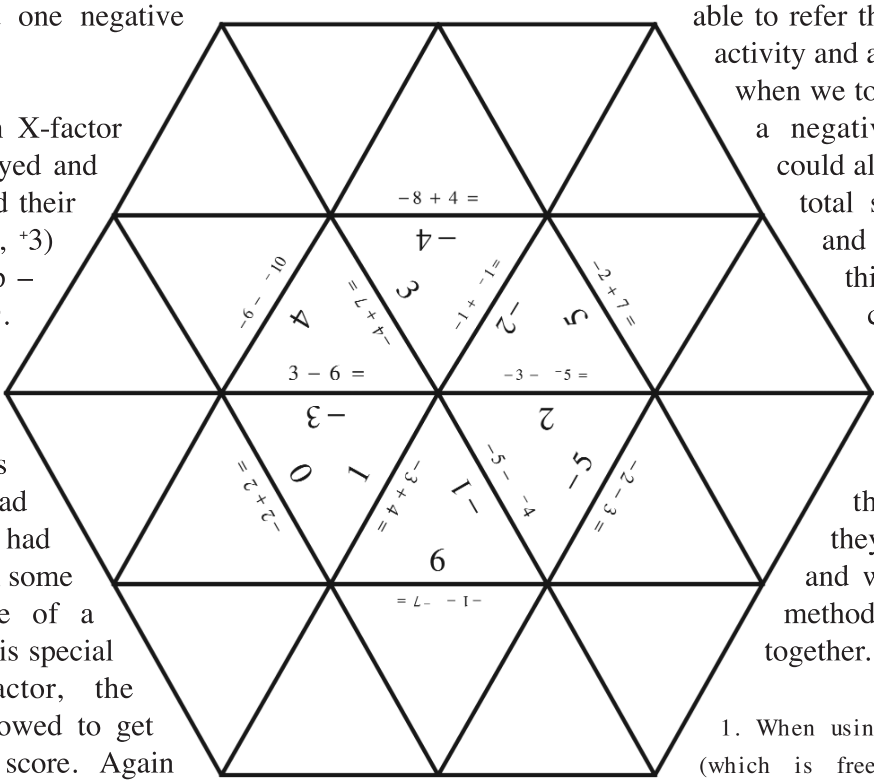
University of Manchester

X-factor starter for negative numbers

Jane Gabb has been collecting teaching strategies which demonstrate what happens when negative numbers are subtracted, trying always to get away from the rules.

Recently I was working with a teacher and a Y8 class who have a tendency to be unengaged, sometimes to the point of rudeness. We decided that we needed to keep the lesson very motivating and get them actively involved from the start. We decided to use the 'X-factor' game as a starting point.

The class was divided into 4 groups – just where they were sitting. One group was 'Simon Cowell' – they could only give scores in the range $^{-}5$ to $^{-}1$. One of the other groups could only use $^{+}1$ to $^{+}5$. The other 2 groups were given the range $^{-}1$ to $^{+}5$. This structure guaranteed that there would be at least one negative score.



A short clip of an X-factor contestant was played and the groups reported their scores. ($^{+}5$, $^{-}4$, $^{+}5$, $^{+}3$) We added these up – the total was 9. This gave us the opportunity to notice that adding negative numbers was a skill they had remembered; they had different strategies, some involving the use of a number line. In this special version of X-factor, the contestant was allowed to get rid of their lowest score. Again we could see that the majority of them understood that $^{-}4$ was the lowest. It was crossed out and the remaining numbers added up – the total was now 13. 'What can you tell me?' was the question, and several pupils gave answers:

- The score has gone up
- It's more now that we've taken the lowest away
- It's bigger than it was before

This gave the teacher the opportunity to emphasise that we had taken away a negative number and this was the same as if we'd added the same amount.

This was repeated a couple of times so that the point could be rehearsed a number of times.

The main activity asked the pupils to work in pairs or groups of 3 on a jigsaw constructed with the Tarsia[1] software. Copies of the jigsaw and the solution are included in this issue; this can be enlarged, (preferably) laminated and cut up to provide the jigsaw. While I was moving around the groups supporting them it was really helpful to be able to refer them to the X-factor activity and ask 'What happened when we took away/subtracted a negative score?' They could all remember that the total score had gone up and began to connect this with the simple calculations they were doing.

For the plenary we asked pupils for those calculations they had found tricky, and went through a few methods of solving them together.

1. When using the Tarsia software (which is freely downloadable at: http://www.mmlsoft.com/index.php?option=com_content&task=view&id=11&Itemid=12 or put tarsia into a search engine and follow the links) you can build a small jigsaw with just 12 pieces by using the extended hexagon jigsaw. When you have opened it, click on the filtering mode icon which looks like a hexagon jigsaw and select **Input mode: simplified hexagon jigsaw.** You can use this for converting any mundane practice task into a jigsaw. Pupils will do far more examples when the work is presented in this way – just try it!

Towards a new primary curriculum: some questions from the Cambridge Primary Review which we urge you to address

From Curriculum Questions 1:
The Cambridge Review (Core Questions)

What should children be learning during the primary phase?
What kinds of curriculum experience will best serve the children's varying needs during the next few decades?
Do notions like 'basics' and 'core curriculum' have continuing validity, and if so what should 21st century basics and cores for the primary phase be?

What constitutes a meaningful, balanced and relevant primary curriculum?

From Curriculum Questions 3
What aims, values and principles should the primary curriculum pursue and enact?
What are the implications of recent research on children's development and the learning on what, as well as how, they should be taught?

How should the curriculum and teaching address children's special educational needs and the circumstances of the nation's most disadvantaged and marginalized children?

These questions could valuably be discussed in future issues of Equals. Why not send us your thoughts to share with other readers?

Similar questions apply to the secondary curriculum, so consider them there also and again share your ideas with others in the pages of Equals.

Pages from the past

When corporal punishment still reigned

On my first schoolday Brenda took me by the hand and left me in my classroom. Our caps and coats hung on hooks on the wall … I think I was smaller than most of my companions and I was afraid. I had good reason to be.
I discovered that my schoolmates were a wild lot. Newcomers were roughed up bullies who ran in a pack.
……………………………………………………..

One day Miss Little sent me with a message to another teacher. When I looked down the massive oak banisters on the main staircase, there wasn't a soul in sight. I threw myself headfirst onto the banister and shot down toward the ground floor. I arrived at the feet of Sister Loyola. Before I could escape, she had grabbed me by the ear and was dragging me back up the stairs to my classroom.

Having let go of my ear, Sister Loyola began to push back her wide sleeves. With my heart thumping, I stood there watching her. Fearing the worst. She tested the rod, bending it with her hands. Then to get the feel of the cane, she struck the air several times. Whoosh, whoosh.

"Put out your hand!" she ordered. Her pale face had reddened. Her lips were set in a hard strength of will.
I raised my arms, offering my palm.
"Higher!" she ordered, lifting the hand with tip of the cane.
I waited, biting my lip.
Suddenly she brought the stick down across my palm and fingers with all her strength. There was a rattle of her cross and beads and a sting of pain as if my hand had been laid open. I stifled a howl.

Extract from William Woodruff, *The Road to Nab End*, Abacus, London, 2006

Wrapping paper - a useful resource for early number and mathematics work

Ruth Smith, a teacher on the Graduate Teacher Programme in Windsor and Maidenhead, reflects on her experience of teaching 2 'lower-ability' key stage 4 classes in an all girls' comprehensive school.

I found this wonderful wrapping paper which has lots of mathematical possibilities (see front cover). Generally wrapping paper can be used to engage pupils who are at an early stage of understanding mathematical concepts. You can find paper which will interest an individual – from the youngest child to teenagers – just by getting an appropriate one. Typically they contain repeating patterns and can be used in all the ways suggested below, just change the questions to match the paper! If the paper is laminated it becomes quite sturdy and can be drawn on with dry wipe pens.

Naming

- Colours
- Animals
- Parts of animals
- Sounds animals make

Counting

- How many pigs can you see?
- How many little ducks in this box? (Point to one of the duck boxes) How many big ducks? How many ducks altogether?
- How many flowers do the sheep have?
- How many feet does one duck have? How many feet do 2 ducks have? Lead on to counting ducks' feet in 2s.
- How many leaves on each flower in the spaces?

Patterns

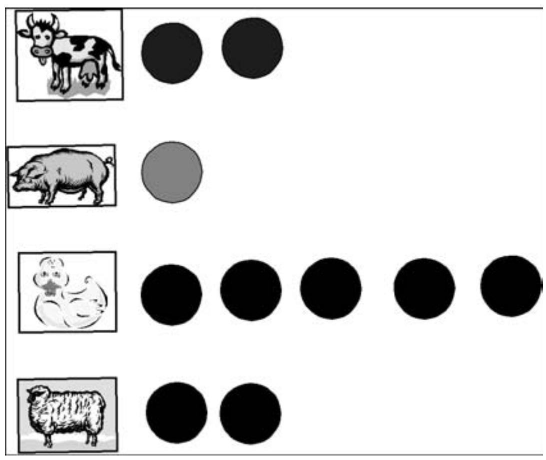
- Look along one of the lines and say the names of the animals e.g. pig, cows, horse, goats, pig – What comes next?
- Look along one of the lines and say the colours of the boxes e.g. green, purple, red, green – What comes next?

Shape & Position

- Can you show me a straight line?
- What shapes are in the corners of the boxes?
- Which animals are like circles?
- Which way is the big duck facing? How many ducklings are not facing the same way?
- Which goat is facing to the left?
- How many animals are facing right?
- Which animals are to the left of the horse?
- What colour flowers are above the cat?
- What animals are behind the pig?
- What can you see above the ducks?
- Which cow has a flower – the one on the left or the right?
- Which goat has a beard and which way is it facing?

Data Handling

- Use counters to match up animals 1-1 to different coloured counters. Then put the counters on paper in columns with pictures as labels.
  E.g.



- How many sheep did we find?
- Are there more pigs or more cows? How many more?

Windsor and Maidenhead

Mathematics and Ancient History

Lucy Sayce led mathematics teachers at schools in Reading in organizing a day at Avebury stone circle. Here she describes the mathematical problems which were tackled by groups of year 9 students.

How often have we tried to answer students who ask "When am I ever going to use this?" and have settled for the reply "In your GCSE exam." Trigonometry is one topic that frequently falls into this category. In reality, it is one aspect of mathematics that is used all the time in the construction industry, an area that many of our "foundation" students may well work in. Although many teachers have little or no experience of surveying techniques a recent cross-curricular day proved that it is easier than expected to put classroom methods into practice in the field. (Quite literally "in the field" in this instance!)

The day was organised for year 9 students and their teachers at the stone circle at Avebury as part of a series of mathematics enrichment days. One focus of the days has been to link mathematics to an unlikely subject in order to help students make connections between mathematics and the real world. The idea originated well before Functional Skills reared its head but the current education agenda makes it even more relevant. It is possible to find mathematics in everything, even where you least expect it.

The structure of the day

Students selected from seven different local schools were split into groups of 4 or 5 and stayed as the same group for all activities (meeting up for a chat with friends at lunchtime). Four different activities were devised (detailed below) and groups were able to choose to do as many of these activities as they wished. Most did three. Interestingly some students found the whole process of choice very challenging; possibly they were more used to receiving direct instructions. The tasks were left deliberately open ended to promote problem solving and higher order thinking. Hint sheets were available, but students were told that reference to these would count against them in the allocation of prizes. Consequently not one hint sheet was asked for. This is clearly a system that could be replicated in the classroom to encourage students to persevere. At the end of the day prizes were awarded for the most accurate answer to each of the activities and also to the students who performed best as a group.

Activity A – How heavy is the largest stone?

Is this the biggest?

Or this one?

Students chose the stone they considered to be the biggest and then took measurements to calculate an approximate volume. Clinometers were used to calculate the height, and the shape was approximated to an appropriate regular prism.

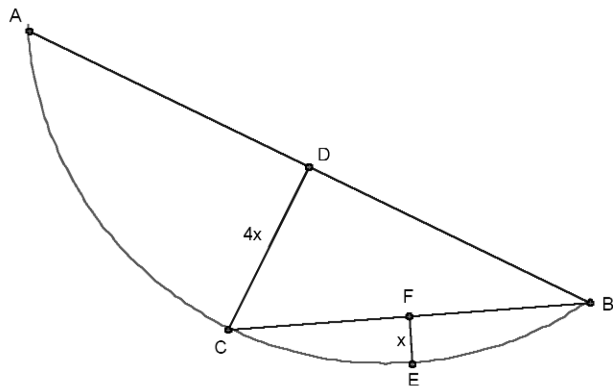
Using the clinometers to measure the height of a stone

Once the volume was known density calculations needed to be done on similar smaller stones in order to convert volume into weight. This led to an interesting eureka moment for one group who didn't realise until it was too late that the stone would displace the water in the measuring cylinder onto the floor! There really is no substitute for learning by doing. A certain degree of inaccuracy crept in, particularly when converting between units, which resulted in one group confidently asserting that the heaviest stone weighed 1300 grammes. (It is actually thought to be about 20 tonnes.)

Activity B – How long did it take to dig the ditch?
The stone circle at Avebury is not actually circular, comprising instead a number of arcs of different radii. However, for the sake of simplicity this fact was not dwelt upon during the day. The ditch encircles the stones and village with an overall circumference of about a mile. It was originally much deeper than it is today, having since silted up, and was white from the cut and exposed chalk. Although now green it is still an impressive sight and deep enough to raise the pulse rate when climbing out of it. Students were provided with a scale drawing of the site and also a diagram of the current cross section compared to the original which was approximately trapezoidal. They were also told how much chalk one man could have removed in one day using only bones and flints as tools. This enabled them to calculate the overall volume of the ditch and hence the number of man hours needed to dig it. A theodolite was borrowed from the local university and set up in a fixed position so that the students could calculate horizontal angles across the ditch and then use triangulation to find the width. Compasses, although less accurate, could have been used for this had a theodolite not been available. Clinometers were used to find the angle of depression to the bottom and hence the depth.

Again, no group had exactly the answer written in the guide books (a staggering 1.5 million man-hours) but then this figure would only be based on approximation. Many were within a factor of 10. Again conversion between units, in this case compound measures, provided considerable practice of a method often needed in real life but rarely effectively consolidated in the classroom.

Activity C – Find the position of the missing stone
A number of the stones are missing from the circle having been broken up for building material or simply removed because they got in the way. In order to replicate ancient surveying techniques as a contrast to today's methods the use of the sagitta (perpendicular distance from midpoint of chord to arc, CD) was outlined to students. When an arc (AB) is halved (BC) the resultant sagitta (EF) is one quarter the original (CD).



By measuring the length of the chord between 2 existing stones and using a rope held as a 3,4,5 triangle to fix a perpendicular it was possible to find points on the arc between the 2 stones where original stones might have stood. If this technique was employed in the original building of the monument, it would have been necessary to know the radius of the arc, but since houses now obscure the centre point students were allowed to take this measurement from a scale drawing.

Using the total station to measure the length of a chord

Measuring with the tape measure

In order to compare ancient and modern surveying techniques students were able to use a laser ranger on a total station (again borrowed) to measure the length of the chord and tape measures for shorter distances. Lengths were also paced out (more converting of units required) to mimic ancient man's use of rods laid end to end and lengths of twine. It was hoped that a hand held satnav could be used to check students' answers to the position of the stone but unfortunately the accuracy at Avebury was only to the nearest 12 metres which was not enough. Position markers could have been used but it was decided on the day to reward method and group work as opposed to final answers for this activity.

Activity D – How long is a megalithic yard?

It is thought that many ancient sites are built to the same basic unit of measurement, the megalithic yard. If this is the case, quite how the information was communicated across vast distances so that the same measure could be replicated to a high degree of accuracy is a mystery. One theory is that the measurement is based on the time that it takes the earth to turn one degree on its axis. This assumes an incredible amount of knowledge about the shape of the earth, its orbit around the sun and relationship to the stars. More detail about this can be found on Robert Lomas website and in Heath and Michell's very readable, if mind stretching, book. In brief, "one degree" on the horizon ($1/366^{th}$ part, taken from the number of days in a year) was marked with 2 stakes. This can be done by marking out a 175:3 isosceles triangle, which does give a surprisingly accurate $1/366^{th}$ of 360°.

Using a pendulum to work out the length of a megalithic yard

The time for the passage of a bright star between the stakes was measured by counting the swings of a pendulum. The length of the pendulum that would give exactly 366 swings during this time was doubled to make a megalithic yard.

Since our trip took place on a sunny day it was hardly practical to wait until nightfall, pray for a clear night and start star gazing. So the time measurement (3mins 56 secs) was given to the students and all they had to do was make a pendulum to swing the 366 times in that interval. Most groups scaled down the length of time and number of swings and managed to get remarkably accurate results. Much discussion resulted from asking if it mattered how hard you swung the pendulum or if the size of the weight was important.

Was it all worth it?

Most definitely! Many students said they would have liked more time on the activities which is a comment rarely heard at the end of a whole day of mathematics. The site itself provided the necessary Wow! factor when students began to realise the scale of the task undertaken to create the monument. The difficulty of working with basic tools was certainly registered but also the level of mathematics that was being used 5000 years ago came as a shock to most and linked our own curriculum to its historical roots. The realisation that Pythagoras was not the first person on earth to use a 3, 4, 5 triangle was a little unsettling to some.

In terms of logistics, Avebury provide a classroom, museum entry and an introductory talk for free. There is an excellent cafeteria which provided lunch, and there is ample space amongst the stones for groups to explore the mathematics. Most of the resources needed are readily available. Long tape measures? Try the P.E. department. Compasses….? Could be borrowed from Geography. Geography may also have clinometers but if not, they are a valuable resource for any Mathematics department and basic models are quite inexpensive. Using the more technical surveying equipment, the theodolite and total station, was surprisingly easy being mostly "point and shoot" like a digital camera. The students responded enthusiastically to their high tech nature and the contrast with the rods and twine of ancient times was stark.

Students came away from the day having had time to think extensively about 4 questions at most. They used a variety of methods and had to draw on knowledge from a number of different curriculum areas and topics. They were given opportunities to apply the mathematics that they had learnt in the classroom in contexts that were both relevant and real. They willingly worked collaboratively and were engaged in deep mathematical discussions as they struggled to find solutions to complex problems. In doing so they were required to persevere beyond normal expectations. All that, and at the end they asked for more. Yes, it was well worth it!

References:

- Heath, R. & Michell, J. (2004) *The Measure of Albion: The Lost Science of Prehistoric Britain*. Bluestone Press.
- Details of how to construct a megalithic yard are available at http://www.robertlomas.com/megyard/index.html
- Avebury can be contacted through www.nationaltrust.org.uk, telephone 01672 539203.

Additional support and funding was provided by the Widening Participation Office at Aim Higher (Berks). Similar days have been run on Mathematics and Music, Mathematics in Art and Mathematics in Sport. To find out more contact Lucy on:
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Reading

Extending Education?

The number of extended schools has grown steadily since their introduction in 2006 when there were 3,277: 117 nursery, 2,328 primary, 734 secondary and 98 special schools. The figures had increased to 10,043 by spring 2008: 272 nursery, 7,542 primary, 1,912 secondary and 317 special schools.

TES: Extended services not reaching the most needy, 6 February 2009