Equals

Realising potential in mathematics for all

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Alice is examined by the White Queen and the Red Queen

MATHEMATICAL ASSOCIATION
supporting mathematics in education

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Realising potential in mathematics for all

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Editors' page

'Can't read, can't write' was the title of a TV programme looking at the plight of several adults of differing ages and backgrounds who had managed to go through the whole school system, and their subsequent lives until today, without learning anything about their own native written language. The letters of the alphabet meant nothing to them. The programme caught up with them where, for various reasons, they had all decided to return to the classroom to attempt to redress this state of ignorance. Very strong emotions were aroused as they came into the open and confessed this lack of a vital skill. These non-readers/non-writers had gone to great lengths to hide their ignorance from those around them. Not being able to understand the written word, besides making life extremely difficult, let alone denying them the delights of great literature, is considered shameful and they were embarrassed as they at last attempted to recognise the letters of the alphabet and learn the meaning of the words they formed.

A much larger number of adults could be found, I am sure, to populate a programme entitled 'Can't estimate, can't use a calculator sensibly'. However, such a lack of any number sense would not necessarily cause shame, there might even be some pride taken in such ignorance. After all, it has in some circles been considered fashionable to be bad at maths – particularly for women.

How many of our pupils will emerge from school unskilled in the field of numbers? And what difficulties will it cause them? An understanding of numbers is not only required for tackling the ordinary activities of everyday life but it is essential for making sense of the state of the world. How then do we interest our most reluctant pupils in number? This is the first question *Equals* has always set out to try to answer. It is the question every teacher should be asking rather than: what is the probability of a child doing well in Sats? A further objective is just as important. It is that all should have a taste of the excitement of the world of number, indeed of mathematics. One of the aims of Phil Beadle (who was attempting to teach the 'Can't read, can't write' group) was to introduce them to the enjoyment of great literature. It is important also for everyone to be introduced to some of the 'big stories of maths' to use the words of Marcus du Sautoy. Du Sautoy, professor of mathematics at Wadham College, Oxford, says that teachers fail unless they present some of these ideas:

> Students should be exposed to the wonders of four-dimensional shapes, the fascination of the primes, the mysteries of topology. One can motivate these as essential tools of modern life: they are key to the way mobile phones change voices into a stream of 0s and 1s, how credit card numbers are kept secure, how Google works.[1]

No doubt du Sautoy was thinking of those who might eventually be arriving at his own lectures, but why should we not have similar ambitions for all students - to see something of the excitement of mathematics, to go beyond the tests and learn to think for themselves? It may mean learning some more ourselves to do this but why not refresh ourselves?

Returning to the 'Can't read, can't write' group, it soon became abundantly clear that each of these late-developing students needed to be given their own individual programmes of work because of the very different ways they tackled their own problems. They needed to interact with each other but also to work on their own.

Lewis Carroll in Numberland: his Fantastical Mathematical Logical Life[2], just published, mentions briefly Charles Dodgson's bouts of school teaching, not all successful or much enjoyed by him, but the book gives some, if not 'big stories', intriguing mathematical puzzles and tricks he set his classes, some of them within reach of our reluctant learners, for example:

Counting alternately up to 100
In this trick we start with the number 1 and then add a new number, never exceeding 10; the person that reaches 100 is the winner. For example, if to the 1 you first add 4 (giving 5) I'll then add 7 to give 12; you might then add 8 to give 20, and I'll then add 3 to give 23.

Continuing in this way we might get the sequence 29, 34, 43, 45, 48, 56, 57, 67, 75, 78, 86, 89. 92, 100 and I win.
How can I ensure that I am *always* the one to reach 100?

This kind of playing around with numbers is a valuable experience for all pupils whether it helps their test scores or not. If you want to win every time you have to work out a strategy. In this issue of *Equals* Richard Cowan reminds us of the importance of an understanding of basic calculation at an early age and other articles discuss later stages of learning needing more complex strategies. In addition, a glimpse into the research of Jo Boaler reminds us that we are all teachers of what I would consider 'the basics' - how to live peaceably and co-operatively in a community. Have we forgotten this in today's frenzy of testing and league tables? And has this forgetfulness helped to develop a more violent society?

1. Marcusa du Sautoy. Without the big maths stories our numbers are plummeting, *The Guardian*, 03.06.08
2. Robin Wilson, *Lewis Carroll in Numberland :his fantastical mathematical logical life*, Alan Lane: London. 2008

Addressing Gaps in Children's Mathematics - A Welsh Approach Part 1

Emma Coates, **a teacher seconded to co-ordinate literacy and numeracy basic skills projects across the LEA of Rhondda-Cynon-Taf (RCT) in South Wales describes her work.**

We have been delivering the DfES Wave 3 Mathematics material for two years in our own special way! I have worked closely with Mary Clark (providing consultancy from Primary National Strategy) who has offered her invaluable advice and support and has asked me to share with you our story of making 'Wave 3' work for us.

The LEA is one of the largest in Wales with 117 primary schools and 19 secondary schools, some teaching through the medium of Welsh or in bilingual settings. I am responsible for the training for both literacy and numeracy projects and I line manage a team of 13 centrally based LSAs who deliver the projects on a one-to-one basis in schools, in partnership with the school based LSAs. Both projects that I co-ordinate are funded through the Basic Skills Agency (BSA). The funding consists of two grants, a Strategic Intervention Grant - £210k and a Training Grant - £115k.

Our numeracy project is based on the DfES material 'Supporting children with gaps in their mathematical understanding' (obtainable from DCSF publications, ref: DfES 1168-2005 G). However, one of the first things we did was change its name and call it 'Spotlight'. I could imagine children saying they were off to 'Spotlight' but not 'I'm just off to 'Supporting children with gaps in their mathematical understanding' ……Miss!'

The project is funded by a BSA training grant that the LEA had secured for 2 academic years, 06/07 and 07/08. The funding was secured with the aim of:

> 'Raising the attainment in numeracy of under-attaining pupils by
> - Identifying and addressing specific gaps in their mathematical understanding.
> - Raising the standard of LSA support for numeracy.'

We would raise pupil attainment through structured one-to-one intervention delivered by Learning Support Assistants. We would raise the standard of LSA support for numeracy by training them to deliver the intervention project and 'up-skill' to improve the quality of support back in the classroom.

Why did we need this intervention project?
Our literacy project 'Catch Up' had been running in schools since 2002 and was extremely successful with recognition from outside agencies along with favourable mentions in many inspection reports as an effective form of intervention. For a number of years schools had been asking for a similar programme for numeracy but we were unable to deliver any support due to limited funding. We were fortunate in April 2006 to secure additional funding from the BSA to enable us to expand into numeracy whilst still maintaining the level of support for literacy.

An analysis of the data provided by BSA showed the average level of innumeracy in our LEA was 56%. This was more than double the average LEA's illiteracy levels of 26% and made us as a school improvement department stop and think- should we have started back in 2002 with a numeracy intervention project instead of literacy? Had this data been available then I am sure we would have seriously contemplated it, although there was almost no material or programmes available for numeracy intervention at this time.

The DfES material contained no record keeping or materials to support gathering views from learners

I began looking at research into numeracy intervention when I took post in September 2005, in particular the work of Dr. Ann Dowker on 'What works for children with mathematical difficulties?' (DCSF research website). This research review looked at the areas of difficulties that children have in mathematics as well as current provision for numeracy intervention.

A key element of Dowker's report of particular interest to me was the use of teaching assistants to deliver intervention programmes. Our intervention project would need to be delivered by the central team of LSAs in partnership with the school LSAs. I was aware from an early stage that the attitude of our central team to delivering a numeracy intervention programme was far from positive. Many openly admitted to being poor at mathematics and felt very unsure of their own ability to deliver effective intervention. If members of my experienced team were feeling this way it was reasonable to assume that many school based LSAs would feel the same. It would be essential therefore, to develop a training programme that would not only educate the LSAs in the delivery of the project but also support them in their own numeracy development.

Developing the Spotlight Project
On securing the 'Training Grant' it was imperative that we found a suitable programme that could be delivered across the LEA *and* for which that I could write a training programme. One of the resources I had been researching was the Wave 3 material from the DfES. The material contained assessments that identified individual gaps in mathematical understanding and provided activities that teaching assistants could deliver on an individual basis. These reflected Dowker's recommendations and provided scope for developing a comprehensive training package. We began piloting this material in just 4 schools for the remainder of the academic year 2005-06. The results of the pilot year were excellent with 92% of the children making progress, 46% achieving a 3C or above.
The material we had chosen would meet our aim of raising standards of numeracy in pupils. We liked the way the material addressed children's 'gaps' to address a variety of learning styles and that the activities focused heavily on the use of mathematical vocabulary and the use of practical resources.
The material would enable us to target a group of under-attaining pupils and provide an individual and tailored program to address their weaknesses and raise standards. However, there was no 'off the shelf' training package available so I had to write our own that would be supportive and responsive to the needs of our teachers and LSAs.

The Structure
As the training grant was only for 2 years we chose to target 60 schools in the first year and 60 in the second, ensuring all 120 schools in the LEA had access to training and support. Due to funding limits schools were offered support on 2 levels.

- 30 schools receiving 'Full Support'; support of a weekly half-day visit from the central team of LSAs and access to all training for an LSA and teacher. They would target a minimum of 10 children.

- 30 schools receiving ‘Training only’; access to all training for an LSA and teacher. The ‘Training’ schools would be given extra support to ensure the LSA had effective support from a teacher ‘mentor’. These schools would receive additional training for teachers and 2 days supply cover to carry out this role. They would target a minimum of 5 children.

The children to be targeted for this intervention would need to be in the ‘Basic Skills’ group and not SEN. Principally, those achieving a 2C at the end of Key Stage 1 or predicted not to achieve a Level 4 at the end of Key Stage 2.

In a new project, listening to the views of the learners is going to be essential

Measuring Success

Our measure of success would need to cover our 2 aims of

- Raising the attainment in numeracy of under-attaining pupils
- Raising the standard of LSA support for numeracy.

Measuring the children’s attainment:

After consulting with advisors I decided to use ‘RM Snapshot’ as baseline. The computer programme would give each child a National Curriculum level and a raw score. Also, as we would only be covering ‘number’ in the project the computer program also enabled us to isolate each child’s scores in this area. The DfES material contained excellent initial and ongoing assessment for learning in the form of tracking charts and super activities for the LSAs to deliver. However, the material contained no record keeping or materials to support gathering views from learners. I created a number of additions to the material to help record and measure success:

- Numeracy interview
- Individual Tracking Charts
- Unit Record Sheets .

In a new project, listening to the views of the learners is going to be essential. The ‘Numeracy Interview’ would enable us to analyse attitudes to mathematics at the beginning and end of the project. As a teacher I know that children who struggle in any area of the curriculum develop a negative attitude and lose the confidence to attempt new tasks. By listening to their views on numeracy it would also give the LSAs an insight into the attitudes of the children before they started delivering the project.

The ‘Individual tracking charts’ would provide a record of the initial assessments and an instant overview of a starting point and next steps. The ‘Unit record sheets’ would form the record of each session with the child, noting strengths and weaknesses. At the bottom, a tick sheet of the learning objectives for the LSAs to conclude if each gap had been filled. As LSAs would be delivering the program I needed to make the assessment process as accessible as possible.

Supporting the teacher

Teachers in the ‘training only’ schools had a greater role in ensuring the success of the project in their schools. They would not have a visit every week from a central team LSA to support their LSA so they would need to provide support and guidance themselves. To help teachers identify their role I provided an additional training course on ‘Mentoring your LSA’. I developed ‘Action Plans’ for the teacher coordinators to follow and we looked at different mentoring models that could be used. The teachers were then able to produce a ‘mentoring plan’ for their individual school. Teachers were also given supply cover to use throughout the year to spend quality time supporting their LSA.

Supporting the Welsh language schools.

All the extra materials we produced were translated for our schools. However our next ‘big job’ is to translate the actual ‘Spotlights’ and will depend largely on future funding. Schools delivering Spotlight through the medium of Welsh needed a little more preparation time to read through the activities and translate them orally for the children.

Measuring the standard of LSA support:

The success of the project on raising the standard of LSA support would be more difficult to quantify. The LSAs would need to be monitored on their use of the Spotlight material effectively and supporting numeracy in the classroom. This would need to be an open and supportive process and not an ‘inspection’.

Also, head teachers would need to reflect on the impact the project has had on the LSAs in the form of a 'Self evaluation'. This qualitative data will be collected in the form of 'CRIS evaluations ' so data could be analysed in a way that could be reported.

The Training Package

All 60 schools each year would have access to the same training with the 30 'Training schools' having extra support for the 'Teacher mentors'. The training programme for the year was planned:

Summer Term: Awareness Raising - A twilight session for head teachers and numeracy coordinators

Autumn Term: 'Initial Training' – 3 half days training for LSAs and teacher co-ordinator

'Improve your basic skills in number' – ½ day training for LSAs to boost confidence

'Mentoring Course' – ½ day training for teachers

'Sharing good practice' – ½ day training for LSAs for top up training

Spring Term: 'Top-up workshop' – 1 day raining for teacher and LSAs

Summer Term: 'Annual Review' – A twilight session for head teachers, teachers and LSAs.

Rhondda-Cynon-Taf Education Authority

For the final part of the description of this project see *Equals* 15.1

Respect and responsibility in the classroom

We are grateful to **Jo Boaler** for permission to introduce our readers to some of her studies of the promotion of 'equitable relations' in classrooms – something sorely needed in these days when we seem to be increasingly surrounded by violence - and perhaps neglected in the testing regime in the UK.

… I propose the term 'relational equity' to describe equitable relations in classrooms; relations that include students treating each other with respect and responsibility. This concept will be illustrated through the results of a 4-year study of different mathematics teaching approaches, conducted in 3 Californian high schools. In one of the schools – a diverse, urban high school – students achieved at higher levels, learned good behavior, and learned to respect students from different cultural groups, social classes, ability levels, and sexes.

In addition, differences in attainment between different cultural groups were eliminated in some cases and reduced in all others. Importantly, the goals of high achievement and equity were achieved in tandem through a mixed ability mathematics approach that is not used or well known in the UK.

I suggest that one route to achieving such relations emanates from a change in the ways core school subjects are taught and learned (see also Matthews & Sweeney, 1997). Eisner reminds us of the simple but seemingly elusive fact that the goal of school is not to do well in school but to do well in life (Eisner, 2004). The notion of equity that I consider in this paper moves the focus away from school outcomes, such as tests scores, and onto the relations, ways of acting and ways of being that students learn in school and that they take with them into their lives. I extend the notion of equity to the relations between students with the assumption that the ways students learn to treat each other and the respect they learn to form for each other will impact the opportunities they extend to others in their lives in and beyond school.

As educators, we need to consider where students may learn such principles and values as respect and responsibility for others in their crowded school days. Two possible sources for such learning are the curriculum, formal and informal, and the pedagogy of classrooms.

The NCC do not convey the idea that core subjects such as mathematics and science have a role to play in the encouragement of such personal qualities. Indeed the location of such important values within a curriculum area that is un-assessed and marginalized (Derricott, 1998, p27) may lead to their neglect and under-representation. Derricott (1998) reports that programmes such as world studies, peace studies, development studies, multicultural education and human rights education have to 'fight for space at the margins of an already cluttered curriculum' (1988 p24) and that their success rests upon the work of particular committed individuals who have the time and the status to give them a place. This means that key values and practices that students need to learn may not be considered the responsibility of teachers of mathematics, science and other subjects.

Nel Noddings has produced an important body of work, over a number of years, arguing that schools need to pay more attention to the teaching of care and compassion among students … she and other scholars consider ways to educate citizens 'for global awareness' and Noddings argues that all subjects have a contribution to make in educating citizens. 'Even mathematics' she argues 'the most closely defined of all subjects – can include a study of birthrates, incomes, comparative health data, war casualties, the cost of social programs, systems of taxation, and appropriate means for collecting and evaluating such data' (2005, p123).

The school that is the subject of this paper did not explicitly relate mathematics to the students' home cultures. Their impressive achievements came about through a mathematics curriculum that was largely abstract but that enabled multiple methods, solution paths and points of discussion and negotiation.

There are many ways in which …equitable relations may be encouraged and formed in classrooms; the version of relational equity that we observed in our study, and that I will describe in this paper, had three important strands:

(1) Respect for other people's ideas, leading to positive intellectual relations
(2) Commitment to the learning of others
(3) Learned methods of communication and support.

I will argue …that these three qualities are important for the production of citizens (MacIntyre, 1984; Schweder, 2003) and that they may be learned through the teaching of a range of subjects. The ways students learn respect for other people are rarely considered by educational researchers, and when they are, they are generally separated from issues of mathematics, or other subject teaching. But if adults are to live and work in a pluralist society and to participate in society in productive ways, then it seems important that such ways of working and interacting should be taught, and modelled in our schools and they cannot be isolated within non academic subject classrooms. The classrooms at Railside school, the focus of this paper, were more equitable not only because they produced more equitable test outcomes, but because they produced more equity-minded students, who engaged in positive intellectual relations and treated each other with note-worthy degrees of respect.

if adults are to live and work in a pluralist society and to participate in society in productive ways, then it seems important that such ways of working and interacting should be taught, and modelled in our schools

Most of the students at Greendale and Hilltop therefore experienced a traditional approach, as named by the schools. The teachers lectured and the students practiced methods, working their way through short questions. Our coding of videotapes allowed us to categorise the ways in which students spent time in their classes. This showed that approximately 21% of the time in traditional' algebra classes at Greendale and Hilltop was spent with teachers lecturing, usually demonstrating methods. Approximately 15% of the time teachers questioned students in a whole class format.

Approximately 48% of the time students were practicing methods in their books, working individually; approximately 11% of the time they worked in groups, and students presented work for approximately 0.2% of the time. The average time spent on each mathematics question was 2.5 minutes.

The second major approach in our study was the approach offered at Railside school in which the teachers posed longer, conceptual problems; students worked in groups and they often presented their work while teachers questioned presenters and other students. Our coding of videos showed that teachers lectured to classes for approximately 4% of the time. Approximately 9% of the time teachers questioned students in a whole class format. Approximately 72% of the time the students worked in groups while teachers circulated the room teaching methods and asking the students questions of their work, and students presented work for approximately 9% of the time.
The average time spent on each mathematics problem at Railside was 5.7 minutes. An additional important difference between the schools was that Greendale and Hilltop employed ability grouping and students were placed into one of three different levels of classes at the beginning of high school. At Railside all students were placed into heterogeneous algebra classes.

The students at Railside started high school at significantly lower mathematics levels than the students in the more suburban schools ($t = -9.141$, $p < 0.001$, $n = 658$), but within two years they were out-performing the other students scoring at significantly higher levels on mathematics tests ($t = -8.304$, $p < 0.001$, $n = 512$). By year 4 41% of Railside seniors were in advanced classes of pre-calculus or calculus (similar to one year of A-level mathematics) compared to approximately 27% of seniors in the other two schools.

The mathematics learning of the students at Railside has been analysed in separate papers (Boaler & Staples, in press; Boaler, in press). This paper will focus upon the unusual values and equitable relations that students developed that are not often the focus of research analyses. Although these were not intended as a focus of our study, it was not possible to spend years in the classrooms at Railside without noticing that the students were learning to treat each other in more respectful ways than is typically seen in schools. In interviews with seniors at the end of high school they told us that the ethnic cliques that were evident in other schools did not form at their school because of the mathematics approach used at the school. Some students explicitly contrasted mathematics classes with other classes in which they had sat in groups but they did not learn to respect, or even know each other. The following analysis will report upon three particular characteristics that students learned and that I argue are important for the achievement of relational equity. These were themes that emerged from our long-term observations and from student interviews. None of these are specific to mathematics and they could be encouraged in any classroom, even though the particular ways that the discipline was introduced and experienced played an important part in the opportunities afforded.

the students were learning to treat each other in more respectful ways than is typically seen in schools

(Extracts from “Promoting 'relational equity' and high mathematics achievement through an innovative mixed ability approach”, to appear in the *British Educational Research Journal*.)

Learn more about what happened at Railside School and why in *Equals* 15.1

University of Sussex

Stained-glass window in Daresbury Church, Cheshire, see page 2.

First steps in word problems

The use of 'cue words' in order to decide whether to add or subtract in a word problem can itself be a source of error. Mundher Adhami writes about another way to approach language in school mathematics.

The difficulty of word problems for many pupils is sometimes explained by suggesting that words and numbers are processed in different parts of the brain. Recent brain research seems to indicate both specialisation and connections between brain parts, e.g. between the regions for processing numbers and those processing words, with obvious advantages for connections. Such connections may be more effortful, at least at the start, and hence might often be avoided for more comfortable thinking, or for relying on rote. But remaining within a comfort zone hinders rather than helps learning.

It seems sensible for us as teachers to emphasise the interplay between mathematics and language and by extension interplay with real life contexts where simple numbers, for example, are used. My view is that that should happen at all levels, from applied advanced mathematics to the simplest addition and subtraction word problems. This goes against some views that favour avoiding the 'confusion of language and context' focusing on 'straight sums' and procedures, something that seems to have prevailed in the last 20 years. Such views do use words, but in nominal ways to avoid difficulty, which themselves may cause more harm than good.

One common 'learned error' by pupils is just adding or subtracting numbers 'in their order' relying on intuition. This comes from the easy word problems teachers give younger or special needs pupils as a kind of encouragement. But that defeats the whole idea of discerning mathematics from the language.

Let's look at wording that places the "answer' as the first number in the sentence to see if it is a manageable challenge for most pupils in lower secondary and upper primary.

> '**Today** I have 12 DVDs. That is after the 3 DVDs I received as a present yesterday. How many DVDs did I have the day before yesterday?'

It is taxing, and would take pupils some time to sort out the sequence, but it is a worthwhile challenge and discussion.

Another common and related learned error is using 'cue words' like 'more' to mean addition, while the full sentence has a different meaning, e.g.

> "I now have 6 marbles, which is 3 more than I had before. What did I have before?

Here is an account of an activity that asks pupils to think of any word problem as a story with a beginning, middle and end, but which may be scrambled in words. It is modified from an account of a real lesson, and presented as a story of a lesson.

The lesson builds on the intuitive knowledge of pupils about the meaning of numbers as labels for collections, e.g. of marbles or books, that can increase or decrease. It also encourages pupils to check the truth of sentences; things have to fit each other in mathematics!

Episode 1: Three parts of a written number word problem

The teacher has three strips of poster paper with the phrases:

I had 3 marbles at first.

Then I found 4 more marbles

Now I have 8 marbles altogether.

She places the three phrases in one line and chants them with the class, so that all can participate.

She asks whether the 'story' is true: **Do you agree with this?** That was a novelty. More pupils perked up when they realised it is actually wrong. They know enough to know something is wrong!

Sarah gets the class to talk about sentences *making sense* or not. **Which number shall we change to make it right?** She talks through *changing the total* and how we can be sure about it, including writing the sum in order 3+4=7. Then she also discusses the possibility of changing the start i.e. 4+4=8 and the middle i.e. 3+5=8. **Which change is easiest? Why?**

Sarah uses a blank sticker for one of the numbers on the same sentences they have been using, and a sticker with a different number for a second to play the game of **finding the missing number**. She asks pupils to write number sentences on their mini-boards to show the class.

She then asks the class to tell each other in pairs what ideas they have about the lesson so far. She manages to get some pupils talking about:

- 'If you know two of the numbers you can find the other',
- 'Not all sentences make sense',
- 'It works the same way for ice lollies'.

In the 20 odd minutes so far number bonds have been practised in this Y4 class, which could be a weak Y7 class. Number sentences have been created, with a bit of freedom and fun. Pupils felt empowered to check the truth of number sentences. And they have been introduced to the idea that a number problem can be looked at as a story.

Episode 2: Choosing the right question

Sarah removes the last phrase i.e. 'Now I have 8 marbles altogether', and changes the numbers:

I had 8 marbles at first

Then I found 4 more marbles

and writes these four questions on the board:

- How many marbles did I lose?
- How many more marbles did I find?
- How many marbles did I start with?
- How many marbles do I now have altogether?

Sarah gets the class to talk about how they know which question to choose.

She then hides the first sentence and adds the third. She asks them to choose the right question this time.

I had 8 marbles at first

Now I have 12 marbles altogether

She crosses out the word 'Then' to use the middle sentence, also asking for the right question.

I found 4 more marbles

Now I have 8 marbles altogether

She again asks them to choose the right question.

In looking back, after about quarter of an hour, the kind of ideas that emerged were:

- There are always three-parts. One part is a question.
- The numbers can change. They can be any.
- No. The numbers have to fit. The total cannot be less.
- The question comes out of the story. You have to understand.
- You can put the parts in any order. You still can solve it

Episode 3: Choosing the middle phrase (both action and number together)

Sarah says the three phrases are now to be put underneath each other, but the middle one is missing. They have the start and the end. They have to choose the right middle phrase from options.

I had 3 marbles at first.
-?--------------------------
Now I have 8 marbles altogether.

The four options are:
Then Sally gave me 3 more marbles.
Then I gave Sally 5 marbles.
Then I got 6 more marbles.
Then I lost 4 marbles.

Pupils produce a variety of responses at this point. Some choose 'Then I gave Sally 5 marbles ' because they know the middle sentence needs a 5. Sarah encourages the class to read the resulting 'story' and comment on it. Several children point out that this does not make sense and can articulate why, talking about 'you would have to take it away, not add it because it says 'I gave the marbles away'. Sarah asks what they could change to make it make sense.

Some children say ‘It’s impossible’ and when asked to give their reasons, say that none of the sentences makes sense for the middle one because none of them give the answer at the end. Sarah asks how the sentences could be changed so that they would make sense and explores a few suggestions.

She then presents children with the following problem which again has 4 possible answers (one of which is correct this time). Children try this out in pairs and then report back. Most have the correct sentence in the middle.

I started with 5 marbles.

(Put the right sentence here)

Now I have 11 marbles altogether.

Then I lost 4 marbles.

Then Sam gave me 3 more marbles.

Then I gave Sam 6 marbles.

Then I got 6 more marbles.

Sarah now asks them to vary the start number in three different ways so that each of the other three middle phrases can be correct. She asks them to use numbers that are different from the examples they have worked on before. They could stay with easy numbers including large ones such as 20, 30, 15 etc, or chose ‘harder numbers’. For one pair Sarah suggested they make the first phrase the missing one.

The lesson ends with no further discussion, with pupils working at their levels. Some were still struggling with the small numbers, and explaining how to work out both the numbers and the words to make the sum correct. Others were flying high on their own ability to make up word problems that make sense, with ‘hard numbers’, and in different orders.

The lesson allowed pupils to explore word problems in small steps. They learned to translate the words into arithmetic by making them into a story. They used a frame of beginning, action and end to work the correct words and numbers into sensible sentences. This frame looks like a procedure, if so it is one of the simplest and most flexible. It is a frame for thinking, rather than for mindless processing. Crucially it allows pupils to find missing numbers and correct errors, i.e. to rely on the ‘overall-ness’ of mathematics to check the bits.

There is a more challenging lesson linking language with mathematics in the *Let’s Think through maths 6-9 pack* published by nferNelson, in which guidance for this lesson is given. In the more challenging lesson two problems are given, one on shapes and one on numbers, which need to be ordered according to clues given in words. Pupils have to decide which clues are useful or useless, solve the problem then discuss the nature of language in word problems.

Careful handling of word problems is needed at all levels of ability and all ages. And it seems at times that some students may switch on to mathematics when it is linked to language and used in contexts.

So word problems may not be as problematic after all.

Cognitive Acceleration Associates

How do we deliver EQUALITY in the 21st century?

The wealthiest 1% of the population owns 21% of the nation’s wealth

The bottom 50% own 7%

Recently it has been shown that health inequalities have grown

The government faces controversy over abolition of the 10p tax rate

This year’s budget pledged an extra £1.7BN in the fight against child poverty: whilst a recent report warned that child poverty could double over the next 2 decades

67% of ethnic minority communities live in the 88 most deprived wards

The median gender pay gap has reduced from 17.4% in 1997 to 12.6% in 2007.

Make EQUALITY matter.

Compass e-mail April 2008

Saving the Planet

Here are some figures quoted by Brighton and Hove

Last year the average household recycled...

260
glass bottles and jars

364
plastic bottles

260
drink and food cans

260
newspapers and magazines

How much does your household recycle?

What does your school recycle?

Food Waste Facts

- Food haulage accounts for 25% of HGV miles in the UK
- 6.7 million tonnes of food waste is thrown out by households in the UK every year. Most of this could have been eaten
- The energy used to produce, package, transport and deliver food to our homes in the UK is at least 15 million tonnes of CO_2 every year
- If we could eliminate food waste, this would have the equivalent impact of taking 1 in 5 cars off UK roads

28%

of our city's household waste is recycled

The equivalent weight of 7300 elephants per year

Saving the Planet Plans

It would be useful to all readers if we were able to publish some responses to this exercise in the next issue of *Equals*. Please send us samples of your pupils' work.

The Harry Hewitt Memorial Prize

Sadly this year we have had no entries for this prize. We know that in past years winning it has caused great delight for some pupils who have never before had any chance of winning a prize in school, so **we will extend the deadline to the end of October and urge you to look for a late entrant**. Remember it is awarded to pupils who have previously taken no interest in mathematics but have recently decided to give it a go and that you the teacher must write a recommendation describing how attitude has changed.

Equals Professional Development Day

If you are able to arrange to be out of school for a day and in London, put **Thursday 22nd January 2009** in your diary now. We shall be holding an *Equals* PD Day at SENJIT, London Institute of Education. During the day we shall be considering what mathematics is needed to be a participating citizen.
More details and an application form can be obtained from SENJIT, London Institute of Education, 20 Bedford Row, London WC1H OAL.

Some of the queries and theories put forward in this issue of *Equals* may be discussed:

- Remaining within a comfort zone hinders rather than helps learning.
- There's nothing worse than a bunch of bolshie adolescents'.
- Children take ownership of their education and use functional processes to plan collaboratively.
- The chance to organise individualised programmes for a class of adolescents varying widely in ability. Class teaching was of course out of the question.
- Students should be exposed to the wonders of four-dimensional shapes, the fascination of the primes, the mysteries of topology.
- How would you explain to someone else, at the other end of a phone, how to draw a star?
- Where do you feel you are most in need of help?

APP - Assessing Pupil Progress: a Primary perspective

Jane Gabb introduces us to a wealth of material which is part of the Renewed Primary Framework.

The materials have been piloted by schools all over the country and other schools are now beginning to use them. There are materials for reading, writing and mathematics and in early 2009 there will be materials for speaking and listening.

The materials serve a number of purposes:

- They provide a structure for teacher assessment which gives teachers confidence in their ability to assess children without testing
- Their use promotes a deeper understanding of progression within the subject
- They can be used as part of an Assessment for Learning strategy to inform planning and teaching

This is what some pilot schools have found before and after introducing APP in mathematics:

Practice before APP	Practice after APP
• define progress through tests including commercial tests and optional National Curriculum tests	• assess progress in relation to key Assessment Focuses for mathematics, identified as relevant for a child or group of children • have a clearer idea of children's' strengths and weaknesses and gaps in their experience
• assess through children's written mathematics exercises	• gather evidence in starters and plenaries and as children work in groups • use spoken and written evidence • observe children selecting the mathematics to solve a problem • talk to children to find out more about how they tackle problems as well as seeing if the answer is correct • observe how children use their mathematics in other areas of the curriculum such as design and technology and science
• few Ma1 assessments made	• start to include more problem solving in lessons to teach skills and processes, as well as assess them • become aware that assessing Ma1 may give a different insight into children's' understanding of the content

On the primary framework website there is a series of video extracts from schools and teachers where the materials have been used and this could be a good place to start to find out more about these extensive materials. Local Authorities will be running training on APP throughout the next academic year to support teachers with the use of these materials

The basis of APP is a grid which details the level of mathematics which is characteristic of a particular level for all the strands of the subject. Each grid covers two adjacent levels and enables the teacher to decide which side of the level boundary a child is working at. As areas of achievement for a particular child are highlighted on the grid it then becomes clear the areas of mathematics which need more emphasis and those where there is little evidence of which level a child is working at.

When beginning to use the materials it is suggested that a small group of children is chosen to work with in this detail. Typically this might be a group of six – two higher attaining, two average and two lower attaining children, but a teacher might also choose to focus on children who they are unclear about their levels of achievement and progress. Through using the grids the teacher becomes much more familiar with progression within each strand and as the use develops it becomes easier for the teacher to work out levels without having to go into great detail for each child. In filling out the grids teachers can also identify mathematics which they need to develop for their class.

Senior staff need to support this initiative, and it has not been difficult for head teachers in the pilot schools to see the benefits. Time needs to be set aside in staff meetings for teachers to moderate work and to share their experiences of working with the materials. There is support on the primary framework site in terms of exemplar 'standards files' which can help with standardisation and moderation of teacher assessment.

To find out more visit:
http://www.standards.dfes.gov.uk/primaryframework/assessment/app/

Royal Borough of Windsor and Maidenhead

Pages from the past

Educational basics have not changed

Rachel Gibbons considers the relevance of an influential report of nearly a quarter of a century ago to education opportunities for all in 2008 and beyond.

Peter Mortimore, writing in *the Guardian* (3 June 2008), asked whether the decision to abolish the Inner London Education Authority (Ilea) 20 years ago was intelligent planning or an act of educational vandalism. This set me thinking of what Ilea achieved in meeting the needs of children with extra difficulties during the 20 years and more that I worked in the Authority. **It became obvious during that period that all teachers need the teaching skills for helping pupils overcome difficulties in various mathematics topics. These skills are generic and relevant to all pupils, which is not the case with the relatively simpler teaching skills needed for those pupils who do not find mathematics difficult or scary.**

This in turn took me back to my copy of *Educational Opportunities for All? (1985)*, the report of the Fish Committee set up in 1984 to review provision for special educational needs. (It is interesting to note that the Committee was chaired by a former HMI and Staff Inspector for Special Education – what other LEA was then or is now powerful enough to command such resources?.) I found many marked passages some of which still seem worth quoting almost a quarter of a century later. Here they are with some of my thoughts as I re-read them:

- **The 1981 Education Act defines special needs in relative terms.**

> A child has "special educational needs" if he or she has a learning difficulty which calls for special educational provision to be made for him or her.

It is clear from the beginning of the report that the Committee recognises that "special needs" of an individual come from two different directions – from the individual so assessed but also from the attitudes and expectations of the community in which he finds himself.

- **Children start out unequal.**

Such an important starting point. To my mind, any class with two or more pupils is a mixed achievement group and should be treated as such. I frequently write about the horrors of 'lockstep', that military marching together of a mathematics class through the pages of a text book.

- **The aims of education for children and young people with disabilities and significant difficulties are the same as those for all children and young people.**

Have we today the same aims for those who struggle as for those to whom learning comes more easily?

- **Handicaps arise from the mismatch between the intellectual, physical, emotional and social behaviour and aspirations of the individual and the expectations, appropriate or otherwise, of the community and society at large.**
- **Schools and colleges should strive to provide for more individual needs and to offer equal opportunities to all. Separate provision outside them, however good, should now be seen as an interim solution.**

This is a valuable reminder for today of the importance of accurate assessment of needs, abilities and interests when choosing mathematical activities for each of our pupils.

The belief in the importance of inclusion has continued. This does not mean of course just collecting all children into the same classroom but making sure that, in that classroom, we enable them all to learn at their own levels and in an appropriate way for them, while sharing in the common community life.

- **The important human needs are common to all and of greater significance than the special needs associated with disabilities and difficulties.**

A welcome reminder of what are really 'the basics' – not curriculum subjects but the ability to live and communicate peaceably with other human beings. Perhaps with all the stress on testing we have forgotten common human needs. Has that omission had an effect which has emerged in the present flaws in society? Has it anything to do with the violence in our society?

- **The one common factor associated with special educational needs is the present inability of schools and colleges to meet the wide range of individual needs in their population.**

Is this true in your school today? If so, what support will help you to meet the needs of the whole range of pupils in front of you? Why not write abut it in *Equals*? Sharing problems brings more resources into play.

- **We recommend that all teachers accept responsibility for meeting as far as possible the special educational needs which may arise in their classrooms and be enabled to do so in collaboration with the network of advisory and support services available to them.**

Here there is a clear recognition that responsibility for meeting educational needs should not be borne alone. Teachers must indeed recognise their responsibility but they need to find all the support available and professional development, where appropriate.

- **The concept of handicap implies that such needs arise from situations and climates in homes, schools and colleges and not solely from individual difficulties or limitations.**

More important questions suggest themselves here - just as relevant today as they were in 1985. Are our expectations of some of our pupils inappropriate? What should we expect from them? What mathematics do they need to prepare them to be actively participating citizens?

All these statements seem to me still to have validity for everyone working with special educational needs of any sort today and they persuade me to conclude with Peter Mortimore that the abolition of Ilea was 'a grave error which has ill-served the learners of London and indeed of the whole country'.

Fulham, London

Mathematics and dyslexia

Here is the start of the Inclusion Development Programme, director Mary Daly. Please send an SAE if you want a copy of the rest.

Mathematics requires the acquisition, organisation and access to knowledge, rules, techniques, skills and concepts with the aim of solving problems. It can be seen as a combination of organised knowledge and creative activity. It requires abilities in visual and verbal skills, spatial skills, linear and sequential skills. It requires large amounts of information to be stored and retained ready for swift and automatic access and retrieval from long-term memory. The language of mathematics is full of challenges – offering no support from context and often using words which have a completely different meaning in mathematics than in English.

It may, therefore, place huge demands on the dyslexic student. This table can only provide some limited general information on the teaching of mathematics to dyslexic students. For more detail see Chinn, S. and Ashcroft, R. (1998) *Mathematics for Dyslexics: a teaching handbook*, 2nd edition. London: Whurr Publishers. You may also like to visit the British Dyslexia Association website for information on dyscalculia, dyslexia and maths: www.bdadyslexia.org.uk/dyscalculia.html

Institute of Education, London

Factors to take into account and characteristics of some dyslexic students which will affect learning	**Effect on learning in the mathematics classroom. Students may have difficulty with the following**	**Strategies which may be incorporated into classroom teaching**	**Additional support which may be needed**
Memory • Working memory (the ability to hold and juggle information in short-term memory) – if overloaded, information may be lost • May be inaccurate representations in long-term memory • Manipulating and sequencing information • Difficulty with rote learning.	• Remembering and carrying out instructions • Remembering recently-learned vocabulary • May know the answer but cannot verbalise it • Remembering facts, figures, technical/ specialised vocabulary • Placing things in order.	• Present new information in small chunks • Set limited but realistic targets • Make learning multisensory • Allow plenty of time for recall.	• Provide support materials to be used at home or with teaching assistants • Provide concise revision notes with lots of visual rather than textual information – flow charts, diagrams, concept maps.
Sequencing • Putting things in order –alphabet, letter order in words, word order in sentences, etc. • Answering questions which require events/ information to be put in order.	• Find it difficult to hold information in their heads and reorder it.	• Have charts, lists of vocabulary, diagrams, etc. on display in the classroom • Teach strategies to aid memory and sequencing, e.g. list the information and cross it off as it is used.	• Make use of e-learning tools such as calculators • Encourage the use of memory strategies such as mnemonics.

The Development and Importance of Proficiency in Basic Calculation

A small but essential component of number proficiency is basic calculation, **Richard Cowan** reminds us: the addition of whole numbers with sums less than 20 and corresponding subtractions.

Many children can solve some basic calculation problems when they enter school. Most improve markedly in accuracy and speed during the primary years. Individuals vary considerably. Understanding basic calculation development is important because differences in proficiency are related to differences in more general mathematical attainment (Durand, Hulme, Larkin, & Snowling, 2005), deficits in basic calculation are common in children with number difficulties (Jordan, Hanich, & Kaplan, 2003), and number difficulties at primary school predict difficulties in adulthood (Parsons & Bynner, 2005).

Despite this, ideas about what drives the development of basic calculation and why variation in this small component is strongly related to more general attainment in primary mathematics are not firmly grounded in evidence.

What drives the development of basic calculation?

Four answers are commonly advocated: rote learning of number facts, strategy development (Siegler, 1996), deriving facts from known ones in conjunction with principles (Askew, 1998), and applying number patterns and rules (Baroody, 1999). All except for rote learning emphasise aspects of conceptual knowledge. Children's knowledge of number facts is correlated with their understanding about number but whether this is a causal relation is disputed: some suggest the correlation results from both being influenced by other factors. Even those who assert the relation is causal do not agree about causal direction.

Establishing why and how fact knowledge, strategy development and understanding of principles are related is important to advance understanding of number development and to inform teaching and curriculum design.

Why does basic calculation correlate so strongly with general mathematical attainment?

Basic calculation might correlate with mathematical attainment because proficiency in basic calculation is vital for mathematical progress, because calculation is overrepresented in National Curriculum and standardised mathematics tests, or because the factors that influence proficiency in basic calculation affect general mathematical progress too. These factors include basic number knowledge and skills, processing speed, mathematics instruction, gender, working memory, socio-emotional functioning, reasoning, language, and social class.

Thanks to the ESRC and to the collaboration of Royal Borough of Windsor and Maidenhead we are carrying out a study to address these issues. We start with the children in Year 3 and follow up by seeing them in Year 4.

Institute of Education, London

Askew, M. (1998). *Teaching primary mathematics: A guide for newly qualified and student teachers*. London: Hodder & Stoughton.
Baroody, A. J. (1999). Children's relational knowledge of addition and subtraction. *Cognition and Instruction*, 17(2), 137-175.
Durand, M., Hulme, C., Larkin, R., & Snowling, M. (2005). The cognitive foundations of reading and arithmetic skills in 7-to 10-year-olds. *Journal of Experimental Child Psychology*, 91(2), 113-136.
Jordan, N. C., Hanich, L. B., & Kaplan, D. (2003). A longitudinal study of mathematical competencies in children with specific mathematics difficulties versus children with comorbid mathematics and reading difficulties. *Child Development*, 74, 834-850.
Parsons, S., & Bynner, J. M. (2005). *Does numeracy matter more?* London: NRDC.
Siegler, R. S. (1996). *Emerging minds: The process of change in children's thinking*. Oxford: Oxford University Press.

There was more to basic calculation than meets the eye

Stewart Fowlie describes more number approaches he has used in his own teaching

310 NO COUNTING, NO TABLES
It was only when my own children went to school that I met real children who counted on their fingers, I also remember we were never asked to memorise number bonds or chant tables.
By the age of 5 most children recognise the number showing on a dice not by counting but by the pattern of spots on each face. Here are patterns for the numbers from 1 to 10 which were pinned up round the classroom. The keyboard on a modern telephone is based on the same principle.

```
1  o          2  o o       3  o o o

4  o o o      5  o o o     6  o o o
   o             o o          o o o

7  o o o      8  o o o     9  o o o
   o o o         o o o        o o o
   o             o o          o o o

10 o o o
   o o o
   o o o
   o
```

Looking at, say, 7 it is clearly made up of 3 and 4 or 6 and 1. Then one notices 2 and 5. Subtraction results can also be seen. That 7 is odd is also clear.

(SPLITTING, PUTTING TOGETHER

2 1,1 3 2,1 4 3,1 2,2 5 4,1 3,2

6 5,1 4,2 3,3 7 6,1 5,2 4,3

8 7,1 6,2 5,3, 4,4 9 8,1 7,2 6,3 5,4

10 9,1 8,2 7,3 6,4 5,5

In every case, the first number can be split into the second and third numbers, and either number taken away from the first leaves the last.)

Crossing 10
Undershooting.
You first get up to 10 and then go on.
7 and 5, or 7 and 3 and 2, or 10 and 2, 12.
13 take away 5, or 13 take away 3 take away 2, or 10 take away 2,8.
Overshooting
Go up or down 10 and correct back.
7 and 5 or 17 take away 5,12.
15 take away 8 is 5 and 2,7.
Another way of finding for example 6 + 8 is to think of 6 as 5 + 1 and 8 as 5 + 3; so 6 + 8 is 10 + 4 = 14
Multiplication
Assuming a child knows:
1 and 1 make 2,
2 and 2 make 4,
3 and 3 make 6,
4 and 4 make 8,
5 and 5 make 10,
We call, for example, 5 and 5 two fives, written 2(5).
Notice that 2(5) = 5(2)
2(5)+5=3(5)   5(2)+2=6(2)
2(2)+2(3)=2(2(3)=2(5)
2(2)+3(2)=5(2)
Most children try to memorise a new table as a whole before beginning to use it. It is easier to get each individual result from the previous table. Thus 3(7)=2(7)+7=14+7=21

Edinburgh

Saving the NHS

Unused prescription medicines cost the NHS some £2 million per year here. That could pay for:

- 400 hip replacements, or
- 280 heart by-pass operations, or
- 2,800 cataract operations, or
- 60 community nurses, or
- 360 knee replacements.

Brighton & Hove Newsletter, April 2008

Hadrian's Wall

Hadrian's Wall originally stretched across the 73 mile neck of England. The middle section of 20 miles remains the best preserved. It took three Roman legions – in total around 15,000 men - about six years (between AD121-AD128) to quarry the stone and build a structure on average 4.5m high (add 1.8m more for the parapet)

Brighton & Hove Newsletter, April 2008

Take another look at nrich

Jane Gabb considers it worth our while to look in more detail at the nrich materials

If you thought that the nrich website (http://nrich.maths.org/public/) only contained puzzles for the more able mathematician, then it's time to take another look. Increasingly there is something for everyone, and some very helpful features are emerging.

The people who run nrich are constructing curriculum mappings for Key stages 1-4, one for content and one showing how the activities support process skills. These can be accessed from the home page by clicking on '**Curriculum mapping**' on the left hand side. You can register to keep up to date with these developing documents.

I have taken as my example part of the KS3 content mapping for Integers, powers and roots for Years 7 and 8, partly because of the need for KS3 teachers to begin teaching the new National Curriculum from September. However all the information applies to the other key stage mappings which go from Foundation to 'A' level.

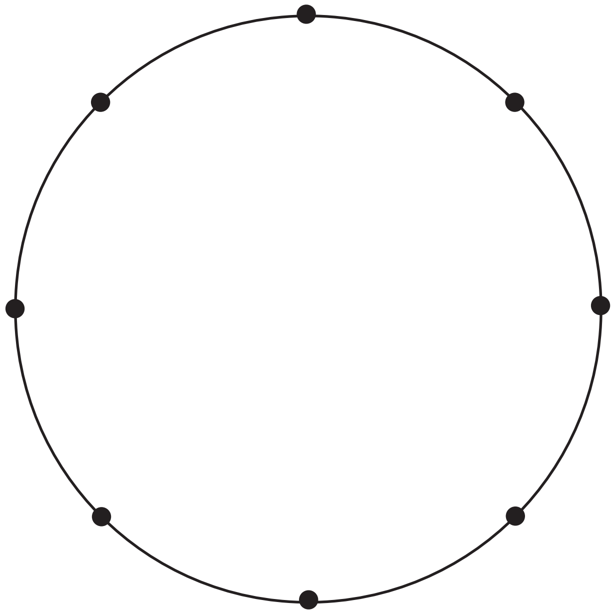
The activities appear in a table with the appropriate objective and are hyperlinked so that clicking on the activity automatically takes you to that activity on the website.

Year 7	**Year 8**
Integers powers and Roots	
Understand negative numbers as positions on a number line; order, add and subtract positive and negative integers in context.	Add, subtract, multiply and divide integers. **NRICH: Connect Three** **NRICH: Playing Connect Three** ✓ **NRICH: Weights** **NRICH: Consecutive Negative Numbers** ✓ **NRICH Article:** **Adding & Subtracting Negative Numbers**
Recognise and use multiples, factors (divisors), common factor, highest common factor and lowest common multiple in simple cases, and primes (less than 100); use simple tests of divisibility. **NRICH: Stars** ✓ **NRICH: 14 Divisors** ✓ **NRICH: Dozens** ✓ **NRICH: Factors and Multiples Game** ✓ **NRICH: Factors and Multiples Puzzle** ✓ **NRICH Article: Divisibility Tests**	Recognise and use multiples, factors (divisors), common factor, highest common factor, lowest common multiple and primes; find the prime factor decomposition of a number (e.g. 8000 = 26 x 53). **NRICH: American Billions** **NRICH: What an Odd Fact(or)** **NRICH: Product Sudoku** **NRICH: Product Sudoku 2** **NRICH: Integrated Product Sudoku**
Recognise the first few triangular numbers, squares of numbers to at least 12 x 12 and the corresponding roots. **NRICH: Sequences and Series***	Use squares, positive and negative square roots, cubes and cube roots, and index notation for small positive integer powers. **NRICH: Sissa's Reward**

The problems can be printed out using the '**Printable page**' option. Those activities which are ticked include **Notes** for the teacher on ways of introducing and using this activity in class. They include suggestions for extension and support.

Here is an example: 'Stars' (from Year 7 Integers, powers and roots). This can either be worked on interactively on the site or on paper – downloads offer circles with from 3 to 24 dots (how's that for differentiation?)

Stars – the problem



When a circle has 8 dots you can move around the circle in steps of length 1, 2, 3, 4, 5, 6 or 7.

If you move around the circle in steps of 2, you miss some points

If you move around the circle in steps of 3, you hit all points.
How else can you hit all points?

When a circle has 9 dots you can hit all points in 6 different ways.
What step sizes allow you to do this?

Now consider 10 points. Can you find the 4 different ways in which we can hit them all?

Explore what happens with different numbers of points and different step sizes and comment on your findings. How can you work out what step sizes will hit all the points for any given number of points?

Now consider 5 points. You can hit all points irrespective of the step size. What other numbers have this property?

Can you find a relationship between the number of dots on the circle and the number of steps that will ensure that all points are hit?

If you would rather work on paper, click below to open PDF files of the circles.

Each PDF file contains 12 identical circles with a specific number of dots. You can select any number of dots from 3 to 24:

…………………………………………………

Teachers' notes

Why do this problem?
This problem encourages students to see the mathematics underpinning a situation. In this case it is the importance of factors and multiples in what is at first glance a geometrical setting.

To avoid spoiling the surprise, it may be worth doing this activity at the start of work on the topic, without telling the students what the topic is...
Possible approach
You will find sheets of different dot-circles for printing out at the bottom of the problem page.

Ask students to draw a five pointed star starting and ending at one of the "points" or vertices of the star. They must do this without taking their pencil off the paper and without drawing over a line they have already drawn. Many learners will have met this before.
Ask the group to discuss in pairs a description of what they did that they can share with the rest of the group. "How would you explain to someone else, at the other end of a phone, how to draw the star?"

Look out for ideas such as step size and ways to describe positions.

When ready, demonstrate a five pointed star with the interactivity and discuss the notation that has been used (going anticlockwise, stepping by two leaves two gaps between the points on the circle). Alternatively, you might get a group of students to stand in a circle and make the stars with string (by passing a ball of string).

Discuss points of interest including:

- What happens if you move clockwise.
- What constitutes a star (in these notes a polygon created from a step size of 1 is not a star).
- There is only one star on a five-dot circle.
- Complementary step sizes produce the same star (step size two is equivalent to step size three in this five-dot context)

Ask students to make as many stars as possible on a seven-dot circle:

- How many stars can they make?
- How do they know they have them all?

Challenge them to conjecture how many stars will be possible on a nine-dot circle (without drawing them at this stage). Discuss in pairs before sharing what they can offer as a convincing argument.

Students can now focus on generalising their results for any dot-circle.

Key questions

- What are the things that affect the number of stars you can draw?
- Can you find one rule to determine the number of stars or do you need different rules for different circumstances?
- Can you write a rule, or set of rules, that someone who had never seen the problem, could understand?

Possible extension

How many times would the string pass around the circle for different stars in different dot-circles?
Can you find the angles at the vertices of any star?

Possible support

If working with a small group - ask each person to create a star based on a different step size and compare the group's results, encouraging the students to identify what is the same and what is different about their stars and putting them into an order they can justify.
Encourage individuals to draw a star and, without showing it to their partner, give instructions to draw the same star. Are the two stars the same?
The activity could be carried out using string on peg boards.

…………………………………………………

I think you will all agree that this makes nrich a superb resource for teachers. Populating those new Schemes of Work with rich mathematical tasks has become a lot less daunting and makes the need for buying a new textbook even less necessary. It is also worth looking at the process mappings which show which activities can be used to teach each of the key processes in the new Programme of Study.

Enjoy your search on this regularly updated site.

Royal Borough of Windsor and Maidenhead

High Hopes

Dora Bowes shares with us the excitement of her discovery of Billy Hopkins.

Due to my ageing eyesight and limited time, I bought a MP3 player and started to enjoy audio books whist travelling to and fro from work. High Hopes by Billy Hopkins was one of those on a recommended list and I soon wondered how on earth I had overlooked this book previously. I laughed, cried and reminisced so much that I had to ‘read’ it twice and now have bought the whole series in paper back to enjoy on holiday.

After an exciting two years of teacher training in London, Billy takes up his first teaching post in Manchester in 1947 where he is given Senior 4.

> ‘These kids I’m trying to teach are angry, not with me but with Rab Butler and his Education Act; with the government for raising the leaving age and stopping them from going out to work. But they’re taking out their anger on me and I’m having a bumpy ride. I really want to teach them but they won’t let me. There’s nothing worse than a bunch of bolshie adolescents.’

Billy burns the midnight oil to devise a new program to engage the students in their local area but with the important difference that the lessons are designed to meet their individual needs.

> ‘The class fell into a happy routine. Mornings were devoted to hard graft – the basic curriculum of the three Rs, science, art, and social studies. The afternoons were given over to broadening their horizons through a series of outside visits and invited speakers, plus their weekly ballroom dancing lessons.’

In my role as a consultant, I have the pleasure of presenting the new curriculum that has an emphasis on process skills and functional mathematics. Yet lo and behold that is exactly what Billy was doing in his early days as a teacher in Manchester!

His book depicts children taking ownership of their education and using functional processes to plan collaboratively, organise and run events such as their cycle ride to Tatton Park. This gave Senior 4 the curriculum opportunities to develop their confidence to apply their learning in unfamiliar contexts.

children taking ownership of their education and using functional processes to plan collaboratively

And to cap it all, this was achieved with the Head teacher working at his extended office set up at the back of Billy’s classroom!

Wilfred (Billy) adds:

> ‘In those far-off days I was faced with classes (in those far-off days, we had a practice called General Class Teaching, which meant I was responsible for teaching "General Subjects" except for Art, Music and Science. This practice gave one the chance to organise individualised programmes for a class of adolescents varying widely in ability. Class teaching was of course out of the question.

I was first attracted to the Dalton Plan (see my attached summaries) but found a variation of this, the Winnetka Plan was better suited to our needs. It meant a great deal of work in preparation of individual programmes and marking of progress. But it worked.

Those were enlightened days (and I had an enlightened headmaster!) and in my particular school I was given the chance to use my own initiative in the rest of the curriculum. As well as the individualised programmes of work, we were able to take the pupils on hikes in the Peak District, visit to places of historical interest, build model towns, and unbelievably – do ballroom dancing!

You may find a few other items of education interest scattered throughout my books. For example:

Kate's Story: Extreme examples of traditional instruction. My mother did her calculations on her chin for the rest of her life up to the age of 90!

Our Kid - examples of very traditional teaching.

High Hopes: Individualised programmes

Going Places: Teaching of English in Manchester: novel writing.

Anything Goes: Teacher training.’

I thank Wilfred for his kind permission to use the extracts from *High Hopes* and his commentary.

Winnetka Plan: Carleton W. Washington and Helen Parkhurst

Washington and Parkhurst were associates of Burk's and, while Washington was superintendent of the Winnetka, Illinois public schools, they created the Winnetka Plan.

The Winnetka Plan included:

- self-paced, self-instructional, self-corrective workbooks
- diagnostic placement tests in which learners were tested to determine which goals and tasks they should tackle
- self-tests that students could take themselves to determine if they were ready for testing by the teacher
- a simple record-keeping system which tracked the progress of each student

Only after performing satisfactorily on the teacher-administered test could a student go on to new material.

The two main tasks for faculty were:

- to analyse course content into specific objectives

- to develop the plan of instruction to allow each learner to master the objectives at their own rate

Group activities were not overlooked: approximately half of each morning and afternoon were devoted to activities such as music, plays, student government, and open forums for discussion. Under the Winnetka plan, "classrooms became laboratories or conference rooms, and teachers became consultants or guides."

The Dalton Plan: Helen Parkhurst

The Dalton Plan was originally developed by Parkhurst to use in an ungraded school for crippled children.

The Dalton Plan included "Contract learning." Having agreed to a contract, students were free to complete them at their own pace. No new contracts were permitted until the current one was satisfactorily completed.

Helen Parkhurst, after experimentation in her own one-room school with Maria Montessori, developed what she termed the Laboratory Plan. It called for teachers and students to work together toward individualised goals. The Laboratory Plan was put into effect as an experiment in the high school of Dalton, Massachusetts, in 1916. From this beginning, the Laboratory Plan and The Dalton School eventually took their names and their mission.

To read more about the current Dalton School, associated with Columbia University: http://www.nltl.columbia.edu/about_Dalton/Admissions/admissions.html

Review by Rachel Gibbons

***Nadia Naggar-Smith, Teaching Foundation Mathematics: a guide for teachers of older students with learning difficulties*, Routledge**

This book contains a very large number of worksheets all ready for the teacher to copy, duplicate and use in the classroom. They give practice in elementary uses of number (85 pages), measure (76 pages) and shape (28 pages). The activities are designed for those whose grasp of these concepts is pretty tenuous. Indeed the book starts with activities devised to develop an understanding and use of the numbers one to nine only. Every 'lesson' (there are ten in each section) has a reference to the adult pre-entry curriculum framework for numeracy and to the P scales

In the 'Introduction to Number' Naggar-Smith reminds us of the Cockcroft Report's definition of basic numeracy as:

> an ability to use mathematical skills which enables an individual to cope with the practical mathematical demands of his or her everyday life.

It is therefore disappointing to find that many of the exercises do not have this practical slant. There does not seem to have been enough thought about what the practical demands of everyday life – or the concerns - might be of the students for whom these activities are designed. There is one lesson, 'introduction to bar charts' concerning gathering, sorting, recording information and using and interpreting it through the construction and use of bar charts. Would it not have been more relevant to present pie charts as these are much more frequently used when data is presented in the news media?

Today Bingo is a part of many people's everyday life – people pay good money to play it. The practical mathematical demand of Bingo is surely that players can recognise the words and symbols used for a range of numbers, yet, although there are references to Bingo, it is not suggested that a simplified version of the game using only the numbers 1 to 9 is actually played. Football is an even more popular activity - all be it watching rather than playing for most – and again the sport is used as the basis for several exercises but, once more, the topic could have been exploited in a much more practical and enjoyable way. Analysing tables of results gives plenty of scope for addition and subtraction with a purpose beyond that of just practising skills – how many goals were scored altogether in a match? How many more goals did the winning team score than the loser? And so on.

The book gives a wealth of practice in the basic skills of number and measure, with a nod to shape, but presented in too abstract a manner for my satisfaction.