Equals

Realising potential in mathematics for all

for ages 3 to 18+

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MATHEMATICAL ASSOCIATION

supporting mathematics in education

There were ten in a bed...

Equals

Realising potential in mathematics for all

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Editors' page

What does 'being numerate' mean?

Only last week a head of a secondary mathematics was quoted as saying that in traditional maths lessons

> "you teach pupils how to do something, do five examples and get them to do the next 20".

Is your aim for pupils who experience the greatest difficulty with mathematics so to teach them rote-learned processes? For *Equals* being numerate means, rather, being able to make sense of applied number, able to interpret the numerical data that makes up a great part of 'the news'. Our priority is that all children are given the tools with which they can become fully participating citizens and this means being well informed. It means being able to make sense of information given numerically. Newspapers are full of statistics. So are books about the state of the planet. Anyone interested in the possible problems of impending climate change cannot begin to understand what may happen, or how we might react to lessen the impending dangers, without some comprehension of the numerical information involved.

Julia Hales tells us:

> Since I was born in 1961 there's been a 10-fold increase in car traffic and a 20-fold increase in air traffic. Yet all this travelling comes at a cost: about a quarter of CO2 emissions come from transport and it's predicted this could increase to a third by 2010. …
>
> Did you know that up to 80 percent of cars travelling to work in the rush hour have only one person in them? Just sharing a lift with one other person would halve the number of cars on the road.[1]

To appreciate, decide whether to accept and how to act on this information demands an understanding of many number concepts, especially a sense of proportion. It follows that, to the *Equals* team, teaching mathematics means making it come alive and revealing its power and its excitement to all.

This editorial is being written on the *Harry Potter* launch-day and the Guardian's 'piece from the past' today is from July 21 1930, a review of Arthur Ransome's *Swallows and Amazons*. Reading it, we all rejoice in the power of the story, a story that, like most good tales, has reality mixed with make believe, so that, for example, the lemonade bought over the counter becomes 'vitriolic grog'. So, as the review points out, many real day-to-day events in the lives of the Swallows and the Amazons connect directly with the lives of young readers while, at the same time feeding their imaginations. This, surely, is what education - right across the curriculum – is all about? Not tests and league tables but the love of reading, of learning, and of filling the imagination with numbers, shapes, ideas of all sorts which will make more sense of, give more hope of managing and make more inviting, the bewildering world in which the children find themselves.

I remember many years ago hearing of a small boy in his first year at school who had just learnt to read and had told his mother with great satisfaction that, now he knew how to read, he had no need to open a book again. Who cares what methods had brought him to this state: phonics or whatever other pedagogical fashion you like to mention? The right effect - a love of books and what they have to offer - was missing. Education in mathematics, as in every other area of the curriculum, should be a journey of exploration rather than a race in which someone has set up a series of man-made obstacles in the shape of hurdles (otherwise known as SATs).

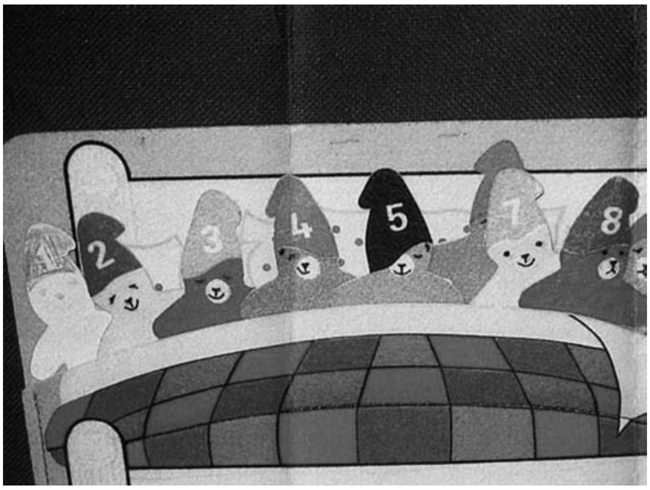
There are few who think mathematics is worth studying for its own sake but all believe that you need it because it is useful. And, as we have already agreed, it is. Therefore let us ensure that all children have enough feeling for the most basic bricks of mathematics – numbers – to fulfil their rights and responsibilities as citizens. To do this, as we have already hinted, their feeling for number must be deep enough for them to understand the essence of the information given, make judgements about its authenticity and act accordingly. More of this aspect of citizenship will be found in the following pages, particularly in "What do the numbers tell us?" (page 14)

1. Julia Hailes, *The New ~Green Consumer Guide*, Simon and Schuster, 2007

There were ten in a bed …

Rachel Gibbons is usually doubtful about whole class lessons but…

Whole class lesson are not to my mind usually a good idea, especially with these children who have all been diagnosed as having disorders in the autistic spectrum, but what an all-involving, fast-paced lesson it is with such imaginative resources to back up the counting songs in which all take part with enthusiasm, Nearly all the resources are hand-made by the teacher, Emily Garner, and her assistants, most with this particular activity in mind. For there is action as well as singing: sight, sound, finger movements, hand movements to imitate the actions mentioned in the songs and the handling of the resources are all involved.



For the old favourite

There were ten in a bed
And the little one said
Roll over …

As you see, there are ten little cut-out teddy bears, all moveable and all fitting under the sheet of the pictured bed. Each has a number displayed and each is made to 'fall out' by a chosen child at the appropriate moment.

Of course the class is very small, only seven pupils with an age range of 4 to 5 years and there are three extra adults to support the teacher, for these are children who need very close monitoring if they are to make progress in the world of normal communication around them. They need more props for learning than other children – which does not of course rule out the possibility that these props would also enhance the learning of any group of children at an appropriate stage.

And the props used with *Ten in a bed* and the other songs abound. There are several fans of cards showing the numerals for the numbers one to ten in which individuals are encouraged to find the number of children still left in bed. The teacher displays the appropriate page of a small spiral-bound book showing numeral and word as each person falls out to indicate the number still left in bed. And at each stage there is plenty of finger counting to reinforce how many people are left.

Three other counting rhymes are used:

1, 2, 3, 4, 5, Once I caught a fish alive;
5 naughty monkeys;
10 fat sausages sizzling in the pan;

with plenty of fish, monkeys and sausages for the activities to be mimed appropriately.

One could question whether it would be better to have a an upper limit of 9 at this stage so that the complications of place value do not intrude and there is no reason why one should not start with only 9 bears in the bed or 9 sausages in the pan. On the other hand, maybe with all the excitement of the general activity, 10 is such an unobtrusive item that it will not cause questions at this stage (it certainly did not in this session) but it is there in the background to be explained later.

The earliest possible introduction to the real meanings of the symbols 1, 2, 3, … can be argued because of the many examples of what we might call their *misuse* in labelling in the world around us. Every vehicle the child sees has an identification made up of a mix of letters of the alphabet and these other nine symbols. In addition, the number 11 bus for instance, is not the 11^{th} in any series of buses. Perhaps it is travelling along the 11^{th} route that was laid out in a particular locality, who knows? It is of no significance. The meaning for which these symbols were invented should be demonstrated as soon as possible.

Queensmill School, Fulham, London

What does inclusive teaching really mean?

Teachers compare two well-tried mixed-achievement approaches in the classroom, SMILE and CAME. **Mundher Adhami** summarises their views and offers some of his own.[1]

Schools can still claim to be *inclusive* while streaming, setting, or otherwise formally labelling pupils. They are differentiating and catering for pupils' different abilities or, more truly, levels of achievement, and allocating equitable resources. This is a formal institutional method of fulfilling the principle of equal worth.

But there is a more genuine sense of inclusiveness that involves pupils across the achievement range working together in the classroom and benefiting from being different, and even recognising that they are complementary to each other.

Arguments about achievement labelling

First let's rehearse some arguments about achievement setting. These are to do with differentiation by attainment in a subject like mathematics or by achievement in general. Let's not concern ourselves with motivation or behaviour although these aspects inevitably enter into setting or streaming decisions.

Many teachers agree that setting or streaming may be valid, or even necessary at the extreme ends of the achievement range. Children, and learners in general, would not benefit from classroom talk or interactions that are either trivial for them or way above their levels of understanding.

In current curriculum terms and NC levels, many teachers think a group of pupils can communicate fruitfully amongst themselves, and benefit from each others' ideas and sorting out of errors, if the range of achievement in the group is 3 national curriculum levels or less, e.g. between NC 2 - 4 or NC 5-8. That is equivalent to a range of 6 years in 'mental age' in the subject, measured by hypothetical average progress of 1 level in 2 years. There is little use in grouping pupils who are working at level 2 with those working at level 8, unless the latter act as teachers, which requires rare pedagogic skills. Therefore the working groups must be near-ability. Hence the formula: 'near-ability' groups in mixed-ability setting. However, there is also the issue that unless there is a range of levels in any group, pupils at the same level and with the same background may reinforce their misconceptions by missing out on sorting them out! So the formula may better be phrased as 'maximum manageable achievement range groups in mixed achievement/ability setting'.

That implies that the full mixed achievement setting is not manageable. I myself have sympathy with this view in terms of teaching mathematics, if the full range includes 4 or 5 national curriculum levels. This is rare in a mainstream school in this country today although this may change as more children with SEN are admitted to mainstream secondary schools.

Working above or below their achievement levels is the main cause of many mathematical misconceptions

The top and bottom 10 percentiles of ability by national standards across the cognitive capabilities are rare in mainstream schools, especially at secondary level. That is because of the many mechanisms which cater for the geniuses/very 'gifted' children and for those judged to have extreme special needs. That makes any narrow ability ranking of pupils in the mainstream school largely artificial since at most it would include the middle 80% of the population where the abilities are diffuse and less susceptible to measurement. The most you can say is that this pupil is 'about average or higher' for his age, or that he is 'about average or lower' and then allow for a possibility of occasional mislabelling.

In my experience with schools since the mid 1970s, including through assessments in the SMILE, GAIM, and CAME projects, a school would most likely have a profile from the 10th to the 70th percentile, or the 30th to the 90th percentile with few, if any, outside this range.

shared experiences of mathematics

Narrow setting by attainment or ability, and labelling by levels, therefore, hides much of the commonality amongst these pupils. That is the case even in cognitive terms alone, quite apart from the equally valid social and emotional terms, with their all-important motivational charge. For example a 'level 4 pupil' may well have reached level 5 in spatial abilities and level 3 in number ability or in algebra, or vice versa, and therefore will work differently in different mathematical topics. Unless classroom work clearly allows a range of level 3 to 5 work and beyond in each topic at the same time, most pupils will either be working below or above their achievement level. That is the main cause of many mathematical misconceptions, temporarily covered up by memorised procedures. It is also the reason for boredom, switching off and even hatred of mathematics, something that does not occur so strongly in any other subject.

These arguments lead to a preference for much of mathematics teaching to be in manageable mixed-achievement groups, at least to age 14 or thereabouts. That is not happening at present. The reasons are partly to do with the focus on teaching for tests and exams, and therefore on procedural and rote learning, which is necessarily level-focused. But the other, and less-talked-about reason, is that teaching generally is a difficult task, and especially where it involves engaging with mixed achievement groups. However, my generation of mathematics teachers did experiment successfully with such methods. And even now there are some who teach in such ways, although, according to HMI reports, rather few.

pupils at the same level and with the same background may reinforce their misconceptions by missing out on sorting them out

Teachers' views on approaches to mixed-ability teaching

Rachel Gibbons and I explored the issue with small groups of colleagues at the Easter conference of the Mathematical Association this year, 2007. First a personalised approach was offered when colleagues were asked to choose one mathematical task from three open tasks from SMILE[2] to work on, individually or in pairs, for about 20 minutes, with the teacher observing the work closely, ready to intervene where it seemed necessary. They then discussed together what they had found useful or problematic. Then the whole group worked briskly on a CAME[3] lesson for another 20 minutes. (Some details of the tasks are given elsewhere). Then followed the main reflection session.

Individualised	Whole-class
Pros	
Cons	

The reflection was based on a simple frame to highlight the advantages and disadvantages of the individualised open-ended work and the whole-class thinking lessons. The two approaches were exemplified in the brief shared experiences the colleagues had earlier in the session, which served to prompt more general observations and ideas.

First colleagues individually jotted words and phrases in each of the four cells for a couple of minutes, then worked in pairs to produce some better phrasing and priorities, then we collected all the ideas on a flipchart, briefly discussing their meanings and connections.

Here is a full list of the ideas as transcribed from the flip chart, with largely verbatim wording agreed by individual colleagues even if not agreed as a group. Each bullet point originated from one or a pair of colleagues.

	Individualised or paired work on open tasks in a class, e.g. SMILE/GAIM/ATM[4]	Whole class in the CAME style
Advantages	• Everyone is involved in their own way. • Less competitive with others in the class. • More open than in normal lessons. • Children who normally sit back will have a go. • Broader range of choice. Children select their own tasks. They target themselves. • More competitive in a good way. • Takes away fear. • Could be done in groups rather than by individuals if they chose. • Personal satisfaction. Sense of achievement. • Better differentiation by ability and pace of work.	• Easy to link with prior knowledge. • Can be linked to real life contexts. • Access by different learning styles • The teacher values all contributions from different pupils. • Links to the rest of the curriculum are possible where needed. • Use of language and symbols explained. • Sharing and discussion of ideas offered in the class. • Shared experiences of mathematics.
Disadvantages	• Doubt and loss of confidence when they cannot do it. • As a teacher you cannot give them help as much as needed. • No sharing of ideas. You are on your own. • Difficult for the teacher to manage. • As a pupil you are stuck at your own level. • Misses out on mathematical vocabulary. • Needs time. Needs patience. The children need to be confident to do it. • One can get lost. Less conversation between students, less exchanges of ideas and practice in articulating thought.	• Some pupils may sit back and not engage. • The lesson goes at the teacher's pace. • The work could be faster or slower than what some people want. • Less satisfaction than when you work on your own. • Differentiation between pupils is difficult. • There is the illusion of covering the topics. • Some children may dominate class discussion.

Do we need to go further than recognising the pros and cons of teaching and learning situations? Isn't it sufficient to accept that almost everything we do has good and bad in it, and the main decision is on the least harmful or most fruitful in the particular case? Can we be sure of anything in teaching and learning? Aren't we operating always on assumptions and hunches that 'something is happening' in the minds and souls of the pupils, but we cannot be sure? My own answer is yes to all these.

But there are *qualifications*, and the awarding of these has to do with acceptance of differences' amongst pupils and teachers, and how catering for these differences has to be part of the entitlement of pupils for their education and the development of their individual potential.

Potential for synthesis

The Cockcroft report recommendations of a quarter century ago (see elsewhere) remain valid. In current terms these recommendations can be formulated as the need for a *balanced diet of mathematical experiences* by all pupils to include:

1. Direct teaching of new material building on prior knowledge that is assumed to have been understood, with exposition and demonstration as in lectures, preferably with opportunities for interactions.
2. Individual or group open investigation and problem solving activities, worked on at pupils' own levels, with the teacher circulating, prodding and offering hints.

3. Whole class *thinking maths* activities. Activities designed to allow collective exploration and discussion of concepts from scratch up to the most challenging logical and mathematical thinking possible in the class. These may start with a motivational 'story' or a 'hook'[5] but then proceed in steps at a responsive pace. The pace should be appropriate to the majority of the pupils in the given class at the start but increasingly moving to challenging even the most able pupils by the end of the lesson, relying on various levels of partial understanding by the rest, and on keeping them thinking after the lesson.
4. *Drill & practice* lessons where the pupils hone techniques and familiarise themselves with efficient methods of solving routine or novel mathematical questions.

In each of the four types of mathematics lesson above a few features could be present in various proportions:

a. Use of *real-life familiar contexts* where possible.
b. Use of *natural language* and how that is linked to mathematical language; encouragement of oral interactions using mathematics in the classroom.
c. Use of *practical apparatus*, audio visuals and technology, without allowing these to create obstacles or diversions.

Professional development as key

It is clear to many of us that the main reason for repeated failure of education reforms, especially with investigations and problem solving in mind, is that the teachers themselves have had little or no experience of this type of learning, and so find it extremely difficult to pursue. We do what has been done to us! And the problem is plainly not the teachers' subject knowledge, narrowly defined as understanding of mathematics as a coherent system with connections. Rather, it is the knowledge of how pupils construct their own mathematical knowledge, and the progression in the subject with its diverse routes. For that kind of subject knowledge to develop in teachers, whether they have honours degrees in mathematics, or are good teachers who have been forced into teaching a subject they dislike, there is no substitute for themselves first engaging in open ended investigation and thinking maths lessons, and reflecting on these experiences with colleagues.

The successes of SMILE and GAIM and many similar experiments in the 70s and 80s were based on teachers coming together and collaboratively creating lessons, trialling them in the classroom and coming back to discuss them. The ILEA of the time, the government, and charitable bodies like Nuffield greatly aided those experiments. But the spirit of experimentation seems to have declined with time in favour of officially promoted, narrow, direct teaching. Educational researchers do not seem to go for large scale experiments.

The CAME approach still adheres to the original principles of the SMILE and GAIM experiments, linking them to the cognitive development theories as a background, but now the schools themselves have to pay for the courses. That slows down the dissemination of good practice until the pendulum swings further, and we enter a new era of experimentation and creativity in teaching and learning.

Back to Cockcroft, I say!

Cognitive Acceleration Associates

1. Based on the proceedings of workshops at the Mathematical Association annual conference at Keele University 11-14 April 2007, run by Rachel Gibbons and Mundher Adhami.

2. SMILE- 'Secondary Mathematics Independent Learning Experience', an ILEA-funded project that started in the 1970s in London. Rachel Gibbons was one of the original teachers and then an inspector involved with the project.

3. Cognitive Acceleration in Mathematics Education, a project developed at Kings College London, based on exemplar Thinking Maths lessons for Secondary and primary classes.

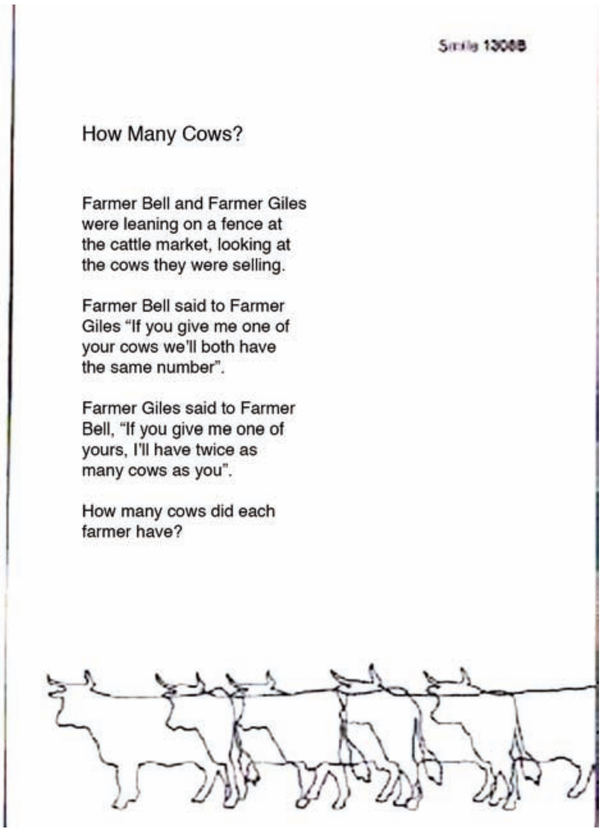
4. GAIM- Graded Assessment in Mathematics, a 1980s project based at Kings College London directed by Margaret Brown, which paved the way to descriptions of the GCSE grades and NC levels. Its GCSE syllabus, serviced by the author, was based on teacher assessment of open-ended work and flexible ways of fulfilment of topic criteria. ATM- Association of Teachers of Mathematics was prominent in promoting open-ended investigations, and Anne Watson serviced a GCSE syllabus based on such activities.

5. Alan Edmiston, a CAME tutor from Sunderland , promoted the idea in the ongoing development of the approach that most or all classes need a starting story on which the thinking lesson 'hangs'. Another CAME tutor, Mark Dawes, from Cambridge, developed the idea further as a need for a 'hook' that can be a story or a puzzle or a dramatic event that galvanises the whole class at the start of the lesson. The notion emphases the motivational factor, as distinct from logical or mathematical terms, and therefore can be seen as a necessity rather than a luxury. In the example given the hook is examining fresh twigs of parsley and mint, offering the idea of mathematising nature.

Resources used in the 13 April 2007 Mathematical Association session on Creating Inclusive Mathematics Classroom, by Rachel Gibbons and Mundher Adhami

SMILE investigations

Copies of the five investigations, all with potential for linear symbolisation, were scattered on the tables for participants to choose and work on, individually or in pairs. The 'teacher' went around to interact with the individual work.



Smile 1300B

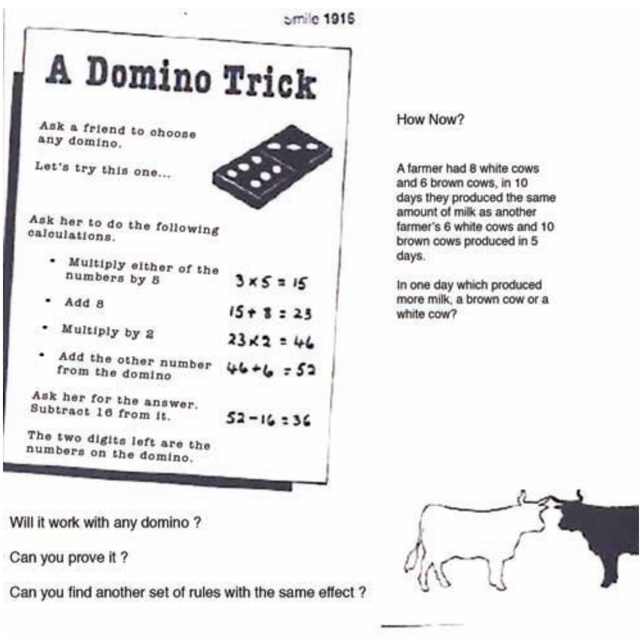
How Many Cows?

Farmer Bell and Farmer Giles were leaning on a fence at the cattle market, looking at the cows they were selling.

Farmer Bell said to Farmer Giles "If you give me one of your cows we'll both have the same number".

Farmer Giles said to Farmer Bell, "If you give me one of yours, I'll have twice as many cows as you".

How many cows did each farmer have?



smile 1916

A Domino Trick

Ask a friend to choose any domino.

Let's try this one...

Ask her to do the following calculations.

- Multiply either of the numbers by 5 — 3×5 = 15
- Add 8 — 15+8 = 23
- Multiply by 2 — 23×2 = 46
- Add the other number from the domino — 46+6 = 52

Ask her for the answer. Subtract 16 from it. — 52−16 = 36

The two digits left are the numbers on the domino.

Will it work with any domino ?

Can you prove it ?

Can you find another set of rules with the same effect ?

How Now?

A farmer had 8 white cows and 6 brown cows, in 10 days they produced the same amount of milk as another farmer's 6 white cows and 10 brown cows produced in 5 days.

In one day which produced more milk, a brown cow or a white cow?

How many cows?

Farmer Bell and farmer Giles were leaning on a fence at the cattle market, looking at the cows they were selling.
Farmer Bell said to farmer Giles, "If you give me one of your cows we'll both have the same number."
Farmer Giles said to farmer Bell, "If you give me one of yours, I'll have twice as many cows as you."
How many cows did each farmer have?

How Now?

A farmer has 8 white cows and 6 brown cows. In 10 days they produced the same amount of milk as another farmer's 6 white cows and 10 brown cows produced in 8 days. In one day which produced more milk, a brown cow or a white cow?

Fishy

A fish had a head 9 cm long. The tail was as long as the head and half the body. The body was as long as the head and tail. How long was the fish?

Reverse digits

Find the four digit number (other than 0000) which when multiplied by 9, gives the original number back to front.

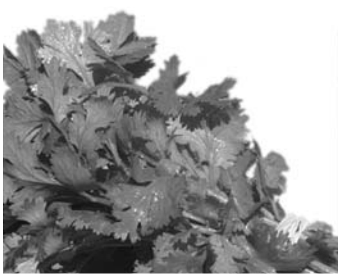
A domino trick

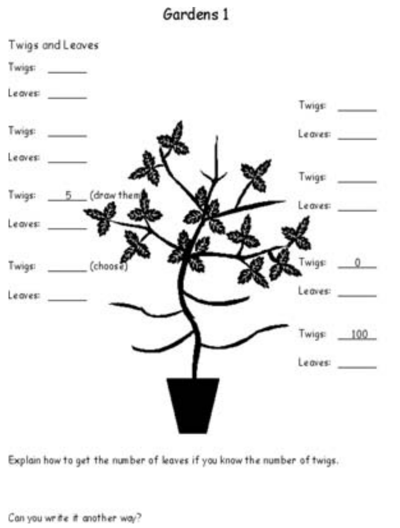
Ask a friend to choose any domino
Let's try this (6 and 3)
Ask her to do the following calculations:

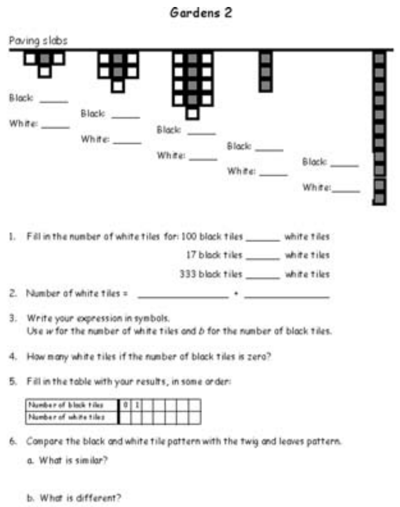
- Multiply either of the numbers by 5
- Add 8
- Multiply by 2
- Add the other number from the domino
- Ask her for the answer, subtract 16 from it. The two digits left are the numbers on the domino. Will it work with any domino? Can you prove it? Can you find another set of rules with the same effect?

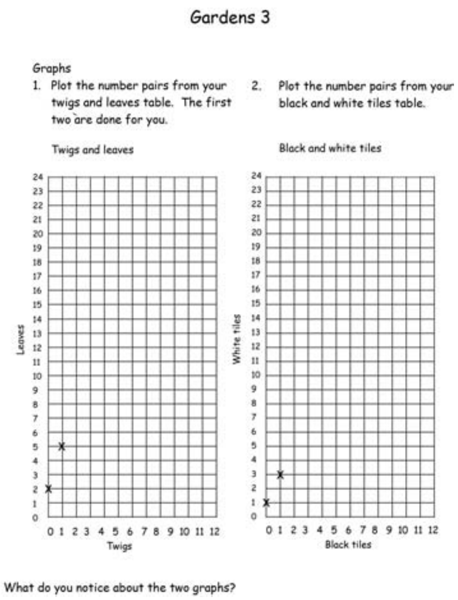
CAME whole-class lesson on linear relationships

This CAME activity for the middle years (Y5-8) starts with participants handling fresh twigs of flat-leaf parsley and mint, and trying to find a pattern in the arrangement of leaves in each.









After sharing their recognition of the patterns in these in their own words, (e.g. a simple pattern of 3s and pairs of leaves at right angles) the participants are given the first of the sheets shown here on articulating and possibly symbolising a two step relationship. The three notesheets are only intended as a structure for whole-class work, and are not be collected or marked.[1]

The group is called together every few minutes to share their responses and reasons for answers. Different wordings and formulations are scripted on the flipcharts, including misconceptions that are then looked at as something that could be 'for good reason' but not appropriate.

In the first episode the key question is on how to find the total number of leaves on a branch if you know the number of twigs. This can be given in different word formulations, then with symbols if possible, including mistakes that are explored. In the second episode a different setting is used for a similar linear pattern and the key question is "what is similar and what is different in the two settings?"

The pace and focus shifts according to the interest of the particular group, but in a rising trajectory of logic and mathematical formulations. In the last episode the differences and similarities between the two linear graphs may lead to discussing gradient (which may be called slope or incline or rise) and intercept (starting or at zero) and could be extended to similar situations.

From the 1981 Cockcroft report *Mathematics Counts* (HMSO 1982)

242 … "Approaches to the teaching of a particular piece of mathematics need to be related to the topic itself and to the abilities of both teachers and pupils. Because of the differences of personality and circumstance, methods which may be extremely successful with one teacher and one group of pupils will not necessarily be suitable for use by another teacher or with a different group of pupils. Nevertheless, we believe that there are certain elements which need to be present in successful mathematics teaching to pupils of all ages."

243 "Mathematics teaching at all levels should include opportunities for

- exposition by the teacher;
- discussion between teacher and pupils and between pupils themselves;
- appropriate practical work;
- consolidation and practice of fundamental skills and routines;
- problem solving, including the application of mathematics to everyday situations;
- investigational work."

1. The guidance and worksheets for this activity can be found on pp131-138 in BEAM's Primary CAME manual. ISBN 1 903142 296. Tel: 0207684 3323. Different contexts are used in the more advanced secondary manual published by Heinemann Education.

Which offer shall I take?

Jane Gabb explains how she uses material from Thinking Maths (CAME)

I recently taught some year 10 classes a lesson based on one of those in the New Thinking Maths[1] book. We have identified that certain aspects of algebra are problematic for them, including being able to find the gradient of a line and the intercept. The students have achieved from Level 3 to Level 7 in the Y9 tests and are in setted groups. This is an account of how I teach the lesson, which incidentally could be taught to year 7 pupils.

I start the lesson by giving them this context: 'There's an elderly person who lives near you that you sometimes run errands for. Can we give this person a name?' Students make suggestions; for the purpose of this article let's settle on Doris. 'Doris isn't too good with some of the new technology, but someone has recently given her a DVD player. She's heard that you can join a club and get DVDs sent to you and she wants some advice from you as to which one to join. We're going to look at 2 DVD clubs and then at the end we'll be able to give Doris some advice about which one to join. Both clubs cost the same and you have to be a member for at least a year.'

I introduce the first club:

Monthly subscription buys you 2 DVDs every month, with a free starter pack of 8 DVDs when you sign up.

I ask students to explore what happens with this club, and particularly to consider how many DVDs Doris would have after 3 months, 6 months and a year. For the lowest attaining classes I give out a sheet with these questions on; for most I just write them on the board. Some students are obviously not used to working in this way with a maths problem and I need to go round clarifying what they need to be doing, just repeating the questions mostly. In all the classes students are able to answer the questions correctly, though some need prompting to remember the 8 DVDs of the initial offer.

They are also able to explain a way of working this out for 3, 6 and 12 months. (Generally this is 3 x 2 + 8, 6 x 2 + 8, 12 x 2 + 8 or variations in terms of the order of these 2 steps.)

I then ask them if they have a way of working this out for any number of months. Again, after a little wait time they are able to come up with: 'Multiply the number of months by 2 and add 8'. In some classes they are happy to introduce a variable and express the formula as 'm x 2 + 8' or '2m + 8' where m is the number of months. At this stage I ask for clarification from several different students as to why this works and how it relates to the original problem – it is very easy to get carried away by the abstract and leave the starting point behind, but one of the strengths of these lessons is that the algebra can always be related back to the original problem. I also check out that the formula works by showing that if we put our original 3, 6 or 12 in the formula we get the answers we have found before. It is worth spending time at this stage of the lesson even if it feels like labouring the point, because you can check that other students are on board with the new ideas.

I then ask them to draw up a table (for the least able I provide a framework) and put in the information we have found:

Months since joining	1	2	3	4	5	6	7	8	9	10
Number of DVDs										

When asked what they notice about the table they say that the numbers go up in 2s. They are able to relate this to the monthly addition of 2 DVDs.

They are then given a graph to plot the above points on. I ask them to tell me about the graph and their responses are:

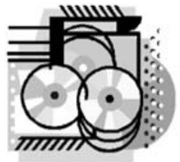
- It goes up in a straight/diagonal line
- It jumps up 2 each time (when I ask why, they are able to tell me that it is because you get 2 more DVDs each month)
- It starts at 8 (again they are able to relate this to the 8 DVDs given out at the beginning)

I ask 'How does this relate to the formula we found?'
Some students are able to 'see' the 2 and the 8 in the formula and relate this to the graph. With a more able group I ask how this relates to the equation of the line, but this is a step too far for most of them.

We then look at the second DVD club:

Wide screen DVD club

Starter pack : 2 free DVDs



Get 3 DVDs every month, with a free starter pack of 2 DVDs when you sign up for a monthly subscription.

They are invited to explore this club using whatever method they like. Many chose to draw a table from the beginning, and note that it goes up in 3s. I ask them if it is the 3 times table and they are able to tell me that it isn't because it starts from 2. After a brief discussion on this which includes writing down an expression for this club's number of DVDs related to the months, they add the graph for this club onto the same axes as the previous one.

When I ask about this graph they say:

- It goes up in 3s (Why?…)
- It starts lower than the other graph (Where does it start? Why?…)
- It's steeper than the first one (Why?…)
- It goes off the edge of the graph
- The graphs cross over

I ask 'What does this mean?' and the responses are:

- Both clubs are the same here
- They are both on 20 after 6 months

My next question is 'What happens after that?'

- You get more with the second club
- The second one is better after that

After a thorough exploration of the differences between the graphs and therefore the clubs, I ask them to think about the advice they would give to Doris and what they would show her to explain what is happening. In most classes they are very clear that they would advise her to join the second club, because you get more DVDs in the end, and they mostly say that they would show her the graphs because they feel that is the clearest way of explaining the differences between the clubs.

However in the class with the lowest attainers, the advice is more thoughtful. One boy says: 'I would leave it up to Doris. She might want to join the first club because she would get more DVDs to start with and that might be what she wants, even if she doesn't get as many in the end. If you haven't got any DVDs you'd get bored if you only have a few at the beginning, watching the same ones over again.' This articulate response is a surprise to his teacher and really shows the advantage of having such a good starting point and being able to relate abstract formulations such as graphs back to such a starting point.

If there is time I invite students to invent their own club and draw the graph for it on a clean page. These are then swapped and students are able to identify the opening offer and the number of DVDs per month from the graph.

There is a good spirit of investigation in the lesson and many students finish with an understanding of how the gradient and the intercept relate to the problem and to the formula we have found. We follow this lesson with one which includes a matching exercise with visual patterns, tables of values, expressions for the nth term, equations of lines and graphs showing the lines. A third lesson, which involves examination questions which need to be marked (many of which have been incorrectly answered), needs some preparation in order for students to fully engage with the process. We are working on this aspect.

Royal Borough of Windsor and Maidenhead

1. Thinking Maths *(Cognitive Acceleration in Mathematics Education)* Mundher Adhami & Michael Shayer, 2006 Harcourt Education ISBN 978-0-435-30780-6

Perspectiva corporum regularium

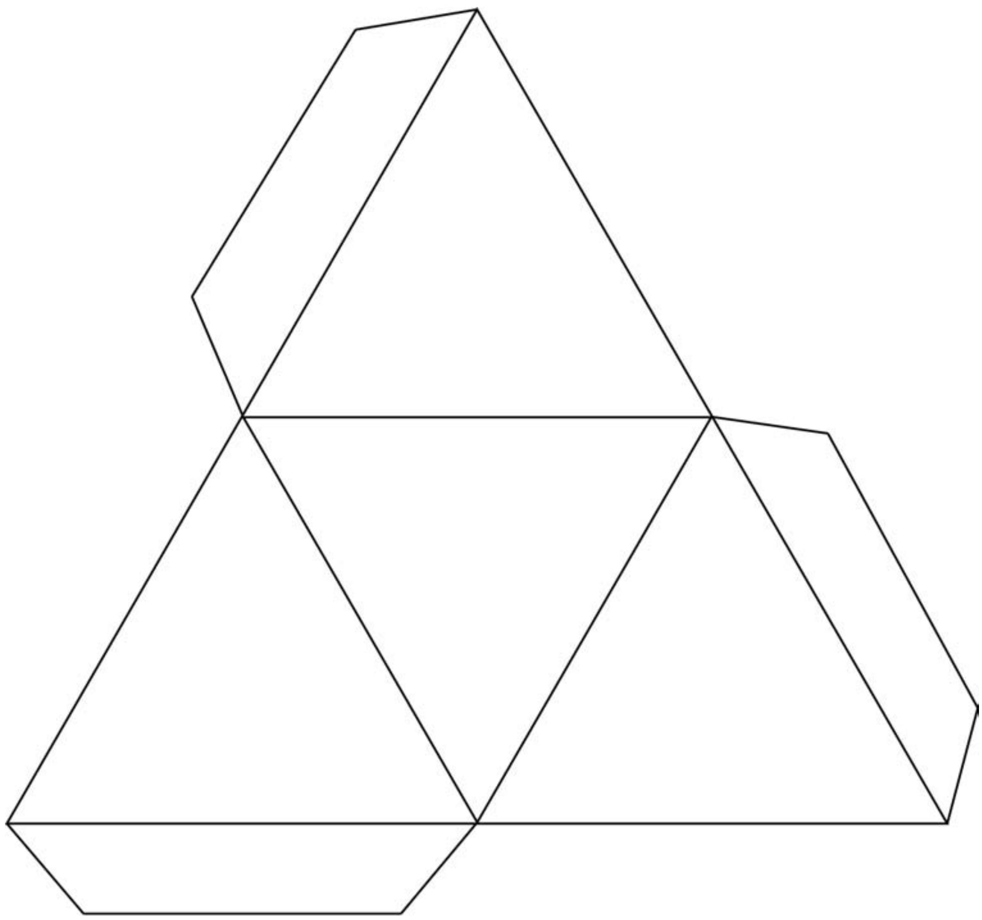
Wenzel Jamitzer 1568

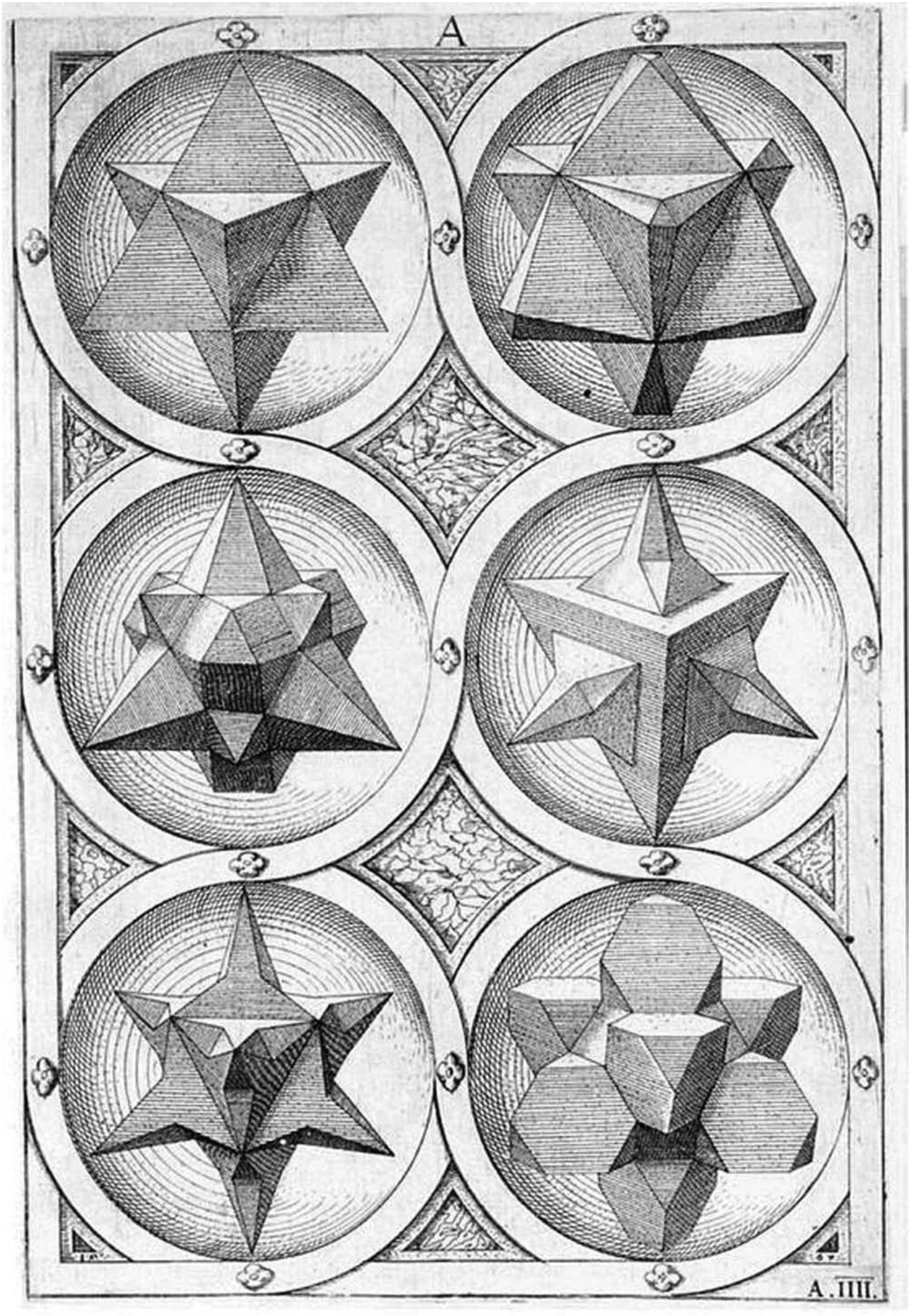
Wenzel Jamitzer was a German engraver and goldsmith who was born in Vienna in 1508 and died in Nurnberg in 1587. The copy of his engraving of 'regular solids in perspective' shown here, as well as demonsrating his skill as an engraver, shows his interest in symmetry.

Below is a net of a tetrahedron, copies of which can be printed on card, so that you can each make a model.

When they are all completed glue them together to make the two large interlocking tetrahedra that Jamitzer illustrates in his first model. If you work very carefully the model will be worth putting on permanent display.

Now you can explore some of the other solids Jamitzer illustrates in his engraving. To understand them better, use 'Playdough' to model some tetrahedra then shave off parts to make the more complex pieces of the models.





We are grateful to the Royal Society for introducing us to this engraving and giving permission to reprint it.

What do the Numbers Tell Us?

All citizens need to be able to comprehend applied number maintains

Rachel Gibbons

For many years we have sprinkled the pages of *Equals* with what we have called ‘significant figures’ – short extracts gleaned from the pages of contemporary newspapers and magazines containing numbers which help us to make more sense of the world we live in - *if we can interpret their messages*. This is because it has always seemed to the editors that, even more important than being able to perform calculations with numbers, is the ability to understand them sufficiently to absorb the information they can give about our environment. This is an area of world citizenship for which the mathematics educators must take responsibility.

even more important than being able to perform calculations with numbers, is the ability to understand them sufficiently to absorb the information they can give about our environment.

I have never forgotten the discrepancies in levels in work numbers I found in my first visit - many, many years ago - to observe lessons in a special school. In one lesson labelled mathematics the pupils were having difficulty deciding how many mini-buses they would need to transport a group of 40 people if each mini-bus could seat 6 passengers. In the next lesson - geography according to the time table - they were considering how many sheep there were in all the farms in Wales. It seemed to me at the time – and still does - that the mathematics lesson should, in some way, have been preparing them for the comprehension of these large numbers.

In this issue of *Equals* we have quoted ‘significant figures’ at much greater length than usual from two newspaper articles, one giving information about recent developments in the production of one of the basic elements of our diet – milk - and the other vividly describing how tourism can not only help to change our climate but also can destroy earthly ‘Paradises’.

It is important that every citizen can be helped to understand the issues raised in the articles quoted below. To do this the questions to be asked will be political, moral or philosophical, rather than mathematical, but to understand both questions and answers to these there must be a basic comprehension of the numbers themselves. The issue of reliability of the numbers is also important. Questions need to be asked such as:

- Where did these numbers come from?
- Who is putting them forward?
- Does that person, or any of the organisations to which she belongs, have something to gain from the argument being put forward?

Later will come the questions more specifically related to the topic under consideration, with perhaps some calculations to understand the topic in more detail. In the story of milk below, for example, one might ask:

> If a cow yields 9,000 litres of milk per year, what average yield per day does that represent?
> And 30 years ago, how much milk per day could a cow be expected to yield?
> How much milk per day do Kemble farms produce?

Subsequent questions will take us into cross-curricular areas and will depend on the direction the discussion of milk yield has taken.

The Story of Milk

Kemble Farms is one of the most efficient dairy operators in the country. The cows give so much milk they are emptied three times a day. Yields are typically 9,000 litres per cow per year, not the highest known since some farms have broken the 10,000-litre barrier, but a long way above the average and spectacular compared with a decade ago. Thirty years earlier, the average yields were usually around 3,500 litres.

The herd size here, usually around 700 cows, puts Kemble in the super-efficient league too. The average number of cows in a dairy herd in the UK is now 100; in 1994 it was 79. A £2m investment in a light and airy, aircraft-hangar-sized shed, where the cows can be kept indoors seven months of the year and fed the concentrated feed they need to maintain such levels of production, has enabled the family business that owns the farm to achieve economies of scale and cut labour costs. And yet Kemble Farms has been selling milk at less than the cost of production. Its costs – fuel, fertiliser, water and feed – have gone up 8% in the last 12 months, but the price it is paid for its milk by Dairy Crest, which processes and packs it for Sainsbury's, has fallen by 8% over the same period. Like most of the British dairy industry it is struggling to make money. …

questions to be asked will be political, moral or philosophical rather than mathematical but to understand both questions and answers there must be a basic comprehension of numbers and measures.

The price of milk in the shops has risen roughly 20% in five years, from just over 44p a litre in 2002 to just over 54p in 2007. Yet the price paid to farmers has fallen.

In 1995 producers got 24.5p a litre for their milk, the average today is 18p a litre, which represents a loss of more than 3p on every litre.

Rising Prices, Failing Farms: the strange story of milk *The Guardian* 24.04.07

In seeking to understand the next article quoted one needs to be able to understand ideas such as:

- how does 15% UK population going on three or more flights abroad each year compare with two thirds of all the journeys begin made by people going on holiday?
- Is the 4,000ft significant to my understanding of the article?
- How far in the future is 2020?

These questions can lead to a more detailed study of the information given below and links then made with teachers of other disciplines to enlarge the pupils' application of number and further widen their understanding of the world in which they find themselves.

- Paradise Lost

4.30am. A low distant drone slowly becomes audible… It's the first plane of the day passing at around 4,000ft over my home in south London on its approach to Heathrow airport. It still has eight minutes and 30 miles to fly before it touches down, but by the time it passes overhead the whine of its jet engines is responsible for the 48 decibels of aircraft related sleep disturbance that the law allows. Just a few minutes or so later another aircraft will follow in its wake. And another, and another. In total, Heathrow airport – the world's busiest - manages a daily average of nearly 1,300 aircraft movements. … I wonder where each passenger passing over me has just come from. …

And then I move on to the consequences of all these journeys. How much fuel is needed to fly 60 tonnes of aircraft through the air for many hundreds, if not thousands, of miles? Is flying hundreds of millions of people around the world each year for their holidays sustainable? …

Fifteen per cent of the UK population now go on three or more flights abroad each year. In 2005, Britons made 66.4 million visits abroad – an all-time record and three times the amount in 1984– with 81% of those journeys made by air, according to the Office of National Statistics.

Two thirds of all those journeys were made by people going on holiday. And the figures just get bigger as you gaze into the future … In 2003 government predictions for the next 30 years predicted that by 2020 the UK's airports would need to be able to handle between 350 and 460 million passengers. That's two or three times the amount in 2002 and at least 10 times the 1970 figure. Incredibly it estimated that one-fifth of the world's international passengers are on flights to or from a UK airport.

Leo Hickman, extract from *The Final Call, g2 The Guardian* 21.05.

So an understanding of numbers and their relative size, and concepts such as fractions, ratio and proportion, along with measures form an important basis for engaging with these key issues of world citizenship. We owe it to all our students to help them gain a firm grasp of these mathematical concepts so that they can be informed and responsible in caring for our world.

Fulham, London

What comes first?

Suggesting that estimation comes before accuracy **Stewart Fowlie** could start us all off on some good discussions on starting points

The ability to estimate the value of such as 863 x 82 roughly is rightly seen as an important part of numeracy. Most will earn to write the original estimate as

$$
\begin{aligned}
700 \times 80 &= 7 \times 8 \times 1000 \\
&= 56000 \\
\text{or } & 60000
\end{aligned}
$$

Surely the ability to realise tht 7 x 8 lies between 50 and 60 should precede the knowledge that it is exactly 56?

Even before a child can count 1, 2, 3, … he knows where a point lies on a number line. Once he can count to 9, he should have little difficulty in naming . 1, . 2, …and writing them on that number line.

. 1 _|__________ . 9 _________|_____

. 2 __|_________ . 8 ________|______

. 3 ___|________ . 7 _______|_______

. 4 ____|_______ . 6 ______|________

. 5 _____|_______

Putting all these points on a single line, we have:

_ | _| _| _| _| _| _| _| _| _
. 1 . 2 . 3 . 4 . 5 . 6 . 7 . 8 . 9

Next think of the line as a path. Suppose it takes, say, 80 steps to walk all the way along the path. First estimate how many steps it would take to get to a certain point and which point will be nearest after a certain number of steps.

I suggest that all this should precede the introduction of multiplication and division.

Edinburgh

Facts in Fiction?

'The whole world's gone mad with fear of avian flu and in Venice we have more pigeons than people.' …
'There's about sixty thousand of us and the current population of pigeons – well, the one given in the paper which is not the same thing – is more than a hundred thousand.'…
'That can't be possible' … 'Who'd count them anyway y, and how'd they do it?'
'Who knows how any official number is determined?'…

Donna Leon, *Suffer the little children* Heinemann: London, 2007

Sewage problems
In China: 10% of Yellow river's flow

Untreated sewage accounts for 10% of Yellow river's flow.
The volume of waste water flowing into the river doubled from 2bn tonnes to 4.3bn between 1980 and 2005.
The 5,464-km (3,395-mile) river supplies water to more than 150 million people and irrigates 15% of the country's farm land, but has lost a third of its fish species and is 70% unfit for drinking or swimming.

Reuters Beijing, *The Guardian* 12.05.07

In Scotland: pump breaks down

170,000 tonnes of raw sewage was discharged into the Firth of Forth when a pump broke down. Sewage had begun to pour into the estuary at the rate of 1,000 litres a second

The Guardian 23.04.07

A page from the past

Revolutionary theories of education in the 1890s

Charles Darwin's granddaughter, **Gwen Raverat**, *born in 1885, gives us a pupil's-eye view of the early education she received*

My mother's theories of education were so revolutionary and sensible that modern thought has hardly caught up with them even now. I find her writing before I was six months old: *'I believe in every girl being brought up to have some occupation when they are grown-up, just as a boy is; it makes them much happier. Gwen is to be a mathematician.'* But later on she, for once, was defeated; and, strange to say, retired baffled before my mathematical idiocy. …

one does not assimilate what one does not find interesting

She also held strongly that the education of the hands developed the mind. But we conservative children did not agree at all; though perhaps that was because the minds of our governesses had not been properly developed. At any rate their efforts to teach us handicrafts were not a success. As usual the theory was right, but the practice went wrong. We objected very strongly from being reft away from proper lessons, such as sums or Latin grammar, to make weak and wiggly baskets, which nobody wanted; …

even interesting lessons can be made incredibly stupid, when they are taught by people who are bored to death with them or do not care for the art of teaching either

We had daily governesses [who] were all kind, good, dull women; but even interesting lessons can be made incredibly stupid, when they are taught by people who are bored to death with them or do not care for the art of teaching either. …

But, anyhow, there was always Miss Greene's Wednesday Drawing Class, which was the centre of my youthful existence. I lived in those days from Wednesday to Wednesday; for it was not only that drawing was an ecstasy, but that Miss Greene's warm generous appreciative nature was a great release and encouragement to me. Besides it was such fun.

We did all kinds of things with Miss Greene; it was not just sitting in the studio - though that in itself was passionately interesting, especially when we had a real model. She took us out to draw buildings and streets and trees and animals, and we learnt about architecture and perspective and anatomy; and she gave us lecture s about the great painters, and showed us reproductions of their work. I can still remember nearly all the lecture on Hogarth; how he ran away with his master's daughter, and the meaning of the queer figures in *Calais Gate* and all about *Gin Lane* and how he hated cruelty. Every week we had homework to do: a drawing from life, and a composition on a set subject. …

I always liked pictures as long as I can remember anything; and I always liked poetry, and knew that id did not matter whether I understood it or not. But music left me quite cold; or worse than that, I thought it a terrible bore … I was so ignorant and uninterested in music that when I went away to school, at the age of sixteen, and was asked in a General Knowledge Paper which were my three favourite composers, I had the greatest difficulty in naming as many as three altogether! And I had no idea what any of them had written. It was my own fault that my development was so lopsided; one does not assimilate what one does not find interesting.

Gwen Raverat, Period Piece: *A Cambridge Childhood,* Faber & Faber, London: 1952fflec.

Attitudes to learning: working with a group of lower achieving Year 5 children

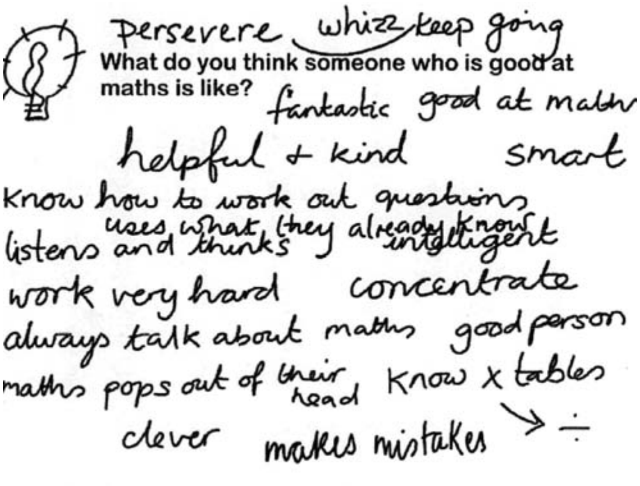
Emma Billington and **Jennie Pennant** look at strategies to develop a positive learning disposition in maths lessons for this group

This collaboration began in the autumn of 2006 when Emma invited Jennie to look with her at the challenges the 17 children in her group faced with mathematics. Conversations with, and observations of, the group – a mixture of boys and girls – revealed that they had begun Year 5 with the feeling that they were poor at mathematics and felt they were likely to fail and get it wrong. As a result the group were unwilling to offer their ideas and emergent thoughts about the mathematical concepts they were exploring.

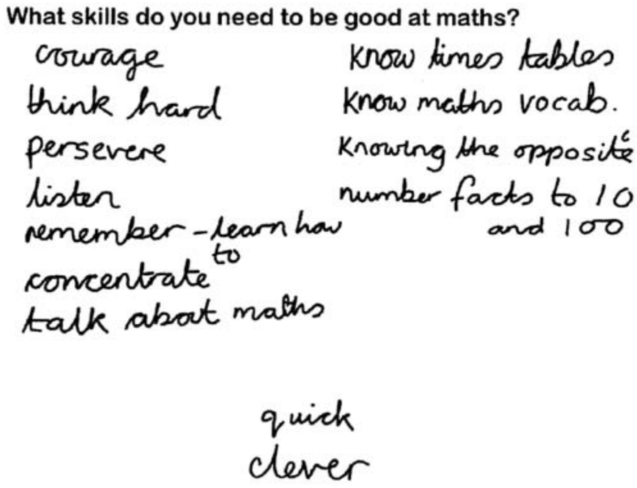
Emma and Jennie spent a session with children looking at three key questions with them

- What do you think someone who is good at maths is like?
- What skills do you need to be good at maths?
- How can we help each other to get better at maths?

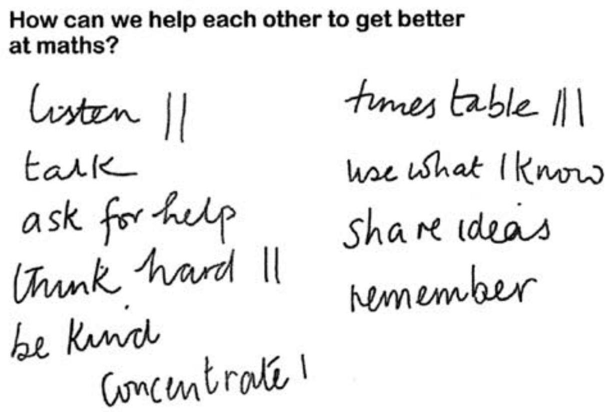
The children worked on their answers individually on paper at first, then shared their ideas with a partner and finally Emma took feedback from the whole class.
Their responses were recorded on the electronic board and are shown below.



The comment 'maths pops out their heads' is rather fascinating. It suggests that to be good at maths requires very little effort. This could be very off-putting for these children who may well find that maths does not come easily.



The idea of needing courage to be good at maths is an interesting one. Sometimes it can be hard for children to offer their ideas and thoughts in the classroom if they are not at all sure about them. It needs a very safe and supportive classroom environment to make this a possible and comfortable activity for children.



The tally marks show where more than one child in the group offered that particular idea. Emma felt that this feedback from the children was very useful and that, as a group, they could work on this in each lesson.

She decided to keep the electronic board notebook so that they could refer back to it in each lesson. The children could find examples for themselves of times when they were managing to carry out their ideas, such as managing to listen, and also discuss ways of supporting each other in developing these ideas. This enabled the children to have a high degree of ownership of the process in the lesson and also meant that progress could be clearly tracked by teacher and children alike.

In December, Jennie and Emma reviewed the progress of this strategy with the children. Emma felt that the explicit focus on the 'attitudes to learning' through the electronic board notebook was helping and that the children were gaining confidence in contributing their thoughts and ideas to the lesson. Jennie and Emma discussed the idea of having a celebration with the children when there was learning as a result of a mistake, so that mistakes became more friendly and positive.

Emma and Jennie also talked about de-cluttering the curriculum and Emma tried to focus on a few significant key skills such as learning multiplication tables and number bonds to 10 and 100. As these key skills developed, Emma then made links between these and other skills, gently providing steps to support the children in progressing. For example, building on number bonds to 100 she took the children into looking at totalling decimals to make 10. From here it then worked well to look at the effect of dividing a number by 10. Emma identified fundamental concepts such as place value that underpin the number system, and worked in depth with these to give the children a confident base from which to explore further mathematical concepts.

By the summer term, the strategy for using the 'attitudes to learning' notebook had developed into focussing on one point per week. At the start of every lesson, Emma looked at the attitude with the children - e.g. being kind to one another, listening, concentrating and together they listed all the things they could do in the lesson to demonstrate that. Emma tried to match the attitude chosen to the maths focus for the week. For example, during the highly practical capacity maths focus she chose the attitude of working together – a development of the children's original list. Emma reported that they have been able to pause mid-lesson and add ideas, or reflect on how well they are doing with the chosen attitude. Very exciting!

In order to develop the listening and sharing of ideas that the children put on their original list Emma worked hard to encourage the children to explain their thinking.
Emma reflects:
'If a child tells that 67 rounds to 70 as the nearest ten, instead of saying "yes" I say "why?". It has helped me to identify where learning is going right/ wrong, and more importantly has shown the children the need to express themselves clearly. Prior to the children explaining, I will often give them the opportunity for a one minute chat with a partner to sort their thinking, before verbalising in front of the class. This has helped them feel more confident in sharing their thinking.'

Reflecting on the project, towards the end of the summer term, Emma is aware of how much more willing the children are to participate than they were last September and that they are now keen and confident to volunteer their ideas. However, her overriding reflection is that, in her experience, success in maths for those who are struggling depends very much on relationships - with the teacher and between the children in the group.
Emma reflects:
'Real progress has only taken place with my group as we have got to know each other, and as trust has developed in each other and in me.'

The question at the beginning of the year – 'how can we help each other to get better at maths?' – has at its heart that need to build trusting classroom relationships so that risk-taking is possible. Without risk–taking how can there be any meaningful learning?

Pinkwell School, Hillingdon and BEAM Education

Starlings

Starlings are the most numerous of Britain's flocking birds. Greater London is the home of probably more than 3 million starlings. The largest roosting sites can contain more than 50,000 birds.

Book of British Birds, Drive Publications Ltd, 1969

Inclusiveness

Jane Gabb defines inclusiveness as all children engaged with and challenged by the mathematics at their own level and describes two very different inclusive classrooms.

Introduction
What I would be looking for in a mathematics lesson that was considered inclusive would be:

- All children engaged with and challenged by the mathematics at their own level, experiencing success and having their successes acknowledged

In order for this to happen the teacher must think carefully about the aspect of mathematics she wishes the children to learn and plan so that each child will have the support he/she needs, while still maintaining an appropriate level of challenge for all.

She must have in mind the TA support available and where this should be deployed (which hopefully differs from lesson to lesson). She must consider what practical resources might help some children to access what could be abstract concepts. She must think about how to motivate and 'hook' children from the start so that they want to have a go at the mathematics proposed.

make sure that children with challenging behaviour are not allowed to dominate or disrupt the learning taking place

She must make sure that children with challenging behaviour are not allowed to dominate or disrupt the learning taking place.

Visiting inclusive classrooms
In our local authority we have 2 teachers who have been identified as leading teachers for inclusion. I was lucky enough recently to observe them both at work, teaching mathematics to their classes. The purpose of the observations was to identify strategies for inclusion.

In both lessons there was evidence of all of the above having been taken into consideration when planning the lesson.

The first lesson I watched was in a special school in a mixed Y5/6 class. The teacher made the transition from literacy to mathematics by using part of the story they had been working on as a context for counting up to 5, then repeating this, and then using that counting to link the familiar ideas of adding with the less familiar concept of multiplication. All the children enjoyed the counting and were involved; some could access the subsequent number work which was to be used as the basis for their main activity.

In the other class, a mixed Y1/2 in a First school, there was also a story type context: the 'witch's brew' problem. This problem involves lots of animals with different numbers of legs which the witch could use to make a magic potion – from zero (snails, snakes) up to 8 (spiders, octopuses). Children have to find different combinations which will ensure that the witch has 10 legs in her potion. They started with taking 2 creatures, and were then challenged to find combinations of 3 or more.

Both teachers had identified which children the TAs would support during the main activity of the lesson, and had provided the appropriate materials for those children. The TAs in each class were very clear about what their role was and there was a high degree of interactive questioning within all the groups they worked with.

In the Y1/2 class, after the initial activities and introduction on the carpet, the children were grouped around 5 tables, with the practical equipment that some might need. (For one child this was a set of cards with the animals on to help her focus on the task.) The TAs were directed to work with particular groups and the teacher also worked with a group (in this case the least able group). In addition an extension activity was listed for one of the groups. (Each of the 2 TAs had a copy of the planning.)

In the special school class, children were in 3 groups, all working on a theme which involved food at their different levels. For one girl, working at the sensory curriculum level, this meant experiencing working with play dough to make things connected with the story. Another group were working with pictures of food, sharing them practically and practising counting. The third group were beginning to look at multiplication and how it connects with adding, using arrays of food and clothes to explore this, together with practical equipment where needed. These groups all came together for the plenary, and each group's successes were shared and applauded by the whole class. Real food which had been made by the middle group was shared, and some children spontaneously began to talk about halves and quarters.

think about how to motivate and 'hook' children from the start

For the plenary in the Y1/2 class, where they had all been working on the 'witch's brew' problem, children again worked in pairs on the carpet. These were pairings outside their working groups – they showed their partner an example of their work and the partner's role was to check that their solution was correct. The final question was: 'Would it be possible to have a total of 9 legs in the potion? Why not?' This question showed that many children had not grasped that because all the creatures had an even number of legs it would not be possible to have an odd number as the total. The lesson finished with 'thinking thumbs', firstly assessing their partner's work and then their own.

Behaviour strategies

The strategies for managing behaviour in both classes were overwhelmingly positive ones, with children who were doing the right thing acknowledged and praised. Questioning to refocus pupils was used in both classes and in Y1/2 a gentle warning to 2 who were somewhat inattentive: 'I might ask you in a moment. Make sure you're ready.'

In the mainstream class there were well-embedded routines for giving out and managing the individual white boards ('Sit on your bag') and giving out equipment for the tables (each group collected a basket containing their equipment and books – with colour coded strips for the different groups.) This would have been extremely useful for the 3 children on the autistic spectrum in the class. It was the teacher's expectation that even at this young age, children should be able to work independently in their groups, without needing adult supervision. The atmosphere in the classroom was one of purposeful activity. Groups worked well independently or supported by very able TAs, stayed on task and found a number of solutions to the problem. They could be heard discussing the problem and helping each other to check their answers.

In the Y5/6 class one child found sitting with others very difficult and there was a negotiated 'time out' routine which enabled her to stay in the class while sitting in a separate area. Other children had a good routine for moving tables for the group work, and then again for the plenary; this was achieved without any fuss.

Conclusion

Making a classroom inclusive requires a lot of thought and careful planning of the environment, the activities and the resources, both human and practical. Relationships with the children need to be built on trust and routines need to be established so that it is possible for everyone to work and succeed at their own level of challenge.

Many thanks to Trish Wellings at Holyport Manor School and Lindsey Henderson from Homer First School for their generosity in inviting me into their classrooms.

Royal Borough of Windsor and Maidenhead

Average earnings

How extraordinarily misleading economics can be. Talk of "average" earnings or "per capita" wealth is virtually meaningless as a true description of a nation: if Bill Gates moved to Albania it would soar up the league tables without a single Albanian being a penny better off.

Polly Toynbee, Downsizing Dreams, *Review Saturday Guardian* 08.04.06

Harry Potter and the Deathly Hallows

… sold 2.7m copies in its first 24 hours, to become the best selling book in British history … in the week ending July 21 Bloomsbury accounted for nearly nine out of ten books sold in the UK.

The Guardian 30.07.07

Climate Change 3: Some Scientists' Views - a report from the Royal Society

This is a yearly event where pupils can gain insights into some of the puzzles scientists are currently unravelling about things great and small.

Many schools visited The Royal Society's Summer Science Exhibition 2007. This year there were studies ranging from the behaviour of plant cells, to animal language, to the protection of distant planets when space exploration is undertaken. As you would expect, many scientists are concerned with the problems of climate change, looking into the effects of global warming and devising ways and means of coping with it.

The *Equals* team visited too and here, through quotes from some of the literature available at the exhibition and graphs from the IPCC, we try to help you to give your students a glimpse of some of what the exhibiting teams of scientists have discovered about the world's changing climate and some of the plans they are devising to cope with it.

Many of the questions scientists ask in their studies are of the form

How much? How many? How far? How long?

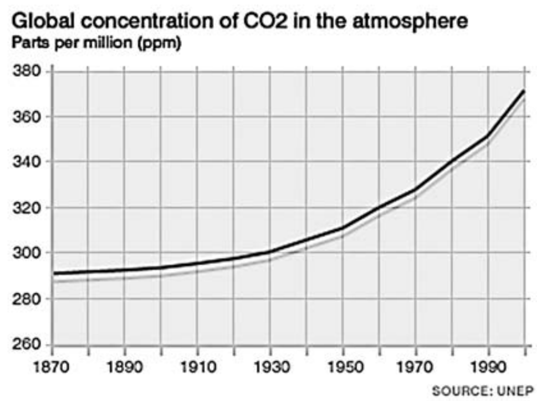
And the answer to such a question is a **number** or a **measure**.
Number is the language of scientists.

Climate change: certainties and uncertainties[1]
The Earth's climate has varied in the past over many different time scales. Right now we are in an interglacial climate system, a relatively mild period between ice ages. But humans are changing the climate system.

The natural greenhouse effect
Some gases in our atmosphere, for example carbon dioxide, methane, nitrous oxide, and especially water vapour, trap heat emitted from the Earth's surface, keeping the planet about 30° warmer than it would otherwise be. This is the 'natural greenhouse effect' and is well understood scientifically,

Human activities, especially burning fossil fuels like coal and oil, have increased the level of these greenhouse gases in our atmosphere. This is throwing the climate system out of balance.

Climate change is the most important environmental issue we face this century. Every one of the hottest 15 years on record has occurred since 1980 – the five hottest since 1997.



Questions:

- What was the CO_2 concentration in the year 2000?
- What is your estimate from the graph for CO_2 concentration for this year? How can you check this?
- What is your estimate for the year 2010 and 2020?

Widely accepted facts

- Carbon dioxide and nitrous oxide levels are rising primarily because of human activities connected with burning fossil fuels

- Methane levels in the atmosphere more than doubled in the last century. Levels are still rising, though the rise has slowed down.

- Carbon dioxide is responsible for 60% of human induced greenhouse warming, methane 20%, and nitrous oxide and other gases 20%.

- Carbon dioxide levels in the atmosphere have increased from about 280 parts per million (ppm) in the mid 18th century – the start of the industrial revolution – to around 379ppm today. You would need to go back millions of years to find another time when carbon dioxide was at such high levels in the atmosphere.

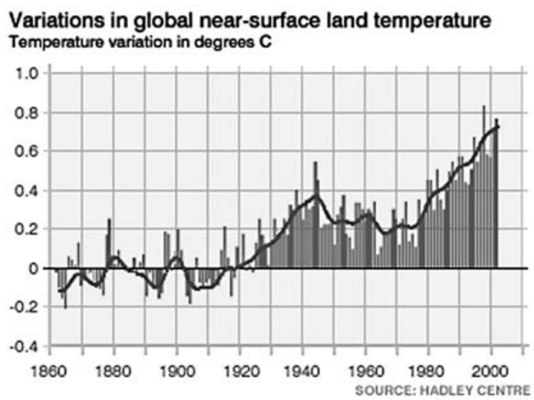
- Nitrous oxide levels are rising by about 10.25% each year.

- Over the last century, the average global surface temperature rose by around 0.7°C. Continents in the northern hemisphere have warmed the most.

- 1998 was the warmest year since 1860, the earliest year for which a precise global estimate was possible. 2002 and 2003 tie for second place.

- The majority of the world's mountain glaciers are retreating and Arctic sea-ice appears to be reducing in both extent and thickness.

- Global sea levels have risen 10-20cm over the past 100 years.



The world heated up by about 0.6 degrees last century, and the 1990s were the warmest decade on record, the International Panel on Climate Change (IPCC) says

Questions

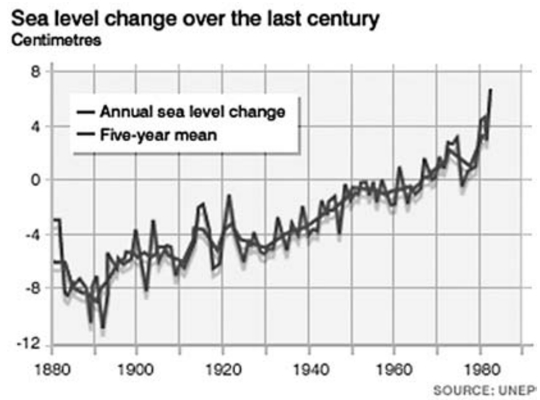
- What will be the rise in temperature you estimate using this graph, by the time you are 20 years old, 30 years old?
- What effects do you think this will have?

The UK countryside[2]

We can expect to see changes in the UK countryside as some types of farming become easier and others harder as the climate changes. Familiar landscapes could look quite different, as crops grown for fuel replace those grown for food. The need to conserve water and to save energy will also alter the face of the countryside.

Temperature, rainfall and the amount of carbon dioxide (CO_2) in the atmosphere all affect crop yields dramatically. Even at the same site, and with the same grass variety, yields may differ by as much as 100% in different years.

A common approach to this research is to learn how plants have evolved to cope with extreme climates, and to use this information to introduce those characteristics into crops. For example, tropical species provide clues abut traits that can help crops to survive the stresses of drought and high temperatures.



Rising temperatures are thought to cause sea levels to rise as the oceans expand and polar ice melts. The IPCC says sea levels rose between 10 and 20cm worldwide during the 20th Century. It predicts a further rise of between 9cm and 88cm by 2100

Questions:

- Think about the 9cm and 88cm? Why do you think we cannot predict better than that?
- Why do scientist worry about seemingly small numbers?

International Polar Year 2007-2008[3]

According to Sir David King, the UK government's chief scientific adviser:

> Climate change is the greatest global challenge of our time.

Only with concerted action based on the best international science can we meet this challenge. There is still much work to be done in assessing the impact of climate change and understanding the conditions at the Earth's poles is fundamental to this.

Polar meltdown[4]

The evidence is overwhelming. Sea levels are rising, the Arctic ice sheet is melting, surface sea temperatures are warming and so is our planet.

1. Natural Environment Research Council, *Climate change: scientific certainties and uncertainties*, Feb 2005

2. The Biotechnology and Biological Sciences Research Council (BBSRC). For more information see www.bbsrc.ac.uk

3. *UK and International Polar Year*, International Programme Office, c/o British Antarctic Survey, High Cros, Cambridge CB3 0ET

4. *Space:uk 22*, June 2007, BNSC (British National Space Centre)

Reviews

John Perry reviews Andrew Jeffrey's *Magic for Kids! (of all ages)*

25 Great Magic Tricks you can do! including Magic Maths: a range of fun and fabulous mathematical tricks to amuse your friends)

The sub-title/blurb almost says it all. *Magic for Kids!* is a collection of simple, clever and fun tricks, around two-thirds of which have a mathematical basis. Aimed by the author (a professional magician and mathematics teacher of some 20 years standing) at KS2 and KS3, the tricks can be enjoyed by children of all mathematical achievement levels. The focus is clearly on the tricks – leading to discussion and learning. Almost mathematics without shouting about it! I cannot claim to have perfected them all, some require practice. It is, however, clear that they work – 'Grey Elephants' worked brilliantly for me and with very little effort, whilst '6081' is a simple twist on an old favourite. Nevertheless, some may question whether there is enough mathematics involved.

Presentation and design is child friendly with an effective use of colour and helpful photographs – the author appears on most pages directly engaging with the budding magician/reader. Divided into four chapters: 'Dice', 'Card', 'Pencil and Paper' and 'Everyday Objects', each trick is well thought and carefully introduced with 'What they see', 'The secret bit' and 'Pro-tip' sections which taken together explain the trick and provide useful advice. The 'Pencil and Paper' are the most mathematical – Less confident learners will as the author acknowledges require a calculator. The text in general requires some reading ability and is not immediately accessible for the struggling reader. The structure and language is nevertheless such that with support most will be able to get into the tricks.

The author's passion for magic and maths is clearly conveyed. A clever touch is the e-mail address providing further guidance and support direct from the author – it works.

In the current strategy driven world it is difficult to pigeon-hole *Magic for Kids!* – is it a KS2 classroom book, a teacher resource book or a book for home? In truth probably elements of all three. In the classroom Magic for Kids! could for the creative teacher at KS2 and KS3 add another dimension to teaching including a touch of fun.

The book can be ordered from www.andrewjeffrey.co.uk for £10. Alternatively, it is free to any schools who book the Magic of Maths show. Copies can then be bought for only £6 by pupils attending the school, of which £1 goes directly to the school.

Health Training and Development Centre, Halifax

Correspondence:

Dear Editors,

Reference Equals Vol. 13 No. 2 p 17.

Here is the connection between the addition sum from Elementary Arithmetic (1850) by Edward Sang and the decimal form of one seventh. It is an interesting exercise, which contains many different concepts, but is hardly suitable for the average pupil.

$$\begin{array}{r} 140000000000 \\ 2800000000 \\ 56000000 \\ 1120000 \\ 22400 \\ 448 \quad + \\ \hline 142857142848 \\ \hline \end{array}$$

Alternatively the summation can be written

$$2\times 7\times 10^{10} + 2^2\times 7\times 10^8 + 2^3\times 7\times 10^6 + 2^4\times 7\times 10^4 + 2^5\times 7\times 10^2 + 2^6\times 7$$

$$= 2^6\times 7\left[1 + \left(\frac{100}{2}\right) + \left(\frac{100}{2}\right)^2 + \left(\frac{100}{2}\right)^3 + \left(\frac{100}{2}\right)^4 + \left(\frac{100}{2}\right)^5\right]$$

$$= 2^6\times 7\left[1 + 50 + 50^2 + 50^3 + 50^4 + 50^5\right]$$

$$= 2^6\times 7\left[\frac{50^6 - 1}{50 - 1}\right]$$

$$= \frac{2^6}{7}\left[50^6 - 1\right]$$

$$= \frac{(100)^6}{7} - \frac{2^6}{7}$$

$$= 10^{12}\times 0.\dot{1}4285\dot{7} - \frac{64}{7}$$

$$= 142857142857.\dot{1}4285\dot{7} - \frac{64}{7}$$

$$= 142857142857 + \frac{1}{7} - \frac{64}{7}$$

$$= 142857142857 - 9$$

$$= 142857142848.$$

Robert J. Clarke 23 May 2007.