Equals

Realising potential in mathematics for all

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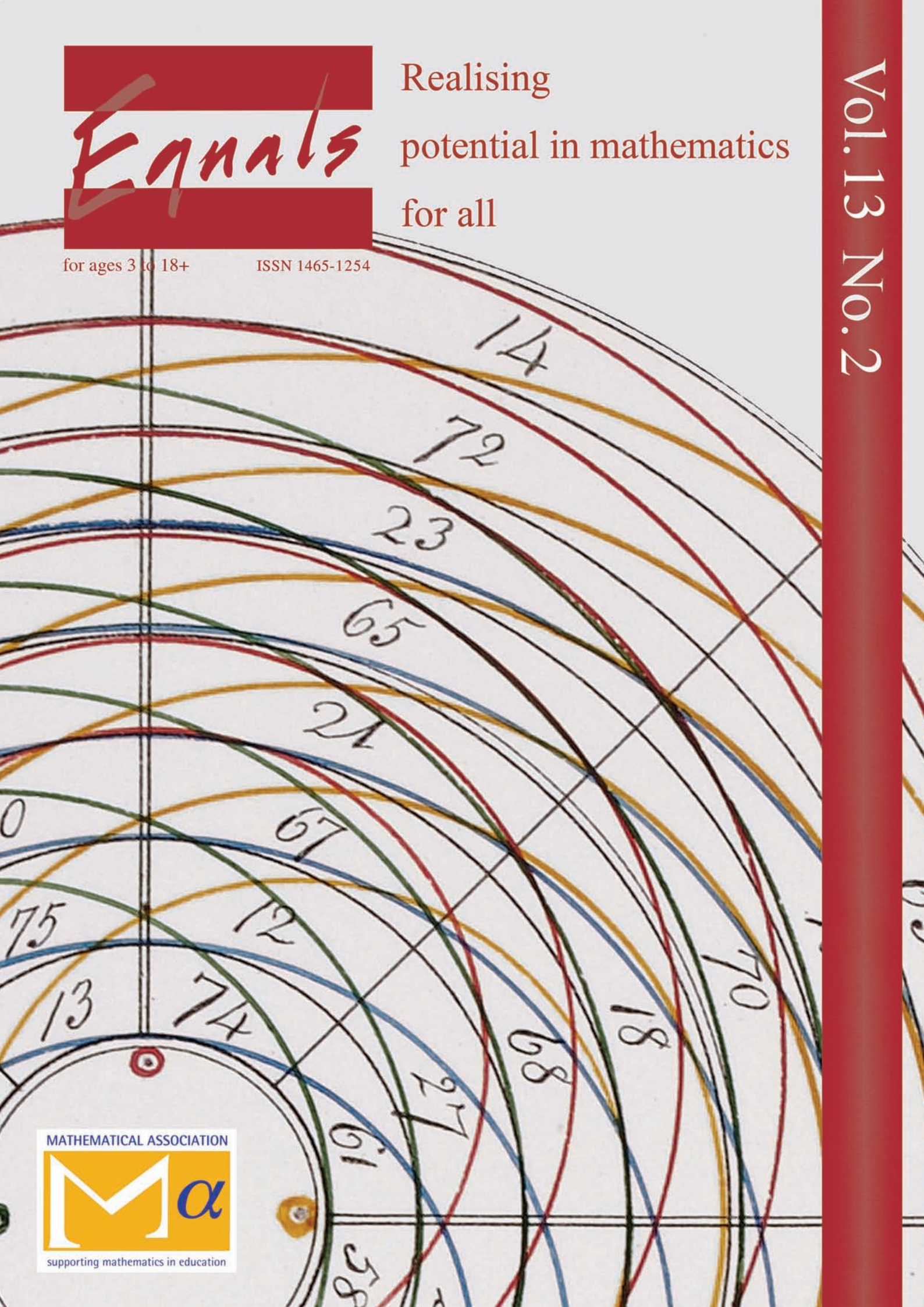

MATHEMATICAL ASSOCIATION
supporting mathematics in education

Equals

Realising potential in mathematics for all

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Editors' page

Personalisation is possible but –

Back in 2004 (*Equals* 10.2) we discussed the meaning of personalisation for teachers, politicians, old Uncle Tom Cobbly and all. Now the word is all the rage again. Christine Gilbert the new chief of Ofsted has recently completed some research into personalised learning and there are now even more queries about what it means. I believe that all good teaching, right across the curriculum, personalises - that is, suits each child's programme to their needs and inclinations and to their place on the (in this instance) mathematical map. But how can you discover what their needs and inclinations really are or what they understand in depth? I suppose results of tests and marks for exam questions do reveal something but they by no means tell the whole story. A more subtle form of assessment and tracking is needed. First and foremost one needs the map of mathematics to be displayed in 2 dimensions rather than in the single dimension of a list. If progress is traced on such a map a pupil's journey is so much easier to track and all assessments are therefore more accurate. More about this in later *Equals*.

Rachel Gibbons

Mind the gap

I am encouraged by the number of schools who are taking up the challenge of using the Primary National Strategy materials 'Supporting children with gaps in their mathematical understanding'. These first came out a couple of years ago as a box of teaching materials. It then became impossible to get hold of the box (you could only order the Cdrom) and questions were asked in the House of Commons about it. As a result the paper version was reprinted and is now available again to primary schools (though not to secondary, who can only order the Cdrom.)

The materials are aptly named and the starting point is a tracking sheet which helps a child's teacher or teaching assistant to pinpoint exactly what gaps there are behind a child's difficulty with calculation. There are some good questions which probe the understanding, and then some teaching materials to help overcome the difficulties. They are designed to be used by TAs working with individuals or small groups on a very frequent basis, preferably daily for around 15 minutes. Schools have different solutions as to how to fit this into a busy day, but the reports are that children really enjoy the sessions and TAs are getting a lot of satisfaction in seeing progress. This is intended as a short term programme (say around 10 weeks), and schools report a good success rate.

The other good news is that these materials are linked to the renewed Primary Framework so that teachers can find appropriate materials for their children who need this kind of differentiation.

Jane Gabb

Helping the pendulum swing

The signs for some time are that the government have recognised the damage of the narrow, fit-for-all moves in teaching of the last 10 or 15 years. Some influential people in QCA and elsewhere have been talking about shifting away from national tests and their league tables, to emphasis on teachers' assessment, pupil's self-planning, and more flexible curriculum and ways of teaching. HMI have also made changes in inspection and are pressing for more life in the classroom. The call has gone out to win over the youngsters who have been switching off maths at an ever earlier age. The education pendulum has started swinging towards more discovery, activity and creativity. Hopefully without scarifying pupils' entitlement and discipline.

The problem nowadays seems to lie in the reluctance of schools themselves to join this hopeful development. Some people, whether in school management or even in advisory positions, seem to have adapted to paper-based, all-done-for-you curriculum, and for various forms of drill and practice. It seems easier and more straightforward than engaging different styles and various paces of pupil's learning.

A time for active and creative re-learning by the teachers is overdue. As with all learning, there is no fit-for-all for teachers' professional development. Teachers need responsive tutors and time for experimenting and for their own enjoyment and discoveries in the classroom, and Professional Development should take many different forms. All teachers with predisposition for active, discovery, and interactive learning can now play a role as peer tutors. *Equals* can help, if readers give us accounts of successful practice in this field too.

Mundher Adhami

Visible numbers are not only for learners with Down Syndrome!

Vikki Horner **argues here that Number Facts to 10, 20, and beyond, are accessible by all children. In the first of two articles, she shows how Catherine Stern's programmes for teaching maths to individuals with Down syndrome can help.**

I have personally worked with structured materials for six years and with Stern apparatus for the past three years, mainly with children with Down syndrome. In this first article I describe the materials and programmes while the second gives accounts of four cases of Down syndrome learners ages from 3 to 36.

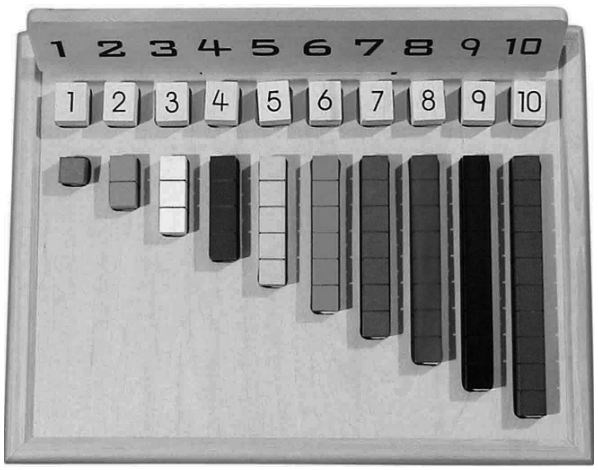
Although the two articles focus on individuals with Down syndrome, please be aware that **Stern** was designed to teach ALL children number and arithmetic from aged 3 to age 11, and for children with learning difficulties/disabilities, these materials are of special importance. They may have difficulty in discrimination, memory, inter-sensory organisation, perceptual processing (both visual and auditory) and the ability to sustain their concentration. They must learn how to receive and integrate information from as many different senses as possible in order to form concepts.

Making the number system visible

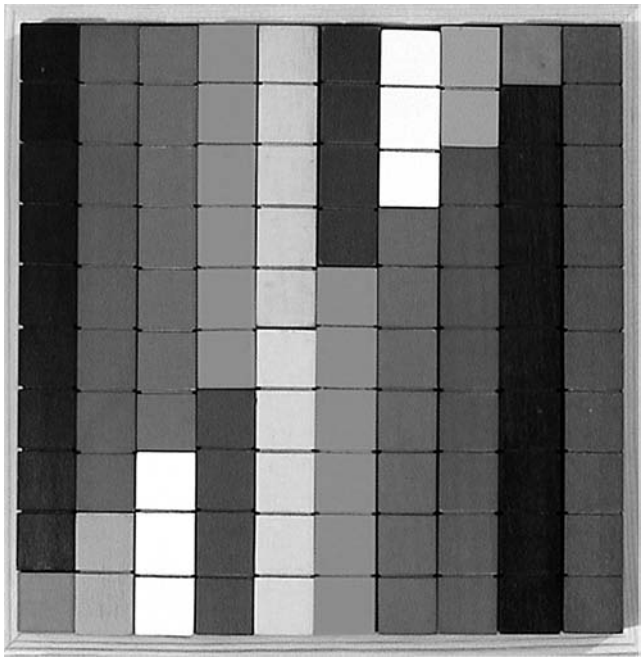
Structural Arithmetic was invented by mathematician Dr Catherine Stern. Her rationale for this teaching approach was that learning should not be based on rote memory, but on visualisation of the structural characteristics of the concept, thus giving pupils insight into the relationships that are to be grasped. It was the founder of Gestalt Psychology – Dr Max Wertheimer who named her approach Structural Arithmetic.

The apparatus was designed to make the number system visible. Learning takes place at the child's pace, and involves all aspects of natural development at this age: *sensori-motor, psychological and social*. Children work through a series of experiments and games using concrete apparatus. It includes two different sets of representations of the numbers 1 to 10 in the form of 2cm graded Number Blocks – size being relevant to a child's sensory-motor development – and pattern boards 1 to 10. The structure of the patterns provides unforgettable imagery and promotes cognitive growth. Children gain solid understanding of concepts through five important areas of development: *visualisation; language; receptive and auditory memory; action and reversibility*. Number names are not used until the second stage, and written work is not introduced until later, so that children with delayed motor skills are not held back.

Stern materials are self-checking devices, specifically designed to encourage children to think for themselves and to ensure learning is successful. Through their experimentation, self-correcting, and increased visualisation ability, children develop mathematical reasoning. For example, if a block is too small or too big for an empty groove, they can see and feel in what way it does not fit and try others until they are successful. One of the most important principles built into **Stern** materials is in their arrangement that keeps many different relationships in view and in the child's consciousness at any given time. The apparatus offers excellent diagnostic tools where assessments can take place simply by watching and listening to what pupils are doing/saying.



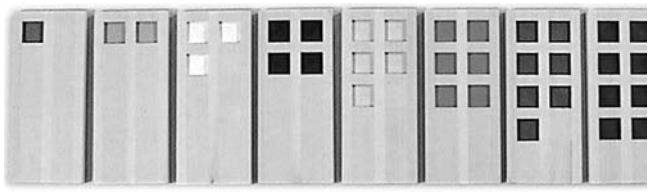
Counting Board



10-box

Addition and Subtraction Facts to 10

Experimenting with Numbers – the first of five programmes – is a multi-sensory approach where children learn the basic addition and subtraction facts with numbers up to and including 10. Taught in small-steps, through three stages, each level broadens previous learning. From the beginning, (level1) simple experiments take place in the Counting Board, 10-Box, and Pattern Boards. Learners discover 'size' relationships, position and sequencing. They fill the 10-Box with pairs of blocks: (later to become the bonds to 10) – the teachers places a random block in the 10-Box; a child finds the block that fits. They find two pairs of like blocks, (preparation for the commutativity of addition). They recognise and match patterns, construct patterns, sequence from the smallest to the biggest, to know where each pattern lives in the sequence.



Pattern Boards

In terms of cognitive growth these experiences can be seen in the following:

- Hand-eye coordination improves as children practice over and over again to fit blocks into matching grooves.
- The ability to scan develops as children search for one block among many scattered blocks.
- The ability to judge sizes is developed when children constantly compare blocks with empty grooves to find a matching combination.
- Left/right directionality and one-to-one correspondence is practised as children fit cubes into the empty pattern boards.
- Spatial awareness is increased.

KIT A

After the 'puzzle' stage of experiments, **level 2** broadens and builds further understanding. Children are keen to talk about what has been discovered so will learn the name of each Block and Pattern Board; learn to count; know which block is meant when described with the words 'one bigger' 'one smaller' 'after' 'before' 'between' and 'equal to'; will add '0' to any number and know that it results in the same number; zero – will know that 10 and nothing makes 10; will add 1 and see that it is the next higher number; add 2 to an even number and learn that it gives the next higher even number; add 2 to an odd number gives the next highest odd number. When subtracting 1 from any number, the apparatus provides unforgettable imagery showing that this results in the next lower number and subtracting 2 from and even/odd number results in the next lower even/odd number.

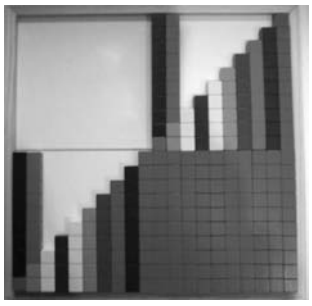
At this stage in the 10-box, with the number names in place, children can now name the familiar combinations of blocks that go together to fill the box and express them orally for example: 8 and 2 makes 10, 10 and nothing makes 10.

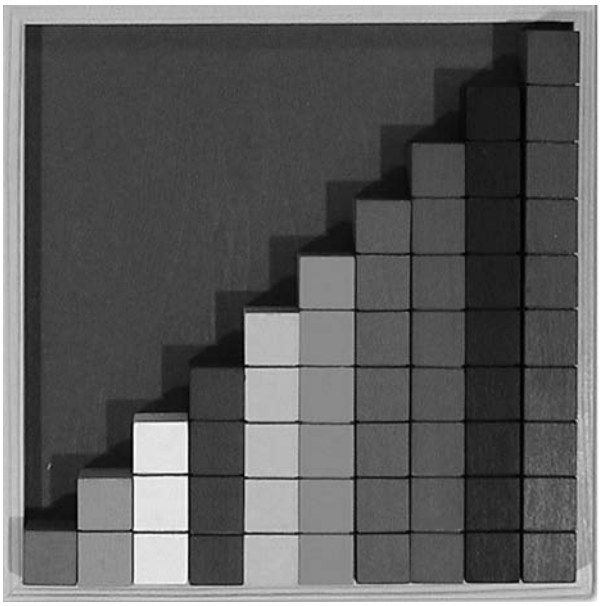
They will become aware that the two like pairs of blocks can be placed in any order and it makes no difference to the sum. "8 and 2 makes 10, 2 and 8 makes 10." Here also, they will begin to understand the concept and language of the 'missing' addend, "8 and what makes 10?" Hide two blocks behind your back and say "I have 10 altogether. In one hand I have 8, (show it) what is in the other hand?" Pupils begin to see the relationship between addition and subtraction that of 'doing' and 'undoing'. When your child gets to be the 'teacher' this is a wonderful way of assessing how many facts are known.

With this knowledge in place it is time to move to level 3 which introduces the numerals 1 to 10 and links them to the named number blocks and pattern boards. Equation work begins. To do this the plus sign and equal sign are introduced. With the pattern boards children are able to act out a subtraction word problem; able to read and understand an equation and use the plus or minus sign in an equation. Back to the 10-box in level 3 children begin to record from memory addition facts with the sum of 10 and record the related subtraction facts. Using the wooden number markers, children will record an equation from hearing an addition or a subtraction story.

Number Bonds to 20

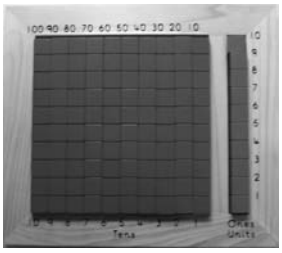
The above levels give the reader an outline of how pupils will develop their numerical and operational understanding with numbers to 10. Once this foundation is in place, the bonds to 20 are tackled using the 20-tray. This is a marvellous piece of apparatus, specifically designed to show relational understanding of what was learned in the 10-box and the combinations to 10. With the size of this piece of apparatus, children can clearly see that the same ordered blocks 1 to 10, are now sitting on top of a base of ten 10-blocks. They 'see' that the same facts to 10 hold true in the structure of the teen numbers.





Transferring Basic Facts to Higher Decades

Learning about place-value using the Dual board really makes this concept visible and enables the transfer of basic facts to higher decades up to 100. It is a wonderful way to show how working with 'ones' or 'units' transfer to working with 'tens'. For example, 3 ones and 3 ones make 6 ones, to 3 tens and 3 tens make 6 tens. It is also an excellent way to demonstrate the concept of regrouping; adding two numbers that make more than 10. By filling the ones column with 10 single cubes it is easy to see that these 10 cubes are now as big as one 10-block. This is then exchanged for one whole ten and is moved into the tens compartment.



Dual Board

KIT B

The number track provides opportunities to see the same topic, taught in the dual board, in a different light, thus aiding the transfer of conceptual understanding. This apparatus is part of the **Stern Kit B**, and this level of teaching can be found in **Teachers Manual Book 3 (www.mathsextra.com).**

Further teaching includes multiplication and division, long division, the structure of numbers to 1,000,000; rounding numbers, fractions, decimal notation, ratio and proportion, the use of percentages and problem solving with these concepts. Books 4 and 5 (Availability 2007).

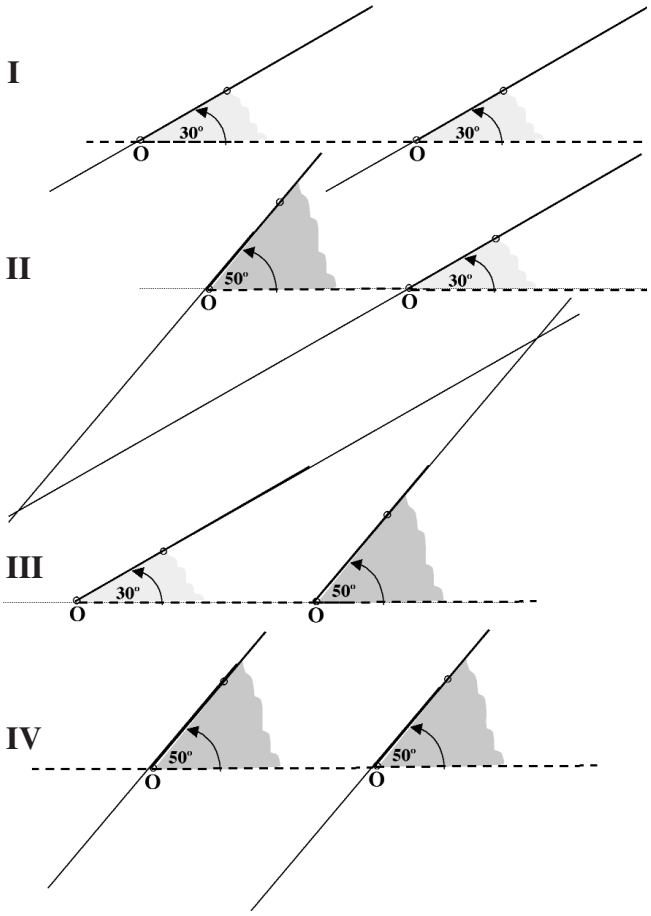
Vikki has actively contributed to her daughter's development, especially for numeracy. She is passionate about helping children develop maths skills which include learning to tell the time and handling money.

Vikki currently advises and provides training using Stern's Structural Arithmetic. She can be contacted on: 01747 861 503 vikki.horner@mathsextra.com

What's in a name?

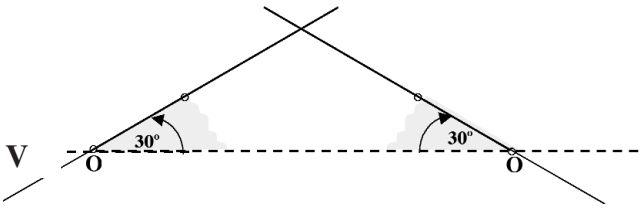
Gerry Rosen builds on his work with angles (*Equals* 12.3) and takes us into angles on parallel lines.

Have some discussion about where lines intersect and about parallel lines based on the following five drawings:

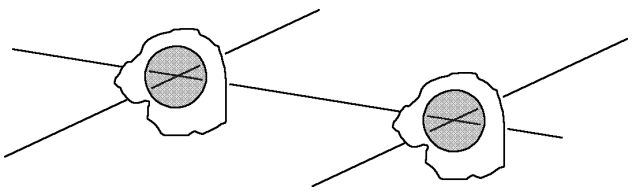


These questions may assist the discussion:

- Which diagrams show parallel lines? How can you tell?
- Which do not? How can you tell?
- Can you calculate any of the other angles?



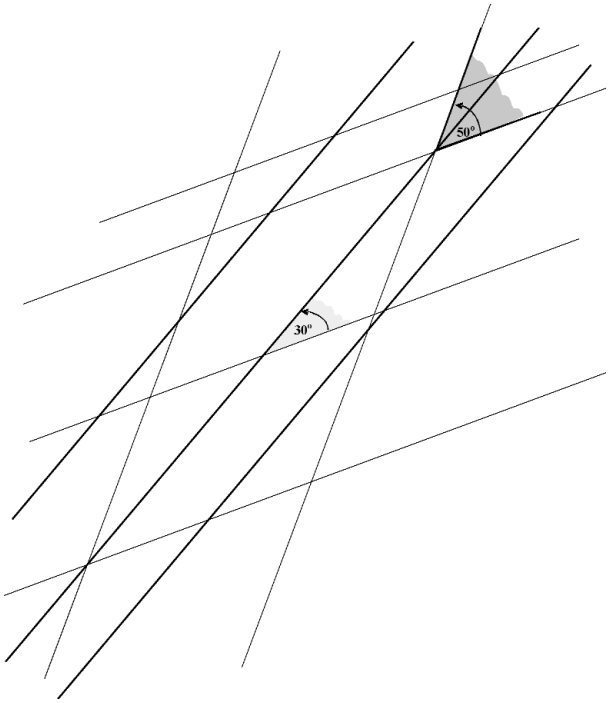
The aim of this discussion is to lead to an understanding that equal angles, leaning in the same direction to another line, make parallel lines.
To enhance this understanding for those that need it and to focus on what in fact is going on, pupils in difficulty are given the holes (surrounded by a bit of paper) like those from file paper with which to look at the combination of angles at each vertex.
And so on to the next worksheet



Worksheet 2: **Parallel Lines**

I went a little bit crazy with those angles of 30° and 50° and produced three sets of parallel lines. (feel free to use the tracing paper angles if you find it helps you)

- Find and mark as many angles of size 30° as possible
- Find and mark as many angles of size 50° as possible
- Find and mark as many angles of size 20° as possible
- Find and mark as many angles of size 150°, 130°, 160° as possible

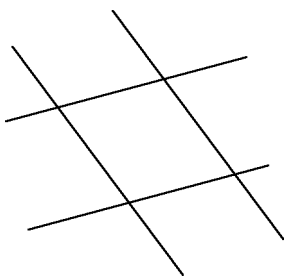


The students were able to move around the worksheet finding equal angles without knowing the names connecting one angle with another.

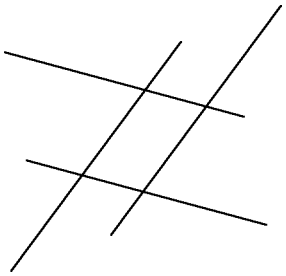
Worksheet 3: **XY FUN**

If you look carefully at sets of parallel lines you can often find the letters of the word **FUN**. See how many of each you can find in the following diagrams where there are 2 sets of parallel lines:

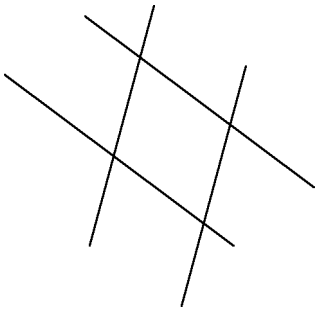
1. Can you find the letter **F** sixteen times in the following drawing? It appears right way up, upside down, on its side, and even back to front.



2. How many times can you find the letter **U** in the following drawing?

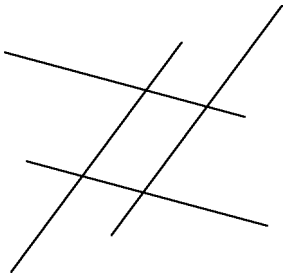


3. And the letter **N**?

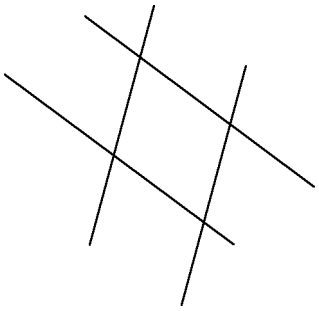


4. The letters **X** and **Y** can be seen here too.

How many times does the letter **X** appear here?



How many times does the letter **Y** appear here?



In conclusion the above worksheets and discussion give the students some success and put them more in control. They become more confident in learning the correct terminology knowing that if the worst comes to the worst they are able to fall back on **XY FUN**.

Western Galilee Reginal High School, Israel

Climate Change 2
Why the Peruvian Alpacas are in danger and what you can do for them

In this second study of climate change Laura Welsh gives you the basis for a discussion with your class on the problems of the Peruvian alpacas and how we can affect their future. Pictures and maps can help the numbers come alive.

Way up in the Andes (mountain ranges of Peru) at heights between 4,000 and 5,000 metres above sea-level lives a community of people called Peruvians and their herd animals known as alpacas. These creatures look a bit like camels, they are fluffy, white, very strong and there are up to 50 in a herd. They help out by carrying much needed food & wool around and they also provide milk & meat. This means that they are very important to the Peruvians for survival; without them these communities would not be able to continue and the people would have to leave their homes. The people and the alpacas co-exist and need each other for survival.

Make a note of the animals that you rely on for food or clothing or comfort.

Recently because of the changing climate of Peru many of these creatures have died; in 2003 when temperatures dropped very low (to -35C), 50% of the alpacas died as they were not used to the cold. The normal temperature for winter is roughly -15C.

How many alpacas do you think might have died in one of the largest herds?

The drop in temperature was particularly significant for these people because they live so high up. This means the land is very sensitive to both extreme hot and cold. These temperature changes are part of a world-wide change in temperatures which will eventually affect everybody. Some areas are getting warmer whilst others, like the Andes, are getting colder. The Peruvians have also experienced severe hail, which destroys the crops.

Have temperatures changed where you live and if so by how much?

Scientists think that temperatures are changing because gases released by human activity are collecting to form a blanket around the earth which is keeping heat in. These gases come from many sources including factories and cars. If we all keep driving so often temperatures will keep changing and many creatures such as the alpacas will soon be extinct.

Do you get driven to school?

How far is it from home to school for each member of the class?

Find the total person-miles travelled each day.

Other ways which could help include thinking about all the possessions you have, being happy with these and not wanting more. Most items can be recycled or reused. This is a good way to stop using up resources and therefore not contributing to climate change quite so much.

How much waste does your household produce in a day, a week?

Work out the volume of waste in a day, a week.

How much of this is recycled?

Calculate the volume of waste for the homes of the whole class.

And the class recycling as a whole.

Everything you do has an effect and if you start thinking about how to slow down climate change now, then we may yet save the alpacas.

Brighton

Creative ideas for helping children develop measuring skills and concepts.

Jennie Pennant shares with us some interesting ideas she has seen recently which could be adapted for different ages and abilities of children.

Do your children need to develop their understanding of length: shorter, longer, how much, dividing, ordering of lengths, estimating? Take a look at an innovative idea a teacher used recently.

Fence weaving

A teacher, at a primary school in Berkshire, helped her class make this wonderful weaving pattern on the fence outside the classroom, to help the children understand these mathematical concepts. As the children measured their piece of fabric, mathematical talk ensued to decide if it was long enough or short enough to fit the pattern. Estimation was necessary to decide how much fabric was needed altogether and division to divide up the fabric into the appropriate lengths. All this mathematical talk and practice resulted in a stunning visual outcome for all the school to admire and discuss.

Which year group do you think undertook this activity? (See page 25 for the answer)

Do your children need to develop their understanding of 2D and 3D shapes and their skills of using a ruler accurately, drawing and measuring straight lines? Here's an idea one London school used recently.

Mathematical hat parade

The children designed and made a hat to show an aspect of maths. This involved a lot of maths talk, looking at nets for shapes such as cubes and cones, accurate measuring and use of a ruler. Some chose calculation ideas for the theme of their hat and others modelled maths equipment such as a die. These were then worn by the children for a mathematical hat parade and much admired. They were able to talk about the shapes involved in making their hat and use accurate mathematical vocabulary. The hats then went on display in the classroom.

Which year group do you think undertook this activity? (See page 25 for the answer)

Digital photographs

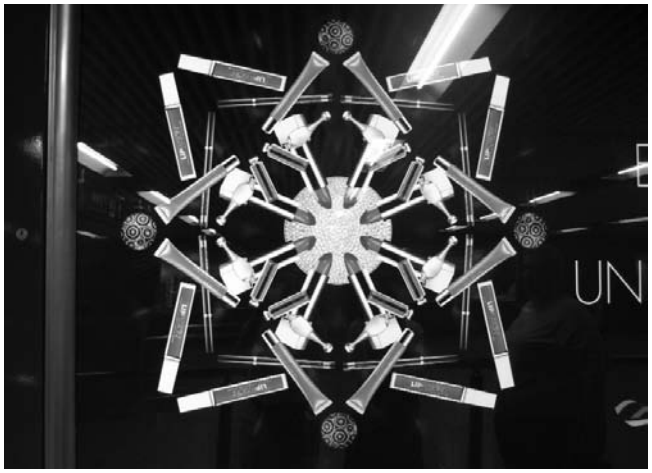
In both these projects, digital photographs allow the work to be recorded in the children's books, so that the learning can be reflected on by the individual children, and success celebrated. The digital photos can be stored on the class computer for display on the interactive whiteboard for the next time the children need to think about and explore measures. They can also be used for a parents' information evening or uploaded onto the school website.

Do you have an idea for creative ways of helping children develop their understanding of mathematical concepts?
Do send your ideas into us at *Equals* by end of July. We will publish the best ideas received in future issues, fully attributed to the author (very good for the CV!) .

BEAM education

News from Hong Kong

On a recent TIPD* visit to Hong Kong to gather information about their Gifted Education programme, the following were seen by Jane Gabb (what is good for the gifted may well be good for those for whom the subject does not seem to come easily):



A wonderful stimulus for talking about symmetry

The mind boggles as to what danger lies here!

Poster in the Hong Kong Institute of Education

Not a lot of fun to be had with the Leisure and Cultural Services Department then.

A stimulus for estimation

A school motto

The gym teacher's T-shirt reads 'No crying allowed.'

A computerised fountain at the top of the Peak on Hong Kong Island.

Typical public housing in the New Territories.

Primary pupils line up at the end of a 10 lesson day – with the bags to prove it.

A Hong Kong school staffroom – not a comfy chair in sight.

Other information

- Class size is generally around 40, with little TA support.
- Primary schools are often very large – 1200 is not uncommon
- Quote: 'We drill children because we believe that even the stupid can pass exams if they get enough drilling'

* Teacher International Professional Development – to find out more visit www.teachernet.gov.uk/professionaldevelopment/tipd/

Royal Borough of Windsor & MaidenHead

Lacking lessons

The Unesco literacy conference were told of the need to reach the 100 million out of school children and 700 million illiterate adults still denied access to basics such as books and writing paper
TES, 23.03.07

A Magical Circle of Circles

Benjamin Franklin 1706-1790, statesman, scientist and inventor, was best known for his involvement in freeing America from Britain and in helping to frame the American Constitution. However his scientific interests ranged wide and is excitement with numbers was great. He invented many magic squares and also this **Magical Circle of Circles**. The illustration here is a copy of the one Franklin sent to the Royal Society. His original drawing and text are still there in the archives, just as he presented them and we are grateful to the Royal Society for giving us permission to reproduce a copy of it here for you to see.

If you read his description carefully you will find the numbers 360 and 180 mentioned over and over again. His diagram is well worth studying.

1. The first thing to do is to see whether all the numbers from 12 to 75 are included and that each occurs only once.

He says that:

the numbers in each Radius, with the central Number, make just 360

2. Is this so?

And that:

all the Numbers in any one of the circles, added together, make, with the central number, just 360, the Number of degrees in a Circle

3. Check that he is right here.

He goes on to say:

Also Half of any of the said 8 Circles, taken above or under the central horizontal double Line with Half the Central Number, make 180, or half the Degrees in a Circle.

4. Check he is right here and then go on to check in the red, yellow, green and blue circles.

5. Make yourself a copy of Franklin's magical circles (without the numbers)

And now comes the really difficult question – how did Franklin manage to 'so dispose' the numbers in this way? It is a question you might like to take away with you to ponder over from time to time. It is difficult so you may never solve it, but it is the kind of question that has intrigued number lovers over the centuries.

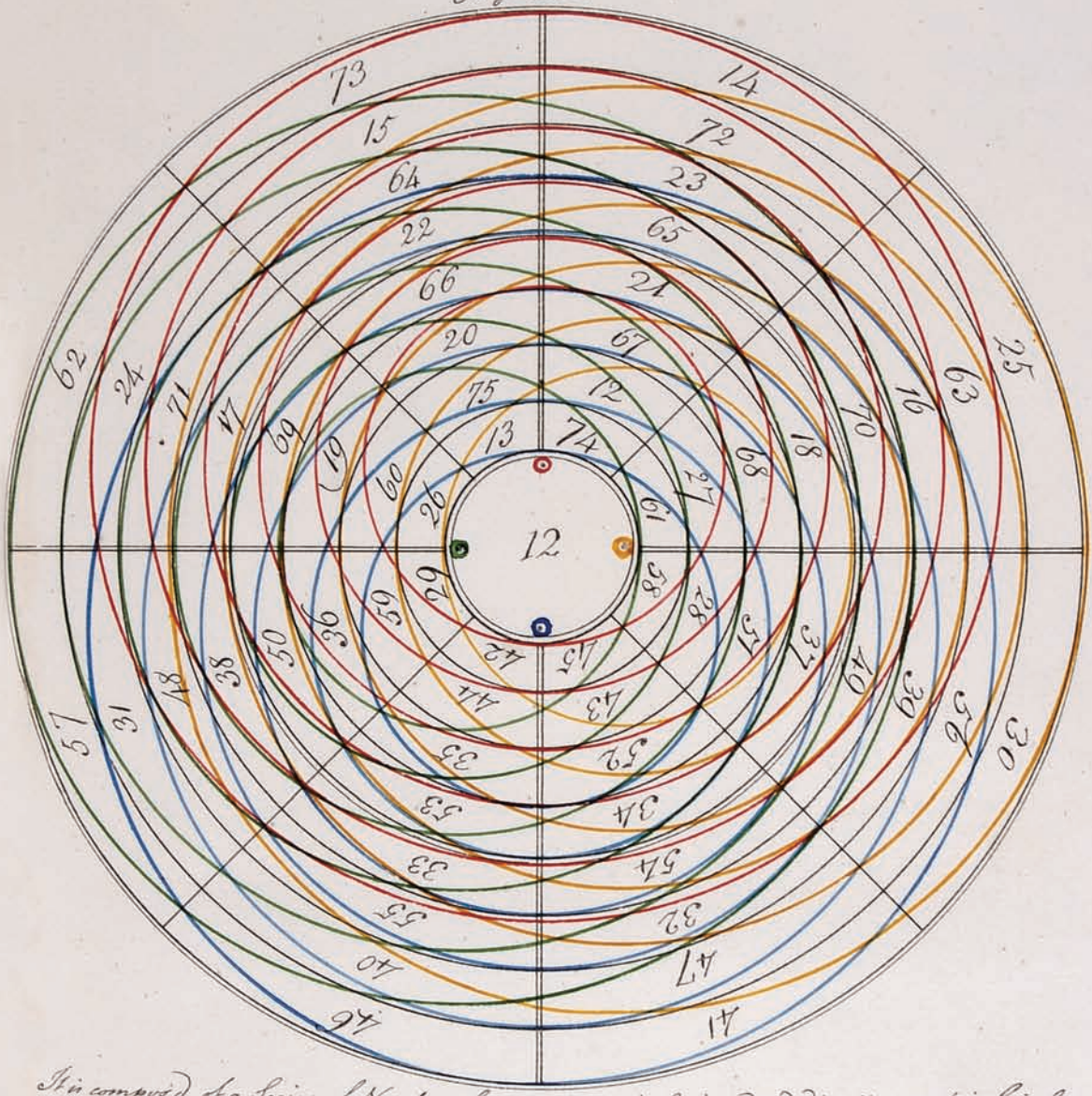
6. Meanwhile you could try something simpler, like 'so disposing numbers' in a 3 by 3 magic square so that the totals of all rows and columns are the same – maybe 18 or 36?

Teachers' notes

It is a complex diagram given here but it presents a lot of practice in simple arithmetic in what seems to us to be an interesting context There is practice too in drawing circles to obtain an unusual but pleasing diagram. The work would best be shared between members of a small group. It is not expected that anyone will discover Franklin's method but "a man's reach should exceed his grasp" and this study may lead to an interest in magic squares with the possibility that a simple one may be devised even if only by trial and error. It certainly is an illustration of the fascination number patterns hold for scientist of all eras.

A Magical Circle of Circles.

By B. F.



It is compos'd of a Series of Numbers from 12 to 75 inclusive, divided in 8 concentric Circles of Numbers, and rang'd in 8 Radii of Numbers, with the Number 12 in the Center, which Number, like the Center, is common to all the Circles and to all the Radii

The Numbers are so dispos'd, as that all the Numbers in any one of the Circles, added together, make, with the central Number, just 360, the Number of Degrees in a Circle.

The Numbers in each Radius also, with the central Number, make just 360.

Also Half of any of the said 8 Circles, taken above or under the horizontal double Line with Half the Central Number, make 180, or half the Degrees in a Circle. So likewise do the Numbers in each Half Radius, with half the Central Number.

There are moreover included 4 other Sets of concentric Circles, 5 in each Set, the several Sets distinguish'd by Green, Yellow, Red, and Blue Ink, and each Set drawn round a Center of the same Colour. These Sets of Circles interfect the first 8 and each other, and the Numbers contain'd in each of these 20 Circles, do also, with the central Number, make 360. Their Halves also, taken above or under the horizontal Line, do, with half the central Number, make 180.

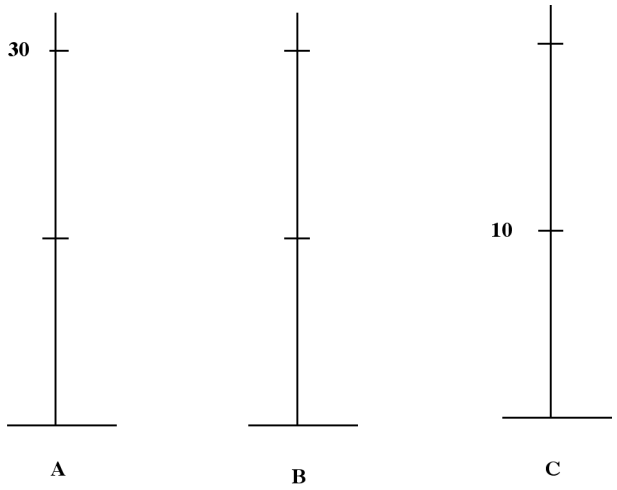
Observe, That there is no one of the Numbers but what belongs to at least two different Circles, some to three, some to four, & some to five; and yet all so plac'd (with the central Number which belongs to all) as never to break the requir'd Number 360 in any one of the 28 Circles.

Teachers learning from children

By listening and allowing time, teachers learn how children build up their maths through confusion, or from scratch. Mundher Adhami thinks this kind of maths may well be more important than the maths in work-schemes or practicing various calculation methods.

Here is, first, an account of some Primary teachers working with a small class of low achieving children age 10. This was part of a course on advanced teaching skills.[1] Three teachers took turns conducting a lesson while the other three observed and interacted with children. Teachers wanted to use the lesson to check if the emphasis on calculation methods has helped children with ideas on number. They experimented with a fresh approach.

Gwen draws three vertical lines with zero and two markers on the board, labelling the lines A, B and C. She asks the class what they think these are.



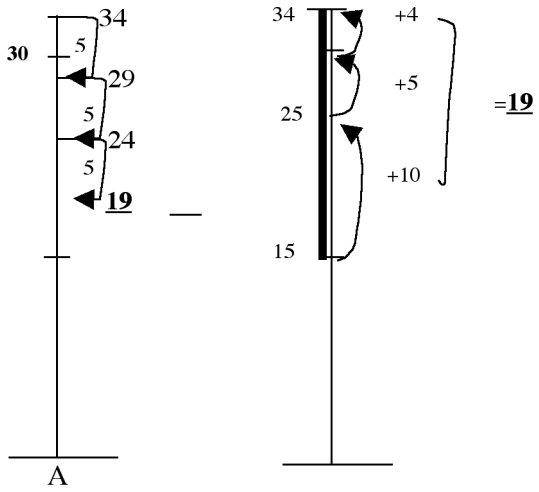
Variously pupils say: rulers; number lines; thermometers; empty rulers, measuring jug. Gwen then asks them to copy the lines roughly on sheets and work with each other to place the numbers 8, 15, and 34 on each of the lines.

The teachers spread amongst the tables to listen and for several minutes prod the children. Some children copied the number 10 from line C to the middle of A, others realised that marker was 15. Some placed numbers seemingly haphazardly. Most have placed 34 at the top mark on the middle line. With questioning they become aware of inconsistencies, and struggled to resolve them. The teachers then ask the children to share their ideas.

T: So what is the most important thing about putting numbers on the number lines?
Ps: Guessing; Guess-estimate; (Ben took pride to explain that this is what a sensible guess means); finding things; you need to know the measure.
T: And how to find the measure?
Ps: You invent it! You work it out; I drew squares. 15 is half-way; you look at other lines; halving again
T: What about the 34?
Ps: It is at the top; the biggest; you have to imagine it.
T: So what if we can extend the number line up, how far does it go?
Ps: to the edge of the board; you can't do it.
T: Can't the line go up to the ceiling?
Ps: Yes, No, No, Yes! Up to a 100.
T: What about the line going down from zero to floor?
Ps: (Puzzled, then venture) Minus, Negative; I am lost.

That first episode took about 20 minutes. There was a friendly atmosphere about the exchanges, and Gwen accepted all ideas as having good reasons, even the wrong ones. The pupils had worked largely in pairs. The pupils were told not to bother about neatness and to say whatever comes to their mind, but to rehearse first with each other. One teacher was scribing what the pupils said, often verbatim, on a flip-chart. The class seemed well predisposed to this way of working.

A second episode followed conducted by Kate, on the sum 34 -15. She said the scorching day temperature of 34 dropped 15 degrees at night. So where is it now? Some boys drew a line and wrote all the 15 numbers down, but of course got confused: should you count the start and the end marks, or only the start, or only the end, or neither.



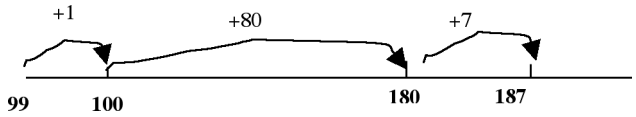
Some girls first went down 5 then 5 then 5 with arrows. They 'sort of knew' they can come down in 10s, but didn't. No one used the currently well rehearsed way of 'add on' from 15 to 34. After a while, after the ideas were shared, Kate asked them to imagine the vertical line made horizontal, placed 15 on one end and 34 on the other; so now they went to the 'add on' method. The two meanings of subtraction were compared.
Much of that went over the head of the class, probably partly because we are *adding* while the temperature *dropped*. But the teachers were exploring the limits of their understanding. The episode took over 15 minutes and the time become short.

A third episode was conducted by Julie who explored with the class how this work links with their other calculation methods. She asked them to give her two-digit numbers to add and they offered 99+88.

Glen, a lively boy occasionally working with his friend Tylor, offered quick answers working things in his head: 187, then 168, and 178. He later said we can just add a hundred then take away 1, and there is no need for writing. Tylor was showing off his HTU headings and making mistakes, which were difficult to understand at the time..

H T U	H T U
8 8	8 8
9 9	9 9
7	7
4 2 1	7 1 1

Ben, of the 'guess-estimate', said he used his fingers to add 10s and 1s.
Rachel, a seemingly more able pupil who was working with two other girls, showed the currently taught method:



Julie rounded off the lesson by asking the class what they thought of what they have done. They talked about it being different from normal, not knowing what you are going to be asked to do, enjoying working with each other, not having to write much, different ways of doing sums, liking the challenge.

Teachers' reflection on the shared practice
Back at the teachers centre we debriefed and looked back at the lesson, and how it progressed in light of the planning we did before the lesson. A few ideas were aired:

- The idea that a number can be both a position on the number line and a movement up and down is important but neglected.
- We agreed that this largely low-attaining Y5 group can still handle the idea of scale, which they called 'measure'. The scale notion was intuitive and aided by finding the unit measure, largely based on halving and halving again and estimating positions.
- The idea that a number line 'goes on forever' is also accessible providing the challenge is made at the right time, and through gradual extension of the imagination, first to the edge of the board, then to the ceiling then through the ceiling, 'up in a balloon'.
- Emphasis on calculation methods seems to stand in the way of knowing how big a number is. Glen's sense of size of the answer was an exception, even if he didn't get the right answer. Compare that with Taylor's focus on learned procedure for place value while not knowing how sensible the answers were. We speculated on the possible reasons for his errors, and the many sources of confusion about where to put the 'carry'.
- Each class needs some such experimentation slots for the teacher to know what they really understand.

The class teacher herself, who observed, was surprised at how children show themselves differently in the CAME thinking lessons from their performance in 'normal' lessons. She thought the lessons good at diagnosing children's strengths and weaknesses.

Some of these ideas were aired with the other groups of teachers who have tried other lessons in other classes. The atmosphere of sharing and recording a few points on the flip charts felt similar to the atmosphere of thinking and sharing by the pupils in the classroom. Parallel processes of learning seemed to occur.

Tutors' reflection on teachers' work
During this teachers' sessions and later, the tutors were discussing the proceeding and adjusting their own roles.[2]

- Teachers seldom have the chance to probe children's understanding of number, and such lessons give them 'permission' to do so.
- Number work has much scope for confusion for both teachers and pupils, and the models and equipment we use, such as the counting stick, the number line and the 100 square may at times be as confusing as helpful.
- Some of the teachers recognised that they themselves have not made conscious comparisons between the two meanings of subtraction on the number line, i.e. the difference between two fixed points, and the movement from a point.
- Teachers' difficulties with number are examples of difficulties in all topics of mathematics. That may explain the reluctance of teachers to stray away from pre-planned lessons in many topics.
- The CAME approach may not suit teachers who are teaching at a 'survival level' in mathematics, even though they would greatly benefit themselves from the lessons by exploring their mathematics as well the that of the pupils.
- The more confident teachers are challenged by the insights the approach reveals as to the richness of pupils' own mathematics and reasons for misconceptions, and the fact that seemingly weak children may actually think at a higher level than those who are high achievers in formal tests.
- There is much hope that a core of open-minded and collaboratively creative teachers may gradually emerge, with this style of course, to help other teachers.
- There may be a useful link between this approach and the revised Primary mathematics framework in that a CAME thinking lesson can act as a diagnostic tool at the start of a block of work. Hopefully this is not going to be made into a paper procedure, so we need to think about it.

'Nested' work of the pupils, teachers, and tutors
The account above exemplifies a feature of the professional development work of CAME in which the tutors of the course, the participant teachers and the pupils are engaged in shared work.

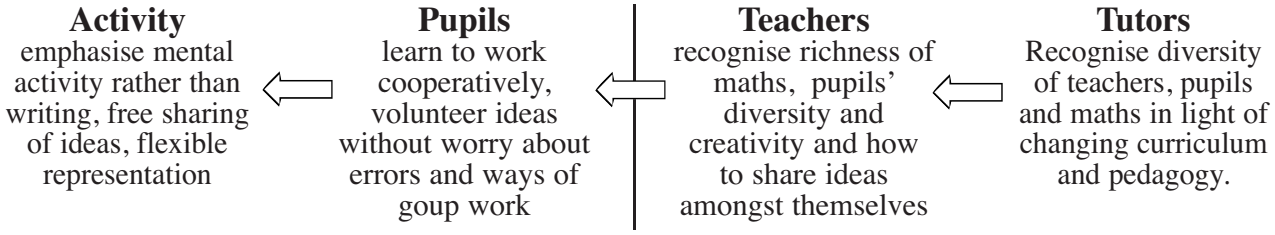
In respect of **ideas in the mind**, it may be presented as follows (the left two slots occurring simultaneously in the classroom, the right-hand slots occurring in the teacher's session):

Activity	**Pupils**	**Teachers**	**Tutors**
on e.g. number sense, highlighted in questions in a planned but open activity	handling maths ideas in their own words, including confusion and misconceptions	handling pupil's ideas in light of typical progression in understanding	handling teachers ideas on pupils and the maths in light of wider curriculum

In respect of **social interaction** between participants

Activity	**Pupils**	**Teachers**	**Tutors**
Presented in episodes, each with introduction, work on materials and sharing	Interacting with teacher, each other and materials or models of maths	Interacting with tutors, each other and models of pupils' maths	Interact with each other and models of teachers' work and wider contexts.

In respect of the **learning culture** within which the whole work is being carried out, including the prevailing values and attitudes to mathematics and learning, it may be presented as follows:



It is clear that we are in a situation where similar and nearly simultaneous processes of learning and development are happening in the classroom, at the teachers' centre, and amongst the tutors and researchers working in the field. We are all learners now, and do not have the onerous mantle of authority over anything or anybody. About time, thanks God!

Cognitive Acceleration Associates

1. The Buckinghamshire Primary CAME course for consultants and coordinators. This course is based on two full days per term over two years, interspersed with colleagues practicing in their classroom in collaboration with others. Each day of the course has slots for hands-on simulation, preparation, local classroom trial, and reflection, with a focus and pace dictated by the particular group. The scene in this article was in an Aylesbury school on 1st February 07. The cluster is one of several run by CAA tutors, see. CognitiveAssociates.co.uk. (CAME=Cognitive Acceleration in Mathematics Education)

2. The co-tutors of the course are the author, Gill Potter of Oxford Brookes University (OBU) and Cathy Tracy the Bucks Mathematics Advisor. Anne Price of OBU attended most sessions and offered academic support. All CAME courses fulfil requirements for a postgraduate certificate in advanced education practice at the OBU, equivalent to one third of a full Master degree.

A page from the past

Teaching calculation before the days of technology

Edward Sang (1805-1890) was born in Fife and became a writer on mathematical, mechanical, optical and actuarial topics. His Elementary Arithmetic, published in 1850, is obviously more advanced than such a title would indicate today. Florence Yeldham writes in 1936[1] :

that the book is interesting and refreshing. Sang must have loved numbers, and he was evidently an expert calculator. The majority of his readers, however, would not have concentration enough for such computing as he now and again recommends. To be appreciated the book should be read with a knowledge of the limitations of those before it rather than the extensions of those since. For instance we know well nowadays that counting and the elementary rules can be taught with material things, beans, nuts, acorns and the abacus. Again, where Sang could teach that, as a merchant has goods coming and going out of his warehouse all day, the details can be entered as they occur and all be collected at the end, we can use a plus (+) and a minus (-) sign and add and subtract the terms unsorted as they happen to fall. We can retain the point (.) in multiplication and division of decimals, and keep the digits of the multiplier in their original order in contracted work. But to see these and numerous other changes all made in one small book marks that book as refreshing. …

There are many strange connexions shown in numbers. Incidentally in the chapter on multiplication he builds up the multiples of 7, pushing each one two places to the right, and finds on addition the set of figures which recur in decimal equivalent of $\frac{1}{7}$.

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  14
    28
      56
        112
           224
             448
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142857142848
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Can you explain this?

1. Frances Yeldham, *The Teaching of Arithmetic through Four Hundred Years (1535-1935)* George Harrap, 1936,

Interdisciplinary Lesson Planning: Integrating Special Education and Mathematics Content

Joanna Caniglia & John Palladino reveal some of the beliefs with which students approach special needs teaching in the United States

The purpose of this study was to investigate the nature and extent of secondary mathematics pre-service teachers' beliefs and knowledge of the need for and types of accommodations necessary for students with disabilities. In order to address the challenges created by legislation, an interdisciplinary team of special education and mathematics methods faculty at a large Midwestern American university created a case-study project. Qualitative data from interviews and essays showed that although future mathematics teachers have a knowledge of general instructional trategies, they lack skill in connecting specific accommodations to disabilities. Although pre-service teachers used a variety of mathematical representations, they failed to translate among and between representations. In addition, it was discovered that many pre-service teachers held strong beliefs about the nature of effective instruction for students with disabilities, the role of caregivers, and the implication of differentiated learning.

With the implementation of the Individuals with Disabilities Education Improvement Act (IDEIA, 2004), and the No Child Left Behind (NCLB)[1] legislation, students with disabilities are not only to participate in the general education curriculum, but must also be assessed as well. Meaningful understanding of mathematics has become a gatekeeper to student success.

Through the efforts of the National Council of Teachers of Mathematics[2], the mathematics curriculum has changed dramatically over the past 30 years from a back- to basic procedural approach to one that emphasizes conceptual understanding.

The National Research Council (NRC)[3] synthesizes the types of competencies necessary to become proficient in mathematics. They include conceptual, procedural, strategic competence (problem solving), adaptive logical reasoning, and productive disposition (value of mathematics). Most states have adopted standards and benchmarks that reflect these documents.

The challenge for teachers is to provide meaningful mathematics instruction consistent with these standards and reforms set by state and national organizations while meeting the needs of students with disabilities. To help prepare pre-service teachers to meet this challenge, a team of mathematics content and special education faculty created and implemented an interdisciplinary, case study assignment to assist pre-service teachers as they search for accommodations and tasks that will make mathematics accessible to all students.

"James" had repeated third grade and was having increasingly significant problems with inattention, over-activity, and impulsivity

This article describes pre-service teachers' responses and beliefs to meeting the needs of a case study "student," James, with attention deficit disorder (combined type), oppositional defiant disorder, and with reading and mathematics deficiencies.

While the laws cited above require teachers to use evidence-based practices in their teaching, special educators and researchers have not yet identified criteria for evidence-based practice nor whether special education has a solid base of evidence-based practices[4]. In addition, the teaching strategies that have been researched are difficult for teachers to access. These difficulties are precisely the challenges that the mathematics and special education faculty desired that their students encounter as they identify best practices for "James."

METHOD

The Case

The purpose of the case-based activity was for students to collaborate with a colleague or colleagues to discuss and determine specific strategies they would use to instruct a middle school student with special needs. The case[18] describes "James," a 12-year old African American male living with his 67-year old maternal grandmother. He had repeated third grade and at the time of referral was having increasingly significant problems with inattention, over-activity, and impulsivity in the form of disruptive behavior and talking. James was below grade level in both reading and math, and according to the teacher, was noted to have problems with short-term memory and following directions.

Based on the identified problems and need, the following recommendations were made regarding treatment interventions[19]:

1. A trial on stimulant medication, such as methylphenidate, to address target symptoms of inattentiveness, distractibility, impulsivity, poor personal organization, and over-activity.
2. Short-term individual therapy to improve self-monitoring and self-efficacy.
3. A High Risk Intervention worker for the home on a weekly basis to help with improving problem-solving and effective communication between James and his grandmother, and to aid the grandmother in development of more effective parenting skills.
4. A classroom program of clearly defined behavioral expectations and systematic reinforcements for positive responses.
5. An Individual Educational Program written by the Learning Assessment Team according to IDEA process and guidelines.

The assignment involved teaming a special education and mathematics major to develop a response to #4 above.
At the conclusion of meetings, each team is to type a two-page (single spaced) report that details:

1. Physical accommodations for James.
2. Instructional materials that you will use to teach James about percentages.
3. Types of lesson modifications that could be used during each of the units' instruction.
4. Types of activities ensuring transitional needs.
5. Assessment accommodations.
6. The role of the special education and mathematics teacher.

Participants

Thirty-four pre-service mathematics teachers from two mathematics methods classes participated in the study. Fourteen teachers are undergraduates with twenty teachers as post-baccalaureates. The participants are students in one of the nation's largest producers of professional educators with a long history of teacher education. This Midwestern metropolitan University with an enrollment of 25,000 annually produces slightly more than 1% of the nation's professional educators. As such, this University requires one course in special education for general education pre-service teachers and one methods in secondary mathematics course.

Analysis

The data for the study were collected from student essays and interviews. The analysis of the data involved three phases. In the initial phase that occurred while math and special education pre-service teachers met, the authors attempted to understand and characterize the pre-service teachers' strategies and conceptualization of the case by noting comments, questions, and concerns from pre-service teachers. At the close of this meeting, the authors conducted a two-phased analysis[19] based on these characterizations. In the first phase, any essay that recommended instructional accommodations was identified and categories of these data were developed. Then, during a second phase of analysis, the authors examined the categories to delineate overlapping categories and to combine non-distinct ones. Comments from brief and informal interviews added clarity to comments.

Qualitative Solutions and Research (QSR) Software[20] was used as a tool for content analysis, enabling the authors to locate information for cross-analysis of cases with ease. Using the constant comparative method ensured that characteristics of pre-service teachers' beliefs emerged. "Comparing as many differences and similarities in data as possible...tends to force the analyst to generate categories, their properties and their interrelations as he (she) tries to understand his (her) data"[21]. The authors independently read and re-read essays and interview transcripts to identify common themes and styles. The themes were compared and a comprehensive list developed.

RESULTS

A frequency of accommodations pre-service teachers described in their essays and during class discussions provide a sense of the types of strategies they intend to use as teachers. No list was presented to mathematics pre-service teachers before the case-study activity. Math pre-service teachers used their special education colleague and a research article on effective instruction for special needs students in mathematics to prepare an instructional plan for James.

Using the essays and interviews as data sources, seven categories of specific strategies and approaches that pre-service teachers would utilize as they teach emerged. An additional category on beliefs toward teaching special education students was found from their comments about the assignment. Each will be will be described with excerpts from students' teaching.

Physical Accommodations

All pre-service teachers agreed that James' seat is to be placed away from distractions such as doors and windows (n=34). Seven students agreed that he should be seated near the teacher or close enough that a special sign/symbol could help James and the teacher communicate without any other student needing to know. Most of the students (27) wanted James to be seated in the front of the class. However, they differed on whether James should decide to sit with friends (5) or to sit in a small group of students who would be a positive influence both academically and socially (8). Based on the case description, James' social skills were positive. Although pre-service teachers recommended that James be able to pass out papers, choose his own groups, or have the freedom to roam the classroom at appropriate times (10) they did not connect James' social skills with their accommodations. A buddy system (with a student who is more capable) was the most often cited configuration (6). Differences in future teacher approaches are evident from responses:

Seat James in the front of the class. Away from windows or doors to eliminate any outside distractions. Seat him by his peers that he likes. (Pre-service Teacher #30).

Sit closer to the teacher, Eliminate distractions. Get a study partner (Pre-service Teacher #1).

Other physical accommodations (mentioned only once) included: silly putty or stress balls to release stress, take down all distractions around the room, insist on tables versus desks in the classroom, and to get tutors to help James (this was not supported in the assignment).

Content Accommodations

Pre-service teachers used a variety of mathematical representations to express percents:

Money (7)
Fraction Circles (4)
Time
Sports Data
Flash Cards
Real Life: Tips, Sports, Taxes, Voting Food, Coupons(18)
Pie Chart (6)
Area Models (3)
Calculator (13)
Colored Paper Plates (1)
Folding paper
Number lines (3)
Multiplication Tables
Mental Math (1)
Manipulatives (7)
Finger Math. Touch Math (2)
Hershey Bars (2), M&M's (5) and Pizza pie (2)
Interactive websites (5)
Grid paper /Hundreds Chart (4)
Sheet with Percentages and Decimals (5)
Human Percentages (1)
Verbal (2)
Class Store Ideas
Tiles (2)

The ability to translate among and between the various expressions of percents is non-existent in pre-service teachers' essays

Although many mathematical representations were suggested, very few pre-service teachers made connections among and between these representations. The ability to translate among and between the various expressions of percents is non-existent in pre-service teachers' essays. For example, students may have used pie charts or manipulatives to demonstrate the concept of percents but did not suggest how to relate that understanding to symbolic or real-life experiences. Overall, all suggested what materials they would use, but none described specific explanations and connections.

It is not only imperative that students learn to translate among representations but to work within the same representation. For example, some percents can be expressed as circle graphs and others as hundred squares. To some children, these are very different concepts. This was apparent when pre-service teachers (n=6) suggested James learn percents through meaningful real life experiences by "going shopping with his grandmother." They also suggested finding gratuity and sales tax. Although these are essential life skills, there is a lack of interest or relevancy to a 7th grader as two pre-service teachers describe:

- Understand percentages for real life…Such as pay check deductions, tips at a restaurant, sale price, taxes and government taxes.
- Real life calculations such as sales tax, calculation of gratuities, estimating time,interpreting some statistics.

In addition to their lack of connections, pre-service teachers were vague in terms of what specifically would be helpful:

- James will use as many manipulatives as possible. The more hands-on he gets the better. If there are virtual manipulatives, even better (FT#13).
- Instructional materials should reinforce concrete learning. There should be a lot of visual aides that capture attention. If the attention of James can be captured, it is certain that the attention of all other students will be also (FT#20)

there are going to be 20 individuals in each class and each individual will have their own way of learning

Role of Caregiver

According to the pre-service teachers' essays and interviews, students saw the grandmother as a possible helpful ally, but many spoke in a condescending tone (n=14). They often believed that they needed to convert her thinking about James.

- James' grandmother also needs an advisor or social worker to help her brainstorm and develop certain interventions for James while he is at home. Her role is of REINFORCER (Future Teacher #30).
- Change her mind. The teachers will then send a letter home each Monday and Friday (Future Teacher, #28).

Seven students saw the role of the grandmother as part of a team to help James. She was perceived to be an integral part in James' progress as the following quote:

- The teacher should approach these conversations with the clear intent that the teachers and the grandmother must be a team in helping James to succeed with both parts equally important in the process.

Three students included impressions or recommendations for the grandmother which were never included in the case study. These students assumed that the grandmother had little or no education and that she was unemployed. As the following students concluded:

- James' grandmother would be asked to help. The sister would also help. Grandmother would help with REAL LIFE transition skills.

Role of Mathematics and Special Education Faculty

Pre-service teachers expressed two types of roles for the special education and mathematics teachers. Four students felt that a collaborative model should be adopted by math teachers and special education teachers.
Two such comments include:

- I would use a collaborative model. Every person needs to be a part of our team. Expectations of grandmother, special education, and math teacher are to be closely linked.
- The two teachers will meet outside of classes. Both will work together to develop his assessment strategies and teaching plan.

Most of the pre-service teachers (n=10) believed that the roles of the special education and mathematics teachers are different. In particular, they see the mathematics teachers' role as one of conductor and special education as consultant. Roles also separate along the lines of content and strategies. Math teachers are perceived to be responsible for delivering content, while special education teachers are to assist in developing strategies and accommodations.

- Math teacher: gather all ideas and choose. The math teacher decides which ideas to use in the classroom. Set up a self-monitoring system for James. Special education student is to meet with a therapist for the suggested short-term therapy to improve self-monitoring and self-efficacy. The special education teacher should also act as a continuing consultant for Jamesand teacher. The special education teachers should come into the math class for visits to monitor any progress and make changes when necessary.
- Math: Find a way in which to best instruct and assess James. Special Education Role: To assist the math teacher in exploring successful teaching strategies, as well as keeping the math teacher informed about anything that might be affecting James performance in themath classroom.

Definitions of "Fairness"

Two pre-service mathematics teachers demonstrate the differences in their definitions of the concept of "equality."

- The problem I am having is I need tohelp this student, but I can't deny the help from the other students. I need to give them much of my time too. I am also having trouble because if I make modifications for this student is it fair to another student who is failing my class. I am really having a tough time with this case study.
- It seems like a great deal of work for one child, but if this helps James to succeed, then it is well worth it. I think the effort alone put forth for him could raise his self-esteem. The only thing that seems for sure is that you have to keep trying different ideas until one of them works. This is a profession where there are going to be 20 individuals in each class and each individual will have their own way of learning and dealing with their environment.

you have to keep trying different ideas until one of them works.

Discussion

The challenge for teachers is to provide effective mathematics instruction for all students. To assist pre-service teachers in becoming aware of accommodations and to utilize strategies to teach mathematics is central to the No Child Left Behind and IDEA legislation. Through data obtained from essays and interviews, pre-service mathematics teachers exhibit knowledge of many accommodations and strategies. However, their list of accommodations lacks connections among representations and often is unrealistic.

Although the theoretical knowledge that mathematics pre-service teachers exhibit is consistent with the developmental nature of novice teachers (RESOURCE), the beliefs of pre-service teachers in respect to special needs populations is relatively unknown. The data discussed in this article uncovers beliefs that are not only contrary to recommendations by the NCTM, but also will not meet the needs of students.

Three such beliefs —attitudes toward caregivers, the roles of mathematics and special educators and definitions of "fairness" give pause and resolve to mathematics and special education methods faculty. The majority of pre-service teachers view the roles of mathematics and special educators as separate versus a team and collaborative approach. This lack of colleagal cooperation extends to caregivers as well. Pre-service teachers made assumptions about caregivers and exhibited a condescending nature to their recommendations. Most disturbing was the belief that "fairness" in teaching translates to pedagogical and content strategies that are applied consistently to all students.

The results give reason for teacher educators to give pre-service teachers extensive field experiences to not only balance theoretical knowledge with realistic experience but also to provide curricula that translate among representations. Opportunities to confront and challenge beliefs that are contrary to NCTM recommendations and Special Education legislation are essential.

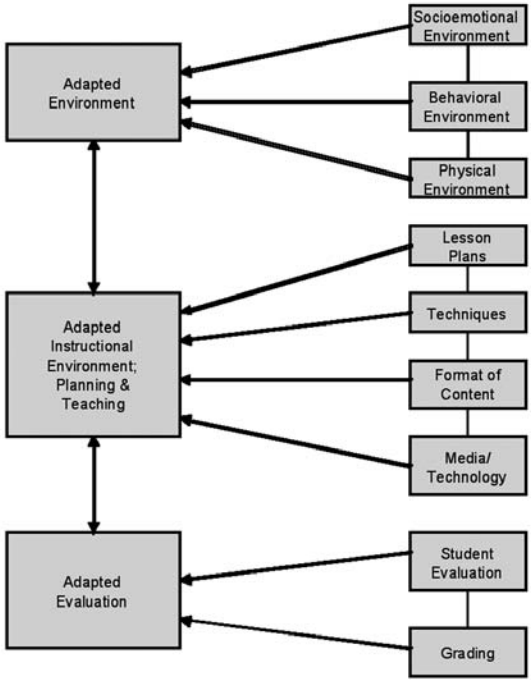
The data are disturbing. As Pajares[22] indicated, "the earlier a belief is incorporated into the belief structure, the more difficult it is to alter. Newly acquired beliefs are more vulnerable to change" (p. 325). The beliefs these pre-service teachers hold have not been tested in the classroom. This research is only the first step in evaluating the preparation of pre-service teachers with respect to special education. During future research, the teachers are supported through mentoring and coaching on their classrooms and through building a collegial community of beginning teachers. Only then will the success of the program be determined.

REVIEW OF RELATED LITERATURE

Relevant to this project, the review of the literature will examine how research has informed practice in two domains: general instructional strategies and mathematics curriculum adaptations for special needs students.

General Instructional Strategies

The interdisciplinary project was based on the validated SAALE (Systemic Approach for Adapting the Learning Environment) model[5]. Although there are numerous models for inclusive instruction[6,7] SAALE best adheres to the strengths and needs of pre-service teacher education preparation as discussed in collaborative meetings throughout the formation of this project. The following illustration conceptualizes the SAALE model:



Source: Wood, J. (2002). *Adapting instruction to accommodate students in inclusive settings*. Upper Saddle River, NJ: Pearson Education, INC.

The model emphasizes that areas of accommodations/ modifications (e.g., format of content, grading) must match individual/group learners' area or areas of adapted need (environment, instruction, evaluation). Thus, teacher education classes in both mathematics and special education must emphasize the triangulation of (a) specific research-based accommodations and modifications (e.g., direct instruction, stratified lesson planning, and differentiated learning), (b) individual/group learner needs, and (c) mathematical content standards:

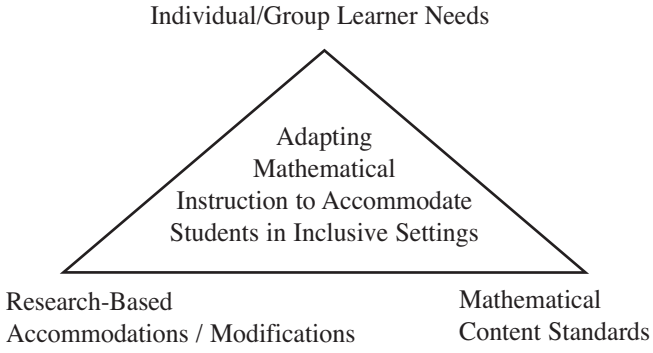


Figure 2. Essential elements in Special Education and Mathematics Methods Instruction.

With the intention that pre-service mathematics teachers will expand their knowledge about and ability to implement a variety of accommodations and modifications, the assignment required students to apply this knowledge to assess individual/group learner needs. Jorgensen [8] reported that effective instructional planning for students with and without disabilities must address questions that include: What is the central problem, or question? What will interest students? What are the students' learning experiences? What assessment approaches can be used? This process applies to a variety of mathematical delivery models for diverse learners that include: inclusive classrooms, self-contained special education programs, team-taught classrooms, and special education consultation delivery models. It was most applicable to the case and most situations that pre-service teachers will encounter as they teach.

Mathematical Curriculum Adaptations

In mathematics education for students with learning disabilities, a significant amount of research focuses on direct instruction emphasizing fact- and basic-skill attainment[9]. According to the U.S. Department of Education, Office of Special Education Programs, and the National Research Council[10,11] what is needed is a synthesis of many types of knowledge to become mathematically proficient, including conceptual, procedural, strategic competence, adaptive reasoning, and productive disposition.

Hiebert and Carpenter[12] have emphasized meaningful teaching and learning in mathematics. Knowing about mathematics means that the special education student comprehends the basic principles of a mathematics problem, knows there is more than one way to explain the mathematics of the problem, and knows that there is frequently more than one acceptable answer[13,14].

This is in contrast to procedural knowledge in mathematics, which means the student can apply a number of different strategies and mathematics principles to complete an item.

Although research in mathematics for special education students emphasizing conceptual knowledge is sparse and most research concentrates on direct teaching with specific instructions, common ground appears to be in the construct of representation theory[15,16].

Mathematics is often used to represent the world in which we live. Representations can be viewed as the facilitators that enable linkages between the real world and the mathematical world. For special education students this is a critical skill. It is therefore essential that pre-service teachers not only understand different representations, but also have a sense of how students translate among representation modes[17]. The translation process among representations offers flexibility and completeness in students' understanding of concepts. Figure 3, depicts Lesh's model of translation. Recognizing specific and appropriate representations and connecting them to others is a necessary process in understanding mathematics.

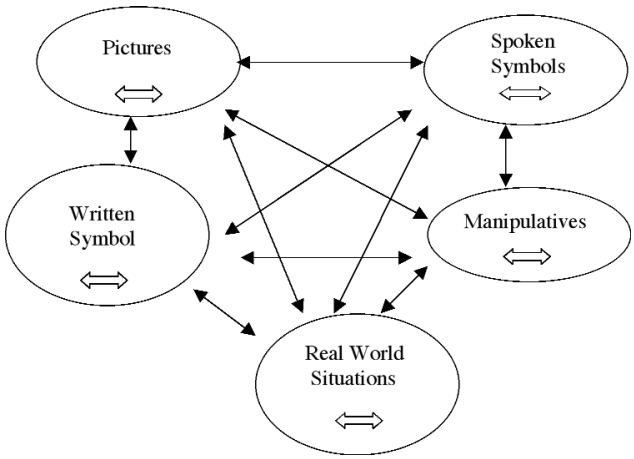


Figure 3. Lesh's model of Representations[15]

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Reviews

A review of
Children's Mathematics
Making Marks, Making Meaning
By Elizabeth Carruthers and Maulfry Worthington
ISBN 1-4129-2283-6
Published by SAGE Publications
www.sagepublications.com

This book is very readable and thought-provoking. It should be read by all early years teachers of mathematics. I suspect it will make you examine your current practice and consider ways to incorporate the ideas which are laid out clearly in the chapters.

My first impressions were not good as the beginning of the book looks at findings the authors noted solely in their own children - the instances of maths taking place in the home and the thinking and language that sprung from them. I couldn't imagine that these episodes were representative of what goes on in households across the land.

However the subsequent chapters deal with data and research that took place in various settings with groups of children and the authors cleverly conclude each chapter with a review of what has just been read and a taster of what is to come. There is an abundance of examples of mark-making in various settings and their research covers a wide age span.

I was fascinated by the comparison with teachers' enthusiasm over children who make marks in English writing compared with their lack of interest in mark making in maths. The authors clearly show how mark making can be incorporated into everyday situations in nursery/school settings and how experiences of role play can shape children's mathematical understanding.

Most apparent was the teachers' interest in what the child had written and how this must be valued by all concerned with the child's development. Allowing children to discuss what they have written gives teachers more insight into a child's understanding than they will get by asking them to select the correct answers on worksheets. Mark making allows children to think and experiment, which helps them towards a better mathematical understanding.

One of the most interesting chapters dealt with teachers modelling how to write calculations. From their research the authors discovered that children often found that this modelling restricted their own thinking. The children believed that the teacher was showing the correct way and that they should therefore write their calculations in exactly the same format. Many of the children in the study found this conflicted with their own thoughts on how to show their thinking. This stopped them getting the correct answer. Whilst there is an obvious need for modelling standard mathematical calculations this should not come so early that children don't develop their reasoning. They should be able to make their own marks and explain their thinking.

The authors also show how including other members of staff and parents has a beneficial effect on the children. When everyone is clear about the objectives it becomes easier for children to talk about their marks and for parents to see mathematical meaning and development.

The final chapter deals very helpfully with the frequently asked questions that are asked in settings where mark making is encouraged. These are from other teachers, heads, parents and the children themselves.

If you encourage emergent writers in English then this is a must read for you. Let's all encourage them to make marks in maths too.

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Answers from page 9:

Activity one - *Reception*
Activity two - *Year 2*