Equals

Realising potential in mathematics for all

for ages 3 to 18+

ISSN 1465-1254

Vol.13 No.1

MATHEMATICAL ASSOCIATION

supporting mathematics in education

Equals

Realising potential in mathematics for all

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Designed by Nicole Lane
Printed by GPS Ltd. Unit 9, Shakespeare Industrial Estate, Watford WD2 5HD

Editorial

Some of the articles in this issue of Equals demonstrate the importance of mathematics in helping us to understand the universe in which we find ourselves. Mundher Adhami calls the search for pattern in the world around us ‘mathematising nature’. If we are to encourage our pupils to understand anything about the chief problem facing the human race today - climate change – we have to familiarise them with the language of number and measure and help them to interpret the information about what is happening to our planet with which they are being bombarded. All this information is expressed in terms of numbers. Do we understand what those numbers mean? Some of them occur in measures – a 2° increase of temperature – others are comparisons – the summer ice in the Arctic has shrunk by 25%.

Geometry of course is important too when we are analysing our surroundings, as Robert Hooke’s images and words remind us in our centre spread activity. Not many children today see the patterns of frost on window panes which regularly delighted those of us who are older in our childhood. Here is a chance to study the beauty of the snowflake as Hooke saw it so long ago and analyse its symmetries. Because of this example of geometry from the natural world in winter, we have left the second half of Gerry Rosen’s more theoretical study of geometry to *Equals* 13.2.

As usual, beside looking at mathematics, we also invite you to consider what characteristics the classroom needs to become a good learning environment. Certainly an atmosphere of trust is the first requirement and, as you will see from ’Standing in front of the class,’ at least one member of the team has been disturbed by hints that the requisite atmosphere of trust - in which children are free to think and develop in their own individual ways - is disappearing. The next essential requirement for steady progress is of course a detailed assessment of where each child is and we consider the final part of Dylan Wiliam’s paper essential reading for all teachers if they are to help children make progress mathematically. Richard Cowan stresses the value of delving into the reasons behind mistakes which might be considered a part of assessment but one we sometimes miss out and Colin Foster then takes us on to the next pertinent question: when should we intervene to help children out of their difficulties? It is no easy job trying to be a good teacher, especially if our pupils lack confidence. The little confidence they do have can so easily be destroyed, especially if mathematics is seen as a discipline where there is only one right answer to a question and only one right way of getting there.

We therefore hope that you will join in the discussion that some members of the *Equals* team have started concerning questions in education about which there is serious concern today. Writing will help you to clarify your thoughts and it will help the rest of us to extend our thoughts further. There is much more in the following pages which we think will help you improve your teaching but we will leave you to find it for yourselves.

Finally, we must apologise to Jennie Pennant for mistakes in 12.3 with the presentation of her useful recommendations for extending the mathematical education of teaching assistants and recommend you all to take note of the amended version which appears in this issue.

Some Current Concerns

At a recent *Equals* meeting members of the editorial team expressed some of their current concerns. Some of those opinions are recorded below. We hope this is the start of an ongoing discussion that will continue in the pages of *Equals* in which both editors and readers take part.

Who should do more mathematics in the lessons: the teacher or the pupil?

Unless pupils are engaged in mathematics, they will not be learning. Recent observations show that teachers are so keen on ‘scaffolding’ children’s learning that there is a tendency not to let them struggle with mathematics, but to give them the answer.

This is leading to a generation of pupils who demand spoon-feeding: 'Just tell us the answer, Miss' rather than being prepared to have a go at something they initially feel is too difficult. This in turn leads to more of the same and a vicious circle ensues. Of course those who have been involved in CAME type approaches have a very different style in the classroom – and not just in CAME lessons – the way of teaching transfers to other lessons.

In the best of lessons all pupils should be doing mathematics all of the time, which is more than the teacher can manage because pupils and resources have to be dealt with: behaviour has to be checked, groups have to be set up, resources have to be managed. But, once a regular routine has been established between a teacher and a class, the teacher should make herself as free as possible to do mathematics with individuals or groups for the majority of the time available. Most of the organisation can be taken on by the pupils themselves.

Is the three-part lesson a good idea?

It is only a good idea if the three parts are of the same activity. In such a case the first part would be an interactive introduction and clarification, the second part independent work by pupils, and the third part a sharing and refining of the ideas generated. Apparently much of current practice the 3-parts lesson has degenerated into a ritual in which the three parts do not link into a hierarchy of progression or challenge.

I think this is a red herring. There's nothing intrinsically wrong with a 3-part lesson meaning there is a starter activity, a main part of the lesson and a plenary. The difficulty comes when this is interpreted as a formula and teachers feel that the structure is more important than the content or the pedagogy. I don't necessary agree with Mundher that all 3 parts have to be related; I think it is perfectly legitimate to have a starter which reinforces or rehearses or revises something previously learnt, or indeed is used as an assessment activity to determine the starting point for something to be covered in the future.

To me the three-part lesson implies a whole-class approach focussing on teaching rather than learning. As soon as you have more than one pupil in your class you have a mixed-achievement group - which implies a variety of different starting points and ways of approaching ideas. Should these all be forced along the same pathway? A common starting point is possible but should the teacher be encouraging uniformity rather than finding the most appropriate path for each individual? And if the latter, what organisation encourages this? Does the fear of mathematics arise from a knowledge, at least in the early stages of arithmetic, that there is a unique right answer and that you may be the only one in the class not to reach it?

What do you think of the Unit Plans in the strategy?

Less experienced teachers do need good lesson materials to rely on, whether for whole class teaching or for groups. We cannot expect all teachers to produce good lessons (or even valid, or even not-harmful!) in all topics and levels. So we do need good texts books and edited materials. The problems arise if the approach to devising lessons is too rigid e.g. 'telling and practice', or reactionary - relying more on top-down curriculum analysis rather than bottom up construction from experience. So the Unit Plans can be an obstacle to the professionalization of teachers, while actually still useful for some. This is not an ambiguous attitude, rather it is recognising differentiation amongst teachers.

Some people believe that there is place for the neo-behaviourist schemes such ABA. What do you think?

Differentiation amongst children requires many of them to be trained or taught in behaviourist ways at times. But we are in trouble if we confine our teaching to these. Even training children to recognise symbols and follow procedures can be accompanied by slots to talk about why we do name things in this particular way, how to make distinctions, how to link the steps, and how some things are similar or difference. Of course even making such connections can be achieved in semi-behaviourist ways, but constructivism is a difficult route in teaching and learning, so half-way houses are welcome.

I think we do need a debate about the relationship between behaviourism (training) and constructivism (learning through experience and build up), including their meanings. In simplistic terms I think training is essential whenever the learner is clearly able to assimilate or learn things that have been missed out in their upbringing, or that are conventions or mere tools, and unrelated to essential cognitive systems. Learning use of the keyboard or the computer is an example.

Compare that to finding percentages and using graphs, which need careful building up and connections. Otherwise some learning can be harmful. Through things being memorised in various desperate and confusing ways The debate will eventually go to whether learning the tables by rote is a good idea, in which I would ask: what does 'by rote' mean?

What differences, if any, there are between delivering and conducting a lesson?

I would maintain that a lesson cannot be delivered. A lecture can be delivered although it is questionable what its hearers gain from it. If they are listening they will assuredly gain something, although it is unlikely to be all that the lecturer intended. If the lecturer is not a good deliverer that is understandable but even if the lecturer is a good deliverer it is still unlikely that the audience will listen throughout because any particularly good thoughts the lecturer may impart will be grabbed by listeners and then chewed over so that they miss the next few points and when their minds come back to the lecture hall the lecture may have gone on to a point where they can no longer pick up the thread. At the very least they will have missed several of the lecturers well prepared pearls of wisdom.

On the other hand you can conduct a lesson much as someone conducts an orchestra, ensuring that all in what equates to the orchestra pit are making or doing mathematics and that each member of the class makes mathematics in a way that blends with what mathematics others are making. Back to music, if the music is of the non-classical variety music-makers have plenty of opportunity to add their own variations to the music being played. This should have its equivalent in mathematics surely.

What does Variety in mathematics lessons mean?

Essentially a variety of types of approach i.e. some whole group work, some small group or pair work, some independent work. Some demonstration by the teacher, some feedback from the pupils. A variety of open and closed questions, some targeted at individuals, some at the whole class using individual white boards or some similar approach. Some activities which are essentially oral/aural, some which involve putting pen to paper and some which involve activity either at a table moving things around e.g. dominoes, jigsaw puzzles, or whole body activities such as locus using the class members as points on a locus or as co-ordinate points.

To what extent should the teacher decide the aim of the lesson?

It depends on the skills of the teacher. When observing mathematics education many, many years ago in the States I had many conversations with Oscar Schaaf, an education lecturer At Oregon State University. Oscar maintained that the teacher should be so thoroughly prepared for a lesson that no mathematics could crop up in that lesson that had not have been foreseen. I did not agree that this was possible but have since reached the conclusion that American teachers in those days were so ill-prepared mathematically that they could only cope with the mathematics for which they had prepared themselves and therefore had to dictate the mathematical aims of the lesson, whereas at least some teachers in this country had sufficient mathematical background to be able to tackle whatever mathematics a pupil might bring up.

I came across some transcripts from discussions with SMILE pupils at N Westminster in 1985 which seem relevant to the question of who decides what in lessons. Here's a quote:

I think it's great because the teachers like they trust you, and you learn to depend on yourself, because the teacher's not going to say anything, he's not going to say anything to you, so you're cheating yourself and you have like, you're marking it and you have to use your conscience and from the time you're 11 you'll learn that you have to be fair, because like it's your future, and the teacher all he's going to do is guide you, he's not going to sit there and tell you, look you've got this right, you got this wrong, there you see, look I've made mistakes and you see for yourself the things you've done wrong and so the next card you do it better, so you learn that you know, you can't cheat yourself

A fair society?

One in 10 of those who draw job–seeker's allowance has spent six of the past seven years on benefits
No night buses - a taxi costs three hours' work at minimum wage
Only 10,000 are hard cases and the job-seeker's allowance is a pathetic £57.45 a week, not enough to survive on.
The real value of that £57.45 has halved since 1979: it's now worth 10% of the average wage.
The city reaps £9bn bonuses
Polly Toynbee, *The Guardian*, 19.1206

Mathematising Nature, in imagination

A new Thinking Maths lesson is currently making the rounds of trials in the classroom. **Mundher Adhami** shows how it helps pupils to make sense of the world mathematically with intuitive reasoning in measurements, data handling and variation.

Place yourself in a Y4 or Y9 class or any other. You are asking the pupils to imagine the Lake District or Switzerland or some other nice place they know. You list the things they saw or can imagine focusing on nature: water, trees, lake, mountains, cliffs, birds, boats, climbing, cable cars, and so on.

Some odd, idiosyncratic or made-up offerings by children add to a pleasurable few minutes of the start of the lesson. Everyone is engaged at their level, gradually focusing their attention, all thinking about an imaginary place with some common features. You tell them the lesson is about linking a few of these features: water and mountains, birds and trees. You may then produce a picture, OHP or poster of a place with a prominent water and mountains with some relief in between. Get them to accept that is just an example.

A whole class more mathematical episode follows.

Draw a straight line on the picture from the water surface to the mountain top, then a sketch of a rough profile that shows a few features they can recognise. (A diagonal line with lake shore on the bottom left would make a better fit with the sketch given below for the independent activity). You ask some children what different points on your sketch would refer to on the picture. They are now mentally moving from a real life 3D space to a 2D profile, with some correspondence of positions, distances and heights. You write these three words, (position, distance and height), on the board and discuss them briefly with the class.

The pupils are now ready to look at other natural features linked to them.

They can imagine some wild–life volunteers like themselves walking up the side of the mountain on a route following the line. They want to suggest rest places for ramblers and what to do for younger and older people, and make a record of what they see.

They walked from the lake to the top, recording their observations.

Here is a rough ***profile*** of a different but similar mountain, not accurate, and some of the data they have collected under six headings (variables).

Discuss with the class some or all of the 6 features and numbers below according to their age and levels of understanding or interest. Ask questions like:

- What does Horizontal Distance mean and how is it different from actual distance?
- What does time of walk mean and how to record it so that it is always measured from the lake non-stop?
- How do we know the height of points on the profile?
- What can bird variety mean? What variety is there is in their own town or estate? What kinds of birds do they normally expect where they live?
- What bout the trees? Should we count shrubs or just the ones with some trunks?
- What high trees they know, and how high? Where do we expect high trees to be?

Pupils, by consensus, may want to change the heading of the feature keeping to the meaning, for example use ‘ How many types of Birds’ instead of ‘ Birds variety’. This type of changing of labels tends to empower pupils while not interrupting the logical flow of an activity.

Independent work episode:
Giving out the worksheet and some graph paper and rulers, ask pairs, or groups of pupils to choose any two of the six features. They are then to draw a graph or picture to show how these two features are related for the four positions A-D.

You may wish to simplify the choice:

- Either choose one from the first set of three features and another from the second set of three features.
- Or choose from a list on the board such as: distance-time; distance-bird variety; distance-tree variety, distance-highest trees; time-height; time-birds variety; time-tree variety, time-highest trees, height-time; height-bird variety; height-tree variety, height-highest trees.

You should spend time first explaining that the profile itself shows the distance vs. height graph, and that there is a different scale for each. They can discuss why there is need for different scales for the two features, e.g. 'to fit on page', 'clarity', 'you are forced to'. That is important since all graphs will need attention to different scales. Pupils would recognise that they must choose pairs of features other than the distance-height.

It is likely that many pupils will take time trying to figure out how to approach the problem, and you could offer advice or probing questions starting: what features you like? What is the biggest number and how to place it on paper.

Deciding on the scale is quite difficult for most pupils since it requires fitting the overall range of values in a feature with the available size of paper, then finding the unit. In the CAME approach this is what we call 'Concrete Generalisation level' which is typical of about level 5 in the Using and Applying strand of the National Curriculum. That is due to the need to fit an overview of a situation with its parts. But it is still accessible for most pupils with the help of the teacher, since each part is readily understood in practice. But a lot of help is needed to allow pupils to work on other, perhaps more important ideas and patterns.

Encourage pupils to make sketches and improve them to show on A3 with felt-tip pen, and not to worry too much about accuracy or neatness. They should realise that we are after ideas and patterns rather than grades or marks. With such an attitude the time for this episode could be reduced to about 10 or 15 minutes.

Sharing episode 1

Reviewing the results is more than 'show and tell', even though that is part of it that allows a sense of achievement for all. You could group the outcomes and talk about the choice of the scale in some cases again, and what each scale actually means. Then you switch to patterns:

- Is there a pattern in bird variety with height or distance? Why would that happen?
- Is there a pattern in tree height? And what may cause that?

This is an episode in which the pupils, having worked on a data handling task, interpret the results back in the real (in this case 'imaginary-realistic') world.

With some classes, in the same or in another session, more mathematics can be achieved using the same sketches. Many pupils would have chosen bar charts, and a few would have linked their tops. The move from bar chart to points that are linked with lines is worth lingering on. That allows the pupils to notice and talk about the top of the each bar, and how to link them to see the pattern. It is almost worth asking the pupils to imagine the bars fading away with only the tops remaining, and in fact switch further to the overall patterns of a the line graph, whether straight or smooth, and what it may mean. This is the move from discrete or separate pairs of numbers to continuous variation. For example pupils may estimate the height of trees in places between the positions where we know about them, and explain why that estimate is reasonable. A useful discussion can be had on why is that useful, and on the fact that most work in data handling is of this kind, even though we sometimes carry out exact calculations such as finding an exact middle number. In real life nothing is exact, but we use numbers to give an approximate picture of the world. All pupils should find ideas of 'reading from the pattern' empowering. No need for them to use terms of logic such as 'interpolation' and 'extrapolation'.

With older or more able classes

A second round or a teaching cycle on the same context and data could focus on one or more of the following lines:

- Looking at other pairs of features, to compare them with the first pair they themselves handled. For example pupils who had worked on distance v bird variety would look at distance v height-of-trees. The comparison is then of two separate graphs or using the graph but with different vertical scales.
- Where two features from the second set (bird variety, tree variety, and height of trees) are chosen a scatter graph may emerge, and some idea of a more indirect relationship can be discussed. In most cases the notion of correlation or a line of best fit need not be discussed. But discussion of the need for more data and what to expect can arise.

Since the context and data are imaginary but realistic, in many classes a creative episode can be attempted. For example 'what can the bird variety be if the tree variety is 10?'. Pupils would appreciate that while they cannot be sure, they can still say it should between 1 and 7 and most likely to be about 5 or 6.

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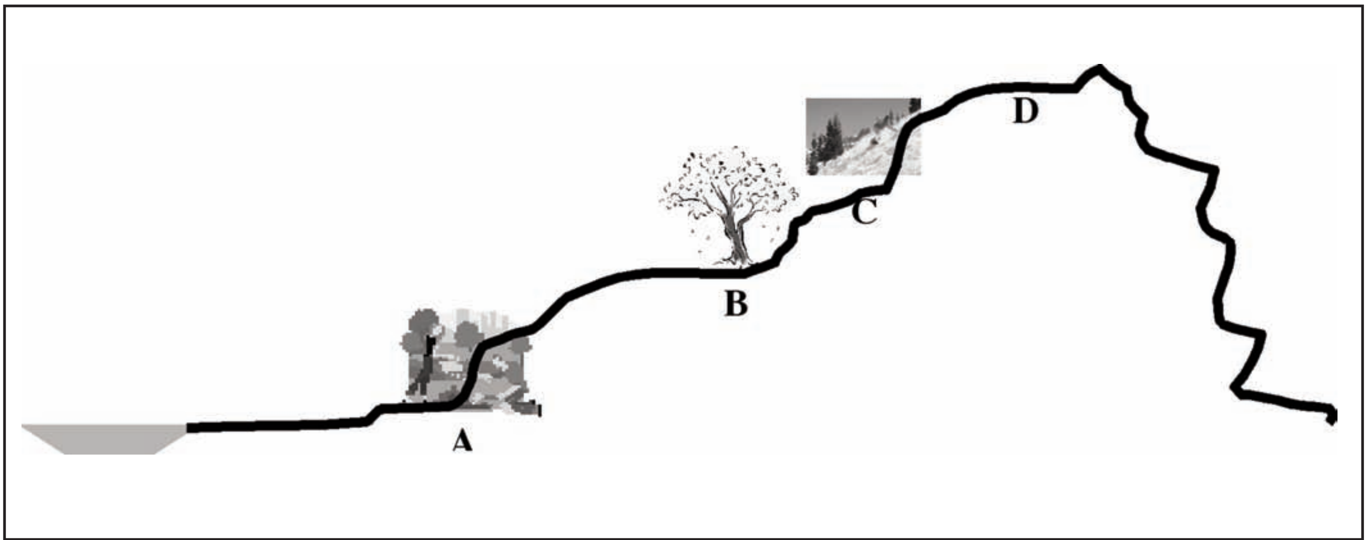
It should be clear from this that the imaginary-realistic context allows many variations. Teachers may think of other features, such as temperature, clarity of air or view or animals or how slippery the route is. They can even change in the profile, for example stretching it so that there is a depression between B and C, values for the feature that do not conform to the linear patterns would move the activity towards correlation.

It is not clear to us in CAME why we still regard activities, such as this one, which do not have identifiable cognitive challenge a Thinking Maths lesson. It is not a trivial labelling issue since we do need some clear distinction from good instruction lessons and good investigation lessons. Both these still allow pupils to work at their comfort level, while we are intent on making pupils work beyond them with teacher-mediated help from their peers. Perhaps it is the aspect of having a 'low floor' or 'access level' and a high roof or possibility of extension, so that it allows a wide range of ability to work at the same time in cycles of independent work and sharing. Perhaps it is allowing many chances for challenge according to different classes rather than one specific challenge.

Thinking lessons are always experiments for both the pupils and the teachers. May all teaching be like this.



Horizontal distance:
Start at lake, distance to A 1500 m, to B 3000m, to C 3600m and to D 4000m

Time of walk:
Start at lake; reach A in 20 minutes; B in 1h and 20 minutes, C in 2h, 50 minutes; and D in 4 h and 30 minutes

Height
Lake is taken as 0m; A is at 50m, B is at 600m; C is at 1100 m; and D is at 1500 m

Birds variety
The mix of birds changes gradually up the mountain. The greatest mix is at near the lake where 30 types nested. At the foothill A, 22 types nested, at B, 15 types nested, while at C, 7 types nested and there is only one type of birds nesting at D.

Trees variety
The mix of trees changes gradually over the route. 50 types were seen near the lake and the foothills at A, 24 types at B, 12 types at C and 6 types a the summit.

Highest trees
The height of the tallest trees gradually changes. This is about 4m near the lake, 6 at A, 13m at B, 20m at C and 3 metres at D.

Cognitive Acceleration Associates

Let's look further at 'seeing' and 'doing'

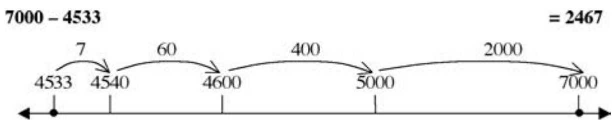
Tandi Clausen-May **expands on a review in *Equals* of her recent work on pupils' learning styles. She offers ideas on how visual images of mathematical tools such as the number line and a novel use of bead strings may relate to personal physical activity.**

Mundher Adhami puts his finger on a key issue in his review of *Teaching Maths to Pupils with Different Learning Styles* when he points out the danger that

> 'the new mathematical models we offer our pupils in the process [of] activity based teaching… may either be ignored or turned into procedures like others, whereas they should be used to allow further refinement and elaboration, by maintaining coherence.' (Adhami, 2006, p 24)

It is certainly true that any mathematical 'model to think with' may be used effectively – or it may be abused by being turned into a meaningless method for getting right answers. A number line, for example, may be reduced to just another routine, to be learnt by rote and followed blindly without any understanding of the meaning of each step. Used like this it will be no more helpful, and it will be considerably less tidy, than a numerical algorithm.

V,A,K – Visual, Auditory, Kinaesthetic – See, Hear, Do – whatever you call it – is just a convenient way to think about the mathematical activity that goes on from day to day in the classroom. Nearly all pupils can benefit from an approach that presents new concepts in different ways, so VAK offers a useful framework with which to classify the *activities* – not the pupils. A number line is just one example out of many.



But Mundher asks,
'Does it really need a number line to think, say, that "4533 add 7 makes it 4540, then 60 makes it 4600 then 400 then 2000, so…"?'

He questions whether a mental image of a number line is actually helpful – and, indeed, for many children, the string of squiggles and instructions may be perfectly meaningful and manageable as they stand. But others may lose the plot at the second digit of the first squiggle. The important numbers to hang on to here are the jumps – the 'add 7', 'then 60', 'then 400', and 'finally 2000'. The 4533, 4540, 4600, 5000 and 7000 are just stopping points on the route. Or if you want to go the other way, of course, you can – 'take 4000', then 'take 500', and so on. This gives different stopping points, but either way, the numbers to keep your eye on are the jumps. The number line gives me a picture in the mind to hang all these numbers onto, so I can keep track of what is going on. My mental image is crude, and not to scale – I have a sense of gathering speed, in fact, so that my first, tentative little hop is just a few units long, but then I am jumping in tens, leaping in hundreds, and finally flying in thousands. But I could not agree more that there is a danger that the mathematical model may be turned into yet another 'procedure' – meaningless, cumbersome, and eminently forgettable!

Interplay between the visual and the kinaesthetic
But my image of the number line is just that: my own mental picture, very personal and perhaps individual to me. As Mundher explains,

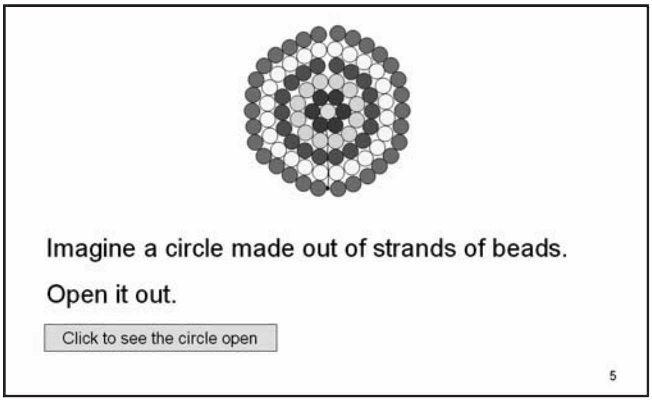
> 'unless what you see makes sense to you in terms of action that you yourself have done, or can easily imagine yourself doing, it is no different from any other phrase or symbol.' (pg 23)

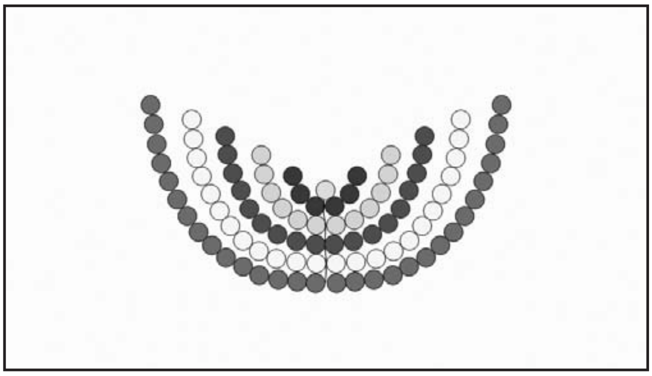
This interface between the visual and the kinaesthetic – the 'seeing' and the 'doing' – is one that we explore every time we ask our pupils to engage in an activity designed to develop their understanding of a mathematical concept. Whether we use the, perhaps fashionable, but long-established and easily grasped 'VAK' terminology, or the language of constructivism to describe what we do, our object is the same. We want to develop the links between the experienced activity and the mental images that offer the models to think with that pupils can use to support their mathematical understanding.

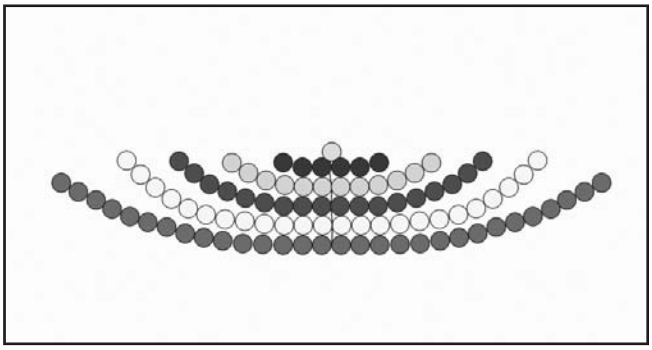
Certainly, direct action by the learner is sometimes essential.

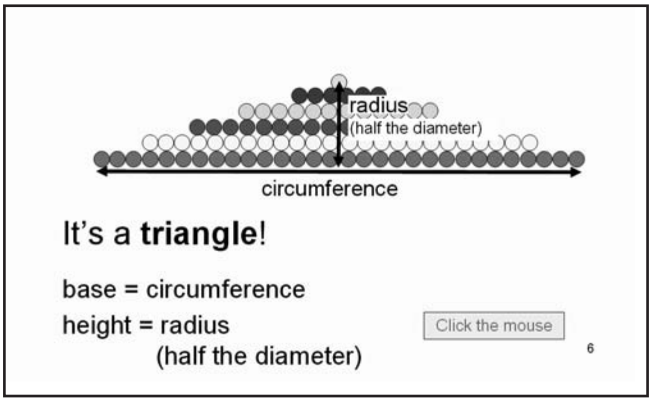
Children who have had extensive experience working with the Japanese Soroban, for example, can learn to work mentally, using an imaginary soroban in their heads to carry out complex calculations with astounding speed and accuracy – but their fingers twitch as the physical memory of the kinaesthetic experience with a real soroban supports their visualisation of the movement of the beads on the frame (Markarian et al, 2004, p13).

Similarly, a model for the area of a circle as half the circumference multiplied by the radius that is based on a set of concentric circles of beads does require the learner to have handled a string of beads at some point in their lives. On the other hand, the beads do not have to be present when the model is explained – a dynamic 'movie' can be shown on a computer screen, and can be understood through the visual, rather than a kinaesthetic, experience. (Clausen-May, 2006, pp 42-44)









What these dynamic visual images offer, though, is not a formula, to be learnt by rote, and then forgotten. It is not a set of symbols: it is a moving picture in the mind, from which an effective approach to the problem of finding the area – first find the circumference, then multiply that by the radius and divide by 2 – may be based. But the pupil does not try to remember a technique: rather, they remember the dynamic model, and then work out the method directly from that. This, I think – and I hope Mundher would agree – is what 'seeing' and 'doing' mathematics is all about!

NFER

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One in 10 teenage girls harms herself each year, … according to a survey of 6,000 15 and 16-year-old pupils in Oxfordshire, Northamptonshire and Birmingham. Girls are four times more likely than boys to harm themselves, with 11 per cent of girls and 3 per cent of boys reporting they had done so in the previous year.

TES 25 August 2006 quoting *By Their Own Hand*

The poorer rather than the richer sections of society have suffered more from the surge in oil prices … with the poorest 20% of households facing an average inflation rate of 2.8% compared with the national average inflation rate of 2.4% and the estimated rate of 2.1% for the richest 30% of households.

The Guardian 07.09.06

Big Brother serves up the values of a 2am night club as if they were the moral norm .

Geoff Barton, Is Big Brother bad for kids? Yes *TES* 28.07.06

"Stop interrupting!"

Teachers are inclined to interrupt. Knowing when and when not to intervene, Colin Foster reminds us, is the core of good teaching.

Choosing when to intervene and when to hold back is, for me, one of the most important and difficult elements of working with learners on mathematics. Gauging someone's mathematical and emotional needs at a particular time is something that I cannot be confident about doing, but there is no alternative to trying. No doubt my interventions (or lack of them) are as much affected by my own feelings and thoughts in the moment as they are by my perceptions of the pupils'.

What I do feel sure of is that I get it wrong more often than I get it right. On one occasion, I stand back and watch someone struggle with a mathematical task in a manner akin to watching someone starting to drown, and feeling no compulsion to help! Their confidence and motivation are slipping away, but I am standing on the side-lines doing nothing about it. On the other hand, there are situations when I find myself fussily butting in, pushing a learner to do something the way it is in my mind rather than going with the grain of their own thinking and allowing them the time and space they need to construct their own understanding of the situation. With all the pressures that teachers find themselves under in the current educational climate, I think that for most of us the second of these – the 'back-seat driver' – is much our greater danger: I don't know many lazy teachers.

Offering these things sooner would 'save time' … but what would the learner

I have been trying to find a middle way between these extremes of doing too little and over-helping that allows me to be supportive and encouraging to learners emotionally but significantly less directive mathematically than I might tend to be instinctively. For instance, I have tried responding to opening cries of "I'm stuck" or "I don't get it" or "I'm confused" with the slightly shocking answer of "Good". It is important that this is said with a smile in the context of a good relationship with the learner, and with body language the very opposite of "Do I care?!"; for example, sitting down beside the pupil and appearing available and supportive without actually taking over the task and 'leading' them through it, which is perhaps what they are expecting. I want to communicate that this is *their* task but that I am here to be with them as they tackle it, if they want me.

Initially when I began to try this there was surprise: "He said '*Good*'!", "What do you mean '*Good*'!?" And I sometimes explain that for me getting confused is great – it means you're probably about to learn something. Getting stuck and getting confused are a normal part of making progress in anything. If you never get stuck you're probably not learning much.

Being over-helpful is often motivated by a wish to help learners' confidence. But it can so easily have exactly the opposite effect, since even when you feel you have helped in a very limited fashion, the learner frequently complains afterwards that they could only do it because "You did it for me," and I find myself saying, "No, I didn't really do anything", but with doubts in my mind that what might have seemed trivial input to me was in fact viewed as vital by the learner. Such incidents reinforce in the learner's mind their uselessness and the teacher's cleverness and indispensability. Human beings are good at telling when they are not being trusted, and are sensitive to a problem being taken out of their hands and taken over by someone more responsible, even though they – the learner – may still be holding the pen and writing it down.

So I have been trying the approach used by counsellors, in which they try to help someone make sense of their thinking or the problems they are encountering by reflecting back to them the gist of what they are saying. Obviously if this is done in a rigid and automatic way it becomes obvious and irritating, but done carefully it does not attract attention to itself and can allow the person to slow down their thinking and focus on their thoughts more easily than they might be able to do by themselves. It can be very tempting to 'lead the witness', slip in an idea of your own, deliberately misunderstand or interrupt them in a 'helpful' way, and no-one is saying that these are always illegitimate in the classroom. But teachers seem to develop 'hinting and nudging' habits that get pupils through tasks and enable plenaries to be completed more quickly, but which, I believe, inhibit learners' progress in mathematical understanding in the long run, and I have been keen to try to overcome my tendencies in those directions.

So you could envisage the following sort of conversation between a learner (L) and her teacher (T):

L "I'm confused!"
T "Good! [smiling and sitting down – looking expectantly] Tell me about your confusion."
L "It doesn't make sense!"
T "OK."
L "It says 'Katie thinks of a number', but *what* number?!"
T "What number?"
L "It just says 'a number', but how are you supposed to know *what* number?!"
T "That's not very helpful!"
L "I know – this book is so annoying!"
T "Mmm."
L "... And she adds 6 and timeses by 2 and she gets 26."
T "... OK ... So what are you thinking?"
L "It could be *anything*!"
T "Anything?"
L "Yeah – how am I supposed to know?!"
T "OK. Like what could it be?"
L "What?"
T "Well what might Katie's number have been?"
L "You mean like 10?"
T "So you think it might be 10 that was her number?"
L "No, I'm just saying it could have been *any* number!"
T "OK ... So what can you do?"
L "... So say it's 10. She thinks of 10 and adds 6 ... [gets calculator, works it out] ... 16 ... Oh, yeah, I knew that! ... I'm not stupid! ... Now what? ... Times by 2 ... [does it on the calculator] ... 32 ... So it's *32*?"
T "It's 32?"
L "Oh no, I mean ... She gets *26*."
T "Right – but you got 32."
L "Right. So it *wasn't* 10. I *get it* – 10 *wasn't* her number ... So that means I've got to try all the numbers! [exasperated] This question is so stupid!"
T "Very frustrating!"
L "OK, so I'll try 11."
T "OK." [I felt a strong temptation to intervene here, but I'm very glad I didn't!]

This progresses, with the student trying 11, getting 34, seeing that that's too much, deciding to try smaller than 10, then noticing that 11 gave 2 more than 10 did, so realising that she wants 6 less than what 10 gave, so therefore going for 3 less than 10 and trying 7 and getting it, with much satisfaction!

This thinking impressed me very much in view of my perceptions of her ability as a mathematician. Had I led the way through this problem, I probably would not have done it this way, and if I had used 'trial and improvement, I would not have expected a consideration of the differences between the values produced to inform so precisely the trying of 7 at the end. Probably the only thinking required of her would have been the arithmetic: "So what is 26 divided by 2," etc. I would have done all the strategic thinking and left the 'sums' for her. Ironically, this is exactly the bit that she chose not to engage with – turning to the calculator here, even though in other situations I know her to be capable of calculations of this kind. Somehow, her mind appears too full thinking about the other dimensions of the problem to leave any room for her to do the calculations as well, and I can identify with this myself (I recently found myself doing '10 + 10' on a calculator because I had had to think so hard about each of the 10's!)

the time to intervene is now, after the problem is completed

It seems to me that the time to intervene is now, after the problem is completed (rather than during) – before the pupil moves on to another problem – to look back at what has happened and what can be learned and whether a reverse operation strategy would have any advantages. Offering these things sooner would 'save time' and lead to a quicker, more efficient solution of the problem, but what would the learner learn? That teachers know everything, that she is useless on her own, that questions don't make sense unless someone else talks you through them, that you can't do anything unless you 'know the correct method'.

pupil-teacher conversations are often assumed to be a solved problem

Much has been and is being written regarding pupil-pupil discussion in the mathematics classroom. But pupil-teacher conversations are often assumed to be a solved problem. Teachers are helped to develop good questions for plenaries, but are given relatively little guidance on how to handle one-to-one mathematical conversations with pupils. I for one have a lot to learn regarding my informal discussions with pupils about their mathematics, and at the moment my end of term report is telling me that I need to stop interrupting and do much more careful listening!

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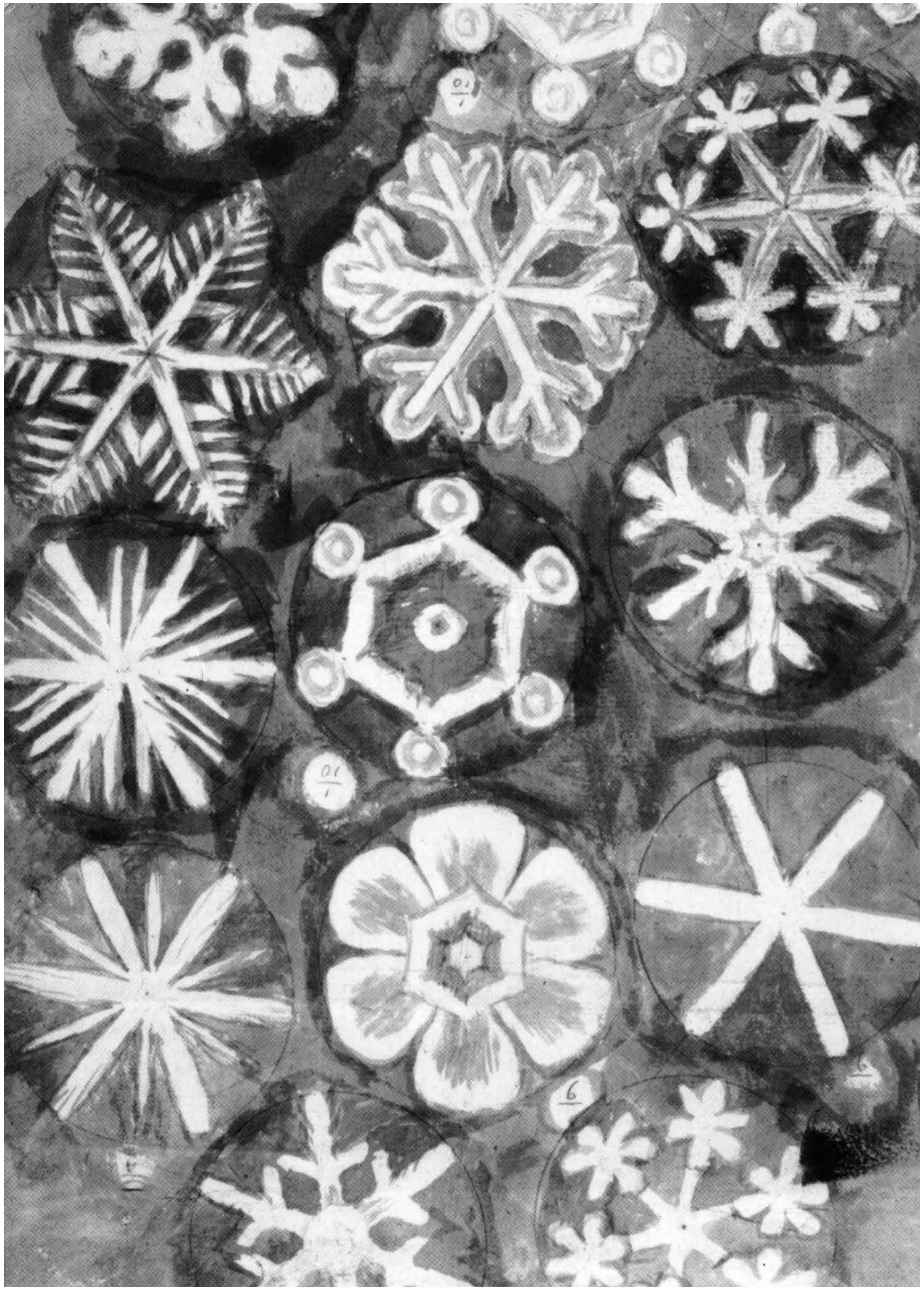

Figures observ'd in Snow.

By Mr Hook

Exposing a piece of black Cloath, or a black Hat to the falling Snow I observ'd such an infinite variety of curiously figur'd Snow, that it would be almost as impossible, to draw a Scheme of every of them as exactly to imitate, the curious and Geometricall Mechanisms of Nature in any one. Some course draughts, such as the coldness of the weather and the ill provision I had by me for such a purpose would permit me to make, I have here added.

In which I observ'd that if they were of any regular figure, they were always branched out, with 6 principall branches; which as all of the same Flake were much of the same make, so of differing Flakes, was there observable a strange variety. But for the most part they were conformable to the Rules observ'd before in the figures of Urine. That is, the branches from each side of the Steme were parallel to the next stem on that side. And if the Stemms were terminated in plates, the branches likewise were of the same kind.

Founded in 1660, the Royal Society is the independent scientific academy of the UK, dedicated to promoting excellence in science

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Figures Observ'd in Snow

The image is by Robert Hooke FRS and it is the drawing that accompanies his description of snowflakes that appeared in the Royal Society's Register Book Original, Volume II, 1662, pp 62 + 63.

The Register Book contains fair copies of scientific papers that were submitted to the Society to establish their precedence for a particular discovery or idea.

Here is Robert Hooke's text in modern script:

"Exposing a piece of black cloth, or a black hat, to the falling snow I observ'd such an infinite variety of curiously figur'd snow that it would be almost impossible to draw a scheme of every of them as exactly to imitate the curious and Geometricall Mechanisms of Nature in any one. Some course draughts, such as the coldness of the weather and the ill provision I had by me for such a purpose would permit me to make, I have added.

In which I observed that if they were of any regular figures they were always branched out, with 6 principall branches; which as all of the same Flake were much of the same to make, so of differing flakes was there observable a strange variety. But for the most part they were conformable to the Rules observ'd before in the figures of Urine That is, the branches from each side of the stems were parallel to the next stem on that side And if the stemms were terminated in plates, the branches likewise were of the same kind."

Have you ever seen a snowflake, studied it in detail, as Robert Hooke did?. There may be no snow around you this winter but Hooke's *figures* enable you to analyse their shapes and maybe "design" some more.

Try.

What is the shape they are all based on? Draw it.

Do you know its name?

Where else do you find this shape?

How many lines can you find to "fold" a snowflake onto itself?

Can you rotate the snow-figure so that it fits on to itself?

Teachers' notes

Hooke's figures give a good informal introduction to the study of a hexagon, or an exploration of symmetry more generally.

You could photocopy the whole of the above and use it as an activity sheet or, alternatively, you could display Hooke's drawing and text – with its modern script version - and discuss it with the class before setting them to work.

If you choose to take your class or group the way of the hexagon, it would be useful to have a picture of a honeycomb, some chicken wire and maybe some of the images from the book on Islamic Art reviewed in *Equals* 12.3.

Or, if the theme to be followed is symmetry and tessellations, how about flowers, leaves as well as more tiles of varying shapes?

(The book mentioned above could be useful too, as could Sharan-Jeet Shan and Peter Bailey *Multiple Factors: Classroom **Mathematics** for Equality and Justice)*

Coaching, a powerful tool

Jane Gabb writes about the experience of learning to become a coach, and recommends a useful book.

It began in April 2005 when I attended a day's course on coaching (and later another day on mentoring). I could see how coaching built on my way of relating with the people I work with in school, and would be a natural progression for my work. As mathematics adviser I am often in the position of having to observe lessons and feedback to teachers, luckily more often for the purposes of professional development than performance management or making judgements. Over the years I have developed a way of feeding back which consists much more of questions to the teacher about the lesson, than statements of my own view of it. This has the effect of involving the teacher much more in the process, rather than 'doing observation' to them.

Coaching takes this several steps further and provides a framework for working in all sorts of different contexts, not just observation and feedback. I will try and describe some of the important features and explain why coaching can be such a powerful tool for school improvement.

The starting point for a coaching relationship is that the coachee (i.e. the person being coached) has the capacity to affect the changes needed (as distinct from a situation where mentoring might be more appropriate). The coach's role is to facilitate the process of identifying what needs to happen and how it will happen, but not to make suggestions about these. This last requirement on the coach proves to be the most difficult to hold to, especially for those of us who are so used to being expected to advise!

At my training I was introduced to the GROW model of coaching, and in the book *Coaching Solutions* the model used is the STRIDE model. The table below illustrates the similarities and differences between the models:

GROW model		STRIDE model	
G	Goal	S	Strengths
R	Reality	T	Target
O	Options	R	Real Situation
W	Will	I	Ideas
		D	Decisions
		E	Evaluation

Each letter represents a phase in the coaching session, though there can be cycles within it, especially if the initial goal is adapted as a result of the discussion. As you can see the main differences are at the beginning and end of the process. The STRIDE model specifically suggests identifying strengths at the beginning of the process; with the GROW model this would come out in the 'Reality' section, but it is useful to have a specific pointer to it. At the end of the STRIDE model we have 'evaluation' and this has two parts, now and later. The now part is about exploring the commitment to a decision and the later part is agreeing a time for a follow up session.

So what does the coach do?
The coach:

- asks probing questions in order to focus the coachee on the problem and possible ways forward;
- listens attentively to the coachee's responses in order to focus on the coachee's situation and perceptions
- tries to understand the situation in order to frame additional questions;
- summarises to check understanding and to clarify where the discussion has got to
- stays silent to allow thinking time (for both coach and coachee);
- picks up on what is not said e.g. through body language, flippant or 'throw-away' comments and challenges or probes whether these might be important;
- moves through the process outlined above, getting the coachee to generate and evaluate possible (and impossible) options, and then to commit to action and state their commitment as a score out of 10.

As important is what the coach does not do.
The coach does not:

- offer suggestions or give advice
- talk about their own experience
- think about what they would do in a similar circumstance, or have done in the past
- make judgemental (or even evaluative) comments
- become emotional (even if the coachee does)

The emphasis is therefore on the coachee's reflections on their own situation (or their own practice if this is feedback after a lesson) in order to gain fresh insight and ideas about it. Coaching sessions can be as short as 15 minutes or longer than an hour, though an hour is probably enough as an instalment in a lengthy coaching process. For a simple issue, the coaching session might be a one off, perhaps with informal feedback to the coach about how it's going. For a more complex problem there is likely to be at least one follow-up session, perhaps more.

The coach's role is to facilitate the process of identifying what needs to happen .. but not to make suggestions

The connection with school improvement is in the need for everyone to become better at what they do in order for the school to move forward. If this is always driven from above there is likely to be resistance and a loss of morale. Conversely, where a coaching approach is used, the issues come from those on the ground, as do the solutions. This leads to people feeling empowered and in control and consequently to higher morale.

As a trainee coach the main things one needs are an understanding of the process, lots of practice and a good network of support. A good book can be very helpful as well. Getting better at being a coach requires reflective practice on the part of the coach, acknowledgement of which elements or phases need to be improved and a determination to get better at it. Really useful support can be provided by getting together with other trainee coaches and forming triads where each takes a role of the coach, the coachee and an observer for a short coaching session about a real issue. I have developed a format for observing coaching sessions which helps the observer to focus on the process, rather than getting involved in the issue. This is to be found at the end of the article.

Sometimes in a coaching session I have been surprised at how easy it is for the coachee to come up with their own solutions; it certainly feels as if they were inside all ready to come to the surface.

It is worth saying a few words about how coaching differs from mentoring and where each approach should be used. Mentoring contains many of the elements of coaching in terms of the attentive listening and probing questioning, but differs from coaching in that advice and suggestions are offered where necessary. This makes it more appropriate for working with most NQTs, student teachers and those who need to make large improvements in their practice.

There are many books on coaching coming onto the market, as is always the case when something comes into fashion. The one I have found very helpful is *Coaching Solutions* by Will Thomas and Alistair Smith. (Published by Network Educational Press Ltd. Tel 01785 225515 www.networkpress.co.uk ISBN 1-85539-188-0)

It is very readable and provides useful illustrations and case studies including transcripts of coaching sessions. It is a thought-provoking book which always leaves me with the feeling that I have a lot to learn about this process, and a lot of skills to develop if I am to become a really effective coach. In other words it helps me in the process of reflecting on my own practice as a coach. It is important to add that if coaching interests you, you will need more than a book in order to become an effective coach. Try and find out what training is available in your area and find some like-minded people to work with on this.

Royal Borough of Windsor and Maidenhead

Coaching Observation Format

Observer to feed back on the process in terms of highlighting important questions and phases of the coaching conversation:

How is the coached teacher made to feel comfortable and supported?

What questions does the coach ask which help the teacher to focus?

What questions does the coach ask which help the teacher to reflect on their practice?

What questions does the coach use in order to challenge the teacher?

How is summarising used by the coach?

How are new ideas generated?

How is the action decided?

If there are any 'throw-away' remarks, how does the coach respond to them?

Even more about the processes through which mathematics is lost

Misconceptions can be derived by intelligent thought processes. Richard Cowan asks whether teachers take sufficient time to unravel the misconceptions of their pupils.

The title pays homage to two fascinating papers by David Kent (1978, 1979), an expert teacher of mathematics. These papers discuss errors that reveal misconceptions of mathematical ideas. The errors are intelligently derived by the students - they make sense- but impede progress. Misconceptions can lurk beneath procedural competence, e.g. the child who correctly solves 0.3 x 0.3 but thinks that 0.09 must be larger than 0.3 because multiplication makes more. As Kent points out, not all errors are symptoms of conceptual confusion. Some can be errors of application -where someone knows enough to understand what is wrong about what they have done. Learners can detect errors of application in assessing their own work. In contrast, errors of comprehension require an instructor to detect and resolve.

Knowledge of common misconceptions can inform teaching but it is not possible to prevent them from occurring. Some way of following up and diagnosing the ideas underlying students' difficulties is necessary. The constraints on classroom life militate against this: primary teachers in Desforges and Cockburn's (1987) study reported they did not have enough time to teach the way they thought they should. Neither diagnosis nor group discussion, another practice frequently recommended to deal with misunderstandings, were common. Under pressure, there are fewer opportunities to consolidate learning and more children fall behind. They may construe themselves or be construed as having mathematical difficulties.

Learners can detect errors of application in assessing their own work

Non-mathematical difficulties
Another challenge is the use of word problems to assess mathematical learning and develop numerical skills. The difficulties some children have with these may be irrelevant to mathematical understanding (Cooper & Dunne, 2000). Nonmathematical difficulties with word problems are likely if the vocabulary is unfamiliar, if the context knowledge is not shared, or if real world knowledge is drawn on in unrealistic ways. In primary school, shopping games are often used to give practice in arithmetic but these often feature unrealistic prices: my six-year-old's maths book featured a kite for 3p and a football for 2p. Some children comment on the contrast between prices in the game and reality; others are reluctant to buy things they do not want or already have (Walkerdine, 1988). It is not just a primary issue (Boaler, 2003). Consider the following example for secondary students taken from a recent US project:

A new Toyota Corolla costs approximately $19,000. It loses, on average, 18% of its value each year. How much will it be worth in two years? In how many years will it first be worth less than $10,000?

A linear function would be appropriate if you were an accountant but otherwise not. If you know about buying new cars, which few teenagers would, then you know its value drops as soon as it leaves the showroom, that you rarely pay what it costs, and that cost and value are not the same. These concerns will not affect students who are unaware of the differences between reality and the intended mathematical model or who have learnt 'Don't bother with reality, just focus on the mathematics' (Gravenmeijer, 1997). Either way this could lead to difficulties in applying mathematics.

Variations in understanding
Diversity in student knowledge and skills is a major challenge. Individual differences in number sense are marked when children start school and this variation predicts later achievement (Gersten, Jordan, & Flojo, 2005). One view is that initial differences reflect variation in experience. Children who know less simply have had fewer opportunities to develop relevant knowledge and skills. Early interventions can make a substantial difference to number sense, e.g. Griffin, Case & Siegler (1994), but the benefits beyond the first couple of years of formal schooling are unknown.

Another view is that the correlation between early number sense and later achievement results from both being affected by child characteristics such as linguistic and memory functioning. Our recent study of Year 3 children with language impairments, recruited from many different schools, found their number knowledge and skills typically resembled those of Year 1 children (Cowan, Donlan, Newton, & Lloyd, 2005). Despite the group profile, some children within the language impaired group were achieving at age appropriate levels. Is this because their teachers knew more about how to support them? Will they still be thriving in later years? There is still much to find out.

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Climate change 1
Experiencing temperature change

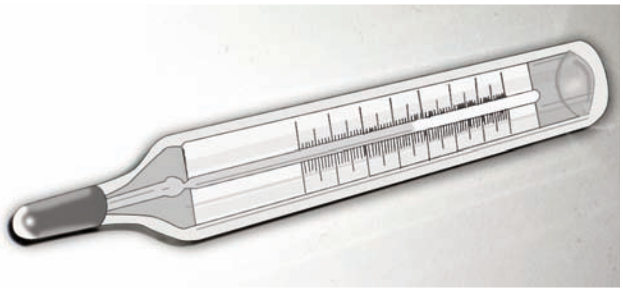
To understand the effects of changes in temperature on the planet **Rachel Gibbons** suggests we first look at ourselves.

> If in the year 2030 carbon dioxide concentrations in the atmosphere remain as high as they are today, the likely result is 2°C of warming (above pre-industrial levels). Two degrees is the point beyond which certain major eco-systems begin collapsing
>
> George Monbiot, *Heat: how to stop the planet b urning*, Alan Lane: London, 2006, p. xi

Pictures from the ski resorts in the Swiss Alps for the 2006 Christmas season show them empty of snow and people. Should we care much about the ski resorts? The United Nations suddenly declares the polar bear an endangered species due to the rapid melting of the arctic ice. Should we worry about the bear or the ice? Scientists talk about a temperature rise between 2°- 3°. Does that really matter much to us as real people, of flesh and bone? After all we do have hot and cold days!

Find out about the range of temperature summer and winter in your town.
What was the lowest temperature recorded last year?
And the highest?

But when we consider the temperature of our own bodies and the changes that may accompany relatively small variations of temperature, we begin to see the possible seriousness of similar changes to the temperature of our planet.



Record the temperatures of all members of the class. Anyone with a temperature 2° or more above this level should not be in school because this is a sign that they are quite seriously ill. With a temperature a degree or so higher our life is in danger.

Fulham, London

Keeping learning on track:

formative assessment and the regulation of learning Part 4

In the final part of his paper on assessment for learning Dylan William discusses the regulation of the learning process.

	Where the learner is	Where they are going	How to get there
Teacher	Evoking information	Establishing goals	Feedback
Peer	Peer-assessment	Sharing success criteria	Peer-tutoring
Student	Self-assessment	Sharing success criteria	Self-directed learning

Table 4: aspects of formative assessment

The regulation of learning
Although at first sight quite different, the four elements of effective formative assessment outlined above form a coherent set of strategies for raising achievement. The coherence of these ideas can be seen more clearly by considering three crucial processes in learning:

where the learners are in their learning
where they are going
how to get there

and the role of the learner, her or his peers, and the teacher in these processes. The result of crossing these two dimensions is shown in table 4.

Rich questioning and effective feedback focus on the teacher's role-first being clear about where we want students to get to (curricular goals), asking appropriate questions to find out where they are, and feeding back to students in ways that the students can use in improving their own performance. Sharing criteria with learners and student self - assessment focus on the learner's role - first being clear about where they want to get to, and then monitoring their own progress towards that goal.

In English, the noun 'regulation' has two meanings; one refers to the act of regulating and the **other** to a rule or law to govern conduct, and so, while it is the former sense that is intended here, the word has the unfortunate connotation of the second. In French, the two senses have separate terms (regulation and réglement) and so the problem does not arise.

The elements in table 4 can be integrated within a more general theoretical framework of the *regulation of learning processes* as suggested Perrenoud (1991, 1998). Within such a framework, the actions of the teacher, the learners, and the context of the classroom are all evaluated with respect to the extent to which they contribute to guiding the learning towards the intended goal.

From this perspective, the task of the teacher is not necessarily to teach, but to create situations in which students learn. This focus emphasizes what it is that students learn, rather than what teachers do. Most teachers appear to be quite skilled at regulating or controlling the activities in which students engage, but have only a hazy idea of the learning that results. This is especially evident in interviews before lessons where teachers focus much more on the planned activities than on the resulting learning (e.g. "I'm going to have them do X"). In a way, this is inevitable, since only the activities can be manipulated directly. Nevertheless, it is clear that in teachers who have developed their formative assessment practices, there is a strong shift in emphasis away from regulating the activities in which students engage, and towards the learning that results (Black et al, 2003). Indeed, from such a perspective, even to describe the task of the teacher as teaching is misleading, since it is rather to 'engineer' situations in which student learn.

However, in this context, it is important to note that the 'engineering of learning environments' does not guarantee that the learning is proceeds in fruitful ways. Many visual arts classroom are *productive*, in that they do lead to significant learning on the part of students, but what any given student might learn is impossible to predict.

An emphasis on the regulation of learning processes entails ensuring that the learning that is taking place is as intended.

When the learning environment is well-regulated, much of the regulation is pro-active, through the setting up of didactical situations. The regulation can be unmediated within such didactical situations, when, for example, a teacher "does not intervene in person, but puts in place a 'metacognitive culture', mutual forms of teaching and the organisation of regulation of learning processes run by technologies or incorporated into classroom organisation and management" (Perrenoud, 1998 p100). For example, a teacher's decision to use realistic contexts in the mathematics classroom can provide a source of proactive regulation, because then students can determine the reasonableness of their answers. If students calculate that the average cost per slice of pizza (say) is $200, provided they are genuinely engaged in the activity, they will know that this solution is unreasonable, and so the use of realistic settings provides a 'self-checking' mechanism.

On the other hand, the didactical situation may be set up so that the regulation is achieved through the mediation of the teacher, when the teacher, in planning the lesson, creates questions or prompts activities that evoke responses from the students that the teacher can use to determine the progress of the learning, and if necessary, to make adjustments. Examples of such questions are, "Is calculus exact or approximate?" or "Would your mass be the same on the moon?". (In this context it is worth noting that each of these questions is 'closed' in that there is only one correct response - their value is that although they are closed, each question is focused on a specific misconception.)

Each teacher will have to find a way of incorporating these ideas into their own practice

The 'upstream' planning therefore creates, 'downstream', the possibility that the learning activities may change course in the light of the students' responses. These 'moments of contingency' points in the instructional sequence when the instruction can proceed in different directions according to the responses of the student-are at the heart of the regulation of learning.

These moments arise continuously in whole-class teaching, where teachers are constantly having to make sense of students' responses, interpreting them in terms of learning needs, and making appropriate responses. But they also arise when the teacher circulates around the classroom, looking at individual students' work, observing the extent to which the students are 'on track'. In most teaching of mathematics, the regulation of learning will be relatively tight, so that the teacher will attempt to 'bring into line' all learners who are not heading towards the particular goal sought by the teacher-in these subjects, the goal of learning is generally both highly specific and common to all the students in a class. In contrast, when the class is doing an investigation, the regulation will be much looser. Rather than a single goal, there is likely to be a broad *horizon* of appropriate goals, all of which are acceptable, and the teacher will intervene to bring the learners 'into line' only when the trajectory of the learner is radically different from that intended by the teacher. In this context, it is worth noting that there are significant cultural differences in how to use this information. In the United States or the United Kingdom, the teacher will typically intervene with individual students where they appear not to be 'on track' whereas in Japan, the teacher is far more likely to observe all the students carefully, while walking round the class, and then will select some major issues for discussion with the whole class.

One of the features that makes a lesson 'formative', then, is that the lesson can change course in the light of evidence about the progress of learning. This is in stark contrast to the 'traditional' pattern of classroom interaction, exemplified by the following extract:

> "Yesterday we talked about triangles, and we had a special name for triangles with three sides the same. Anyone remember what it was? ... Begins with E ... equi-..."

In terms of formative assessment, there are two salient points about such an exchange. First, little is contingent on the responses of the students, except how long it takes to get on to the next part of the teacher's 'script', so there is little scope for 'downstream' regulation. The teacher is interested only in getting to the word 'equilateral' in order that she can move on, and so all incorrect answers are treated as equivalent. The only information that the teacher extracts from the students' responses is whether they can recall the word 'equilateral' or not.

The second point is that the situation that the teacher set up in the first place-the question she chose to ask-has little potential for providing the teacher with useful information about the students' thinking, except, possibly, whether the students can recall the word 'equilateral'. This is typical in situations where the questions that the teacher uses in whole-class interaction have not been prepared in advance (in other words, when there is little or no pro-active or 'upstream' regulation).

Similar considerations apply when the teacher collects in the students' notebooks and attempts to give helpful feedback to the students in the form of comments on how to improve rather than grades or percentage scores. If sufficient attention has not been given 'upstream' to the design of the tasks given to the students, then the teacher may find that she has nothing useful to say to the students. Ideally, from examining the students' responses to the task, the teacher would be able to judge how to (a) help the learners learn better and (b) what she might do to improve the teaching of this topic. In this way, the assessment could be formative for the students, through the feedback she provides, and formative for the teacher herself, in that appropriate analysis of the students responses might suggest how the lesson could be improved.

Summary

In this paper, I have outlined some of the research that suggests that focusing on the use of day-to-day formative assessment is one of the most powerful ways of improving learning in the mathematics classroom. In other words, even if teachers do not care about deep understanding, and instead wish only to increase their students' test scores, then attention to formative assessment appears to be one of, if not the, most powerful way to do this.

To be effective, these strategies must be embedded into the day-to-day life of the classroom, and must be integrated into whatever curriculum scheme is being used. That is why there can be no recipe that will work for everyone. Each teacher will have to find a way of incorporating these ideas into their own practice, and effective formative assessment will look very different in different classrooms. It will, however, have some distinguishing features. Students will be thinking more often than they are trying to remember something, they will believe that by working hard, they get cleverer, they will understand what they are working towards, and will know how they are progressing.

In some ways, this is an old-fashioned message - indeed, none of the strategies that teachers have used to put these principles into practice in their classrooms is new. What is new is that we now have hard empirical evidence that quality learning does lead to higher achievement, even when performance is measured through externally - mandated tests. What is also new is the broad theoretical framework of the regulation of learning, which may help teachers to understand how these ideas can be implemented effectively, so that teachers and students can, together, keep the learning of mathematics `on - track'

Learning and Teaching Research Center at the Educational Testing Service

Standing at the front of the class like teachers

What is happening in today's classrooms? Rachel Gibbons suggests that we need to question the role of both the teacher and the teaching assistant.

A teaching assistant, Matthew Thomson, was quoted in the *TES* in October 2006[1] as saying that

> it was inevitable that teaching assistants would be subject to more pupil assaults because they were working beside the students rather than standing at the front of the class like teachers.

If this really describes what is happening in today's classrooms it seems to me we need to question the role of both teacher and teaching assistant before we decide where in the classroom that role dictates they should be.

Is the front of the class (aloof from the rough and tumble of the lesson apparently) the place the teacher should be? and if so why?

You stand in front of people if you want to give them information or 'deliver' orders. Is that what teaching is all about? And where does learning come in in that scenario? I never found that standing in front of a group of pupils struggling to come to terms with mathematics and attempting to give either information or orders was very effective. What was required was a one-to-one approach.If I attempted to give the whole class information, they did not remember what I had said. I had to say it all over again to each one individually. As for orders, those certainly needed to be given and accepted individually. And to learn any mathematics every member of the class needed to be active, manipulating materials, noting and discussing the effects of their actions with each other, or with a teaching assistant or the teacher.

I would suggest that activities for the pupils can be set up without 'standing in front of the class like a teacher'. And, after they have been set up, the teacher's place is right in the middle of the class. That is the place from which you can best organise small groups for activities, encourage and give confidence and advice.

Perhaps it would be useful at this point to turn a 'page from the past'. I remember once being faced with the prospect of meeting once a week with a group of Year 10 mathematical ne'er-do-wells who, it had been decreed, should have an extra mathematics lesson timetabled each week. These single lessons could not be linked with the rest of their mathematics lessons because the members of the group came from different mathematics sets. Having chosen to be the teacher for that group what could I do? Would I be forced to meet them 'standing at the front of the class' asking for confrontation? Not if I could help it.

The belt from The Victorian Classroom at Scotland Street School[2]

Having got together a collection of games of varying levels of difficulty but all requiring mathematical thinking, I laid them out on the bench running along the side of the classroom. As the boys (although it was a mixed school they were all boys) arrived I was somewhere around the place - probably skulking in a corner - but made no comment. One or other of the boys began to ask what they should do. I shrugged my shoulders and pointed to the activities on the bench. It worked. Gradually they got started and then I began to intervene. Their levels of achievement across the class were clearly very different but the variety of activities I had provided presented challenges for every one of them without putting them off. By half term they were all involved with the games and I was being beaten in one of the more sophisticated games by the brghtest lad in the class (who had never done a stroke of work in school before). Clearly this technique is not appropriate for every situation but the scene is given to show that it is possible (and can be very effective) for a teacher to avoid entirely 'standing in front of the class'. Furthermore there were no teaching assistants provided in that situation – just me. If there had been maybe we would all have started off skulking in corners, giving the boys some freedom to choose before we intervened. Certainly getting to know the needs and interests of the individual members of the group and helping them to get down to some mathematical thinking would have been easier with some assistants. Because of course this is the second duty of the teacher - being aware at every instant of what is happening in every corner of the room and influencing it where necessary.

And are those working alongside the pupils in the greatest danger of assault?

Teaching assistants take over some of the responsibility for this awareness of each pupil, as Matthew rightly says working alongside a pupil. Just as the first duty of the teacher is to build up a relationship with the class so, surely, the teaching assistant's priority is to build up a relationship of trust with the individual pupil. Matthew Thomson's comment suggests that he has not managed to form such relationships. If he is in the greatest danger of assault, then , either he is trying to support a child that is way, way out of line and should not be in a mainstream classroom anyway or something has gone badly wrong with the presentation of the curriculum to this pupil or the relationship between him and the pupil he is supporting. Without trust neither knowledge of the curriculum nor the most sophisticated teaching techniques will be of any use.

There must also be trust between the teacher of a class and the teaching assistants allocated to it. They must have time to discuss roles and ruses thoroughly before they can function effectively as a team. They must also discuss the curriculum together, increasing their depth of understanding of it as a team. I would suggest that 'the teacher standing in front of the class' is not the model to adopt for the best results in these circumstances. If the teacher can avoid an over-dominant stance and students have their own individual programmes which they are expected to manage as far as they are capable, they will more readily turn to the adults in the room for help and are much less likely to be aggressive. In such a set up one of the teachers' main roles will be to get small groups of pupils together to discuss the mathematics they are doing rather than standing in front of the class and 'performing'.

Fulham, London

1. Jonathatn Milne, 'We're on a hiding to nothing', *TES* 27.120.06
2. *website:* *http://www.sandaigprimary.co.uk/classes/primary_six_05-06/victorians_at_scotland_st.php*

Basic Counting

In the last issue Jennie Pennant and Becky Teale's article on short training sessions for teaching assistants was printed with the details of the first session on Basic Counting missing. We include it here, together with a picture of a progression sheet from the Models and Images pack referred to.

BASIC COUNTING

- **Resources**
  Flipchart and pens, paper, pens/pencils, Progression sheet from Models and Images (Counting on and back in ones and tens), copies of two handouts - the principles of counting, Raps and Rhythms CD/boook (BEAM) and Models and Images CD Rom (DfES).
- **Aims of the session**
  To introduce/revise the basic principles of counting with TAs. To look at progression, questions and models/images surrounding the concept of counting.

Basic Principles

(TAs need a copy of the two handouts - principles of counting.)

Discuss each principle and give examples:

Stable order:	songs involving numbers - Raps and Rhythms could help here.
One-to-one:	children count objects in a line whilst pointing and saying correct number.
Cardinal:	'I've counted 5 so there are 5 things altogether.'
Abstraction:	use of a feely bag for objects or noises of cubes dropping into a tin.
Order Irrelevant:	bubbles can be counted like this.

Building up to higher numbers

Explain the challenge of the teen numbers, importance of the landmark tens and relative size. Use coins and beadstring to model ideas. *At this point it may be useful to show the clips of Emma Y1, 'Counting on...using coins'; Kathy KS1, 'Counting on...bead string' from the CDRom from the Models and Images pack.*

Potential Difficulties

Discuss how differences can arise. Use the reverse of the Progression sheet for examples of how children can be become confused by counting. Discuss with TAs how they would move the children forward from this difficulty. Record any thoughths on the progression sheet for future use. Encourage the use of concrete apparatus, no matter what age the child in order to embed this concept.

Models and Images

Use the progression sheet to look at the models/images shown. Look at the progress of counting as indicated by the models. When is the jump to adding 10 made and why? (Money is good for this) How is the concrete apparatus helping the child? How else could the beadstrings be used? What other practical and ICT resources do you have in your school that could be used for this activity? Look at the importance of the language that is used and how this helps the image when they are used together.

Progression

Using the Progression sheet, look at the progress of the counting throughout KS1/2. This should be helpful in tracking back for TAs if they have children who are in need of this. Discuss the questions that are posed. Could your TAs use these in the classroom and how would they help the child to progress?

What next?

TAs need to identify an aspect from the session to try out with the children they work with and feedback on their experience at the next session.

BEAM, London

FIVE PRINCIPLES OF COUNTING

- **The stable order principles:** understanding that the number names must be used in that particular order when counting.
- **The one-to-one principle:** understanding and ensuring that the next item in a count corresponds to the next number.
- **The cardinal principle:** knowing that the final number represents the size of the set.
- **The abstraction principle:** knowing that counting can be applied to any collection, real or imagined.
- **The order irrelevance principle**: knowing that the order in which the items are counted is not relevant to the total value.

FIVE PRINCIPLES OF COUNTING

The stable order principle

This means understanding that the number names must be used in a particular order when counting. We help children to learn the order by practising counting in shorter strings to start with, and by emphasing particular 'landmark' numbers such as two, five and ten. The order of numbers can be reinforced through rote counting, 'counting up', number rhymes and songs, and visually through the use of a number line. Teachers and supporting adults model the count by taking opportunities as they arise to count objects or events.

The one-to-one principle

This means understanding and ensuring that the next item in the count corresponds to the next number. We help children with this by encouraging them, for example, to touch or move objects to one side as they are counted. Children should be encouraged to organise their counting in this way since, in addition to keeping track of the items that they have counted, the concept of matching one number name to one object is emphasised through the physical contact and movement associated with saying the number as the item is counted. Moving the items into line as they are counted will also provide a helpful structure for checking. Teachers often develop ideas of one-to-one matching through making sure that there is one cup and one plate for each child at snack time. This can be developed for some children by giving tasks such as 'We have counted that there are five people here. Can you find five cups for them?'

The cardinal principle

This means knowing that the final number represents the size of the set. It is a concept that may take time to develop for children with severe or profound and multiple learning difficulties. There is evidence that it develops in four stages:-

1. Being aware that the last number said in the count is the response expected by the adult without recognising that it represents the quantity.

2. Knowing that the last number in the count indicates the quantity. The transition between these first two stages can be developed through the use of particular intonation on the last number in the count and by repeating, for example, 'There are **five** elephants' after the count.

3. Recognising that if a count is interrupted they can say how many they have counted so far and continue with the remainder of the set to find the total. Children should have opportunities to pause and then continue counting - this can be developed later to a more abstract level by placing the items already counted out of sight before continuing the count.

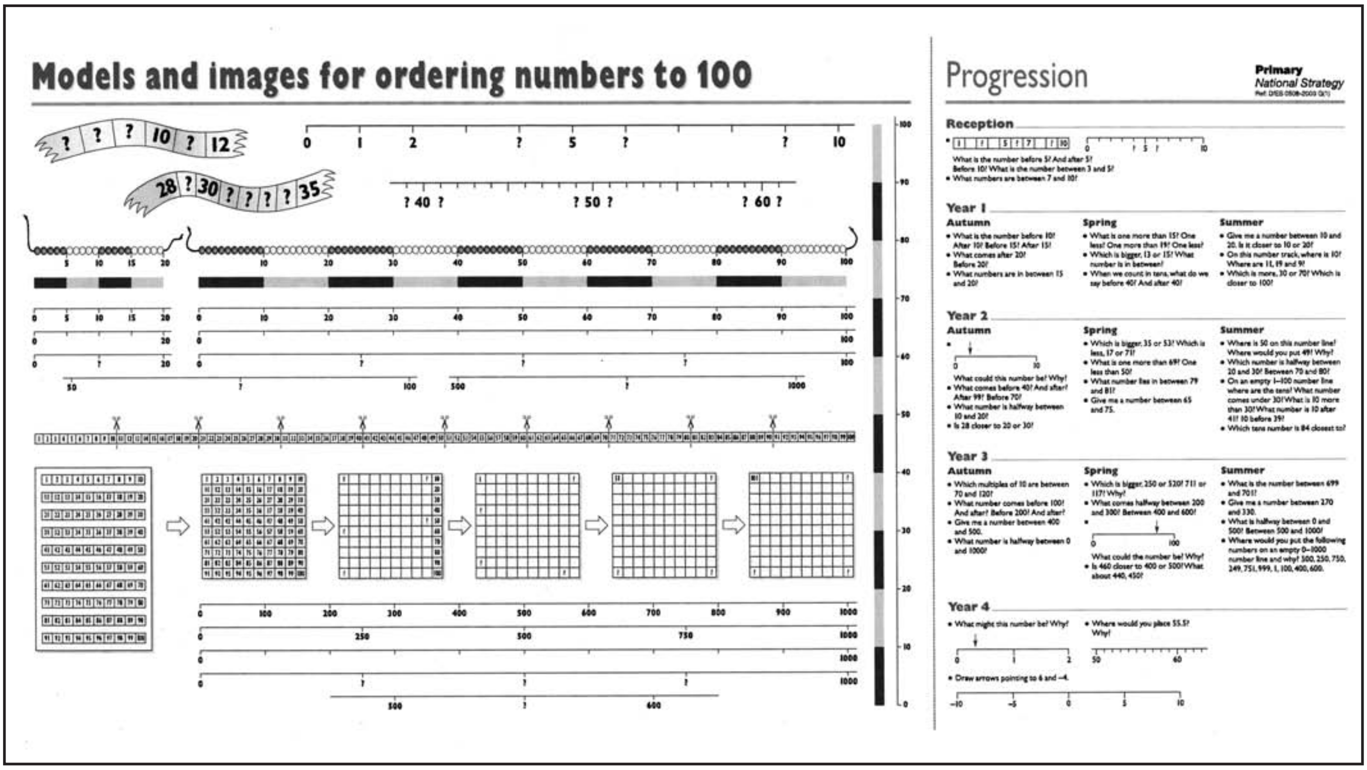
4. Awareness that values, once assigned to sets, can be compared. Without this principle, children are not able to compare the sizes of two sets represented by numbers.

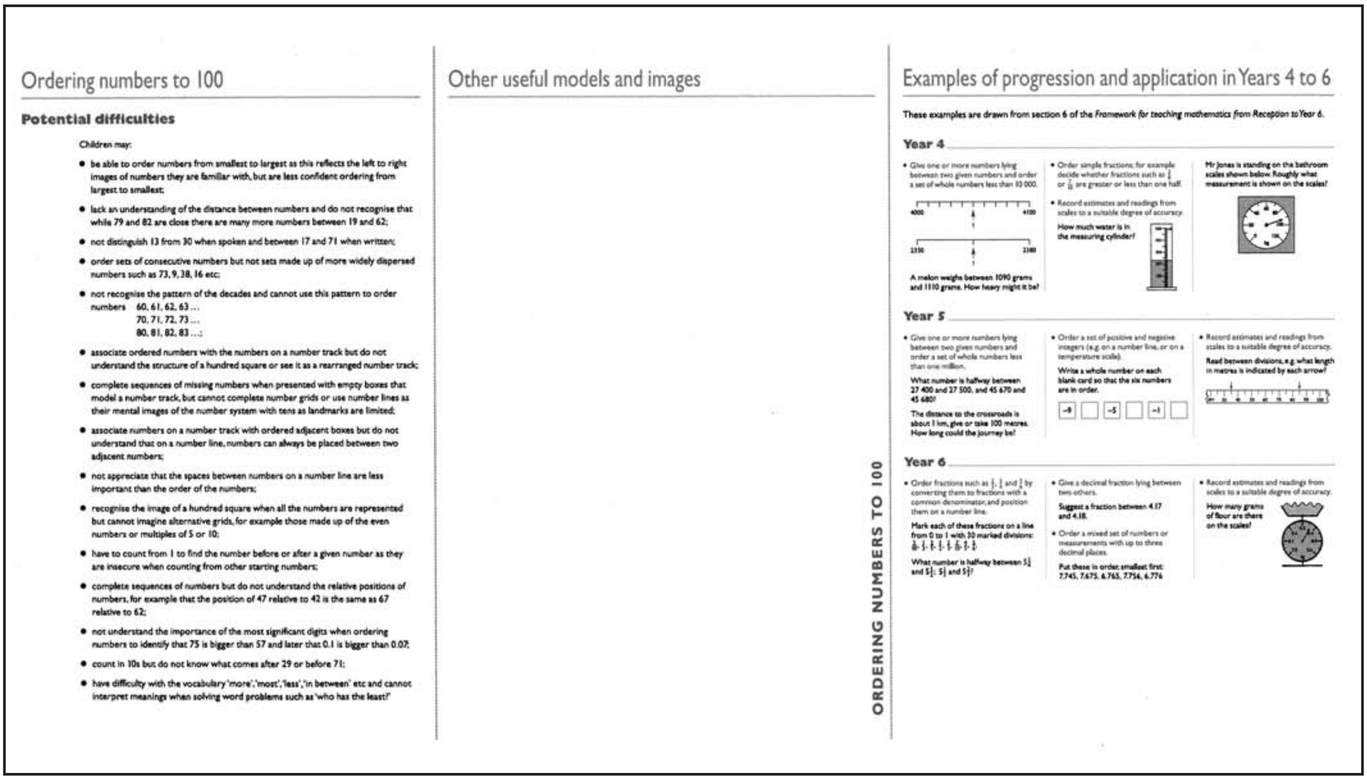
The abstraction principle

This means knowing that counting can be applied to any collection, either real or imagined. Children's early experiences of counting are generally with real objects. Often these are related to sorting activities where they need to consider items which are included or excluded from a particular count. As children develop their counting skills they need to begin to count objects that they cannot see, for example how many people live in their home, and events as they happen, for example how many times the teacher bangs on a drum, leading to counting events that happen somewhere else.

The order irrelevance principle

This means knowing that the order in which the items are counted is not relevant to the total value. In the early stages of counting, children are not able to recognise that the order in which they count objects does not affect the cardinal value. They need to understand that the name tags given to items as they count are not related to the properties of the item but are given temporarily to enable the number of items in the set to be found. Children need to have experience of counting and recounting objects in different orders to establish that the cardinal value remains the same. The same principle applies to the way in which objects are arranged, and opportunities to count objects that are arranged close together and then further apart should be offered to help develop an understanding of the conservation of number.





War Zones Outraqe

Campaign to address the “moral outrage” of 43 million children in conflict zones going without basic education. In 2003 more half armed conflicts used child soldiers under 15.

Bullets but no books, *TES* 15.09.06

Standard Unit

The double decker bus has become a standard unit of measurement of the amount of nuclear waste in the UK, but the numbers offered vary widely, from seven to 1,000.

Andy Martin, *The Guardian* 12.08.06