Equals

Realising potential in mathematics for all

for ages 3 to 18+

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MATHEMATICAL ASSOCIATION

supporting mathematics in education

Equals

Realising potential in mathematics for all

Front Cover image: *Geometric Patterns from Islamic Art & Architecture*

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Editorial

At the time of writing, the TES first leader has the title 'It's time to take a fresh look at SEN'. The select committee chair, Barry Sheerman, is quoted as saying that meeting their needs "should be the hallmark of a successful education system and a civilised society". The very existence of this journal is proof that, in mathematics at least, these needs have not been met over the years. Perhaps we should ask how well have the needs of all our pupils been met. Any flaws in approaches will show more with the pupils who are our particular concern than the others.

Molly Hughes's voice from the past gives us a warning as she records that in her own education in the 1870s "it was in the earliest stages of dealing with Number that the mischief began". Although later she was a leader in educational ideas, her views of the duties of a teacher before she began her teaching career were not enlightened, "mainly consisting…in talking and putting red crosses on other people's mistakes." The continuing stress on testing, so that the pupils of this country are the most tested in the world, suggests that many still believe this is the best way to encourage learning. We are urged to question this belief by Ian Adamson, who asks in his first e-mail, "Is there any properly documented research on the most effective teaching strategies for maths?" As usual in this issue we do include some research, which Adamson later acknowledges. Dylan Wiliam continues to remind us of the central importance of formative assessment in the learning process. How can the teacher give direction for further exploration if she does not know where the pupil is on the mathematical map? And exploration is what the business of learning is all about. As ACME recommends: "Mathematics lessons need to encourage good quality mathematical discussion through increased group and pair work and mathematically rich tasks". You might look, for example, at some of the research evidence from the Freudenthal Institute in Holland showing that working with the empty number line provides such rich experience and plenty of experience and common sense suggest that other methods support understanding and are therefore preferable to the absorbing of algorithmic processes. Tim Coulson seems to be of the opposite opinion in his response to our criticism of the new primary framework, still insisting on the importance of learning procedures in maintaining that all children "have the right to leave primary school with at least one secure and efficient written method of calculation for each of the four operations". Mundher Adhami and Tandi Clausen-May also point out the usefulness of the empty number line as a thinking tool for working with numbers. And Alan Wood's timely reminder that a mathematics lesson is about more than mathematics - that it is in the classroom that children learn how to associate peaceably and tolerantly with their fellows - should make us question the organisation of our classrooms. Are they places where everyone is subservient to the leader, learning how to produce the accepted forms, with no room to think for themselves, organise their own work, share resources or respect the needs of others?

For it is not only how we teach but what we teach that matters. The argument that appeared in *Equals* 12.2 concerning the new primary framework addressed just this question. Concepts or algorithms? Ideas or methods? It depends whether you think learning is for life or for testing I suppose. After they have left us, our pupils will have to come to terms with much that is new, that we have not yet dreamed of. If they have not learned how to learn, how to reason things out for themselves they will not face up satisfactorily to the problems pervious generations have created for them.
We want teachers to engage in the debate and then having made a professional informed judgement take their pupils forward.

Malcolm Swan, in describing the work he has been doing for the post-16s with the DfES Standards Unit Mathematics Project team[1], quotes a learner:

The good thing about this was, instead of like working out of your textbook, you had to use your brain before you could go anywhere else with it. You had to actually sit down and think about it. And when you did think about it you had someone else to help you along, if you couldn't figure it out for yourself, so if they understood and you didn't they would help you out with it.

1. The DfES leaflet *Towards more active learning approaches*, National Research and Development Centre for adult literacy and numeracy lists some of the best research from the last 20 years. (www.maths4life.org)

There were 26 murders using firearms reported between 2004 and 2005.

The Big Issue No. 694, London 22-28 May 2006

What’s in a name?
Part 1

Frustrated by difficult mathematical language which he feels gets in the way of mathematical understanding, Gerry Rosen suggests a way forward when looking at angles on parallel lines.

'**Tis but thy name that is my enemy;**
……
O, be some other name!
What's in a name? That which we call a rose
By any other name would smell as sweet.
Shakespeare, Romeo and Juliet Act II scene 2

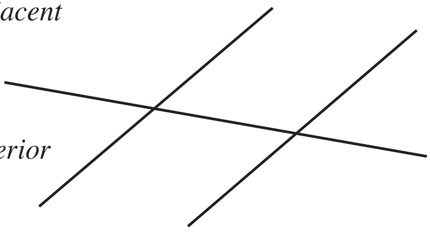
So why in mathematics do we insist so often on words that do not convey exactly the same meaning as they do in everyday speech? Words that need to be defined in order that others know what we are talking about. And what of our pupils? Do the definitions help us to communicate with them or them with us? Sometimes the names we give mathematical concepts are their own worst enemies interfering with our pupils’ ability to assimilate them.

A large majority of mathematics is visual, (yes even arithmetic and algebra), and words often get in the way being too clumsy to express quickly and efficiently what our eyes see and our brains process. This often leads to frustration. Our task is to develop spatial awareness and help pupils identify the relevant from the less relevant. Geometry is “right brain” orientated and is so often hampered by “left brain” constraints.

Names such as:

Vertically opposite (why not horizontally opposite?)
Supplementary adjacent

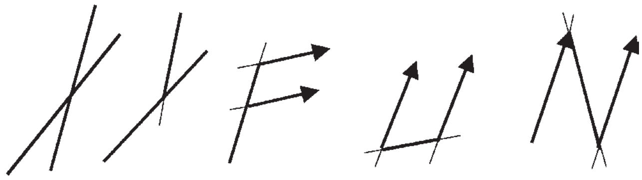
Alternate
Corresponding
Supplementary interior



Who uses these words apart from mathematics teachers? In addition each of the above is often taught initially independently from the others and practice given in solving problems before moving on to the next one.

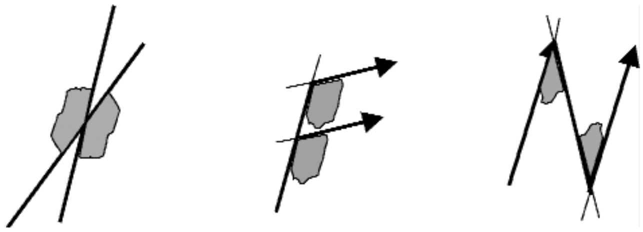
It is a question of what or who is in control – the geometrical terms or the pupil. If the pupil has mastered their use then there is no problem. But what if not?

This is where XY FUN comes into play.

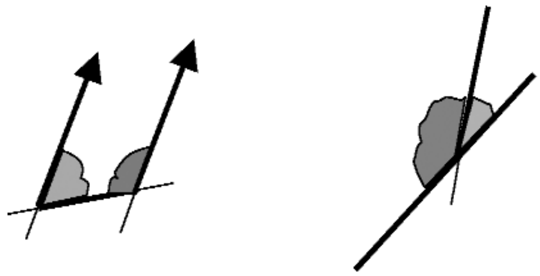


Instead of the confusing names the pupils in my class use the letters of the above slogan to identify the appropriate connections. **XY FUN** gives them all the connections – the whole picture.

At a later stage the letters are sorted into those that indicate equal angles,



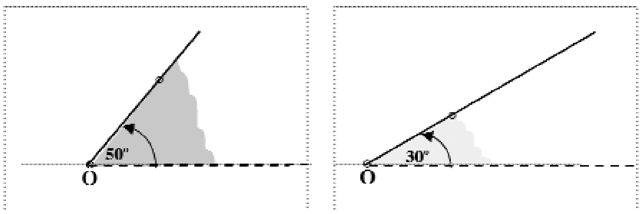
and those whose pair of angles add up to 180° or two right angles if you prefer.



At a still later stage, once the pupils are in control, **XY FUN** will have their accepted names introduced.

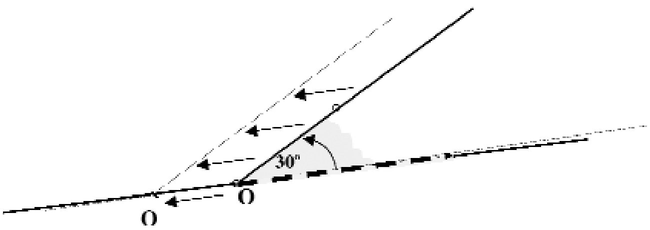
To see how this can come about consider the following worksheet.

WorkSheet
In addition to the worksheet the pupils were provided with two pieces of tracing paper with the following two angles. (Note: the two small circles on each diagram, one at the vertex of each angle and the other on the rotated arm that forms the angle, are holes for a pencil point)



CONSTRUCT ANGLES

1. Draw a straight line
2. Mark a point **O** on the line to show where you want the vertex of the angle to be.
3. Take the tracing paper and place the dotted line on the straight line you have just drawn.



4. Slide the angle along the line until the point you have marked appears in the hole at the vertex of the angle.
5. Mark a point using the other hole in the tracing paper.
6. Remove the tracing paper and draw a straight line through the point you have just marked and the point at the vertex.
7. Mark the angle 30°.
8. Mark more points on the same line and repeat the process as shown.
9. Tell me something about the lines you have drawn.
10. Draw another line and this time construct angles of 50°.
11. Now construct the following angles using your angles of 30º and 50º -

    100º 20º 180º 60º 130º 10º 40º 90º

    discuss with your neighbour how this might be done.

(Lines on which to construct these can be given if such support is needed.)

In the next issue I will show you how to move from this starting point into work on angles and parallel lines.

Western Galilee Regional High School, Israel

Key issues for the future of primary mathematics learning and teaching

Following on our criticism of the new draft primary guidelines it seemed to the editors that it might be useful to readers to review briefly the recent paper from ACME (Advisory Committee in Mathematics Education) on key issues for the future of primary mathematics learning and teaching.

The committee gives eight recommendations that might prove useful when considering the revision of the *Primary Framework for Teaching Mathematics*: four in the area of Curriculum planning and four in Pedagogy. The paper also notes that the issue of primary mathematics CPD is being addressed separately.

Curriculum Planning
Early on the paper recognises the important effects of assessment procedures on learning but pronounced them beyond the scope of the paper. The committee expresses concern that teachers' confidence may have been reduced by the top down decisions about what topics should be taught and how they should be presented.

This leads them to their first recommendation:
Headteachers need to encourage and support teachers in actively identifying appropriate curriculum planning for their pupils.

The second recommendation follows from the attempt they see to cover all topics in the framework at the expense of consolidation of knowledge:
Teachers need to be given the opportunity to plan sequences of lessons where pupils experience a carefully developed progression of mathematical concepts and ideas, so that they develop as mathematical thinkers.

The committee expresses concern that mathematical ideas are not integrated into the general framework of learning in the primary school and they recommend:
Teachers need more guidance within school on the ways in which mathematics can be creatively integrated with other areas of the curriculum without losing the rigour of mathematical learning.

They advise careful consideration of the use of developments in technology such as interactive whiteboards:
The use of IWBs alone is not likely to improve learning and teaching. Teachers need to be encouraged by their schools to reflect on the impact that IWBs have on their own pupils' learning of mathematics and consider this in relation to the use of other teaching strategies.

Pedagogy
There is a suggestion that the three part lesson structure has been used without reflection and that a wider discussion of pedagogy is needed:
Mathematics pedagogy should be more widely discussed during teacher training and in-service and should be better related to end purposes. It should be informed by national and international research.

Should there be so much whole-class direct teaching? The example of an alternative model mentioned is of the extension of Cognitive Acceleration in Mathematics (CAME) to primary children. This stresses mathematical discussions and the recommendation here is:
Mathematics lessons need to encourage good quality mathematical discussion through increased group and pair work and mathematically rich tasks.

After a reminder that initially the Primary Framework for Teaching Mathematics listed eight features for whole-class direct teaching: directing, instructing, demonstrating, explaining and listening, questioning and discussing, consolidating and evaluating, it mentions the importance of encouraging pupils to develop their own modelling skills, summarising:
An agreed understanding of the term 'modelling' needs to be developed amongst teachers so that they can use this as a tool for developing mathematical thinkers in their classrooms.

Finally the importance of the Foundation Stage is noted to ensure that young learners are stimulated:
Schools need to ensure that teachers understand the difference and importance of appropriate pedagogies associated with Early Years mathematics education.

A copy of the paper can be found at www.acme-uk.org
The Advisory Committee on Mathematics Education (ACME), established in January 2002, is an independent committee which acts as a single voice for the mathematical community, seeking to improve the quality of education in schools and colleges. It advises Government on issues such as the curriculum, assessment and the supply and training of mathematics teachers. ACME was established by the Royal Society and the Joint Mathematical Council of the UK with the explicit backing of all major mathematics organisations, and is supported by the Gatsby Charitable Foundation.

Lake Baikal

Lying over the fault line between two tectonic plates, whose separation is gradually dropping its floor lower, the waters plunge to a depth of over one mile …It harbours nearly one fifth of all the fresh water on the planet. … Of the 2,000 species inhabiting its depths, 1,200 are unique to it… Some 250 aquatic plants endure only here. … 500 pound sturgeons take two decades to reach maturity and carry up to 20 pounds of caviar each. Minute, red-eyed gammarid shrimps live a mile down packed 25,00 to the square yard.

Colin Thubron, *In Siberia*, London: Penguin, 2000

Danger – Falling Rock

Two million cubic metres of the Eiger (twice the volume of the Empire State Building) is rapidly working its way loose. The crack is now widening at 75cm a day. In 2004 three lumps of the dolomites in northern Italy came loose. The biggest chunk – 74 metres high- fell more than a quarter of a mile to block a hikers trial.

The Guardian 08/07/06

Why are the pipes leaking?

Leakage in London remains unacceptably high. But we are doing all we can to get it down, at a cost of £500,000 every day. …
Over 3,000 miles of the water pipes are more than 150 years old. … We are working as fast as possible, with new plastic pipes being fitted in 23 areas in London. By 2010 we will have replaced 1,000 miles of our oldest and leakiest iron pipes.

Letter from Thames Water, 4 May 2006

Keeping learning on track:

formative assessment and the regulation of learning Part 3

In this third part of his paper on assessment for learning Dylan Wiliam considers the importance of students being involved in the assessment of their own work and understanding before they begin work what the criteria for assessment are.

Sharing criteria with learners

Frederiksen and White (1997) undertook a study of three teachers, each of whom taught 4 parallel year 8 classes in two US schools. The average size of the classes was 31. In order to assess the representativeness of the sample, all the students in the study were given a basic skills test, and their scores were close to the national average. All twelve classes followed a novel curriculum (called ThinkerTools) for a term. The curriculum had been designed to promote thinking in the science classroom through a focus on a series of seven scientific investigations (approximately two weeks each). Each investigation incorporated a series of evaluation activities. In two of each teacher's four classes these evaluation episodes took the form of a discussion about what they liked and disliked about the topic. For the other two classes they engaged in a process of 'reflective assessment'. Through a series of small-group and individual activities, the students were introduced to the nine assessment criteria (each of which was assessed on a 5-point scale) that the teacher would use in evaluating their work. At the end of each episode within an investigation, the students were asked to assess their performance against two of the criteria, and at the end of the investigation, students had to assess their performance against all nine. Whenever they assessed themselves, they had to write a brief statement showing which aspects of their work formed the basis for their rating. At the end of each investigation, students presented their work to the class, and the students used the criteria to give each other feedback.

the criteria will provide a focus for negotiating with students about what counts as quality in the mathematics classroom

As well as the students' self-evaluations, the teachers also assessed each investigation, scoring both the quality of the presentation and the quality of the written report, each being scored on a 1 to 5 scale. The possible score on each of the seven investigations therefore ranged from 2 to 10.

The mean project scores achieved by the students in the two groups over the seven investigations are summarised in table 3, classified according to their score on the basic skills test.

Group	Score on basic skills test		
	Low	Intermediate	High
Likes and dislikes	4.6	5.9	6.6
Reflective assessment	6.7	7.2	7.4

Note: the 95% confidence interval for each of these means is approximately 0.5 either side of the mean

Table 3: Mean project scores for students

Two features are immediately apparent in these data. The first is that the mean scores are higher for the students doing 'reflective assessment', when compared with the control group—in other words, all students improved their scores when they thought about what it was that was to count as good work. However, much more significantly, the difference between the 'likes and dislikes' group and the 'assessment' group was much greater for students with weak basic skills. This suggests that, at least in part, low achievement in schools is exacerbated by students' not understanding what it is they are meant to be doing—an interpretation borne out by the work of Eddie Gray and David Tall (1994), who have shown that 'low-attainers' often struggle because what they are trying to do is actually much harder than what the 'high-attainers' are doing. This study, and others like it, shows how important it is to ensure that students understand the criteria against which their work will be assessed. Otherwise we are in danger of producing students who do not understand what is important and what is not. As the old joke about project work has it: "four weeks on the cover and two on the contents".

no-one is seriously suggesting that students ought to be able to write their own school- leaving certificates

Now although it is clear that students need to understand the standards against which their work will be assessed, the study by Frederiksen and White shows that the criteria themselves are only the starting point. At the beginning, the words do not have the meaning for the student that they have for the teacher. Just giving 'quality criteria' or 'success criteria' to students will not work, unless students have a chance to see what this might mean in the context of their own work.

Because we understand the meanings of the criteria that we work with, it is tempting to think of them as *definitions* of quality, but in truth, they are more like labels we use to talk about ideas in our heads. For example, 'being systematic' in an investigation is not something we can define explicitly, but we can help students develop what Guy Claxton calls a 'nose for quality'.

low achievement in schools is exacerbated by students' not understanding what it is they are meant to be doing

One of the easiest ways of doing this is to do what Frederiksen and White did. Marking schemes are shared with students, but they are given time to think through, in discussion with others, what this might mean in practice, applied to their own work. We shouldn't assume that the students will understand these right away, but the criteria will provide a focus for negotiating with students about what counts as quality in the mathematics classroom

Another way of helping students understand the criteria for success is, before asking the students to embark on (say) an investigation, to get them to look at the work of other students (suitably anonymised) on similar (although not, of course the same) investigations. In small groups, they can then be asked to decide which pieces of students' work are good investigations, and why. It is not necessary, or even desirable, for the students to come to firm conclusions and a definition of quality—what is crucial is that they have an opportunity to explore notions of 'quality' for themselves. Spending time looking at other students' work, rather than producing their own work, may seem like 'time off-task', but the evidence is that it is a considerable benefit, particularly for 'low-attainers'.

Student peer- and self-assessment

Whether students can really assess their own performance objectively is a matter of heated debate, but very often the debate takes place at cross-purposes. Opponents of self-assessment say that students cannot possibly assess their own performance objectively, but this is an argument about *summative* self-assessment; no-one is seriously suggesting that students ought to be able to write their own school-leaving certificates. What really matters is whether self-assessment can enhance learning, and in this regard, accuracy is a secondary concern.

The power of student self-assessment is shown very clearly in an experiment by Fontana and Fernandez (1994). A group of 25 Portuguese primary school teachers met for two hours each week over a twenty-week period during which they were trained in the use of a structured approach to student self-assessment. The approach to self-assessment involved an exploratory component and a prescriptive component. In the exploratory component, each day, at a set time, students organised and carried out individual plans of work, choosing tasks from a range offered to them by the teacher, and had to evaluate their performance against their plans once each week.

The progression within the exploratory component had two strands—over the twenty weeks, the tasks and areas in which the students worked were to take on the student's own ideas more and more, and secondly, the criteria that the students used to assess themselves were to become more objective and precise.The prescriptive component took the form of a series of activities, organised hierarchically, with the choice of activity made by the teacher on the basis of diagnostic assessments of the students. During the first two weeks, children chose from a set of carefully structured tasks, and were then asked to assess themselves. For the next four weeks, students constructed their own mathematical problems following the patterns of those used in weeks 1 and 2, and evaluated them as before, but were required to identify any problems they had, and whether they had sought appropriate help from the teacher.

Over the next four weeks, students were given further sets of learning objectives by the teacher, and again had to devise problems, but now, they were not given examples by the teacher. Finally, in the last ten weeks, students were allowed to set their own learning objectives, to construct relevant mathematical problems, to select appropriate apparatus, and to identify suitable self-assessments.

Another 20 teachers, matched in terms of age, qualifications, experience, using the same curriculum scheme, for the same amount of time, and doing the same amount of inservice training, acted as a control group. The 354 students being taught by the 25 teachers using self-assessment, and the 313 students being taught by the 20 teachers acting as a control group were each given the same mathematics test at the beginning of the project, and again at the end of the project. Over the course of the experiment, the marks of the students taught by the control-group teachers improved by 7.8 marks. The marks of the students taught by the teachers developing self-assessment improved by 15 marks—almost twice as big an improvement. Now the details of the particular approach to self-assessment are not given in the paper, and are in any case not that important—Portuguese primary schools are, after all, very different from those in other countries. However this is just one of a huge range of studies, in different countries, and looking at students of different ages, that have found a similar pattern. Involving students in assessing their own learning improves that learning.

'low-attainers' often struggle because what they are trying to do is actually much harder than what the 'high-attainers' are doing

Frederiksen, J. R. & White, B. Y. (1997). Reflective assessment of students' research within an inquiry-based middle school science curriculum. Paper presented at the *Annual meeting of the American Educational Research Association*. Chicago, IL.

Fontana, D. & Fernandes, M. (1994). Improvements in mathematics performance as a consequence of self-assessment in Portuguese primary school pupils. *British Journal of Educational Psychology*, 64, 407-417.

Gray, E. M. & Tall, D. O. (1994). Duality, ambiguity and flexibility: a 'proceptual' view of simple arithmetic. *Journal for Research in Mathematics Education*, 25(2), 116-140.

Learning and Teaching Research Center at the Educational Testing Service, USA

Supporting children's mathematical learning – some short training sessions for Teaching Assistants

Teaching assistants need help to do their job properly says **Jennie Pennant**

Sometimes, as a teaching assistant, it can be hard to start working alongside children in their mathematics lessons when the mathematics seems so different from what they experienced at school. These three sessions all have the same format and are intended for maths subject leaders, or the person responsible for Teaching Assistants in the school, to carry out with TAs to strengthen their understanding of the mathematics that children are experiencing.

Each session is intended to last between 20 and 30 minutes and allow you focus on some key skills that children need to acquire:

- basic counting
- addition and subtraction facts to 20
- ordering numbers to 100.

To use the sessions you will need a copy of the DfES material entitled 'Models and Images'. This is obtainable, free of charge, from DfES publications – tel 0845 60 222 60 or by email dfes@prolog.uk.com. The reference code is DfES 0508-2003GCDI

It is suggested that the sessions are run over three consecutive weeks and that TAs carry out a task with the children they work with, based on the session covered, in between each session. This helps them embed their learning.

FIVE PRINCIPLES OF COUNTING

- The stable order principles: understanding that the number names must be used in that particular order when counting.
- The one-to-one principle: understanding and ensuring that the next item in a count corresponds to the next number.
- The cardinal principle: knowing that the final number represents the size of the set.
- The abstraction principle: knowing that counting can be applied to any collection, real or imagined.
- The order irrelevance principle: knowing that the order in which the items are counted is not relevant to the total value.

ORDERING NUMBERS TO 100

- **Resources**
  Flipchart and pens, paper, pens/pencils, copies of Progression sheet (ordering numbers to 100), NNS mathematical vocabulary book and Models/Images CDRom.

- **Aims of the session**
  To introduce/revise the basic principles of ordering numbers to 100 with TAs.
  To look at progression, questions and models/images surrounding this concept.

Basic Principles
The use of numbers all around us in the classroom helps with ordering. Language at lining up time, PE and registration can support this aspect of maths. The adult use of language then backs up the other work done in class. Pencil and paper strategies of empty number lines need to be used here. At this point it may be useful to model this on the flipchart with the extended use of the empty number line for ordering decimals. Do we have alternative displays of number squares so that the numbers go up as they get bigger? Could this be something we could change? *Look at the vocabulary book to check Y1/2 for the appropriate language to use.*

Potential Difficulties
Discuss how difficulties can arise. Use the reverse of the Progression sheet for examples of how children can become confused by ordering to 100. Discuss with TAs how they would move the children forward from these difficulties. Record any thoughts on the progression sheet for future use. Encourage the use of concrete apparatus, no matter what age the child in order to embed this concept.

Models and Images
Use the Progression sheet to look at the models/images shown. Look at the progress for ordering as indicated by the models. How is the concrete apparatus helping the child? What other practical and ICT resources do you have in your school which could be used for this activity? Look at the importance of the language which is used and how this helps the image when they are used together. *Use the CDRom – Kathy KS1-using the ITP Ordering…*

Progression
Using the Progression sheet, look at the progress of ordering to 100 throughout KS1/2. This should be helpful in tracking back for TAs if they have children who are in need of this. Discuss the questions which are posed. Could your TAs use these in the classroom and how would they help the child to progress?

What next?
TAs need to identify an aspect from the session to try out with the children they work with and feedback on their experience at the next session.

ADDITION AND SUBTRACTION FACTS TO 20

- **Resources**
  Flipchart and pens, paper, pens/pencils, copies of Progression sheet (addition and subtraction facts to 20) Models/Images CDRom and NNS mathematical vocabulary book
  (DfES 0313/2003)

- **Aims of the session**
  To introduce/revise the basic principles of addition and subtraction to 20 with Teaching Assistants.
  To look at progression, questions and models/images surrounding this concept.

Basic Principles
Using the Progression sheet, discuss the need for the basic principles of counting to be covered before this is introduced. Much of the addition and subtraction is achieved through practical apparatus and things with which the child will be familiar – eg. hands to look at number bonds to 10. The language of doubling is introduced at quite an early stage – do TAs see the need for this and the need for cohesive language throughout the school? Look at the vocabulary book to check Y1/2 for the appropriate language to use.

Potential Difficulties
Discuss how difficulties can arise. Use the reverse of the Progression sheet for examples of how children can become confused by addition and subtraction to 20. Discuss with TAs how they would move the children forward from these difficulties. Record any thoughts on the progression sheet for future use. Encourage the use of concrete apparatus, no matter what age the child in order to embed this concept.

Models and Images
Use the Progression sheet to look at the models/images shown. Look at the progress for addition and subtraction as indicated by the models. How is the concrete apparatus helping the child? Note that the images are reflecting the calculation (eg.5+2 is represented by 5 blue spots followed by 2 blue dots and not vice-versa.) What other practical and ICT resources do you have in your school that could be used for this activity? Look at the importance of the language that is used and how this helps the image when they are used together. Use the CDRom, Mandy Y1- using fingers.. Kathy KS1- using counters…, Kathy KS1 - using key rings.

Progression
Using the Progression sheet, look at the progress of addition/subtraction to 20 throughout KS1/2. This should be helpful in tracking back for TAs if they have children who are in need of this. Discuss the questions that are posed. Could your TAs use these in the classroom and how would they help the child to progress?

What next?
TAs need to identify an aspect from the session to try out with the children they work with and feedback on their experience at the next session.

BEAM, London

Flag waving and a Sense of Proportion

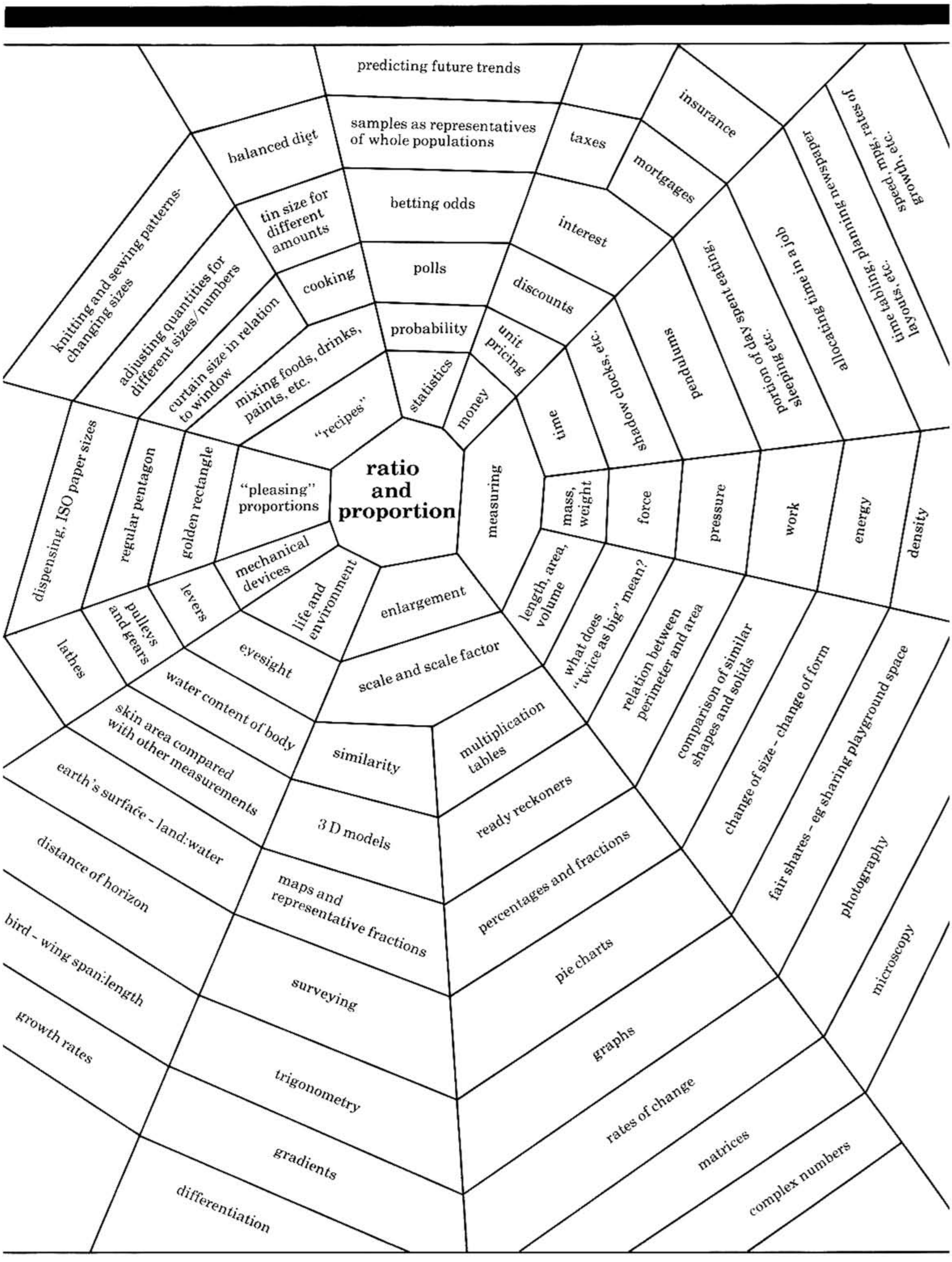
"Were you waving a flag last week at Queenie's 80th Birthday bash?" asked Gerry Rosen, writing from Israel. He went on to say that the radio (BBC World Service) of course was full of it. "What particularly caught my attention was an interview with someone who defined himself as a historian and professional pollster. He was asked about the country's attitude to the monarchy. He works for one of the famous polling firms and said that he had conducted two questionnaires – one on the monarchy in general and the second on Her Majesty!"

"He stated that, when asked about the monarchy in general, one in five were against but the reaction to the Queen was much, much better – eighty percent believed that she was good for the country."

"Last time I checked 80% for was 20% not!! Have things changed since I've been away?"

How many people have difficulty understanding ratio and proportion and interpreting percentages or other forms of fractions? In the *Equals* Workshop at the Mathematical Association Conference in April this year, Jacqui Bowers told how even mathematics advisors have been known to have problems and have been known to insist that 3/7 was the same as the ratio 3:7.

At the Workshop, run by Jacqui Bowers and Rachel Gibbons, we discussed ways of encouraging pupils to develop a sense of proportion and used all sorts of activities, mainly from the SMILE resource bank, to find new ways in.
Our posters included the one shown, which is taken from the booklet *A Sense of Proportion* from *Checkpoints* published by the ILEA in the 1970s and 80s, sadly now long out of print.


ratio and proportion
"pleasing" proportions
"recipes"
statistics
money
measuring
enlargement
life and environment
mechanical devices
golden rectangle
regular pentagon
dispensing, ISO paper sizes
mixing foods, drinks, paints, etc.
curtain size in relation to window
adjusting quantities for different sizes/numbers
knitting and sewing patterns-changing sizes
cooking
tin size for different amounts
balanced diet
probability
polls
betting odds
samples as representatives of whole populations
predicting future trends
unit pricing
discounts
interest
taxes
mortgages
insurance
time
shadow clocks, etc.
pendulums
portion of day spent eating, sleeping etc.
allocating time in a job
time tabling, planning newspaper layouts, etc.
growth, speed, mpg, rates of
mass, weight
force
pressure
work
energy
density
length, area, volume
what does "twice as big" mean?
relation between perimeter and area
comparison of similar shapes and solids
change of size – change of form
fair shares – eg sharing playground space
photography
microscopy
levers
pulleys and gears
lathes
eyesight
water content of body
skin area compared with other measurements
earth's surface – land:water
distance of horizon
bird – wing span:length
growth rates
scale and scale factor
similarity
3 D models
maps and representative fractions
surveying
trigonometry
gradients
differentiation
multiplication tables
ready reckoners
percentages and fractions
pie charts
graphs
rates of change
matrices
complex numbers

Reflections of reflections of reflections …

Parallel mirrors

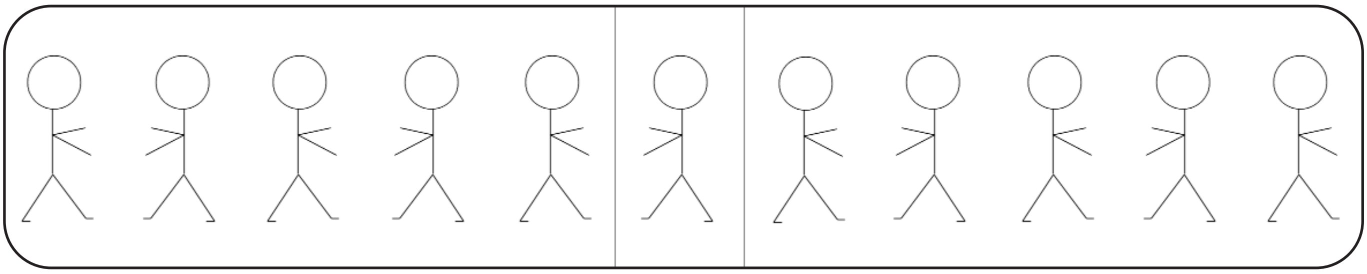
“She loves the mirrors,” says the woman with the pram waiting to get into the supermarket lift. Like the small girl, I too love the mirrors. Two form the side walls of the lift and another, at right angles, forms the back wall. Because the side-wall mirrors are parallel, they give a series of reflections of reflections of reflections … and on and on and on and on. If there is such an exciting lift in your area, is it too much to hope that your enterprising teacher will take your class to the supermarket so that they can take a ride up or down in the lift? Or maybe ride up and down several times as long as they don’t disrupt the other customers too much. Perhaps it would be better to suggest you go with parents or carers when they are shopping in the local supermarket. What a start to a study of reflection.

If no such trips are possible can you set up two parallel mirrors somewhere in the school which are large enough to give reflections of your whole self, or at least the top half of you? After reflection each of you can try to draw a simplified version of what you saw.

Photograph by Barrie Stead

The easiest way to do this is to use tracing paper,

- show yourself as a stick figure somewhere between the two parallel mirror lines (make one arm do something different from the other one)
- fold the paper along one mirror line
- trace your reflection on the folded paper
- unfold and then fold along the second mirror line and repeat the process
- unfold and then fold along the first mirror line again and repeat the process
- go on using the mirror lines alternately as fold lines …
- go on until you run out of space on the paper.



Kaleidoscope

Another exciting example of multiple reflections can be seen when looking down a kaleidoscope. The name is derived from three Greek words kalos (beautiful), eïdos (form) and skopeïn (to view). The mirrors in a kaleidoscope are at an angle of 60° or 45°. They do give very beautiful patterns with symmetries that are interesting to study and draw.

- The tracing paper method will help here too.

Poet's vision

Louis MacNeice wrote a poem entitled 'Reflections' which describes a mirror reflecting the reflected:

The mirror above my fireplace reflects the reflected
Room in my windows; I look in the mirror at night
And see two rooms the first where left is right
And the second, beyond the refected window, corrected
But there I am standing back to my back. The standard
Lamp comes thrice in my mirror, twice in my window,
The fire in the window lies one room away down the terrace,
My actual room stands sandwiched between confections
Of night and lights and glass in both directions
I can see beyond and through the reflections the street lamps
And home outdoors where my indoors rooms lie stranded,
Where a taxi perhaps will drive in through the bookcase
Whose books are not for reading and past the fire
Which gives no warmth and pull up by my desk
At which I cannot write since I am lefthanded.

Are his windows of the room opposite to the fireplace, giving the same effect as the lift mirrors, or not? Can you draw a plan of MacNeice' room showing the reflections he describes?

You may like to study patterns in MacNeice's poem, looking at its form and then maybe reading some more of his poems.

Then you might also enjoy writing a poem about reflections in the lift mirrors.

Teachers' notes

There are some quite difficult concepts underlying all this investigation of reflections but there is no reason why those who are not famed for their mathematical achievement should not make an approach to them through practical demonstrations and experiences. Williams and Shuard wrote that making shapes that "balance is a fascinating game … and leads to one of the most important concepts of mathematics – symmetry".[1] None of the mathematical principles of symmetry are mentioned in the suggestions of what to do here and teachers can analyse the resulting drawings of the objects and their images discussing them to whatever level of theory they think is appropriate for the individuals who have drawn them.

Williams and Shuard also remind us that it takes a long time to learn about left and right and that experience with reflection is useful in gaining a left/right understanding.

1. E. Williams and H. Shuard, *Primary Mathematics Today*, Harlow: Longmans, 1982

Insights for the past and present

More quotes from Molly Hughes. Molly (b.1865) was an innovator in the education of girls and her insights seem relevant today, even though some may seem outdated, or at least in need of attention.

Why bother with the letters?

In her first autobiographical volume Molly Hughes describes some of her own early difficulties: The joy of saying goodbye to arithmetic was tempered by fear of the unknown. What if mathematics should turn out to be worse? The completely unknown is never so fearful as the partially known. At my private school we had 'begun a little algebra'. This involved spending some two hours a week in turning complicated arrangements of letters into figures, with preliminary notes that *a* stood for 5, *b* for 7, *c* for 3, and so on. When the turning was done we added and subtracted as required and got an answer. One day I asked the teacher why they bothered to use these letters when figures were just as good, and there were plenty of them. She told me I was impertinent.

A London Child of the 1870s, OUP 1934 (reprinted 1977, 78, 79 as first part of the trilogy *A London Family 1870-1900*) p. 77

Putting red crosses on other people's mistakes

Her own introduction to teaching was an eye opener for her:

The majority of us who matriculated faced the fact that we should have to become teachers. It seemed a fairly pleasing prospect, mainly consisting, as far as work went, in talking and putting red crosses on other people's mistakes. But we now heard that you could be taught how to teach – a funny idea. … Along with some other enthusiasts Miss Buss was trying to raise teaching into a real profession, like Law or Medicine. p.112

The training colleges already in being did not satisfy their ideals, and they looked around for some appropriate place and for some appropriate person. Undoubtedly Cambridge with its colleges for women as encouragement, was the right background for general culture. And a certain Miss E. P. Hughes, one of the most brilliant of Newnham's graduates, was exactly the right one to be the Principal. pp. 113-4

The number of new ideas, new friends, new experiences, all coming to me in a rush, made this time at Cambridge seem very long in retrospect, in fact a large slice of my life. I can hardly believe that it lasted only two terms, from September '85 to April '86. To make the utmost use of such a short course Miss Hughes arranged for us to pay visits to different types of schools in the neighbourhood of our homes during the vacation. … One of my expeditions was to Croydon High School, where I was told: 'We have no rules and no punishments.' This staggered me, after the rigours of the North London. 'Oh it works all right,' said the charming Headmistress, 'the girls behave quite well without them; after all nobody wants to forget books or to be untidy, and really there's no fun in being unruly if you're not punished for it, is there?'

A London Girl of the Eighties OUP 1936 p. 154

Intuiting numbers:

Later Molly was involved in planning a course for women mathematics teachers in training:

The mathematics man … offer[ed] to give six talks. One would not be of much use, he said, for it was in the earliest stages of dealing with Number that the mischief began, and he would like to talk about these difficulties as well as the later ones. 'I find that my women students here in college cannot be broken of their school habit of shirking fresh thought, and waiting for some "rule" or "dodge" and then learning it by heart. Real grappling with a problem had become almost impossible for them. People ought to begin with realities in the cradle. …

His plan of teaching little children to intuit numbers by the use of playing card patterns was now become a common-place, but in those days it was a striking change from learning the multiplication table by heart.

A London Home in the 1890s OUP 1979
(first published 1946) p. 23

All angles the same:

Then she described the reactions of her own children to the studies she set them:

I don't know whether the love of measurement is common to all children, but the boys seemed to have a passion for it, and the eldest enjoyed even angles and the use of a protractor. I told him to draw any number of triangles he liked, all shapes and sizes; then to measure the angles in each very carefully and add the results. I went about my own business and after a long time came the surprised report: 'it's so funny, mother, they all come out the same!'

A London Home in the 1890s OUP 1979 (first published 1946) p.189

Renewing the Primary Mathematics Framework

The renewed primary mathematics framework is being launched during the autumn term of 2006. **Tim Coulson** of the Primary Strategy outlines some of the opportunities it provides and responds to an article in the June edition by some members of the Equals editorial team.

Teachers deserve the credit for the rise in mathematics attainment since 1997. Much has been achieved and the National Numeracy Strategy Framework for Teaching Mathematics has played an important part in this. Evidence from many sources shows the success of primary schools in making use of the drive on improving teaching and children's attainment in mathematics including use of the Numeracy Strategy teaching framework. These include:

- nearly 100,000 more children every year (15% more than was the case) achieve the nationally expected standard when they leave primary school than was the case in 1998;
- the near eradication of primary mathematics teaching described as 'unsatisfactory' by Ofsted;
- an international study in 1995 that put England's 10 year olds below international averages in mathematics whilst the parallel study published in 2004 showed that England's 10 year olds were well above international averages, and that the improvement in England between 1995 and the second study far outstripped any other country.[1]

So why renew the mathematics framework?

In discussions about developing the renewed framework, we have seen ambition from practitioners, teachers, heads and local authorities to improve further, particularly the teaching and attainment of children that find mathematics difficult but that teachers believe have the potential to succeed.

What are the issues? A review of Ofsted's evidence suggests that we need to attend to:

- too great a concentration upon coverage of the detail of the mathematics curriculum at the expense of ensuring that children understand the detail;
- weaknesses in planning for progression in learning mathematics;
- weaknesses in teachers' use of assessment;
- teachers still insecure about subject knowledge
- the quality of teaching mathematics in nearly one third of lessons is no better than satisfactory.

The renewed teaching framework in mathematics provides slimmed-down objectives in seven strands of learning that are designed to help practitioners and teachers see the 'wood for the trees' and give a clearer sense of the important aspects of mathematics that need to be taught to children each year. The strands of learning are set out to support discussion across settings and schools, Foundation Stage to Year 6, about progression in the specific aspects of mathematics. Assessment is integral to all the guidance on planning, and a particular focus is given to 'probing questions'.

'Teachers in Years 3 and 4 face the challenge to address the fall off of children achieving national expected levels'

For some, primary teachers' weaknesses in mathematics subject knowledge can sound rather repetitive from the mathematics community. A key message of the renewing of the mathematics framework is the importance of structuring children's learning not only across a lesson (and this is now much more successful than previously) but across a series of lessons. This is where the importance of teachers' own subject knowledge is crucial and we were delighted to see the new National Centre for Excellence in Teaching Mathematics open in June, and look forward to it stimulating greater Continuing Professional Development (CPD) provision for primary teachers.

Where teaching is 'satisfactory', there are children making progress in the lesson – but there are also children who find mathematics difficult that are unlikely to be making progress.

Headteachers are clear about this and feed back the importance of helping teachers for whom mathematics is not their strength. Whilst the renewed framework is for all teachers, we intend it to be most useful for teachers for whom mathematics is not their strength.

We are particularly keen to support teachers in Years 3 and 4 who face the challenge of addressing the fall-off of children achieving national expected levels. In 2005:

- 90% of children in Year 2 achieved level 2;
- 78% of children in Year 4 achieved level 3;
- 75% of children in Year 6 achieved level 4.

In the June edition of this publication, the editorial team shared its views on proposals in the draft of the renewed mathematics framework about calculation. Years 3 and 4 are crucial years for building on earlier work on number, early addition and subtraction and developing a clear sense and feel for number. The aim is for children to have a versatility and confidence with numbers that enables them to 'see' the best way to manipulate, combine and operate with numbers.

The draft proposals included a discussion paper on the development of mental and written calculation. We are at a stage when we can look back at the great improvements there have been in mental calculation and schools gaining greater clarity about progression in calculation. We can also see, however, significant weaknesses. The TIMSS study showed that whilst elsewhere 75% of 10 year olds answered correctly

15 x 9

only 59% answered it correctly in England. Our observations suggest that whilst mental calculation is considered important by most teachers, too many children do not receive any systematic teaching in it, beyond some practice in the starter part of lessons. We intend to do all we can through the renewed framework and the associated training to promote further the importance of consistent and systematic teaching of mental calculation.

'Whilst mental calculation is considered important by most teachers, too many children do not have any systematic teaching'

During the consultation on the draft version of the renewed mathematics framework, we received many comments about references to the term 'standard methods' – the 1999 Framework stated that: "Standard written methods are reliable and efficient procedures for calculating which, once mastered, can be used in many contexts but they are of no use to someone who applies them inaccurately and who cannot judge whether the answer is reasonable. For each operation, at least one standard written method should be taught in the later primary years but the progression towards these methods is crucial, since they are based on steps which are done mentally and which need to be secured first." We continue to expect that children have the right to leave primary school with at least one secure and efficient written method of calculation for each of the four operations.

One of the key steps taken by many schools has been the adoption of a calculation policy – no longer should children experience Mrs X's method and then a completely different approach with Mr Y. These policies have made clear the progression in understanding that children will be assisted towards and the particular procedures that the school would like children to master by the time they leave primary school. Many local authority mathematics teams have also developed helpful routes of progression to support schools. In the consultation, we have asked for views on whether the large numbers of children that move between schools should also be able to expect this consistency as they move from one school to the next. We will be working closely with local authority mathematics teams to support greater consistency between schools.

The renewed framework will refer to 'efficient' rather than 'standard' written methods but we will be working with local authorities and schools to consider the case for greater consistency of approach. We are keen that where teachers have successfully helped children acquire and develop an expanded method for a particular operation, these children should not be restricted from being able to refine this to an efficient method they have 'under their belt' by some local orthodoxy. In the new framework there is further detail on the steps and stages that children need to meet to secure a better understanding of written methods of calculation.

We see the renewal of the mathematics framework as a huge opportunity to refresh and renew the determination to support children who find mathematics difficult. We are pleased to have the opportunity to contribute to this publication and look forward to further dialogue about how best to crack the big challenges primary teachers face.

Primary Mathematics Strategy

i TIMSS, NFER 2004

What place mistakes?

Jennie Pennant discusses how changing our attitude to children's mistakes could help them to change their attitude and become more successful at mathematics, and learning in general.

I wonder what message we give to children either directly or subliminally about the place of mistakes in their maths learning. It is so sad to hear children say that they are 'rubbish at maths'. Their feeling of failure may well have been brought about by all too frequently feeling that they have 'got it wrong'.

Does 'not getting the right answer' constitute getting it wrong? Thomas Edison said, 'I've never made a mistake, I've only learned from experience.' So could it be that whatever the outcome of a child's attempts at a mathematical question it can be seen as progress and that some learning can be gathered from it? This would certainly make trying and experimenting with ideas a more productive and positive process. It could also give the child a more positive self image of themselves as a mathematical learner. Robert Schuller asks, 'What would you attempt to do if you knew you could not fail?' and if mistakes are viewed in the light of Thomas Edison's remark then it could be a lot of mathematics!

It may also help children's mathematical learning to be aware of the anonymous saying 'Only those who do nothing make no mistakes'. Once mistakes are seen as inevitable and welcomed because of the insight they bring about the process of solving a problem or calculation, then we are freer to have a go and see what happens.

However, I wonder how often as teachers, we model making mistakes, trying and readjusting our approach in the light of things learnt from our previous attempt, in front of the children. Sadly, to make a mistake can be seen as a slip up and not to be welcomed into the mathematical dialogue by the teacher. Yet surely we want children to see us at work as mathematicians so that they can copy our processing. You only have to watch Foundation stage children playing doctors and nurses, or similar role play activities, to see how important copying is to their learning. Do we model for them that cognitive conflict and dissonance as we seek to solve a puzzle and get stuck and frustrated whilst we search out the breakthrough strategy or thought?

It is interesting to reflect that we are discouraged from telling a child that an answer is wrong or from putting a cross in their book. Yet surely they soon realise that requests like, ' Tell me how you got that answer' can well be another way of telling them they have actually got it wrong and need to think again. It is a very thin disguise!

Correcting mistakes, or teasing out misconceptions, is a very important part of the learning process as we grapple to discover a more successful path through the problem. As Confucius said, 'A man who has committed a mistake and doesn't correct it is committing another mistake'.

'A man who has committed a mistake and doesn't correct it is committing another mistake'.

Do let's encourage children to welcome their mistakes as learning opportunities and help them to understand that the traditional saying, 'Failure is the mother of success' means that just because you don't immediately get the right answer you must have failed is untrue. Failed to get the right answer may be - but failed to learn something useful about the process you are exploring to achieve the right answer – never!

Happy making of mistakes……..

BEAM, London

Water shortage

Water use in UK gardens accounts for less than six percent of the average water consumption of a typical household (we all use a bout 150 litres a day, twice the amount of 20 years ago). …
Averaged sized gardens with a planted area of 200 sq m evaporates 4,820 litres every 10 days – established plants need the equivalent of 25mm rainwater every 10 days in dry spells.

The Garden, Vol 131, part 5 May 2006, London, RHS

More Proverbs from E. F. O'Neill

In recent issues we published the thoughts of **E.F. O'Neill** and some of his proverbs. We promised you some more, so here they are.[1]

Let teachers be spacious

Educate for difference

The passing of exams is not education - ask him something he does not know and see if he can find out

Real poverty is lack of imagination

The best way to learn is to live

What do the children do when the teacher is out of the classroom?

Children are only little devils when they cannot find something legitimate to do

Let teachers be human
They are not parrots – let them get off their perches

Simultaneous work means the suppression of individuality
Simultaneous work means the suppression of initiative

1. *E.F. O'Neill of Prestolee* as recorded by Gerald Holmes, London:Faber and Faber, 1952, see *Equals* 11.3 p. 6 and 12.1 p. 20.

The Harry Hewitt Memorial Prizewinner 2006

Sam Henderson (aged 12)

I am a mathematics teacher at St.Joseph's RC Middle School, Hexham. This is a small middle school with 343 pupils of varying ability. Our motto "Striving for Excellence" means we encourage all children no matter what ability to achieve their best. This is especially true for the children who have learning difficulties.

Sam started at St. Joseph's in September 2003 with a variety of learning difficulties. His psychologist report read:-

> "Sam has a diagnosis of ADHD and is on medication. He has a history of delayed spoken language skills but these are now appropriate for his age.
> His literacy skills are also delayed and these are still at about a 7 year old level.
> His numeracy skills are also well behind his peers.
> He has attention and concentration difficulties."

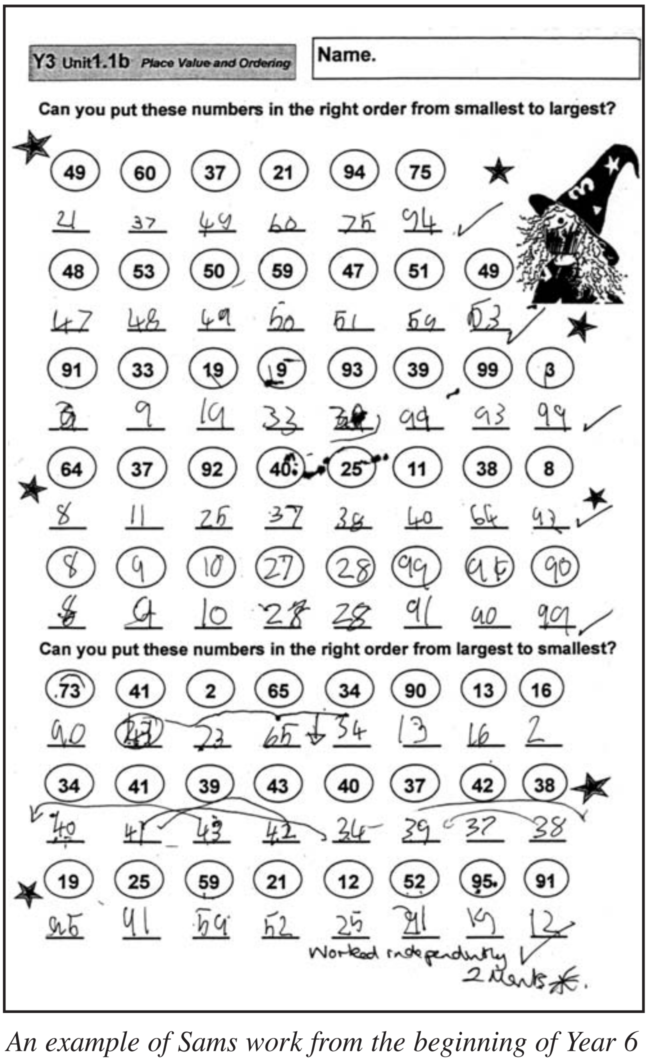
In Year 5 our mathematics groups are organised so that we have two parallel groups, a middle group and a fourth group for pupils who find maths very difficult. I took Sam in the fourth group with 11 other pupils. Sam entered St. Joseph's at National Curriculum level below 2. One of the first lessons I remember is when I asked Sam which number I was pointing at on the number square. The number was 27 and Sam said it was a 2 and a 7. He could not at this stage recognise numbers over 10. He had a teaching assistant with him every lesson to help him and he relied greatly on her to the extent that he wanted her to do the work for him as it was too hard. He was quickly given a special programme of work as he could not tackle the work of the rest of the class. The work of the class at this time was revision of level 2 touching level 3 from *Target Maths* Year 4. Sam did have strengths however, his spatial skills were more developed so when we were doing shape or graph work he did well and this gave him a boost.

In Year 6 he had improved, but not enough to catch up with the class so he continued to work on his special programme. I have included a sheet from his Year 6 book. Mrs Storey (his teaching assistant) and myself decided we would try to make Sam a more independent learner so gradually over the year she would leave him to work by himself until he was working almost totally by himself and even started puting his hand up and answering questions. We saw a marked improvement by the end of Year 6 when he moved on to another teacher, Mrs Garraghan and a new teaching assistant, Mrs Stanley.

In Year 7 he has continued to improve and the optional Year 7 national tests showed that he was now working at a National Curriculum level 3A. he is now able to work with the class and he is no longer the poorest in the class. I have included Sam's thoughts on his recent maths work from Year 7 to compare with the work from the beginning of Year 6 and a letter from his parents.

We are delighted with Sam and think that to move nearly 2 levels in as many years is fantastic and for this reason I would like to nominate him for the Harry Hewitt Memorial Prize.

Mrs Maureen Stevens



An example of Sams work from the beginning of Year 6

In year five it Was hard doing maths
but now I can do my ten times
table
I can now do the same work as
the Class but it used to be difficult
maths is all right.

Sam Henderson

Woodside
Slaley
Hexham
Northumberland
NE46 1TT

Reference: - Samuel James Henderson
St Josephs Middle School
Hexham

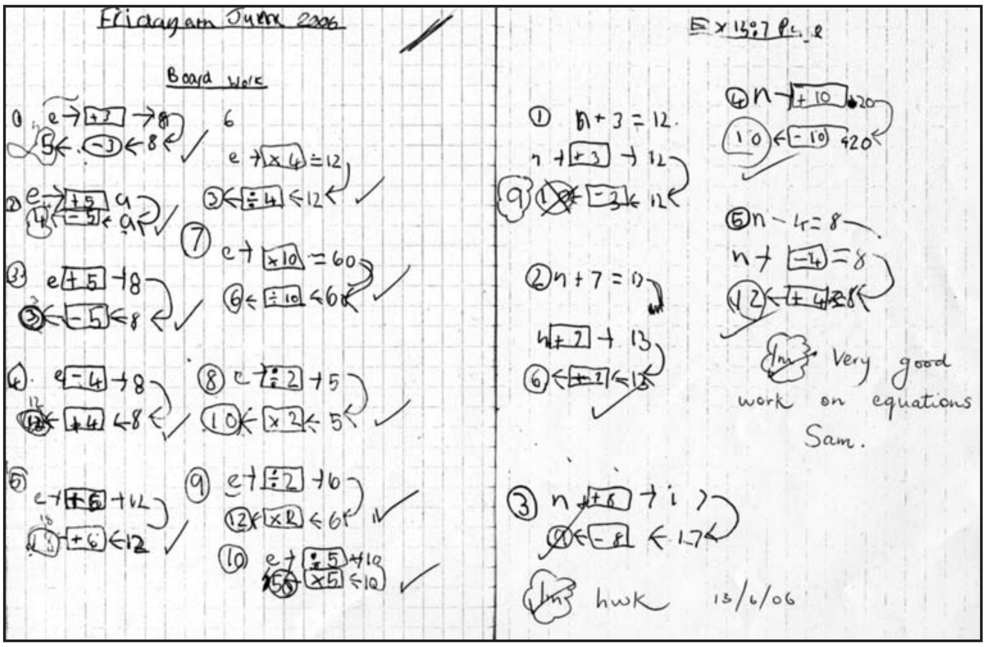
We are writing to give our support to Sam regarding the special math's award.

When Sam started St Josephs his understanding of math was very poor. Over the 3 years he has been at St Josephs he has made significant improvement thanks to the hard work of his teachers and classroom assistant. Thanks to their help Sam has become more confident and is willing to try hard to succeed.

Yours faithfully

MAHenderson

R E & M A HENDERSON



An example of Sams work from the beginning of Year 7

Correspondence:

We were not the only ones to find flaws in the new primary framework. Robert Clarke explains his unhappiness with a statement the draft made

11 June 2006

To the Equals Team,

Ref. Vol. 12 No. 2 p 5 eg. 563 — 278

I am unhappy with the initial statement

$$\begin{array}{r} 500 + 60 + 3 \\ -200 + 70 + 8 \end{array}$$

which actually means 563 — 200 + 78 and is not what is intended. Strictly it should be

$$\begin{array}{r} 500 + 60 + 3 \\ -(200 + 70 + 8) \end{array}$$

but to avoid the brackets

$$\begin{array}{rr} & 500 + 60 + 3 \\ - & \\ & 200 + 70 + 8 \end{array}$$

would be more acceptable.

This is not trivial since at a later stage expressions like the following will occur $2x + 5y + z - (x + 7y + 2z) = 2x + 5y + z - x - 7y - 2z$,

In general it is bad practice to use a rule early on which needs correction later.

I hope this comment is not too pedantic, but it is a good illustration of the great care which is needed with mathematics.

Best wishes
Robert J. Clarke.

An e-mail from Ian Adamson took us to task for our lack of a research base for our comments on the new primary framework. We hope his comments inspire a response form many readers for our next issue:

Dear Equals editorial team,

I am a recently PGCE'd teacher of maths at Croydon college. The various theories of learning which were explained to us were of no practical use, since it seemed that there was no evidence to support any one of them over any other, in any circumstances. It was all opinion and hearsay. Your description of the recent consultation email (of which I would dearly like a copy, please) indicates that it too promulgates ideas which are merely personal preferences, with no citation of any evidence that any of them works at all.

However, your response is no better. It resorts to repeated use of the word 'surely', rather than a documented explanation of your position. Why is there no evidence-based research cited? Is there any properly documented research on the most effective teaching strategies for maths? I realise that it is difficult to organise, but is not the Mathematical Association in a position to promote it?

The recent Scottish study of the use of synthetic phonics in the teaching of reading shows that such proof of efficacy is possible - the question remains why it was not done many, many years ago. and why are educational 'experts' still querying it. What is being done in a similar vein in the area of maths teaching?

The reference to groups of teachers working together to share best practice is most encouraging. However, unless these groups can demonstrate (and I mean with statistical certainty, not anecdotally) that 'their methods' are superior (not necessarily absolutely, but at least in the circumstances of a controlled trial), why should the 'experts' on high listen to them any more than the experts are listened to?

My lack of experience may mean that I have missed out on a whole body of best practice that has been proven to be so in statistically validated controlled trials. This is what I would expect the Mathematics Association to be promoting. Please prove me wrong.

Yours sincerely,

Ian Adamson

Date: June 28, 2006

However, the next day he reflected further and his next e-mail started

Dear Equals editorial team,

Having just taken you to task for your non-mathematical approach to the above, I must absolutely commend you in the highest possible way for the very next article 'Keeping learning on track'. It contains some of the most mind-blowing research that I have ever come across - in particular the finding that revealing marks undoes all the good of formative comments blew me away.

Finally we received a letter from Alan Wood taking us to task for using the phrase "Getting the Buggers to add up". Certainly the words are not ones that one would have been found anywhere in the educational scene a few years ago, but they do of course vividly express feelings of frustration which we all must have experienced at some time or other. However, Alan Wood suggests we should all respect the feelings of others and reminds us that

> mathematics proficiency is not the only responsibility of those in education, and it is without question the too ready use of swearing by celebrities, sports stars and parents which leaves some children to follow suit and which hinders the re-introduction of discipline (without violence) in schools.

We are adding a letter from one of the editors which appeared (without the footnotes) in the TES in response to Anita Straker's input there into the primary framework consultation. (An earlier letter from some of the editors did not get printed but its text was used in an article discussing the proposed new framework.)

Concept or algorithm?

Dear Letters Editor TES[1]
As one of the editorial team of *Equals*: who did refer to a return to "the dark ages", realising potential in mathematics for all ' I would still argue with Anita Straker's plea for all elements in the new primary framework. Yes, methods of calculation have always been there but the extra stress on them is alarming.

It was as far back as 1979 that Stuart Plunkett wrote:

> 'For a lot of non-specialist teachers of mathematics (the vast majority of primary school teachers), as for the general public, the four rules of number are the standard written algorithms. Concept and algorithm are equated. So to teach division you teach a method rather than an idea. …[2]'

It is still the ideas that matter. Also around that time Michael Girling, HMI, defined numeracy as 'the ability to use a four function calculator sensibly'[3]. This surely is the simplest, most straightforward method for any calculation. It is the method I use and I guess Anita does too and we both use it sensibly because we both have a sound understanding of the underlying concepts. Like Michael Girling I would say
'The most refined methods of long division, for instance … need not be taught.'
And with Stuart Plunkett would ask, 'Why teach any refined methods?'

1. 'Maths without understanding', Letters, *TES*, 23 June 2006
2. Stuart Plunkett, 'Decomposition and all that rot', *Mathematics in School*, Vol. 8 no. 3, May 1979
3. Michael Girling, 'Towards a definition of basic numeracy', *Mathematics Teaching* no. 81

Should we take a plane?

Aviation is the fastest growing source of greenhouse-gas emissions – more than 10% if the UK total. The sheer rate of growth is staggering: as 12% per year, aviation is growing faster here than even in the boom economy of China. … aviation emissions will double buy 2020 and quadruple by 2050.

‚Fly and be damne' Mark Lynas, *New Statesman* 03.04.06

20 years ago …

20 Years ago, reactor 4 of the Chernobyl Power Plant exploded. Around 135 km2, an area the size of Greece, was heavily contaminated with caesium-137. Today, still 5.5 million people live there. The Ukrainian government reported in March 2002 that 84% of the 3 million people who had been exposed to radiation, were registered as sick. In 2000, the number of adults with thyroid cancer was 48% higher than in 1986.

Petition www.million-against-nuclear.net

Reviews

Mundher Adhami reviews Tandi Clausen May's latest book and adds some thoughts of his own

Teaching Maths to Pupils with Different Learning styles
Tandi Clausen-May
Paul Chapman
London
ISBN for paperback 14129 03599

This rich, small volume has a clearly formulated call for teachers to recognise the learning differences amongst pupils. Rather than seeing difficulties, we should see preferences that call for adjustment to the curriculum and pedagogy. The book offers several practical activities and resources across the curriculum that teachers might try in the classroom. About half of the suggestions and resources are shape, space and measurements, and data handling

Tandi Clausen-May, a researcher at the National Foundation for educational Research (NFER), has several years of experience in primary and secondary school mathematics curriculum and assessment. She is one of the many educationists who are critical of the continued dominance of formal methods of teaching and reliance on procedures and rote learning, however disguised. She rightly regards her experience with what are commonly known as learning difficulties as being applicable to the mathematical education of all pupils in the mainstream.

The rationale for teachers taking into account the differences of learning style across the ability range is powerful in the introduction and elsewhere. A nice start for that is a quote from Albert Einstein stating that the elements of thought in his case are 'of visual and some of muscular type', while 'conventional words or other signs have to be sought for laboriously only in a secondary stage.' Tandi rightly highlights the fact that this well established difference of processing mathematics, at higher levels sometimes described as algebraic vs. geometric, is ignored in the current curriculum. The emphasis on written assessment dominated by written texts and symbols lies heavily on teaching at all levels. On page 6 she notes the potential of using Cognitive Ability Test scores in highlighting differences between verbal, non-verbal and quantitative reasoning of pupils so that teaching can cater for them.

The approach advocated in response to these problems is labelled VAK: Visual, Auditory and Kinaesthetic. This has become fashionable of late, not a bad development in itself, however some of us tend to be suspicious of fashions in education.

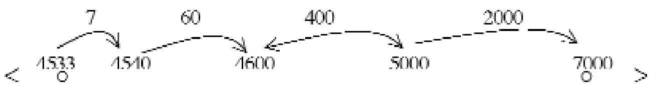
Let me pose a couple of issues here, hoping they engage some colleagues and perhaps enrich the debate. The first is to do with ***seeing***, and the way it is confused with ***doing***, while both are opposed to hearing. Is that really justifiable? It seems to me that unless what you see makes sense to you in terms of action that you yourself have done, or can easily imagine yourself doing, it is no different from any other phrase or symbol. Anything that is not pictorial or iconic of something familiar is of no use as a substitute to physical activity. So the three – seeing, doing and hearing - are not of the same status in learning and cognition, and it is seems best to stay with the psychological distinctions and debate around the action and the word, and which comes first.

There has been a rich historical debate about that including within the constructivist tradition, e.g. between Piaget and Vygotsky with the latter in the end coming to the position of the former. Each of us builds our mental map of the world and the interrelations within it through activity, always mediated by words, images, icons and interpretations. The bits, whether primary and physical or symbols and cultural, have to fit each other to make sense. This 'fitting-together of bits' allows us much flexibility in finding the bits themselves through any number of routes since all is connected. Otherwise we must rely on memory of fragments and procedures that are much more prone to confusion or forgetfulness for not being coherent or consistent. In schools constructivism means activity-based learning and mediation by teachers and peers. I think that is a better description than VAK.

One example is 'seeing' located within the simplest mathematical models, e.g. the empty number line. What follows should not detract from the value of the number line, but rather raise a question as to whether we are not undervaluing it, and even changing it to a simple procedure, leading to discarding it.

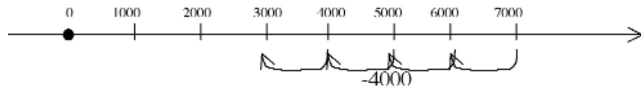
Of the many excellent models and examples Tandi has offered I think the ones on the number line are rather weak, reflecting the current practice that seems in need of salvage.The issue is skipping what I think is a necessary step of imagining a rough scale while using the number line.

It is not really clear what the number line adds to understanding of subtraction, or ease of calculation in the example (page 4) for **7000-4533=2467**



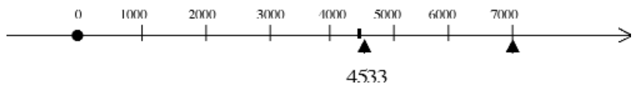
Does it really needs a number line to think, say, that '4533 add 7 makes it 4540, then 60 makes it 4600 then 400 then 2000, so …'? Probably even better in reverse order? And what does it do to the sense of quantity to have the distances on the number line shown in such a way, so that the 60 is only slightly different from the 400 or even 2000?

I think there is a great advantage for pupils to always see the number line with a scale – at least a rough kind of scale. The preference is to have it vertical with a zero line so that going up and down, higher and lower is associated with larger and smaller numbers, and makes the negative numbers possible early. But even as a horizontal line with the convention of the numbers going up to the right (not automatic for languages that are from right to left), the zero and the rough scale seems advantageous. For the example above, **7000 - 4533 = 2467**, compare the visual image with the following two:



Each move seems more visually meaningful: subtraction is 'going back', each of the 1000s a step, and the equal scale, however approximate, is maintained. The answer is imaginable on the number line as being below the 2500 and can be calculated by first going down 30 then another 3.

A second image is possible by asking the student first to find the two numbers on the number line, then find how much to move between them, i.e. the difference meaning of subtraction. The answer again is visual in a rough scale and easily workable.



The thrust of Tandy Clausen-May's book is clear: to go for activity based teaching. And that is a direction that requires much thought. Otherwise the new mathematical models we offer our pupils in the process may either be ignored or turned into procedures like others, whereas they should be used to allow further refinement and elaboration, by maintaining coherence.

Jane Gabb reviews Geometric Patterns series from Tarquin: *Geometric patterns from Roman Mosaics* (ISBN 0-906212-63-4), *from Churches and Cathedrals* (ISBN 1-899618-13-9), *from Patchwork Quilts* (ISBN 1-899618-41-4), *from Islamic Art and Architecture* (ISBN 1-899618-22-8). All by Robert Field. Available from www.tarquin-books.demon.co.uk at £3.70 each.

These little books are a delight and a great resource for cross curricular projects. They may well get you talking to your Art and Design colleagues and encourage you to set up an imaginative project based around mathematical pattern. The books contain a wealth of information about the patterns to be seen in real life situations. There is encouragement to try and draw them, for it is only by trying to reproduce a pattern that one begins to understand its construction and its different parts. This is very valuable.

The author first began to take an interest in Roman patterns while on holiday in Southern Spain. He was not allowed to take his camera into a museum where there was a wonderful floor and decided to sketch the pattern in order to remember it. The book contains lots of photographic examples taken from places in England as well as other places in Europe. Detailed grids support the reader in constructing the patterns.

The churches and cathedrals book has patterns from ceilings, columns, windows, arches, ironwork and floors from churches all over England, France and Italy. The patterns here contain more curves than the Roman ones, which makes them more difficult to construct but the author has drawn useful diagrams to show the elements clearly.

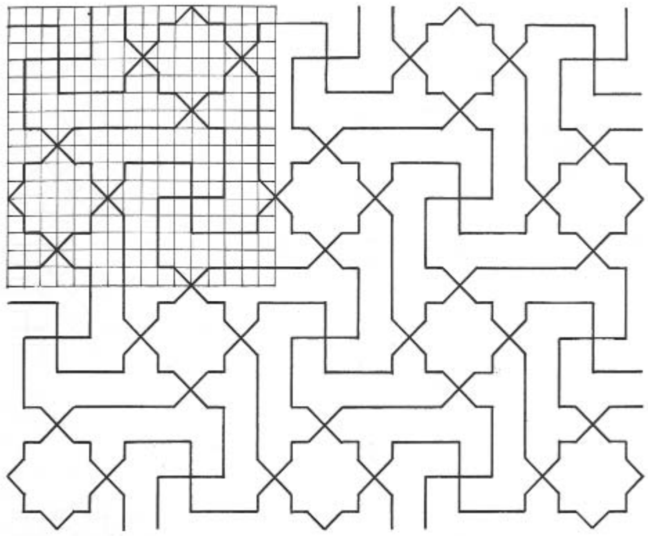
The Patchwork quilts book is very attractive and colourful as you might expect, with lots of useful grids which show how the different patterns are made up. The visual effects of using different patterns are stunning and make one want to reach for a needle and thread immediately. I can envisage a cross curricular project combining design technology (textiles) with mathematics to the enrichment of both subjects.

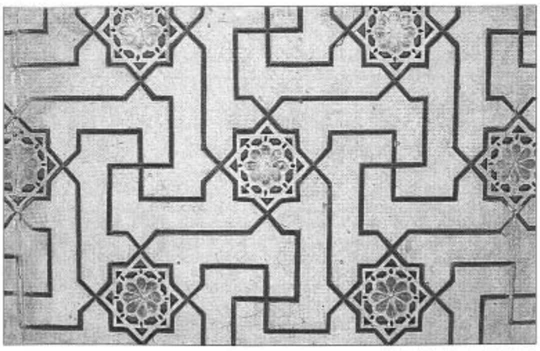
My personal favourite is the one on Islamic art and architecture. I find these patterns have a real interest because they often link shapes which are common with those which are unusual. Different patterns based on circles or stars are explored with many photographs of real places from Istanbul and Syria to Woking! The use of squared overlays is helpful in understanding how the complex patterns are built up.

So get planning for activities and this could be an eye-opener for everyone, not just your students!

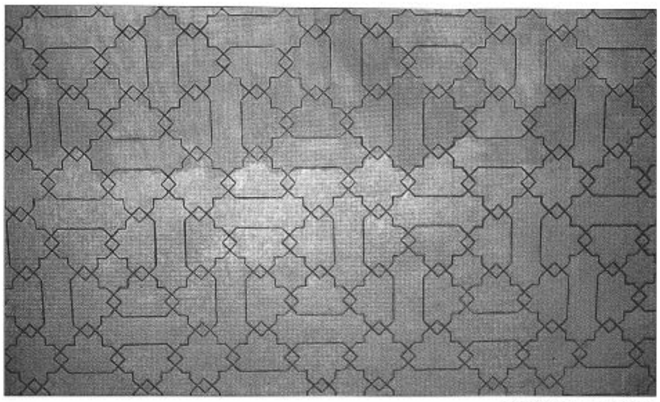
Royal Borough of Windsor and Maidenhead

The Alhambra Palace has an astonishing range of repeating patterns in the tilework and stucco decoration which cover its walls. Both of these patterns come from the Lion Courtyard and both are ingenious developments of squares set obliquely upon a square grid. Look carefully at the way they are achieved.





The Alhambra, Granada



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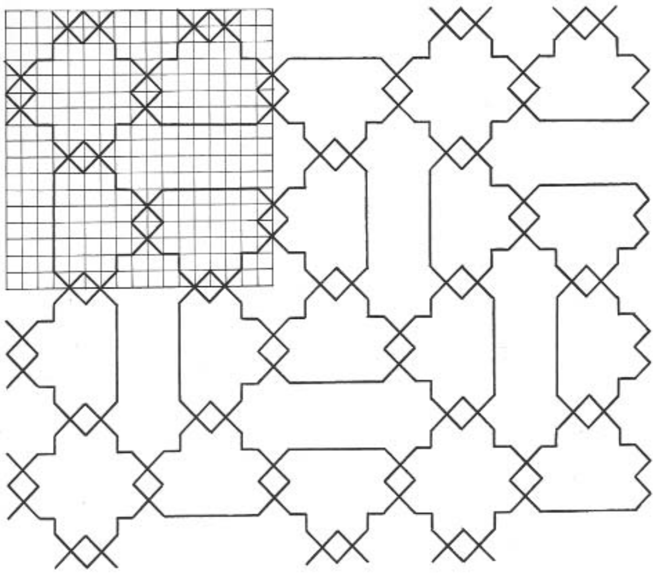

This excerpt from *Islamic Art and Architecture* illustrates how the patterns are taken from photographed and then extracted onto a grid to make the understanding of the pattern clearer and the drawing easier.

Ratty returns to the riverbanks

The release of 500 young water voles (Toad's long suffering companion in The Wind in the Willows) into a river should go some way to halt the fall in population. … The conservation project, launched by the Game conservancy Trust, will see voles released in 50 separate colonies along 18 miles of riverbank.

The Guardian 01/07/06