Equals

Realising potential in mathematics for all

for ages 3 to 18+

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MATHEMATICAL ASSOCIATION

supporting mathematics in education

Realising
potential in mathematics
for all

Front Cover image: Pair of Black-browed Albatrosses (Diomedia melanophris) in flight, by Michael Gore.

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Editorial

Yet more changes for teachers. While educators think of their work in term of life times, the politicians' long term view is only of four or five years duration. We now have to come to terms with the draft of the 'Renewal of the mathematics framework' (or refuse to do so maybe). On its first page we are told that advice has been incorporated on 'effective assessment'. We would suggest you could not better the advice given by Dylan William in the pages of *Equals* - in the last issue, this one and the two to follow, we have separated into 4 digestible pieces his latest thoughts on assessment for learning.

The chief problem some of the editors have with this new framework is the uniformity it is advocating. If the idea of a school having a one and only method of 'doing long division', say, is shocking, how much more is having a **national** method for long division? When does one want to use such a method? What is more important is that one should understand what it is one is doing. In 'Keeping things in proportion' Martin Marsh suggests a method of visualising processes which involve long multiplication and long division which would be best tackled with a calculator where the numbers get difficult. 'Speaking and listening' shows the importance of talking together about mathematical ideas, the final methods chosen by those involved in the discussions are not thought to be of such great importance. In the study of the albatross in the centre spread what is important to understand is what calculations are needed to study the distances, speeds, sizes under consideration. *How* you perform those calculations is of secondary importance.

The problem of just learning the rules was felt over 100 years back. The London girls of the 1880s, Molly Hughes and her friend Winnie, were both "unable to fathom the reason for turning a division 'upside down and multiplying'." They "agreed to 'never mind but just do it'." With this stress on standard methods in the new framework are we in danger of similar situations arising in the classrooms of today?

Mundher Adhami in 'Frogs and co-ordinates' has some fresh ideas about introducing the language of graphs and co-ordinates with a new view of numbers and number lines. This could give us another entry into proportion and a quick solution of many proportion problems rather than resorting to standard methods of long division and long multiplication. It is important to let pupils follow, as far as possible, their own chosen methods for all these calculations and in 'Speaking and Listening' we are further reminded that teachers listening to pupils and pupils listening to each other allows the children to increase their understanding of the problems being studied.

None of the contributors to this issue of *Equals* advocates the blind following of rules. Nor do they attach much importance to what method comes out in the end, but we are all exploring, in one way or another, how to increase our pupils' understanding of mathematics. I have told the story many times of a course I ran with a colleague many years ago for primary teachers who wanted to improve their own mathematical understanding. They all came wanting us to re-teach them the rules. "How do you do long division? Isn't there something about counting up the numbers before - or is it after - the decimal point?" We refused to answer these questions and persuaded them to tackle some mathematical puzzles and problems. After a few weeks they were no longer interested in listening to a 'performance' by either of us. They found it much more interesting and meaningful to work singly or in groups making up their own rules as they went along, in other words doing mathematics and only checking with a 'teacher' when they chose.

Heather Scott's article, 'seeing the bigger picture' does concentrate more on results – exams and all that final measurement stuff, but even so, she talks of the excitement of 'seeing the penny drop'. It is understanding that is the priority.

Women in Prison

In September 2005, the prison population was 77,807 4,768 of whom were women
In the last decade the female prison population has increased by 200%
61% have no qualifications
25% have been in care as children
Over 50% report that they have suffered domestic violence
37% have previously attempted suicide
Over 60% are mothers
Nearly 60% are held away from their home area
The majority of sentenced female prisoners are held for non-violent offences

Liberty Winter 2005: sources; Social Exclusion Unit. Prison Reform Trust and the Prison Service

Renewing the Primary Mathematics Framework

The renewed primary mathematics framework is about to be launched in September 2006. The *Equals* team have received a copy of an e-mail sent to all local authority mathematics teams as part of the consultation process which has prompted this response from Rachel Gibbons, Mundher Adhami, Martin Marsh, Jane Gabb and Nick Peacey, particularly in relation to the major contentious issue - that of the approach to calculation.

If you had put a group of 20 people connected in any way to primary mathematics education and asked them to come up with something that would improve standards of mathematics education in primary schools, the idea of renewing the primary mathematics framework would have been a long way down the list of priorities and might well not even have been on the list. And looking at what has been done, particularly in relation to the approach to calculation, the time might have been better spent. We believe the new framework will pressurise teachers even further into becoming automatons, so obsessed with following the rules and 'delivering' lessons and methods that they lose all the flexibility to respond to their pupils' individual needs. As the *Equals* editorial team attempts to absorb the 35 closely printed pages several of us wonder what indigestion problems teachers are going to have with this new electronic version.

There is initial encouragement. On the second page of this document teachers are urged to be flexible. However, this is soon dispelled as not being the reality by the following pages that contain mountains of advice and guidance that seems to assume that teachers do not have minds of their own. It is rather condescending given all of this information to state that:

'A high performing school … should not throw its planning away and start again.'

What self-respecting school would contemplate doing such a thing?

The 'new' framework will be electronic and has, we are told, been introduced to 'support learning in mathematics and strengthen pedagogic practices'. It is intended to give teachers' 'a 'clearer picture of progression' and 'generate further momentum to address the one in three mathematics lessons that are no better than satisfactory and not sufficiently challenging to ensure progress for all children'[1]. These are very noble aims but could a document, electronic or otherwise, be the best vehicle for bringing this about? We do note, with relief, that "the 'using and applying mathematics' strand will be given the greatest attention" and look forward to seeing how this is going to be accomplished.

To coincide with the launch there will be five days of training for mathematics managers. The first 'option' of the training package is entitled 'Calculation, mental and written strategies – practice and application. Planning and pedagogy.' It is the proposed approach to calculation that causes us greatest trouble and with which the rest of this article is concerned, in particular written methods of calculation.

'The most refined methods of long division, for instance … need not be taught.'

Tim Coulson poses the following question: 'Should there be a national approach to teaching calculation?'[2]. Of course not. The suggestion that schools should 'adopt a common approach to calculation that all teachers understand and work towards' is very prescriptive and open to all sorts of misunderstandings. If the idea of a *school* having a one and only method of 'doing long division', say, is shocking, how much more is having a **national** method for long division? Unless that method is 'use a calculator and perform some checks to confirm you have pressed the right buttons'. Michael Girling, HMI, defined numeracy years ago as the ability to use a four function calculator sensibly'[3]. The new framework's stress on the 'right method for various calculations' seems to leave out all consideration as to whether being numerate should include any understanding of algebra or geometry. As for the idea that there should be a national approach to teaching calculation with "**the** standard written method, which will be taught for each operation", we are amazed at such a backward step.

Surely, we want all pupils to develop a repertoire of well understood strategies that they can apply in different contexts. Confidence will not be gained by trying to swallow whole the extraordinary didacticism of the framework guidance, it will be brought about by respecting our pupils, their views and their preferred ways of working.

With this new version of the framework we are in danger of returning to the days before May 1979 of which Stuart Plunkett wrote:

'For a lot of non-specialist teachers of mathematics (*the vast majority of primary school teachers*), as for the general public, the four rules of number are the standard written algorithms. Concept and algorithm are equated. So to teach division you teach a method rather than an idea. …[4]'

If the idea of a *school* having a one and only method of 'doing long division', say, is shocking, how much more is having a national method for long division?

Plunkett also goes on to remind us that Michael Girling wrote
'The most refined methods of long division, for instance … need not be taught.'
"Why", asks Plunkett, " teach any refined methods?"

Consider the guidance given on subtraction.
The approach to written subtraction calculations is broken down into three hierarchical stages:

- using the empty number line by counting up,
- partitioning,
- expanded layout leading to the standard method of decomposition.

The aim is 'that children use mental methods when appropriate but for calculations that they cannot do in their heads they use *the* standard written method accurately and with confidence'[5]. It is the word '**the**' with which we have the greatest problem. The rationale for its use runs:
'typically children's readiness for the standard methods is about:

- late Year 3/4 for addition,
- Year 4 for subtraction,
- Years 5 and 6 for multiplication, and
- Year 6 and beyond for division'.

'The most refined methods of long division, for instance … need not be taught.'

What a petty 'goal' for the primary mathematics curriculum - that pupils work towards these written methods at whatever level of mathematical ability.

Suggesting these age-related expectations is very dangerous both for pupils and teachers and particularly for those pupils who struggle with mathematics. It surely goes against Cockcroft's 'seven year difference'? The argument is based on the false assumption that the standard method is some sort of gold standard that every child has a right to achieve. There is a clear message that a child who calculates 613 – 237 using the empty number line - or some other method - is somehow not as advanced numerically as a pupil who calculates it using the vertical method of decomposition. Is s/he two stages behind?

The empty number line method is described as 'a staging post' and the standard method as an 'end point'. We strongly disagree with this point. Not only is the empty number line a highly efficient method for doing this calculation (as is any method which gets the correct answer in a reasonable amount of time) it supports mental methods because of its strong visual image, develops pupils' understanding of place value and always treats the numbers as whole numbers rather than bits of numbers. Surely, the major priority of the mathematics curriculum should be to develop a number 'sense' for pupils of all ages. The notions of approximation, estimation, rounding etc. and how these link with the number line, measurements and accuracy are vitally important in accomplishing this. So rather than downgrading the number line, its use should be expanded by encouraging equal intervals and approximate proportions. The beauty of the number line is that it can become more sophisticated with age, reaching continuity, infinite subdivision and in exploring rational and irrational numbers. The limitations of the number line at the moment are that its use is confined to calculation.

The 'gold standard' of decomposition is only more efficient in the sense that there is less writing, but this does not necessarily mean that it should always be the end point in subtraction. The confusion associated with the expanded method of decomposition leading to the standard method has resulted in hardly any schools to our knowledge even attempting to explain it to pupils.

Is there really any chance that Year 4 pupils, and a good number of Year 4 teachers, will understand this?

Example: 563 - 278, adjustment from the hundreds to the tens and the tens to the ones

$$\begin{array}{r} 500 + 60\ + 3 \\ \underline{-\ 200 + 70 + 8} \\ \end{array} \qquad \begin{array}{r} 400 + 150\ + 13 \\ \underline{-\ 200 + \ \ 70 + \ \ 8} \\ 200 + \ \ 80 + \ \ 5 \end{array} \qquad \begin{array}{r} 400 \quad 150 \quad 13 \\ \cancel{500} \quad \cancel{50} \quad 13 \\ \cancel{500} + \cancel{60}\ + \cancel{3} \\ \underline{-\ 200 + 70 + 8} \\ 200 + 80 + 5 \end{array} \qquad \begin{array}{r} 4\ 15\ 13 \\ \cancel{563} \\ \underline{-\ 278} \\ 285 \end{array}$$

Here both the tens and the ones to be subtracted are bigger than both the tens and the ones you are subtracting from. Discuss how 60 + 3 is partitioned into 50 + 13, and then how 500 + 50 can be partitioned into 400 + 150, and how this helps when subtracting.

The argument that 'presenting all calculation as a linear movement in one or two directions…does not lay the foundations for later calculation methods or secure an understanding of algebra as generalised arithmetic' is a very puzzling statement. How do any written methods support the understanding or why $\frac{3}{4} - \frac{1}{8}$ is $\frac{5}{8}$ or that $3x + 4x = 7x$?

In conclusion, we read, 'the Strategy will…promote the use of the empty number line to support mental calculation…However, it is a staging post that should not hinder or forestall the introduction of more efficient written methods of calculation.' We thought the Strategy was not proposing a national method of calculation!

The approach to multiplication and division also has problems associated with it. The grid method of multiplication and long multiplication appears to go down well with pupils. One of the reasons given for using the standard algorithm is that pupils can apply it in complex products in algebra.

$$\begin{array}{r} 4\,7 \\ \underline{\times\ 2\,6} \\ 240\ +\ 42 \\ \underline{800 + 140\qquad\ } \\ \underline{800 + 380\ +\ 42} \end{array} \qquad\qquad \begin{array}{r} 4N + 7 \\ \underline{\times\ \ 2N + 6} \\ 24N + 42 \\ \underline{8N^2 + 14N\qquad\ } \\ \underline{8N^2 + 38N + 42} \end{array}$$

Have any of our readers ever seen this link before? (the one for division is even more bizarre!) And what is wrong with the grid method to make this association.

x	20	6
40	8000	240
7	140	42

x	2N	6
4N	$8N^2$	24N
7	14N	42

It is not surprising that in the progression for multiplication using the standard algorithm there is an error. Can you spot it?

- Reduce the recording, showing the links to the grid method above.
- This expanded method is cumbersome, with six multiplications and a lengthy addition of numbers with different numbers of digits to be carried out. There is plenty of incentive to move on to a more efficient method.

$$\begin{array}{rl} 286 & \\ \times\ \ \underline{29} & \\ 4000 & \quad 200 \times 20 = 4000 \\ 120 & \quad\ \ 80 \times 20 = 1600 \\ 350 & \quad\ \ \ \ 6 \times 20 = \ \ 120 \\ 1800 & \quad 200 \times\ \ 9 = 1800 \\ 720 & \quad\ \ 80 \times\ \ 9 = \ \ 720 \\ \underline{42} & \quad\ \ \ \ 6 \times\ \ 9 = \ \ \ \ 54 \\ \underline{8294} & \\ 1\ \ \ \ \ \ \ & \end{array}$$

How is it possible to easily check if you are correct? In the grid method, the sense of size and the unit digit in each cell should surely make checking far easier.

The proposed stages in progress in division and the approach to 'chunking' need lots of consideration also, too much to be included here.

The fundamental aims of the re-launch must be to engage teachers in professional dialogue about the teaching of mathematics in their schools. This dialogue can only happen if schools and teachers are empowered to make informed decisions about what works for their pupils and not just do what they are told to do. Through Primary Learning Networks schools are beginning to realise the power of groups of teachers working together on coming to solutions that work for them not which are dictated to them 'from above'.

We urge that the Primary Strategy should not go back over a quarter of a century to “Decomposition and All That Rot”. We recommend that schools and teachers should decide what are the best approaches to help their pupils (and themselves) gain a deeper understanding of mathematics rather than having to try to remember processes which someone tried to force on them and which they soon forgot. Don’t tell them what to do - even under the guise of ‘guidance’. Learn from the mistakes of the past.

The consultation for the renewed framework continues until June 2006. We urge you to respond to the consultation. A website has been set up especially for this purpose.

www.standards.dfes.gov.uk/primary/features/frameworks/consultation/

If you would like to have a copy of Tim Coulson’s e-mail to mathematics teams please e-mail Equals.

1.E-mail form Tim Coulson to Local Authority Mathematics Teams April 2006 pp 1,2
2.E-mail form Tim Coulson to Local Authority Mathematics Teams April 2006 p7
3.Michael Girling, ‘Towards a definition of basic numeracy’, Mathematics Teaching no. 81
4.Stuart Plunkett Decomposition and all that rot, Mathematics in School, Vol. 8 no. 3, May 1979
5.Guidance in teaching calculations – Written methods for subtraction of whole numbers p 20
6.E-mail from Tim Coulson to Local Authority Mathematics Teams April 2006 p7
7.E-mail from Tim Coulson to Local Authority Mathematics Teams April 2006 p10

Pen Portraits

Patricia Hancock describes a way of overcoming pupils’ difficulties in communication

As you stand in the doorway of the headteacher's office at Queensmill School in Fulham, the first thing that strikes you is a large display of photographs of children's faces on the wall immediately behind her desk. The display is both bright and colourful. Every pupil in the school is represented here and they are displayed in their class groups - the background colour of each photograph denoting the class to which each child belongs.

Queensmill is a school for primary-aged pupils with very special needs, as every pupil attending the school has a diagnosis of autism. This means that they have difficulties with communication, social interaction, imagination and they usually have accompanying learning disabilities too - in other words they have great difficulty in making sense of the world.

On closer inspection of this attractive display of photographs - `Pen Portraits', the visitor can see that not only is there a lovely photograph of each pupil, but, surrounding each individual photo there is some personal information about them. This gives the reader a `snapshot of each pupil's likes, dislikes, anxieties, physical needs and means of communication. The information has been compiled by the class teams, but it is written in the first person and shows that the dedicated staff working with these children has a detailed knowledge of each pupil in their class. But what of supply staff or other visitors to the classroom who do not know the children and therefore cannot make necessary judgements when they first need to communicate with them? This is the reason that these Pen Portraits are also clearly displayed in each classroom and any adult new to a class is directed to the pen portraits before they interact with any of the children. The pen portraits give a start in `getting to know' each child and have proved invaluable in preventing the pupils becoming unduly distressed by unfamiliar staff who will have no knowledge of what may or may not cause huge stress levels due to the wrong approach being made.

As well as the wall display, the Pen Portraits are also reproduced and displayed in a file along with each pupil's Statement of Educational Need, medical requirements, their Individual Education Plan and their Current P and NC levels.

The Pen Portraits are proving to be very popular at the school with both visitors and parents alike.

Queensmill School, Fulham, London

I like….

- my box of magnetic letters,

I can write lots of words by myself

- chase and catch
- lots of hugs
- singing to myself
- swimming
- Fimbles and Balamory
- photobooks about me and my friends

I don't like……….

- lots of people close to me
- lots of movement and noise like on the playground
- trying new foods
- touching new textures
- not being understood

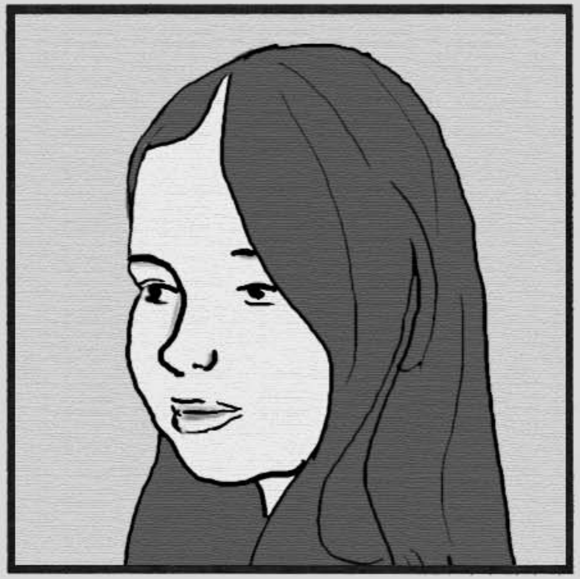
I communicate by………..

- smiling and laughing when I am happy
- holding people's hands to show them what I want
- speaking words quietly
- pushing people's hands away or shouting if I don't want something
- using PECS to request things

I get anxious when…….

- people don't understand me
- there is a lot of noise
- there are people moving around too fast

It helps me if grown ups are patient and quiet around me, give me time and lots of praise



I need help with………

- understanding what people say, I need several seconds to make a response
- putting on my support belt and splints

Keeping learning on track:

formative assessment and the regulation of learning Part 2

In this second of 4 sections of his paper Dylan Wiliam considers the effects of grades, comments and/or praise on the future progress of the learner, giving us all bench marks for evaluating our own formative assessment procedures in the classroom

The quality of feedback

Ruth Butler (1998) investigated the effectiveness of different kinds of feedback on 132 year 7 students in 12 classes in 4 Israeli schools. For the first lesson, the students in each class were given a booklet containing a range of divergent thinking tasks. At the end of the lesson, their work was collected in. This work was then marked by independent markers. At the beginning of the next lesson, two days later, the students were given feedback on the work they had done in the first lesson. In four of the classes students were given marks (which were scaled so as to range from 40 to 99) while in another four of the classes, students were given comments, such as "You thought of quite a few interesting ideas; maybe you could think of more ideas". In the other four classes, the students were given both marks and comments.

Then, the students were asked to attempt some similar tasks, and told that they would get the same sort of feedback as they had received for the first lesson's work. Again, the work was collected in and marked.

Those given only marks made no gain from the first lesson to the second. Those who had received high marks in the tests were interested in the work, but those who had received low marks were not. The students given only comments scored, on average, 30% more on the work done in the second lesson than on the first, and the interest of all the students in the work was high. However, those given both marks and comments made no gain from the first lesson to the second, and those who had received high marks showed high interest while those who received low marks did not.

In other words, far from producing the best effects of both kinds of feedback, giving marks alongside the comments completely washed out the beneficial effects of the comments. The use of both marks and comments is probably the most widespread form of feedback used in the Anglophone world, and yet this study (and others like it—see below) show that it is no more effective than marks alone. In other words, if you write careful diagnostic comments on a student's work, and then put a score or grade on it, you are wasting your time. The students who get the high scores don't need to read the comments and the students who get the low scores don't want to. You would be better off just giving a score. The students won't learn anything from this but you will save yourself a great deal of time.

giving marks alongside the comments completely washed out the beneficial effects of the comments.

A clear indication of the role that ego plays in learning is given by another study by Ruth Butler (1987). In this study, 200 year 6 and 7 students spent a lesson working on a variety of divergent thinking tasks. Again, the work was collected in and the students were given one of four kinds of feedback on this work at the beginning of the second lesson (again two days later):

- a quarter of the students were given comments;
- a quarter were given grades;
- a quarter were given written praise; and
- a quarter were given no feedback at all.

The quality of the work done in the second lesson was compared to that done in the first. The quality of work of those given comments had improved substantially compared to the first lesson, but those given grades and praise had made no more progress than those given absolutely no feedback throughout their learning of this topic.

At the end of the second lesson, the students were given a questionnaire about what factors influenced their work. In particular the questionnaire sought to establish whether the students attributed successes and failures to themselves (called ego-involvement) or to the work they were doing (task-involvement). Examples of ego - and task-involving attributions are shown in table 1.

Attribution of	Ego	Task
Effort	To do better than others To avoid doing worse than others	Interest To improve performance
Success	Ability Performance of others	Interest Effort Experience of previous learning

Table 1: ego- and task-related attributions

Those students given comments during their work on the topic had high levels of task-involvement, but their levels of ego-involvement were the same as those given no feedback. However, those given praise and those given grades had comparable levels of task-involvement to the control group, but their levels of ego-involvement were substantially higher. The only effect of the grades and the praise, therefore, was to increase the sense of ego-involvement without increasing achievement.

This should not surprise us. In pastoral work, we have known for many years that one should criticize the behaviour, not the child, thus focusing on task-involving rather than ego-involving feedback. These findings are also consistent with the research on praise carried out in the 1970s which showed clearly that praise was not necessarily 'a good thing'—in fact the best teachers appear to praise slightly less than average (Good and Grouws, 1975). It is the quality, rather than the quantity of praise that is important and in particular, teacher praise is far more effective if it is infrequent, credible, contingent, specific, and genuine (Brophy, 1981). It is also essential that praise is related to factors within an individual's control, so that praising a gifted student just for being gifted is likely to lead to negative consequences in the long term.

praise was not necessarily 'a good thing'

The timing of feedback is also crucial. If it is given too early, before students have had a chance to work on a problem, then they will learn less. Most of this research has been done in the United States, where it goes under the name of 'peekability research', because the important question is whether students are able to 'peek' at the answers before they have tried to answer the question. However, a British study, undertaken by Simmonds and Cope (1993) found similar results. Pairs of students aged between 9 and 11 worked on angle and rotation problems. Some of these worked on the problems using Logo and some worked on the problems using pencil and paper. The students working in Logo were able to use a 'trial and improvement' strategy that enabled them to get a solution with little mental effort. However, for those working with pencil and paper, working out the effect of a single rotation was much more time consuming, and thus the students had an incentive to think carefully, and this greater 'mindfulness' led to more learning.

The effects of feedback highlighted above might suggest that the more feedback, the better, but this is not necessarily the case. Day and Cordon (1993) looked at the learning of a group of 64 year 4 students on reasoning tasks. Half of the students were given a 'scaffolded' response when they got stuck—in other words they were given only as much help as they needed to make progress, while the other half were given a complete solution as soon as they got stuck, and then given a new problem to work on. Those given the 'scaffolded' response learnt more, and retained their learning longer than those given full solutions.

In a sense, this is hardly surprising, since those given the complete solutions had the opportunity for learning taken away from them. As well as saving time, therefore, developing skills of 'minimal intervention' promote better learning.

Sometimes, the help need not even be related to the subject matter. Often, when a student is given a new task, the student asks for help immediately. When the teacher asks, "What can't you do?" it is common to hear the reply, "I can't do any of it". In such circumstances, the student's reaction may be caused by anxiety about the unfamiliar nature of the task, and it is frequently possible to support the student by saying something like "Copy out that table, and I'll be back in five minutes to help you fill it in". This is often all the support the student needs. Copying out the table forces the student to look in detail at how the table is laid out, and this 'busy work' can provide time for the student to make sense of the task herself.

The consistency of these messages from research on the effects of feedback extends well beyond school and other educational settings. A review of 131 well-designed studies in educational and workplace settings found that, on average, feedback did improve performance, but this average effect disguised substantial differences between studies.

Perhaps most surprisingly, in 40% of the studies, giving feedback had a negative impact on performance. In other words, in two out of every five carefully-controlled scientific studies, giving people feedback on their performance made their performance worse than if they were given no feedback on their performance at all! On further investigation, the researchers found that feedback makes performance worse when it is focused on the self-esteem or self-image (as is the case with grades and praise). The use of praise can increase motivation, but then it becomes necessary to use praise all the time to maintain the motivation. In this situation, it is very difficult to maintain praise as genuine and sincere. In contrast, the use of feedback improves performance when it is focused on what needs to be done to improve, and particularly when it gives specific details about how to improve.

This suggests that feedback is not the same as formative assessment. Feedback is a necessary first step, but feedback is formative *only if the information fed back tothe learner is used by the learner in improving performance*. If the information fed back to the learner is intended to be helpful, but cannot be used by the learner in improving her own performance it is not formative. It is rather like telling an unsuccessful comedian to "be funnier".

As noted above, the quality of feedback is a powerful influence on the way that learners attribute their successes and failures. A series of research studies, carried out by Carol Dweck over twenty years (see Dwck, 2000 for a summary), has shown that different students differ in the whether they regard their success and failures as:

being due to 'internal' factors (such as one's own performance) or 'external' factors (such as getting a lenient or a severe marker);

being due to 'stable' factors (such as one's ability) or 'unstable' factors (such as effort or luck); and

applying globally to everything one undertakes, or related only to the specific activity on which one succeeded or failed.

those who see ability as incremental see all challenges as chances to learn—to get cleverer

Table 2 (below) gives some examples of attributions of success and failure.

Dweck and others have found that boys are more likely to attribute their successes to stable causes (such as ability), and their failures to unstable causes (such as lack of effort and bad luck). This would certainly explain the high degree of confidence with which boys approach tests or examinations for which they are completely unprepared. More controversially, the same research suggests that girls attribute their successes to unstable causes (such as effort) and their failures to stable causes (such as lack of ability), leading to what has been termed 'learned helplessness'.

More recent work in this area suggests that what matters more, in terms of motivation, is whether students see ability as fixed or incremental. Students who believe that ability is fixed will see any piece of work that they are given as a chance either to re-affirm their ability, or to be 'shown-up'.

Attribution	Success	Failure
locus	internal: "I got a good mark because it was a good piece of work"	internal: "I got a low mark because it wasn't a very good piece of work"
stability	stable: "I got a good exam-mark because I'm good at that subject" unstable: "I got a good exam-mark because I was lucky in the questions that came up"	stable: "I got a bad exam-mark because I'm no good at that subject" unstable: "I got a bad exam-mark because I hadn't done any revision"
specificity	specific: "I'm good at that but that's the only thing I'm good at" global: "I'm good at that means I'll be good at everything"	specific: "I'm no good at that but I'm good at everything else" global: "I'm useless at everything"

Table 2: dimensions of attributions of success and failure

If they are confident in their ability to achieve what is asked of them, then they will attempt the task. However, if their confidence in their ability to carry out their task is low, then, unless the task is so hard that no-one is expected to succeed, they will avoid the challenge, and this can be seen in mathematics classrooms all over the world every day. Taking all things into account, a large number of students decide that they would rather be thought lazy than stupid, and refuse to engage with the task, and this is a direct consequence of the belief that ability is fixed. In contrast, those who see ability as incremental see all challenges as chances to learn—to get cleverer—and therefore in the face of failure will try harder. What is perhaps most important here is that these views of ability are generally not global—the same students often believe that ability in schoolwork is fixed, while at the same time believe that ability in athletics is incremental, in that the more one trains, the more one's ability increases. What we therefore need to do is to ensure that the feedback we give students supports a view of ability as incremental rather than fixed.

feedback should contain a recipe for future action, otherwise it is not formative

Perhaps surprisingly for educational research, the research on feedback paints a remarkably coherent picture. Feedback to learners should focus on what they need to do to improve, rather than on how well they have done, and should avoid comparison with others. Students who are used to having every piece of work scored or graded will resist this, wanting to know whether a particular piece of work is good or not, and in some cases, depending on the situation, the teacher may need to go along with this. In the long term, however, we should aim to reduce the amount of ego-involving feedback we give to learners (and with new entrants to the school, not begin the process at all), and focus on the student's learning needs. Furthermore, feedback should not just tell students to work harder or be 'more systematic' - the feedback should contain a recipe for future action, otherwise it is not formative. Finally, feedback should be designed so as to lead all students to believe that ability—even in mathematics—is incremental. In other words the more we 'train' at mathematics, the cleverer we get.

Although there is a clear set of priorities for the development of feedback, there is no 'one right way' to do this. The feedback routines in each class will need to be thoroughly integrated into the daily work of the class, and so it will look slightly different in every classroom. This means that no-one can tell teachers how this should be done—it will be a matter for each teacher to work out a way of incorporating some of these ideas into her or his own practice.

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Learning and Teaching Research Center at the Educational Testing Service, USA

Remarkable trees

… the experience of finding 1,000 year-old-trees …A big oak or beech can weigh 30 tons, cover 2,000 square yards, include ten miles of twigs and branches. Each year the tree pumps several tons of water about 100 feet into the air, produces a new crop of 100,000 leaves and covers half an acre of trunk and branches with a new pelt of bark

I found these 60 remarkable [trees]… Roughly two thirds of the collection consists of ancient native trees, one third of exotic newcomers from Europe, the East and North America. The natives include only six out of the 35 species normally considered by botanists to be indigenous – meaning that they came to Britain unassisted after the last Ice Age. … I include … the champion sessile oak, … 37 feet in girth above the lowest branch.

Thomas Pakenham, *Meetings with Remarkable Trees*, London: Weidenfield and Nicholson, 1996

Save The Albatross[1]

Albatrosses have circled the world for millions of years. Early explorers of the Southern Ocean came home with stories of monsters, sea sprites and giant white birds… The giant wandering albatross, covering 10,000 km in a single foraging trip is the bird with the longest wingspan in the world... Grey-headed albatrosses from South-Georgia have been satellite-tracked twice around the globe in their year-long sabbatical between breeding attempts. The fastest round-the-world trip took 46 days.

Now 19 out of 21 species faces global extinction. Every year at least 100,000 albatrosses are snared and drowned on the hooks of lingline fishing boats. The longline fishing boats kill 300,000 seabirds each year, around one-third of them albatrosses.

Albatrosses are creatures of the southern hemisphere which occasionally wander north of the equator. One lived for 34 years in the Faroes with gannets. They are about 2 ft long, with a wing span up to 8 ft[2].

Birdlife International's Save the Albatross campaign has been adopted by the Volvo Ocean Race[3]. The Race follows the route of the old clipper ships around the world, over a 33,000 nautical miles route used by albatrosses for millions of years. This was, we suppose, the route described by Coleridge in his poem, "The rime of the ancient Mariner". The three verses quoted give us some feel for the bird and for the rough weather likely to be encountered on the voyage.

The ice was here, the ice was there,
The ice was all around:
It cracked and growled, and roared and howled
Like noises in a swound!

At length did cross an Albatross
Through the fog it came;
As if it had been a Christian soul
We hailed it in god's name.

It ate the food it ne'er had eat,
And round and round it flew.
The ice did split with a thunder-fit;
And the helmsman steered us through!

And a good south wind sprung up behind;
The albatross did follow,
And every day, for food or play,
Came to the mariner's hollo!

1. Extracts from 'Albatrosses: life at the extreme', Royal Society for the Protection of Birds, *Birds*, Winter 2005
2. Bruce Campbell, *The Oxford Book of Birds*, London: Oxford University Press, 1964
3. (www.volvooceanrace.org)

Teachers' notes

Extracting the mathematics

A study of the albatross offers some useful explorations of applied geometry.

For example:

1. Consider the earth as a sphere and find the circumferences of the circle of latitude 30° S.

2. Study the shape of birds – Do they all have the same proportions – look at the ratio of wingspan to length for example. Does this ratio vary with the size of birds? How about planes – their ratio of wingspan to length? This may for some lead to a discovery that in enlargement corresponding lengths, areas and volumes change at different rates. Some difficult concepts here but using simple apparatus and building simple solids of the same shape can start an exploration of the changing relationships of length, area and volume as the shape grows bigger. (One of my more academic colleague in the past used to say that the only everyday use she made of her mathematical knowledge was in understanding that if she doubled the amount of cake mixture she was using she did not need a tin of double the dimensions to bake it in.) (See J. B. S. Haldane, 'On being the right size' reproduced in James R Newman, *The World of Mathematics*, vol 2, London: Allen and Unwin, 1960.)

3. Further study might concern speeds:
   What is the average speed of the albatross on its journey?
   What is the direction and strength of the prevailing wind and how does this help the albatross?

Across the curriculum

There is more than mathematics here of course.

Biology:
Consider why the relationship of the mass of a bird – or any other living creature - to its surface area is important (See Haldane and Gibbons & Blofield again).

Geography:
Bring in wider studies of climate, more on winds, a study of fishing patterns, of fish populations.

History:
Start with a study of the development of fishing or the routes of the great explorers.

And then there's **English** of course with a study based on Coleridge's poem.

As an inspirational extra for any of these suggestions the film *Winged Migration*, Writer, Director and Narrator Jacques Perrin, 2001 is recommended. You need 98 minutes set aside to view it but maybe there could be a whole year viewing – or even a whole school. One review does not exaggerate when it comments

The cinematography is breathtaking. At times we seem to be riding on the back of a bird or as part of a V formation of honking geese. You really get an intimate appreciation of the muscular power and the endurance of these creatures as they make their annual migrations from one end of the earth to the other.

...continued overleaf

David Tipling (davidtipling.com)

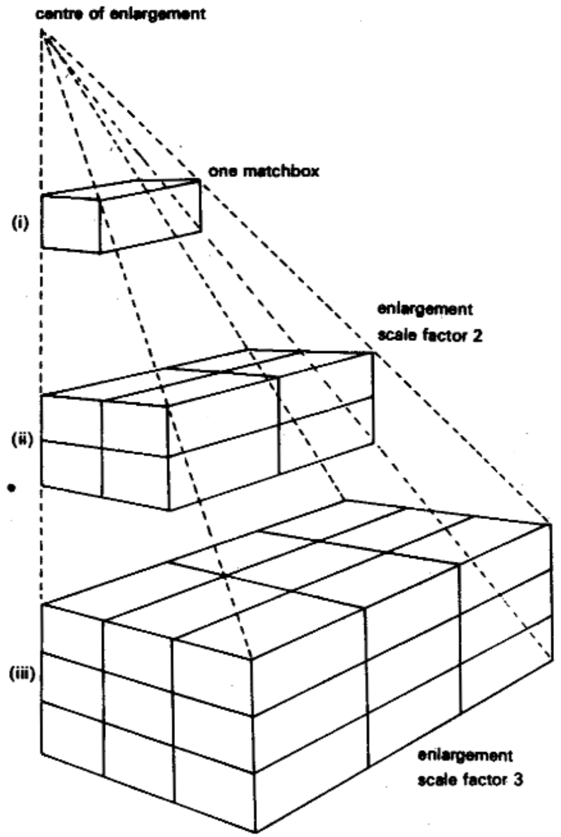
The extract below from Gibbons & Blofield, *Life Size: a mathematical approach to biology*, Macmillan: London 1971, will prove useful here.

ENLARGEMENT

In Figure 7.5 we see two enlargements of a matchbox with different scale factors. How do the heights, surface areas and volumes of (ii) and (iii) compare with those of (i)?

Fig. 7.5

Two enlargements of a matchbox



Let us first consider the surface areas, measuring them as we did those of the arrangements of matchboxes in Figure 7.4.

(i) 2 tops + 2 ends + 2 sides = 1 matchbox surface,
(ii) 8 tops + 8 ends + 8 sides = 4 matchbox surfaces,
(iii) 18 tops + 18 ends + 18 sides = 9 matchbox surfaces.

Considering volumes:

(i) contains 1 matchbox,
(ii) contains 8 matchboxes,
(iii) contains 27 matchboxes.

Tabulating these results

	Height	Area	Volume
(i)	1	1	1
(ii)	2	4	8
(iii)	3	9	27

From these results we see that when something grows, keeping the same shape, its height, area and volume increase at different rates.

GROWTH

Suppose the matchbox solids represent three stages of a growing cell or organism which takes in food and oxygen and gives out waste material through its body surface. In changing from stage (i) to stage (ii) its volume, as we see from the table, gets multiplied by eight. This means that it will probably require eight times as much food and oxygen and produce eight times as much waste at stage (ii). But the surface area is the main region of gaseous exchange, food intake and waste removal and this, at stage (ii) is only four times what it was at stage (i).

Seeing the bigger picture….

Heather Scott explains the benefits of a new scheme aiming to boost underachieving pupils' examination results

Place value, the four operations, fractions, decimals, long division, percentages, algebra, area and perimeter, trigonometry. These, according to educational consultant, Heather Scott, are just some of the things that make pupils struggle with mathematics and lose faith in their ability to perform well at exam time.

"Failure to grasp certain elements of the curriculum can cause even the brightest pupils to lose confidence, which in many cases can result in them becoming disaffected and disinterested in the subject altogether," she explains. "Often, it starts with them not absorbing what has been taught in one lesson but failing to alert their teacher. Then there is a knock-on effect when the class moves on to something else that requires them to have understood the previous lesson. Eventually they become completely lost to the point where nothing makes sense. Untangling the problem is difficult for time-pressed teachers who have perhaps thirty other pupils in the class."

However, Heather recognises that it is not just subject-specific issues that stop pupils from realising their potential. "More often than not, it's a complex set of situations that produces underachievement. A difficult home situation, poor concentration skills, inconsistent teaching (being taught by a number of supply teachers due to staff shortages) can all cause or compound the problem. What we must understand is that it is rarely a simple case of them not wanting to learn."

The consultant will start by trying to establish how much of the curriculum the pupils understand. "For the first session, I usually take a list of all the topics and get the pupils to tell me which ones they are confident with and which could do with further explanation," says Heather. "Because the groups are smaller I can also ask questions to find out exactly where they are struggling and what might be causing the problem."

Consultants use a variety of different teaching methods, but sessions differ from normal classroom work in a number of ways, as Heather explains: "It's important that we are not just providing an extra class, but trying something new to reawaken their interest in the subject. One of the key differences is that normal lessons tend to break a subject down into small pieces, eventually building up to the bigger picture, whereas we work the other way round. So for example, a teacher might teach trigonometry by getting pupils to work out the length of the hypotenuse in one lesson, then they'll spend the next lesson doing the same with the adjacent side, followed by the opposite side. Only after all this has been done will they will start to work on mixed problems. The trouble with this is that it's easy for pupils to get lost along the way. When we teach this, we'll often explain to them the purpose of trigonometry before they start and get them to look at mixed problems first, so they understand what they're aiming for. It's fantastic when they master a seemingly difficult problem in just one session and this really helps to boost their confidence."

For the programme to have a lasting effect on the school, consultants also look at the way pupils are being taught and identify where improvements could be made. "Consultants sometimes sit in on or run lessons with teachers and give them new ideas to try. We might, for example, help a Newly Qualified Teacher learn to control the class more effectively. However, we're not there to criticise or completely change the way lessons are taught, as each teacher has their own individual style." Consultants can also provide teachers with Pupils' Champions! materials, including revision booklets and worksheets.

Since the scheme began it has seen some excellent results, with improvements in the percentage of five A*-C grades in over ninety percent of schools. Some schools have improved as much as twenty percent. "We evaluate how the scheme is working through regular conversations with the school and formal monthly progress reports," says Heather. "This way we get a clear idea of what's working and what could be done differently."

For Heather, however, the most rewarding feedback is that which comes from the students themselves. "It's such a great feeling when you see the penny drop on one of their faces. They're so pleased when they finally get to grips with a challenging problem." Asked why she thinks the programme has had such success, she says: "A lot of it comes down to the fact that we have time to give those who are falling behind the individual attention they need. Also, as we're working in smaller groups, we can get a clearer picture of the individual pupil's weaknesses and establish how to phrase explanations in a way they'll understand. Finally, I'd say our consultants are a persistent bunch, they won't stop until they're absolutely sure what they're teaching has been understood!"

Keeping things in proportion – the power of the visual image

Are your pupils having trouble with proportion? Martin Marsh has some suggestions to offer.

1 How many kg is 1650g?

2 What does an item cost in £ if it is marked at 35 euros?

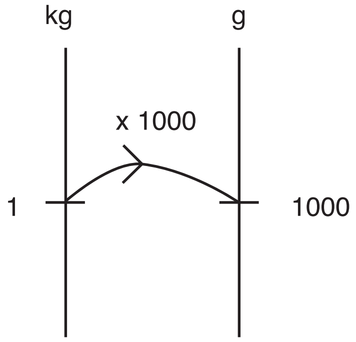
3 If an item has been reduced by 15% in a sale and now cost £75 what was the original price?

4 320 people take part in a survey. What angle on a pie chart would represent 42 of the people?

5 A distance is measured as 47 miles. What is this in kilometres?

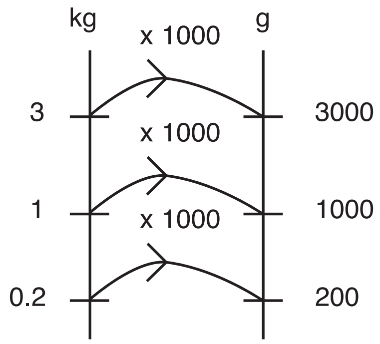
At first sight these questions may seem very different but in fact they all require an understanding of proportional reasoning. They are also problems that pupils have difficulties with often because they don't know what to multiply and divide by what.

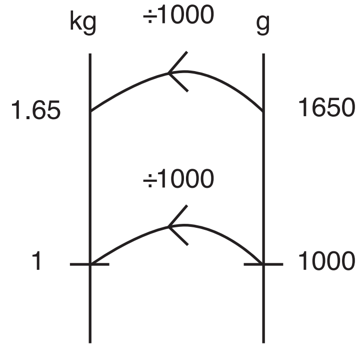
For the first question try the parallel line model - the visual image can help clarify the thinking process for pupils.



The diagram shows that 1 kilogram and 1000 grams are the same because they are at the same level on the scale and that you have to multiply by 1000 to convert 1 kg to grams. The same relationship clearly holds for other amounts

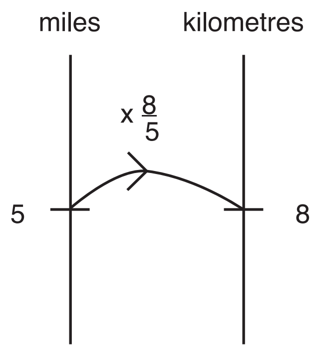
The beauty of this diagram is that it is also clear that to convert from grams to kilograms you 'go the other way' and divide by 1000; you carry out the inverse operation.



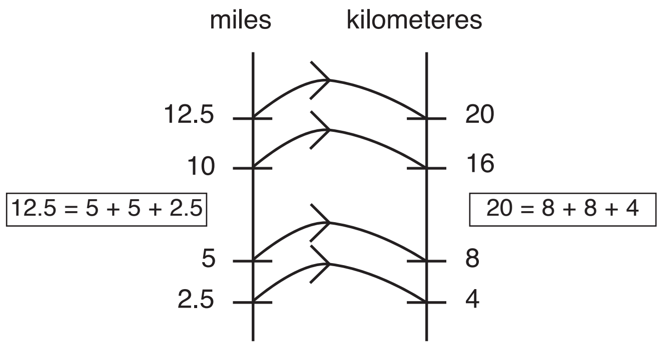


The Secondary Strategy materials (Interacting with Mathematics at Key Stage 3) explore the idea of 'multiplicative inverse' rather than 'inverse operation'; i.e. instead of dividing by 1000 you multiply by 1/1000.

Consider converting from miles to kilometres.
5 miles = 8 kilometres.



Plenty of preparation in understanding this would have to be done on fractions to get to this stage but the diagram is helpful for problems which do not require any multiplying and dividing by 5 and 8 e.g. How many miles is 20 kilometres?



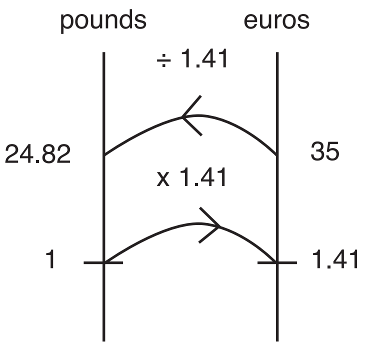
The relationship between numbers **on** the lines is as important as the ones across the lines.

However, once you know that to go from miles to kilograms you multiply by the 8/5, then you also know that to go back the other way you divide by 8/5 (or multiply by 5/8).

Any problem that has a proportional relationship can be done this way. To show the power of this approach I have solved 3 of the problems given at the beginning of this article using parallel number lines.

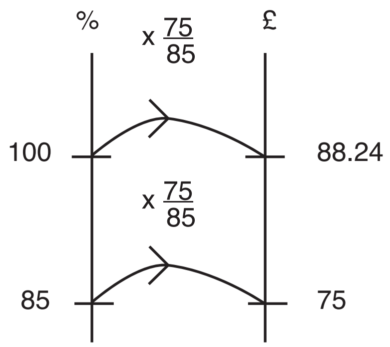
2. What does an item cost in £ if it is marked at 35 euros?

£1 = 1.41 euros



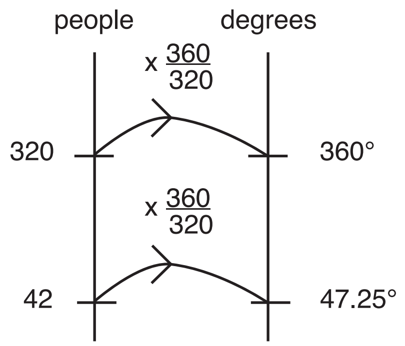
So the item costs £24.82

3. If an item has been reduced by 15% in a sale and now costs £75 what was the original price?



So the original price was £88.24

4. 320 people take part in a survey. What angle on a pie chart would represent 42 of the people?

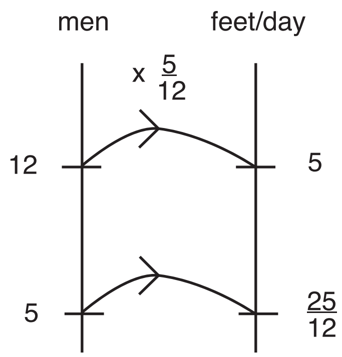


The angle representing 42 people would be : 47.25°

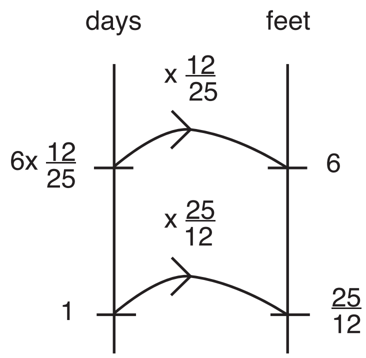
Finally, can you use the lines to solve this problem which my father always used to quote to me.

> 'If it takes 3 men four days to dig a hole 5 feet deep, how long does it take 5 men to dig a whole 6 feet deep?'

3 men digging for 4 days is the same as 12 men digging for 1 day (we will assume that they don't get in each others' way and hence slow each other down!).



1 man digs 5/12 of a foot /day.
5 men will dig 25/12 feet/day.



It takes 72/25 days
= 2 22/25 days

… and do you know I was never convinced of the answer to this problem until I did it this way!

Slough LEA

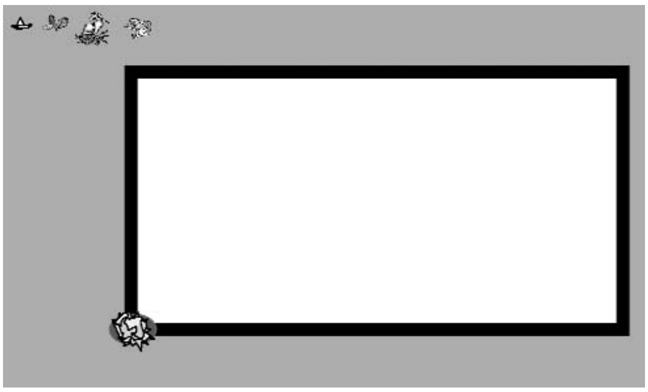
Frogs and co-ordinates

Children's early handling of coordinates may help them avoid some later difficulties – **Mundher Adhami** explores a fresh approach

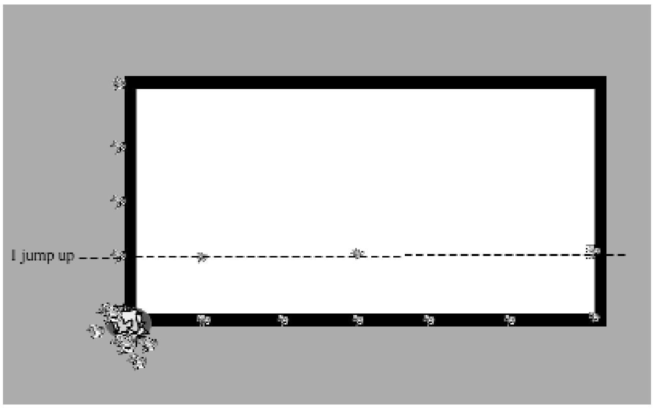
Children as young as 7 seem to be able to understand position on the page using numbers in two dimensions and they build up a model of plane space in their minds, which should reduce their confusions higher up the school. This was shown by a classroom activity trialled as part of a series of thinking mathematics lessons for the lower primary school[1].

At the start of the lesson, the children have to understand the need for a grid and numbers to resolve a difficulty in describing a position where words are not sufficient. Children begin by looking at a large sketch on the floor - or on an OHT - of a pond with a plant growing at one corner. They then try to describe, in words, where some objects are placed: a paper boat, a small flower, a bird, and a frog. (The teacher had torn the pictures from a worksheet and placed them in various positions inside the 'pond') They look at the need for referring to the plant at the bottom left corner and the edges, and having difficulty describing how far one position is from another.

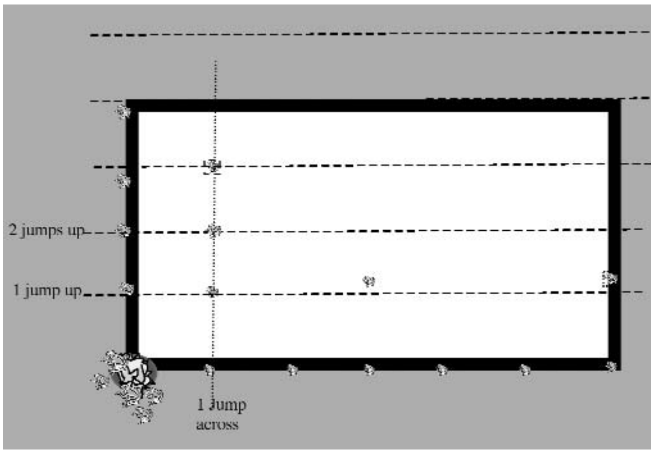
In groups of six they split into three pairs: a pair to give directions to another pair who are separated from them by a screen or a box, for placing objects in the pond, with a third pair to check the descriptions. They use single words like 'middle', or phrases such as 'nearer to the edge', 'long and short edge', 'next to the plant'. They share the finding as to how useful it is to refer to fixed corner and edges when following instructions. They recognise the need for full descriptions referring to the plant, the edges, and indicating in some way the distances from them to fix the position. Points on the edges themselves also need to be referenced to the plant and to whether they are long or short.



Now the teacher introduces the idea of lots of frogs living in the plant. (In the diagram the images of the frogs are much reduced so you need to imagine them!) Knowing that a frog crosses the short edge in four jumps (always the same distance for a jump!) and the long edge in 6 jumps, the children now consider whether they use that knowledge to fix positions inside the pond. They gradually construct the lines for each step, realising they have made a grid. Typical questions are: *What happens if the frog starts at the left bottom corner of the long edge? Where will it be in one jump? How far from the middle or the edge? Other places on that edge? How shall we mark what we are doing?*

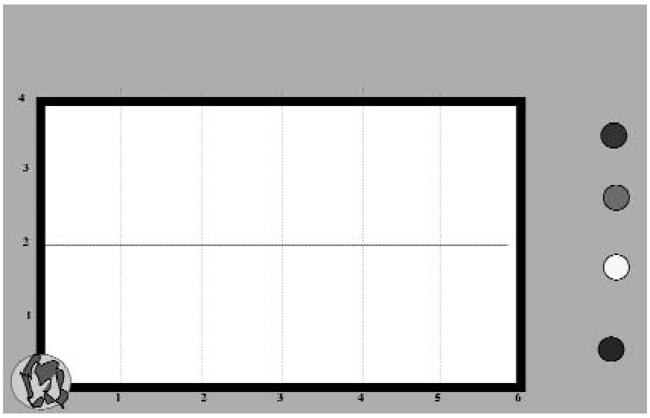


Once a few marks have been made: *What patterns are we starting to see?* (marks for one jump in a line, moving to a line) *What does this line show? What can we call it?*



The teachers then give out to the groups a worksheet with the grid and the four coloured blobs or pencils to mark given positions. Write out the instructions:

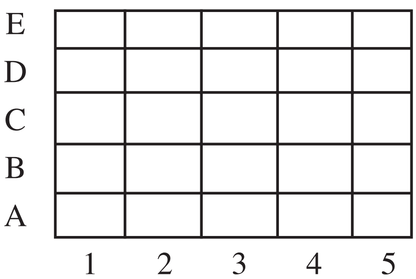
o *Red:* *Two jumps from the tree along the long edge and one jump into the water.*
o *Yellow:* *4 and 2*
o *Blue:* *5 and 3*
o *Green:* *0 and 3*



The whole class now consider together how useful it is to label the lines across and up. The children should notice that it is the lines that are labelled not the squares. Older children may be able to accept the plant as zero across and zero up. In some classes a '3-and -a- half jump' is possible.

Discussion
Nothing is inherently difficult in the topic of coordinates for adolescents. One can see things on the page and relate them to a grid. After all describing a position is much easier than handling many other notions of mathematics. However, we are aware that for some reason confusion abounds in the secondary school, or at least that pupils do not easily use the two-dimensional coordinate grid.

I think the main confusion come from using the labelling of squares rather than the lines. This ends with emphasising the counting number system rather than the real number system. Of course this persists in adult use of street maps, so is dominant in real life, which makes it an even greater obstacle. Sometimes the labelling is from top to bottom, left to right sometimes the other way round. But in all cases each square has two unique labels. That is useful but does not allow the system to be internalised as a general model of space.



Labelling squares disallows zero, fractions and negative numbers.
It restricts the grid to what is labelled.

The main relevant difference between the counting number system and the real number system is that the first is discrete the second is continuous. The counting system does not really have zero, because there is not an object to count as such. It has become common to introduce zero as part of counting, and no one is sure if that is a good idea. Sometimes the 100-square has the numbers 0-99. Is it really helpful to make 1 visually the same as 0?

The real number system is based on measurement on an equal-interval scale. For children this can be made simpler through the action of walking or jumping. In walking, you start from a spot and take steps and the spots are labelled 1, 2, 3, and so on. They should be equal steps, and the starting spot is easily recognised as needing a label. Half a step is also mark-able, as are all the fractions in-between. You can also go back to a position one or more steps before zero.
The use of the real number system in building up understanding of quantity including distance, even though the fractions and negative numbers are not approached, is the real basis of the number line and the ruler. (That is a subject that needs much more discussion). Coordinates then become no more than working on two number lines at the same time, one turned at right angle to the other.

It would be interesting to see how far children in the primary class can go with these ideas.

1. NFER Nelson *Let's Think through Maths 6-9*, by Mundher Adhami, Michael Shayer and Stuart Twiss. This is a teacher' pack with guidance and materials for 19 activities developed in the ESRC funded Kings College research project 'Realising Cognitive Potential of Children through Mathematics', 2001-2004.

Cognitive Acceleration Associates

War

In Dresden on February 13, 1945, about 135,000 people were killed by British firebombing in one night. It was the largest massacre in human history.

Kurt Vonnegut, quoted in *The Review Saturday Guardian* 14.01.06

Etc.

More than 9m plastic bottles are thrown away in the UK every year, 8% are recycled.
Mobile phone users in China exchanged some 12bn text messages over the Lunar holidays.
Sixty-two species of moth are thought to have become extinct in the UK in the past century.
Sixty-two per cent of Norwich residents are registered eBay users.
US airlines lost 10,000 pieces of luggage last year.

The Guardian 25.02.06

More pages from the past

London girls of the 1880s tell us something

In spite of the fact that her views on the position of women strike us today as very outdated, Molly Hughes (b.1865) was an innovator in the education of girls. These pages from her book *A London Girl of the 1880s*, give us insights into aspects of education with which we are still struggling.

Exams as an alternative to 'finishing'

… I saw later that Miss Buss faced a Herculean task. The endless anxieties she caused her pupils were as nothing to her own big anxiety. She was a pioneer, and almost single-handed, in getting some kind of systematic education for girls. She had no school to copy, no precedent of any kind. Her private school had been so successful that she found herself before long with five hundred girls all to be taught something and to be trained along Victorian lines of good behaviour.

To be taught something – but what? Negatively the problem was easy. All the hitherto satisfactory ideals of accomplishments and 'finishing' must be wiped out, but what was to take its place? While the education of boys had been gradually shaped from ancient times, engaging the attention of philosophers, that of girls had as a rule no other aim beyond making them pleasing to men. This idea was to Miss Buss anathema, and she failed to see all its great possibilities when really well done. To be deeply pleasing to a husband, and widely pleasing to other men, seems to me to be as good an ideal as a women can have. But instead of facing squarely the real needs of future wives and mothers, as the vast majority of girls were to be, Miss Buss seized the tempting instrument at her hand – the stimulus to mental ambition afforded by outside examinations. By this means the curriculum was ready made. And thus, for better or worse, the education of girls became a feeble imitation of what the boys were doing for the public examinations, made no distinction of sex, and no woman's voice was heard in the examination boards.

Balancing teachers

… we encountered a classics teacher who was a mental aristocrat. She seemed not only socially superior to the bulk of the staff, but she knew her subject and, more remarkable still, she knew the business of teaching. (p. 72)

Most attractive to some of us was Mrs Bryant, who was by no means a clear teacher, but had the rarer quality of inspiring us to work and think things our for ourselves. With her Irish sense of humour and kindly sympathy she gave a good balance to the masterful spirit of Miss Buss. (p. 72)

Never mind, just do it

Winnie was good at arithmetic , and at last made me able to face a complicated simplification of fractions, and indeed to get fun over seeing it come out. But we were both unable to fathom the reason for turning a division 'upside down and multiplying', … We both laughed and agreed to 'never mind but just do it'.(p. 6)

The joy of saying goodbye to arithmetic was tempered by fear of the unknown. What if mathematics should turn out to be worse? The completely unknown is never so fearful as the partially known. At my private school we had 'begun a little algebra'. This involved spending some two hours a week in turning complicated arrangements of letters into figures, with preliminary notes that *a* stood for 5, *b* for 7, *c* for 3, and so on. When the turning was done we added and subtracted as required and got an answer. One day I asked the teacher why they bothered to use these letters when figures were just as good, and there were plenty of them. She told me I was impertinent. (p. 77)

From *A London Girl of the Eighties*, London: OUP, 1936

How do you count tigers?

Indian conservationists are due to start counting the country's dwindling tiger population this week. The last estimate in 2003 showed a tiger population of between 260 and 280 in the Indian section of the Sunderbans, the world's largest natural tiger habitat.

A century ago there were around 40,000 tigers in India but official estimates now suggest that figure has fallen to 3,700. While trade in tigers is illegal, a whole animal can fetch up to £30,000 on the black market.

The Guardian 7 January 2006

Developing Speaking and Listening within the Mathematics

Lucy Ball and **Heidi Williams** explore ways of supporting pupils to develop their speaking and listening skills in the daily mathematics lesson

What is Speaking and Listening?

Speaking and Listening are the forms of language most frequently used to explore new knowledge, to understand new experiences and to develop new meanings. When children become involved in mathematical dialogue, it helps them explore, investigate, challenge, evaluate and actively construct mathematical meaning.

Why is speaking and listening important within the daily mathematics lesson?

Speaking

- Speaking allows the children to clarify their ideas.
- Speaking allows the children to put thoughts into words.
- Speaking allows them to express themselves in a range of ways.
- Speaking allows them to share their ideas in groups.

Listening

- Listening allows the children to clarify what others have said.
- Listening allows the children to build on what they have heard.
- Listening allows the children to take turns and influence others.
- Listening allows the children to notice significant uses of mathematical language.

Group discussion and interaction

- Group work allows the children to explore ideas and concepts.
- Group work allows the children to build on each other's contributions.
- Group work allows the children to express their ideas in a variety of ways, e.g. questioning, hypothesising, discussing, investigating and consolidating.
- Group work allows the children to develop their language and social skills needed for cooperation and collaboration.

Developing talk

It is important that you restrict the resources available in order to promote talk within the classroom and through group discussions. Ground rules need to be set before you can expect the children to use Speaking and Listening effectively. These should be on display within the classroom and referred back to on a regular basis. The teacher needs to use the correct vocabulary and model the 'talk' before asking the children to do so. The children could be encouraged to 'have a voice in their heads' – this might help them organise their thoughts. It might be useful to model a problem with a learning support assistant badly, allowing the children to recognise the importance and relevance of effective speaking and listening. Allow plenty of opportunities to promote the correct use of vocabulary.

How to develop Speaking and Listening within the classroom

Talk partners – Allow all children to participate in speaking. The children are placed in pairs and allowed time to talk during the lesson for an agreed length of time. Guidance needs to be given to clarify what they should be talking about.
For example,

- One child to think of a shape and describe it using the correct vocabulary while the other listens and questions until the shape has been named. This could also be done using numbers.
- Give the children a calculation. Ask the children to discuss how they could do it. Is there more than one way?
- Re-inforce the objective. Ask the children to explain what they have learned during the lesson, what they need to do at their tables etc.

Barrier games – Focus on giving and receiving instructions. The speaker has to give clear information and explicit instructions to the listener. The listener has to ask questions to clarify understanding and gain information.

For example,

- Sit the children back to back and ask one child to describe a shape while the other draws it. This can also be used for direction. Extend to making tangrams.
- Place a barrier between the children and ask one to give instructions on how to make a repeated pattern.

Word tennis – Involves the children taking turns.
For example,

- One child says a number bond to 10 and the other responds with a different number bond to 10. Repeat until the children can find no further examples.
- One child says a calculation and the other gives the answer.

Snowballing – The children are organised to discuss something or to investigate an issue in pairs. The pairs then join together to form a group and share their findings. The small groups then join together to make a larger one.
For example,

- Give the children a problem to solve.
- Give the children a statement about a number or shape. Is it true or false?
- Gathering numbers to fit with a specific criteria, e.g. collecting even numbers, multiples etc.

Jigsaw – The children are organised into 'home group' and given a number. Assign each child with the same number (i.e. all number 3s) to one area of investigation. Then the children undertake investigations and agree on the main points to report back to their home group.
For example,

- Investigating shapes.
- Investigating ways to measure given areas / objects.

General activities

- Organise the whole class into 4 groups. Give 3 children a number card in each group. Children then organise themselves according to a given instruction, e.g. an odd number. A nominated child from each group will then run to a given area to ring a bell when they have made their number.
- Give the children 3 2-digit numbers on a grid. Prepare a die with 1,2,3,10,20 and 30 on. Roll the die. The children have to discuss with their partner where the number should go. The numbers in each section can be added, multiplied, divided or subtracted. They must reach the target numbers. They are not allowed to rub out or change a number, but can discard a number if they wish. Get the winning pair to explain their method. Are there any other methods?

47	65	76

- Learn raps and poems to help consolidate key facts.
- Mathematical imaginings, e.g. Imagine a rectangle drawn in front of you. Label the corners of your rectangle A, B, C and D. Is the line AD longer than the line DC?

- There are a number of publications promoting the use of Speaking and Listening within mathematics, such as

"*Reasoning Skills in Maths – Talk it, solve it*" By Jennie Pennant, Rachel Bradley and Jacky Walters (This was given out at a Head's Conference in September 2005)

"*Eyes Closed*" published by BEAM

Speaking, Listening, Learning: working with children in Key Stages 1 and 2, Primary National Strategy, DfES 0623-2003 G

Looked after children

are nine times less likely to get five good GCSEs than their classmates and 19 times more likely to leave without a single GCSE. …

Only one in 30 children are in care primarily as a result of socially unacceptable behaviour. But by the age of 19, one in 25 of those who leave school without qualifications are in custody. A further third are not in education, employment or training, and a further one in five have dropped off the radar screen after losing touch with their former carers.

Almost two thirds of those in care are there as a result of abuse or neglect, one in 10 because of a dysfunctional family and 8 per cent because of absent parents. More than a quarter of children in care have statements of special educational need compared to about 3 per cent of the general population. …

Official statistics published last week show more than 8,000 children in care in England have three or more placements in a single year. Of these, almost half are aged 10 to 15.

Jon Slater, *'Help them to play catch-up'*, TES 170206

Reviews

'Talk it, solve it' – reasoning skills in maths.
Published by BEAM www.beam.co.uk

There are three books in this series

- **Talk it, solve it: Years 5 and 6**
  Reasoning skills in mathematics
  Jennie Pennant with John Thompson

- **Talk it, solve it: Years 3 and 4**
  Reasoning skills in mathematics
  Jennie Pennant with Claire King and Jacky Walters

- **Talk it, solve it: Years 1 and 2**
  Reasoning skills in mathematics
  Jennie Pennant with Rachel Bradley and Jacky Walters

Reasoning skills are a fundamental, but often underrated, part of both the mathematics and language curriculum. They are absolutely essential in daily life. BEAM developed these books in conjunction with Bracknell Forest LA to help teachers and teaching assistants get children thinking, and talking, about numbers and shape, while honing their logical reasoning. These books encourage children to be strategic, find examples to match a statement, be systematic, look for patterns, look for ways to overcome difficulties and check results.

The activities are designed so that children identify an unknown item (number, shape, amount and so on) by means of clues or questions and answers. This encourages them to develop the skills of asking relevant questions for clarification, learning to make contributions relevant to the topic, qualifying or justifying their ideas and responding appropriately taking into account what others are saying.

The introduction to each book offers suggestions as to how to use the activities for those who need additional support, more able learners and those learning English as an additional language, thus making them valuable for the spectrum of learners.

Each book includes:

- paired and group problem solving
- photocopiable activity cards and solution sheets
- teachers' notes
- CD-ROM containing all the photocopiable sheets

A Second Starter for Study:
Anne Watson, Jenny Houssart and Caroline Roaf eds., ***Supporting Mathematical Thinking*****, London: David Fulton in association with NASEN, 2005**
introduced by Rachel Gibbons

In *Equals* 12.1 we looked at some of the ideas put forward in this book to "engender innovative thinking". In this second browse we will explore suggestions the writers make about the ways children think and investigate some "barriers to learning" they describe and how these may be overcome. The writers have useful suggestions about the part learning assistants can play in studying in detail individual children's strengths and difficulties in mathematics and sharing their insights with the teacher.

They begin by pointing out that, if the ability to think mathematically and understand mathematical concepts are adaptations of our natural propensities, then, rather than having innate superior abilities, those who are better at mathematics than others must be better at learning from mathematical experiences. The question then arises: do we give enough variety of mathematical experiences for all to be able to learn from them?

Anne Watson reminds us that a few students seem unable to develop a repertoire of strategies. If they do change their strategy for a certain purpose then they change to using the new strategy always and never again use the old one. Indeed, I think I remember during my initial training in the dim and distant past being advised never to give a choice of methods to those who have difficulties: just stick to one standard procedure and drill the students in that. That way they are safe.

The trouble with this approach, of course, is that if you forget a procedure you have no way of getting it back. I remember once, years ago, running a course for primary teachers who wanted to brush up on their own mathematical skills. They arrived saying, "Show us how to …". They had forgotten, for example, how you set out a long division sum and wanted to be reminded of the standard procedure and nothing more. Gradually they were persuaded to tackle some mathematical problems for themselves, either individually or in groups. And it worked. Later when I thought I would like to 'perform' again in lecture mode it was politely indicated that they were not interested. By then they were confident in pursuing their own investigations into mathematics with individual discussion and help when needed.

So maybe all students, if given the chance, are capable of thinking in sophisticated ways. Anne Watson points out that teachers plan on the basis that less successful students need more structured step-by-step approaches but, as she further asserts, "there is no standard recipe for mathematical success." How do you approach mathematics with your less successful pupils? It would be useful in future issues of Equals to have some descriptions of the way your students think and approach problems.

Sometimes confusion can occur because words have different meanings in mathematics and in every day contexts. For example the word 'similar' has a much more precise meaning in mathematics than in everyday usage. Stephanie Prestage and Pat Perks consider some of the barriers to algebra which have 'similar' (in the everyday sense) roots. "What is 2 more than x?" may get the response "z". Prestage and Perks therefore suggest changing the order of the algebra curriculum. They do not consider teaching mathematics differently to students with SEN but finding ways to teach mathematics better to all students. Looking at algebra as a way of generalising arithmetic, they suggest, can help pupils to see underlying patterns.
"If you do not understand how arithmetic generalises, it becomes difficult to see patterns. If you cannot see patterns, each sum is unrelated to any other you have done.

You will find in *Supporting Mathematical Thinking* many analyses of children's problems with certain mathematical concepts and a wealth of suggestions as to how you might attempt to help them overcome their misconceptions. It should be a reference book wherever mathematics is taught. Meanwhile, while you all have it on order we would welcome descriptions of remedies you have used in your classroom

Harry Hewitt Memorial Prize 2006

As we reminded you in *Equals* 12.1:

If you have a pupil who has suddenly made remarkable progress in mathematics – from however low a level - just write to *Equals* BEFORE THE END OF JULY (deadline extended) describing it and put your pupil in for the 2006 prize

See *Equals* 11.3, page 18, for last year's winning entry and 12.1 page 15 for a picture of the winner at the prize-winning ceremony